Trevor M. Young

# Performance of the Jet Transport Airplane

## Analysis Methods, Flight Operations, and Regulations

Aerospace Series

Editors Peter Belobaba, Jonathan Cooper, and Allan Seabridge

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# Performance of the Jet Transport Airplane

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Analysis Methods, Flight Operations, and Regulations

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# Contents

## Foreword

The number of textbooks on aircraft performance published since the end of World War II is overwhelming. Most of them treat propeller as well as jet-propelled airplanes, and a few textbooks pay attention to both fixed-wing aircraft and rotorcraft. Dr. Young has made a conscious choice to concentrate on subsonic jet transport airplanes. One of his arguments supporting this choice has been that most books on the market deal with the topic of performance from a pure flight mechanics perspective and pay little attention to FAA/EASA airworthiness regulations and actual flight operations. Also, the majority of the available books have treated the performance of propeller airplanes cruising at speeds up to 700 km per hour as well as jet airplanes cruising at speeds up to approximately 900 km per hour. After digesting Young's book, I came to the conclusion that he also wanted to pay more attention to the effects of compressibility on high-speed performance than is usually done. It is interesting to note that despite his confinement to a single category of aircraft, Dr. Young has produced a comprehensive text of 650 pages.

In my view, the choice of dealing primarily with jet transport airplanes will be an advantage for attracting the maximum audience since, apart from aeronautical engineering students and professionals, there could be considerable interest from the general public. This becomes clear when it is realized how many passengers nowadays travel each day all over the world in long-range, fast, reliable, and safe jetliners. Since the 1950s, when the Boeing 707 and the Douglas DC-8 appeared in the air, and in spite of several oil and financial crises, the growth of air transportation has been nearly uninterrupted with global annual increases of almost 5%. Although many factors have contributed to this trend, there is no doubt that the steadily increasing safety record and the improvement in fuel efficiency (from approximately 10 seat-kilometers per liter of fuel burnt in 1950 to 40 in the year 2000) were dominating factors.

After the appearance of the B-2 stealth bomber in the late 1980s, NASA confronted the American industry with the question: *Is there a renaissance for long-range transport*? The reply was a diagram showing the slow development of the parameter cruise Mach number $M$ times aerodynamic efficiency $L/D$, which had increased modestly from 13 in 1960 to 15 in 1990. In view of the abovementioned actual fuel efficiency improvement, this apparent contradiction might suggest that the industry had not been creative enough during those years and was eager to start a comprehensive technology development program (to be financed by the U.S. government). But the suggestion that the industry could realize a major technological step forward by the turn of the century has not materialized. And anyone who has digested Young's sophisticated treatment of optimum cruise performance will conclude that the cruise parameter $ML/D$ is a misleading criterion to judge an airplane's cruise efficiency. In fact, the same parameter had been used in 1960 as one of the arguments to start the development of the supersonic British–French Concorde, an airplane that had a fuel consumption per seat-kilometer three times that of contemporary subsonic airliners.

A set of flight safety rules ensuring that a public transport flight is dispatched at an appropriate all-up weight that attains a high level of safety is essential for maintaining the requirements of the commercial air transportation system in modern society. The performance capabilities of a newly developed aircraft type are determined by means of flight testing during the certification process, which is executed according to the stringent regulations of the American Federal Aviation Administration and the European Aviation Safety Agency. Based on these flight tests, operational limitations are prescribed for every flight, so that the risk of unsafe operation is reduced to an acceptable level. In contrast to most textbooks on airplane performance, Young's book pays considerable attention to the close interrelationship between airworthiness and safe operational performance. He has drawn on his knowledge of flying, as a general aviation pilot, to describe the complex procedural and regulatory framework in which flight operations take place.

