RALF SCHULZE

# ANALYSIS A COMPARISON OF APPROACHES





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## **META-ANALYSIS**

A Comparison of Approaches

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## A Comparison of Approaches

**Ralf Schulze** 



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### Preface

Meta-analysis is a method for systematic literature reviews on a certain substantive question of interest. In contrast to the more traditional narrative review it explicitly includes elaborate methods for an evaluation of a corresponding research database. Meta-analysis is one of the more recent additions to the researchers' methods toolbox. It enjoys a growing interest in many research domains beyond subdisciplines of psychology as well as in methodology and statistics. The increasing number of research articles, using and further developing this method, are indicative of its perceived high value for researchers. Yet, there are also controversies still surrounding this method, parts of which are concerned with the implications of meta-analysis for the entire research endeavor, that is, how we should conduct research and how to interpret single study outcomes. Notwithstanding such controversies, meta-analysis has become a standard in the methods canon, at least in psychology but also in other disciplines like medicine as well. Hence, when considering these facts, one might easily think that there must be a consensus on an exactly specified single best way to conduct a meta-analysis, because it appears as such a wellestablished method in widespread use.

At least for correlation coefficients as effect sizes — on which the present book focuses — this is not the case. There are several specific sets of procedures available, so-called approaches to meta-analysis, and the research consumer of meta-analyses is confronted with their application in various contexts. In such a situation one may presume that differences in procedures are inconsequential for the substantive results, or alternatively wonder whether the application of different procedures may lead to differences in results. The meta-analyst who wants to conduct a review of the literature also faces the situation of many available approaches and has to make an informed choice between them.

This book provides an in-depth analysis and evaluation of extant metaanalytic approaches for correlation coefficients as effect sizes. The approaches are described and compared from a theoretical-statistical viewpoint as well as on the basis of the results of a Monte Carlo study. Under which circumstances the approaches produce comparable results and when they differ substantially is evaluated. The adequacy of the specific procedures for the application to a series of potential true situations in a universe of studies is assessed and a comparative evaluation of the approaches is thus provided.

The book is divided into four parts. In Part I, the basics of meta-analysis are introduced. The development and growth of the method is described from a bird's eye view. The basic steps of meta-analysis are explicated and briefly summarized with respect to their function for a review of the literature. In

this first part, the fact is highlighted that several author groups from different research domains in psychology have more or less independently established sets of procedures for meta-analysis. The emergence of these approaches is described to have eventuated partly for historical reasons and also because of the strong interest of the approaches' proponents in certain substantive research problems from the areas of industrial and organizational (I/O) and clinical psychology, respectively. It is shown that most of the basic steps of meta-analysis — from problem formulation to public presentation of results — are in fact common to *all* approaches, but several differences in procedural details at the analysis stage prevail. The analysis step of meta-analysis is the main subject of the second part.

In Part II, statistical methods of meta-analysis are specified. This part of the book begins with a chapter on effect sizes, the data used in meta-analysis. The properties of the most famous families of effect sizes, correlation coefficients *r* and standardized mean differences *d*, are described from a theoretical viewpoint. A clear emphasis is, however, placed on the properties of correlations. Beyond such a description, several characteristics of the effect sizes, supposedly of relevance for the comparison of approaches, are analyzed and the conversion of effect sizes from both families is specified.

After the presentation of effect sizes, general frameworks of meta-analysis are presented. The general frameworks are fixed versus random effects models, mixture models, and hierarchical linear models. These frameworks are very helpful to look at the approaches of interest from a very general perspective to recognize their particularities and limitations. Furthermore, the models are introduced to enable a classification of the subsequently outlined specific approaches to meta-analysis of correlations. The most well-known approaches are specified in detail in this part of the book. Moreover, several refinements of the approaches are presented, some of which can be classified as fixed and some as random effects model approaches.

Furthermore, the series of models leads to certain classes of situations for the application of meta-analysis. The framework of mixture models is used to conceptualize the research situation of meta-analysis and the specific situations under investigation in the Monte Carlo study in Part III. The situations of relevance are the homogeneous case with only one constant effect size in a universe of studies and heterogeneous cases. The first heterogeneous case is specified as a uniform two-point distribution of different universe parameters to be estimated, and the second case is a continuous distribution in the universe of studies.

After having presented and examined the approaches in detail, some consequences of choosing between approaches are pointed out from a theoretical viewpoint. It is shown that such a choice is not inconsequential in general, as is often implicitly assumed. The approaches are finally compared and statistical methods are summarized. The classification and comparison of the approaches is done with respect to the following characteristics: fixed versus random effects models, use of effect size measure (correlation coefficient, Fisher-*z* transformed correlations, corrected versions of correlations, and transformation of r to d), and weighting scheme used. All of these characteristics are proposed to be relevant for the outcomes of meta-analysis and approaches are differentiated along these lines.

In another major part of the book, Part III, the results of an empirical comparison between the approaches by using Monte Carlo methods are presented and differences in results are investigated. The very common assumption of researchers that the choice of an approach is merely a matter of taste and that results from applying different approaches to the same data are not different is once more scrutinized in this part of the book. The refined approaches considered in Part II are also part of the Monte Carlo study so that their quality can be assessed in comparison to more well-known approaches.

The theoretical analyses and results of the Monte Carlo study are summarized and discussed in Part IV. Recommendations for the application of metaanalytic methods to a database of correlations are provided and the implications of using suboptimal methods is discussed.

It is hoped that the presented analyses and results will help to further understanding and evaluation of the methods of meta-analysis. In addition, it is hoped that the present book will be instrumental for the interested metaanalyst and research consumer in making an informed choice and evaluation of the approaches and the corresponding results.

I am much obliged to the following individuals whose support have made this book possible or helped make it better than it otherwise would have been: Dankmar Böhning, Michaela Brocke, Vanessa Danthiir, Heinz Holling, Andreas Jütting, Malte Persike, Bernd Schäfer, and Oliver Wilhelm.

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Last but not least, I would like to thank my wife Claudia, whose incredible patience with me and enduring will to support my work in all conceivable ways continues to amaze me.

> RALF SCHULZE MARCH 2004, MÜNSTER

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# Part I

## Introduction

# The Growth of Meta-Analysis and Implications for Methodological Controversies

The research literature in most fields of science is steadily growing at a seemingly ever increasing rate. Nowadays, it appears to be virtually impossible for a researcher even in a relatively restricted field of study to keep track of all relevant published articles. Hence, there is a strong need for summaries of recent theoretical and empirical results in all scientific areas. Traditionally, there are reviews published in periodicals like the Annual Reviews, for example, where experts of the field are invited to present the current state of a field of study. Besides the function to inform interested researchers about the recent developments and findings such reviews are also relevant for an evaluation of the state of knowledge of a scientific area and even to guide decisions of policymakers to find scientifically well-founded solutions for everyday problems. However, clear-cut summaries of a research field are only easily established with a fairly consistent empirical basis, which is rather an exception than the rule, at least in the social sciences.

As Hunter and Schmidt (1996) have described for the field of psychology, making sense of heterogeneous results can be rather frustrating not only for researchers but also for policymakers. This may have the adverse effect of a negative appraisal of a whole scientific area potentially leading to cuts in funds and bad reputation. This kind of situation characterized the state of affairs in psychology in the early 1970s in the United States, with the negative consequences just described. It was in this climate when researchers became more occupied with the way summaries and reviews were actually carried out. Although the problem of summarizing the state of knowledge was not an entirely new one, the scientific examination of the review process itself was immensely intensified from this time on and ideas on the methods for a syn-