Another important feature of Young's book is a self-contained, detailed, and scientifically rigorous treatment of topics associated with all aspects of the complete mission of jet transport airplanes. If appropriate data are available to define airplane lift, drag, engine thrust, and fuel consumption, the methods presented will provide solutions with adequate accuracy to be applied in industrial projects. And for professional engineers involved in the early and advanced stages of new airliner design, the performance criteria derived in the chapters on takeoff and landing performance, trip fuel planning, cost considerations, noise, and emissions could serve as design objectives. In particular, the data Items published by IHS ESDU (Engineering Science Data Unit) form a major source of guidance for those associated with flight performance, and many derivations in Young's book agree with the relevant publications. Of interest to pilots and engineers responsible for planning flight operations will be his detailed treatment of field performance, which provides a thorough description of this complex subject.

Comparing the present comprehensive and well-written text with many books on aircraft performance makes it clear that many of them have shortcomings in the sense that they tend to emphasize mathematical treatment at the cost of convincing worked examples and excellent illustrations based on realistic data of existing aircraft. This might explain why preparing this book has cost Dr. Young 12 years. I am sure that the reader will agree with me that the author has spent this time in a very commendable way.

Professor Emeritus Egbert Torenbeek
TU Delft, April 18, 2017

# Series Preface

The field of aerospace is multi-disciplinary and wide ranging, covering a large variety of products, disciplines, and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce innovative and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed on to others working in the industry and also to researchers, teachers, and the student body in universities.

The *Aerospace Series* aims to be a practical, topical, and relevant series of books aimed at people working in the aerospace industry, including engineering professionals and operators, engineers in academia, and those in allied professions such as commercial and legal executives. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation, and support of aircraft, as well as topics such as infrastructure operations and current advances in research and technology.

Aircraft performance concerns the prediction of an airplane's capabilities (range, speed, payload, takeoff, etc.) and evaluation of how well it functions throughout its operation. The topic is inherently multi-disciplinary, requiring not only an understanding of a wide range of individual disciplines such as aerodynamics, flight mechanics, powerplant, loads, etc., but also an appreciation of how these topics interact with each other. Such analyses are essential as a basis for the design of future aircraft.

This book, *Performance of the Jet Transport Airplane: Analysis Methods, Flight Operations, and Regulations*, provides a comprehensive overview of the topics required to carry out performance calculations. It is a welcome addition to the Wiley Series' existing content relating to airplane performance, and will prove to be very useful for undergraduate and graduate aircraft design courses. A complete range of relevant technical topics is examined, complemented by sections on cost, operations, and regulations.

Peter Belobaba, Jonathan Cooper, and Allan Seabridge

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I wish to pay tribute to my teachers and mentors. It is they who guided my understanding of this wondrous, fascinating subject of flight. It is they who introduced me to the mathematical models and analysis techniques that can be used to predict and study the performance of airplanes. I acknowledge and thank Tony Streather (formerly of the University of the Witwatersrand, Johannesburg); Adrian Marx (posthumous) (formerly of Swissair, Zürich); Martin Eshelby, John Fielding, and Denis Howe (formerly of Cranfield University, UK); and Walt Blake (formerly of Boeing Commercial Airplanes, Seattle).

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Trevor M. Young
March 3, 2017

# 1

# Introduction

## 1.1 Definitions of Performance

The word *performance* has been defined many times—the following two definitions, provided in dictionaries of general word usage, encapsulate much of what is discussed in this book:

(1) Performance describes the "capabilities of a machine or product [in this case, an airplane], especially when observed under particular conditions" [1].
(2) Performance is the "manner in which or the efficiency with which something [in this case, an airplane] reacts or fulfills its intended purpose" [2].

The first definition highlights the *capabilities* of the product. Identifiable attributes of interest that quantify the capabilities of any airplane include the payload that can be carried over a defined distance, the airplane's stall speeds, rates of climb, turn radii, optimum cruise speeds and altitudes, takeoff and landing distances, and so forth. These are all performance attributes that, for a particular set of conditions, can be established by calculation or measurement.

When describing an airplane's performance, the associated conditions under which these attributes were determined is critical information—without which, the data would be meaningless. Frequently, it is the combination of the airplane's weight, the altitude at which it flies, and the ambient air temperature that must be established to describe adequately such performance capabilities. This set of conditions occurs so often in performance discussions, it has its own abbreviation: WAT (for weight, altitude, and temperature). The significance of this combination will become apparent in the ensuing discussions. The barometric pressure of the ambient air around an airplane is a function of altitude, and, through a fundamental law of gas dynamics, air density is linked to pressure and temperature. Consequently, this combination of altitude and air temperature can be seen to influence significantly the performance of an airplane by affecting the engines' thrust and the aerodynamic forces that are generated by the relative motion of the airplane with respect to the surrounding air. An airplane departing from Denver International Airport, Colorado (elevation 5434 ft) on a hot summer day requires a substantially longer takeoff distance compared to that which would be required at Shannon Airport, Ireland (elevation 46 ft) in mid-winter for a similar takeoff weight, for example.

A complete and accurate description of the operating environment is essential when fully describing the capabilities of an airplane, and this goes beyond the WAT conditions. The type of runway surface—for example, textured or smooth—affects braking distances. The presence of standing water or snow on the runway reduces an airplane's acceleration during takeoff, increasing the required takeoff distance. Winds too impact an airplane's performance. Trip times and fuel usage can be significantly affected by jet streams, which are fast flowing, narrow air currents found in the atmosphere at altitudes of about 30 000 to 40 000 ft. Takeoff and landing distances

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are shortened when operating into a headwind, compared to a nil wind condition. The presence of turbulence is a consideration when a pilot selects an approach speed for landing, and, consequently, this can influence an airplane's landing distance.

The performance capabilities of an airplane naturally depend on the thrust produced by the engines. The thrust of jet engines is not unlimited—it is restricted, or governed, at certain critical flight conditions (e.g., during takeoff) based on such factors as the ambient air temperature, altitude, and airplane speed. The engine's electronic control system governs the engine to ensure that its structural integrity is not compromised when producing the defined, or rated, thrust.

All airplane systems have an indirect impact on an airplane's performance by virtue of their installed weight, which makes up part of the airplane's operating empty weight (OEW). Several systems, however, can also have a direct impact on performance—for example, the cabin pressurization and air conditioning systems extract power from the engines (either as electrical power or as compressed air) at the expense of the available propulsive power. Consequently, the rate of climb for certain airplane types, under demanding conditions, can depend on the air conditioning settings selected by the pilot.

An airplane's performance also depends on its configuration, which, in this context, describes the position or setting of re-configurable parts of the airplane by the flight crew or flight control system—that is, the positions of the flight controls (e.g., flaps, slats, rudder, and ailerons) and undercarriage, and so forth. One important consequence—which all student pilots learn at an early stage of their training—is that an airplane does not have a single stall speed, but rather a range of stall speeds depending on its configuration and the associated WAT values. Interestingly, stall speeds also depend on the pilot's actions prior to the stall (e.g., the airplane's bank angle and the rate at which the airspeed decreases are factors that can influence a stall, albeit to a lesser extent than WAT).

The second general definition of performance, given earlier, extends these ideas by indicating that it is the manner in which, or the efficiency with which, the airplane accomplishes these actions that is a measure of its performance. In other words, performance topics also address the question of how well an airplane accomplishes its task or mission. A definition of performance provided by the United States Federal Aviation Authority is that it is "a measure of the accuracy with which an aircraft, a system, or an element of a system operates compared against specified parameters" [3]. This expands the discussion of performance to the determination of parameters that facilitate comparative assessments of airplane performance to be conducted. An important comparative measure of an airplane's overall performance is its payload–range capability. The standard presentation of this information is a chart that indicates, on one axis, the payload (i.e., the mass or weight of the passengers, baggage, and freight) plotted against, on the other axis, the distance that can be flown for a given set of mission rules (which defines the flight profile and fuel reserves). *Efficiency* is all about achieving the desired result—for example, flying a set distance with a given payload—with the least effort expended or resources consumed. The determination of the optimum conditions, in terms of flight speeds and cruise altitudes, that will minimize the trip fuel or, alternatively, the trip cost are classic airplane performance studies. An extension of studies of this type considers the environmental impact of the flight in terms of the noise generated or the exhaust emissions produced.

Another important attribute that often gets considered under the topic of performance is the manner in which the airplane responds or reacts to control inputs by the pilot (or flight control system) or to external influences such as atmospheric turbulence. The response of the airplane to a sudden vertical gust depends on the airplane's speed and its aerodynamic characteristics (e.g., the lift-curve slope and the wing loading), for example.

The study of the response of an airplane to a set of applied forces (e.g., lift, weight, drag, engine thrust) is known as *flight mechanics*, which is fundamentally based on the application of Newtonian mechanics. The subject of flight mechanics has traditionally covered two separate, but related, topics: airplane performance (which primarily deals with trajectory analysis) and airplane stability and control. This book addresses the former topic, as applied to jet transport airplanes. All elements of a typical flight are considered (i.e., takeoff, climb, cruise, descent, maneuver, approach, and land). For a performance assessment of an airplane to have real value, however, the context and the operating environment need to be clearly defined and understood. For commercial jet transport airplanes, this context is to a large extent imposed by operational and regulatory procedures and constraints. Many of these considerations have traditionally not been addressed in academic textbooks on the subject of airplane performance, but are well known, of course, to those closely involved with the planning and execution of flight operations. In this book, performance discussions have extended many of the traditional, idealized performance representations to include key operational and regulatory aspects. The determination of the maximum takeoff weight that a pilot can safely utilize for a given set of conditions, for example, can be a complex study with many considerations, which include the runway length, slope, and surface condition; the wind and ambient conditions (viz., air temperature and barometric pressure); location of obstacles near to the intended flight path; airplane configuration (e.g., flap setting); and airplane limitations (e.g., brake energy limits, which are important in the event of a rejected takeoff).

Another point that arises from the above-mentioned definitions of performance is that the most important performance characteristics of any product are those that relate most closely to the primary purpose or role of the product. In broad terms, the role of commercial transport airplanes is to transport people and goods efficiently and safely, by air, from one geographical location to another within an aviation infrastructure (comprising airports and related services, air traffic control, ground and satellite navigation systems, and so forth).

## 1.2 Commercial Air Transportation

Commercial air travel is considered to have started on January 1, 1914, when Tony Jannus—flying a biplane designed to take off and land on water—transported the first recorded fee-paying passenger (across Tampa Bay in Florida, United States) [4]. One hundred years later—a time period that is associated with aviation advancements on a previously unimaginable level—saw 8.6 million passengers transported each day on commercial flights around the world [5]. Nearly 1400 airlines serviced 3800 airports [5]. Passenger and cargo air transportation had grown to become an essential part of modern life and a vital component of global economic activity. In 2014, about 35% of world trade by value was transported by airfreight [5].

Commercial air travel continues to grow. Air traffic—measured in terms of revenue passenger kilometers (RPK)—has experienced global long-term growth rates of approximately 5% per year. Informed forecasts by key stakeholders predict average growth rates of 4.5–5% per year for the forecast period (i.e., until 2035) [6, 7]. In 2015, the number of in-service jet transport airplanes, including freighter aircraft, was 22 510 (made up as follows: widebody types 22.4%, single-aisle types 66.0%, and regional jets 11.6% [7]); to satisfy increasing passenger demand, the global fleet will need to double by about 2035 [7, 8]. As might be expected, air traffic growth is linked to economic growth—both at a regional and global level. Interestingly, air traffic has historically enjoyed growth rates greater than the economic growth rate measured by gross domestic product (GDP) change. Economic cycles coupled with social factors (e.g., pandemics) and political factors (e.g., liberalization of aviation markets and wars) have a significant effect on the demand for air travel, which, consequently, fluctuates over time. One of the key economic factors is the price of crude oil, which impacts airline economics directly through the price of jet fuel and indirectly through its influence on global economic activity. Fluctuations in the global demand for air travel about the long-term trend associated with economic downturns or other significant events (e.g., terror threats or attacks) have historically been short lived, typically lasting about one year, after which the upward trend has continued.

The demand for air travel is also a function of its cost to the consumer (reduced prices stimulate demand), which, in turn, is dependent on the economic factors that impact airlines. The largest single cost element for airlines in recent years has been aviation fuel, which can represent about half of the total operating cost for a long-haul flight. Airplane fuel efficiency is thus a critically important performance metric for the airline industry; efficient airplanes are essential for airline profitability and long-term economic success of the industry.

Two important environmental considerations for the operation of commercial jet airplanes are noise and engine exhaust emissions. Noise limits for airplane certification are issued by aviation regulatory authorities. In addition, local authorities (e.g., airports) frequently impose additional noise limits. Compliance, in certain cases, can necessitate a noise abatement operational procedure that has performance implications (e.g., a reduced climb thrust setting after takeoff). The environmental impact of airplane emissions is of growing concern. For every 1 kg of jet fuel that is burned in an airplane's engine, approximately 3 kg of carbon dioxide is produced [9]. Increasing airplane fuel efficiency is thus a powerful direct means to reduce the environmental impact of air transport.

## 1.3 Jet Transport Airplanes: A Short History

What is often referred to as the *jet age* is that fascinating period of aviation history since turbojet engines were first installed on passenger transport airplanes (airliners). The first production jet airliner, the DH 106 Comet, manufactured by de Havilland in the United Kingdom (UK), entered commercial service in May 1952. The airplane offered unprecedented performance. In the cabin, it was quieter than its piston engine counterparts. It was also much quicker and was able to fly higher—thus avoiding bad weather—due to the thrust of its four de Havilland Ghost turbojet engines. The Comet 1, however, was grounded in 1954 when its certificate of airworthiness was withdrawn after two aircraft suffered explosive decompression in flight due to metal fatigue (caused by repeated pressurization cycles).

The first generation of successful turbojet commercial airplane types entered service in the late 1950s. In the Soviet Union, the Tupolev Tu-104 began service with Aeroflot in 1956. The design, which was extensively based on the Tu-16 bomber aircraft, had one Mikulin turbojet engine installed in each wing–fuselage junction. In the UK, deliveries from de Havilland of the extensively redesigned Comet commenced in 1958. The Comet 4 had a longer fuselage than its predecessor (the maximum seating capacity was 81) and more powerful Rolls-Royce Avon engines. In the United States, the Boeing 707 and Douglas DC-8 entered service in 1958 and 1959, respectively. Both types featured four turbojet engines mounted under the airplane's wings. Early variants, however, had limited payload–range capability. In France, Sud Aviation developed the first short- to medium-range jet airliner: the SE 210 Caravelle (later, Sud Aviation merged with Nord Aviation to form Aérospatiale). The design featured two Rolls-Royce RA-29 turbojet engines pod-mounted off the rear fuselage—a configuration that has been extensively used since then in the design of regional jet airplanes.

In the 1960s, newly developed low bypass ratio turbofan engines, such as the Pratt & Whitney JT3D, offered much improved airplane performance. Later variants of the highly successful Boeing 707 and DC-8 types offered true intercontinental capability due to the engine's lower fuel consumption. Convair, a division of General Dynamics, produced the medium-range

Convair 880. Despite its high cruise speed, the airplane failed to attract much interest—this was in part due to its poor economic performance compared to the competition, which included the Boeing 720 (a short-range derivative of the Boeing 707). New types that entered service around the world included the three-engine Boeing 727 and the two-engine Douglas DC-9 (this started a four-decade production run of many derivatives, including the MD-80, MD-90, MD-95, and Boeing 717). Three British airliners entered service in the mid-1960s: the short-range British Aircraft Corporation BAC One-Eleven, the short/medium-range Hawker Siddeley HS 121, and the long-range Vickers-Armstrongs VC10 (which featured a pair of rear-mounted engines on each side of the fuselage). In the Soviet Union, the long-range Ilyushin Il-62, which adopted the same engine configuration as the VC10, commenced service in 1967. The highly successful Boeing 737, featuring twin-turbofan engines installed closely coupled below the wing and a six-abreast seating arrangement, began servicing short- and medium-haul routes[1] in 1968 (to date, more Boeing 737 airplanes have been produced than any other jet airliner). To compete with turboprop airplanes in the regional airplane market, Yakovlev (in the Soviet Union) introduced the small Yak-40 (with 24–32 seats) and Fokker (in the Netherlands) introduced the F28 Fellowship (with 60–65 seats) in 1968 and 1969, respectively.

The 1970s will be remembered as the decade that saw the introduction of widebody airliners, that is, with a fuselage wide enough to accommodate two passenger aisles (typically with seven or more seats abreast). The four-engine Boeing 747, three-engine McDonnell Douglas DC-10, and three-engine Lockheed L-1011 TriStar airplane types began servicing long-haul routes worldwide, with significantly increased payload capacity over the Boeing 707, DC-8, and VC10 types. The Boeing 747 (nicknamed *Jumbo Jet*), which entered service as the world's largest passenger jet in 1970, would remain in production for over 45 years and would dominate this market sector for much of this time. The Tu-154, a narrow-body (i.e., single-aisle) airplane with three rear-mounted engines, began service in 1972. This successful medium-range type, which was designed to operate from unpaved or gravel airfields, saw 40 years of service. The A300, the first product of Airbus Industrie,[2] entered service as the first twin-engine widebody airplane in 1974. The decade will also be remembered as a time when airports were extremely noisy places. The first noise regulations for subsonic jet airplanes developed by the International Civil Aviation Organization (ICAO) came into force in 1972. These regulations led to the phasing out of many noisy first-generation jet airplanes, while others were modified with hush kits to reduce their noise during takeoff and landing (these modifications inevitably resulted in performance penalties). Noise regulations also spurred the introduction of higher bypass ratio, quieter turbofan engines, such as the General Electric CF6, Pratt & Whitney JT9D, Rolls-Royce RB211, and CFM International CFM56. These engine types and the many derivatives

1 There are no standard definitions of short, medium, or long haul (or range). The terms are used loosely in the aviation industry, with different sectors and organizations defining the categories in their own ways. One definition, used for market categorization, is based on trip duration: short (<3 hr), medium (3–5 hr), and long (>5 hr). Another definition, which is also widely used, defines long haul as a trip time exceeding 6 hr. Ultra-long haul is today taken to mean a flight of more than about 12 hr (although, again, different organizations define it differently). Short haul is sometimes used by airlines to describe domestic or regional routes. When based on distance, short haul is defined by one manufacturer as a trip of less than 3000 miles (2607 nm) and long haul as greater than 4000 miles (3476 nm) [10]. Another definition, which is used in market categorization, defines short haul as a trip distance of less than 2000 nm and long haul as greater than 2000 nm [11].

2 Airbus Industrie, with headquarters in Toulouse, France, was established in 1970 as a consortium of European aerospace companies—a Groupement d'Intérêt Économique (GIE) under French law. Aérospatiale of France, Deutsche Airbus (DASA) of Germany, and Hawker Siddeley of the UK were the initial shareholders. Construcciones Aeronáuticas SA (CASA) of Spain joined soon afterwards. In 2001, following several changes to its structure, the airplane manufacturer became a joint-stock company: Airbus SAS. Airbus's parent company, EADS, was renamed Airbus Group in 2014.

that were subsequently developed based on these designs would power much of the western world's commercial jet airplanes in the decades that followed.

The 1970s also saw the start of supersonic passenger transport services. The Aérospatiale/BAC Concorde—which was operated in small numbers (seven each by Air France and British Airways)—commenced commercial operations in 1976 (the type was retired in 2003). The Russian Tupolev Tu-144 commenced passenger service in 1977 (the fleet was grounded after 55 scheduled flights). Both designs, which were capable of cruising at speeds greater than Mach 2, employed a tail-less, delta wing configuration with four after-burning turbojet engines. Supersonic cruise, however, came at a considerable aerodynamic penalty (the lift-to-drag ratios of these airplanes were less than half that of a typical subsonic airliner), and the fuel efficiency was significantly poorer (a feature that became increasingly apparent as improved, higher bypass ratio turbofan engines were developed for subsonic airliners). Other issues that plagued both Concorde and TU-144 operations were high levels of takeoff noise and the sonic boom (which essentially limited supersonic flight to overwater sectors).

In the 1980s, first and second-generation narrow-body airliners were withdrawn from service in large numbers and replaced by more fuel-efficient twin-turbofan airplanes—these included the newly developed Boeing 757, Boeing 767 (Boeing's first widebody twinjet), and Airbus A310 types. Boeing also introduced the 737-300/-400/-500 models (the so-called Classics), with CFM56 engines and a redesigned, more aerodynamically efficient wing. In the Soviet Union, the Yakovlev Yak-42 and the Tupolev Tu-204 (both narrow-body designs) and the four-engine widebody Ilyushin Il-86 entered service. The Il-96, a shortened longer-range derivative, followed a few years later. British Aerospace, identifying the need for a STOL (short takeoff and landing) airliner, introduced the high-wing four-engine BAe 146 (later variants were called the Avro RJ). The first member of the very successful Airbus A320 family (which includes the A318, A319, A320, and A321 models) began service on short- to medium-range routes in 1988. The type, which was developed as a direct competitor to the Boeing 737, pioneered the use of digital fly-by-wire flight control systems on airliners. The Fokker 100, a 100-seat regional jet with two rear-mounted engines and 5-abreast seating, also commenced service in 1988. A shortened derivative, the Fokker 70, with 70–80 seats, followed a few years later.

In the 1990s, the first *large twins*—widebody twinjets with a maximum seating capacity exceeding *ca.* 300—were introduced into commercial service. The Airbus A330 and Boeing 777—both featuring two high bypass ratio turbofan engines installed below the wings—were able to compete on long-haul transoceanic routes that, in the West, had previously been the exclusive domain of the four-engine Boeing 747 and the three-engine McDonnell Douglas DC-10 and its derivative, the MD-11. This was made possible by new regulations that became known by the acronym ETOPS (initially defined as Extended Twin Operations). Airbus also introduced the long-range four-engine A340, which shared many common design features and components with the A330. In the single-aisle market, the Next-Generation Boeing 737 airplanes entered service (replacing aging Boeing 737 Classics and DC-9/MD-80 airplanes). In the regional jet market, the first variant of the Bombardier (of Canada) CRJ family commenced service in 1992. Embraer (of Brazil) entered this market with the twin-engine ERJ family in 1996 (models include the -135, -140, and -145). The Dornier 328JET, a 32-seat jet-engine commuter manufactured by the American–German Fairchild Dornier, began commercial operations in 1999.

This decade (i.e., the 1990s) also saw much consolidation and reorganization in the aviation industry with, for example, McDonnell Douglas merging with Boeing, and Fokker ceasing production of its own designs. The Lockheed Corporation, which had withdrawn from the civil aircraft business, merged with Martin Marietta to form the defense-orientated Lockheed Martin. The dissolution of the Soviet Union in 1991 significantly affected civil aircraft production in the region, which declined by 80% within a few years; a substantive downsizing and reorganization of the industry ensued [12]. In Europe, British Aerospace (which, much earlier, had been created by the nationalization and merger of several UK companies, including BAC and Hawker Siddeley) merged in 1999 with Marconi Electronic Systems to form BAE Systems. In the same year, DASA (of Germany), CASA (of Spain), and Aérospatiale-Matra (of France) agreed to merge to create EADS (European Aeronautic Defence and Space Company), which became the majority shareholder of Airbus.

By 2000, Airbus and Boeing effectively shared a duopoly in the large single-aisle and widebody airplane markets (the Tupolev Tu-204/-214 and Ilyushin Il-96 annual production rates were in the low single figures, and the production of the BAe Avro RJ was coming to an end). Following more than 15 years of development effort, Airbus introduced the A380, the world's largest passenger transport airplane (with a seating capacity of 525 passengers in a typical three-class configuration). The A380, a double-deck, widebody, four-engine airliner, began commercial service in 2007. On the other end of the size spectrum, there was much activity in the small single-aisle and regional jet sectors. Embraer, building on their success with the ERJ, introduced the larger E-Jet family in 2004 (models include the E170, E175, E190, and E195). In the Ukraine, Antonov (which had gained worldwide prominence for manufacturing very large military transport aircraft: the An-124 and the one-off giant An-225) developed a new family of high-wing regional jets: the An-148/-158. The An-148 entered service in 2009 after receiving Interstate Aviation Committee (IAC) type certification.

In 2011, the Russian Sukhoi Superjet 100, a low-wing twin-engine regional airliner, began commercial operations with IAC type certification. European certification followed soon afterwards, demonstrating the airplane's compliance with western airworthiness and environmental standards. This opened up international markets for the newly established parent company: United Aircraft Corporation (UAC), a state-owned conglomerate of Russian aerospace companies (including Ilyushin, Irkut, Sukhoi, Tupolev, and Yakovlev). To meet growing international demand for efficient, long-haul operations—and to replace an aging global fleet, which included many four-engine Boeing 747 and Airbus A340 airplanes—both Boeing and Airbus developed new widebody twin-engine airliners: the Boeing 787 entered service in 2011 and the Airbus A350 in 2015. New materials and manufacturing technologies featured strongly in these designs, which made extensive use of carbon fiber composites to reduce airframe weight. In 2016, the Bombardier C Series began commercial operations. This new two-member family of twin-engine medium-range airplanes, comprising the 110-seat CS100 and 135-seat CS300 models, were developed to compete with the Embraer and Sukhoi regional jets and also with the smaller Boeing 737 and Airbus A320 airplane types. In the same year, the Chinese Comac ARJ21-700, a 90-seat single-aisle twinjet with a resemblance to the MD-80, commenced domestic service with Chinese type certification. The next airplane program for this state-owned manufacturer is the single-aisle twin-engine C919. Also competing in this market, on completion of development, will be the UAC Irkut MC-21. Japan's first passenger jet airplane, the 70–90 seat Mitsubishi MRJ, commenced flight testing in 2015, with an entry-into-service target of 2020. The decade (i.e., the 2010s) also saw Airbus and Boeing launching new versions of their single-aisle airplanes—that is, the Airbus A320neo family (first deliveries in 2016) and the Boeing 737-MAX family (first deliveries in 2017). New engine technologies (in the CFM International LEAP and Pratt & Whitney PW1000G turbofan engines) and aerodynamic refinements were key elements of these improved versions of the industry's two best-selling jet transport airplanes.

Although superficially similar in appearance to the jet airliners of the 1960s in terms of their primary geometry—that is, featuring a tubular fuselage with rear empennage, swept wing, and pod-mounted engines installed either under the wing or on the rear fuselage—the latest generation of airliners have many advanced structural, aerodynamic, system, and engine design features that enable them to operate at much improved fuel efficiency levels compared to the early generation jet airliners. Using the Comet 4 as a baseline, these airplanes are more than 70% more fuel efficient per available seat kilometer (ASK) [13]. Noise levels and exhaust emissions have also been significantly reduced over this time. Overall reliability, passenger comfort, and safety[3] have all been dramatically improved.

## 1.4 Regulatory Framework

The performance of commercial jet transport airplanes needs to be considered within the context of the regulatory framework that applies to the certification and subsequent operation of these aircraft. In the United States (US), these regulations are issued by the Federal Aviation Administration (FAA); equivalent European specifications are issued by the European Aviation Safety Agency (EASA). Additionally, the national aviation authorities of individual countries with significant aviation industries publish their own regulations. In a colloquial setting, these regulations are often referred to as *the rules*. Although substantially similar in many respects, important differences exist between various sets of regulations. Herein, only the US and European regulations are considered. The key documents and the organizations responsible for these measures are described in Chapter 23.

For the purpose of certification and operation, airliners fall within the *transport* category of aircraft. As regards airplane *certification*, key regulations include the US Federal Aviation Regulation Part 25 [15], usually abbreviated as FAR 25, and the European counterpart, EASA Certification Specification 25 (Book 1) [16], abbreviated as CS-25. In many instances, these regulations are identical (the result of many years of effort to harmonize technical details). Common regulations are often written as FAR/CS 25—a practice that has been adopted herein. As regards the *operation* of these airplanes, important regulations are the US Federal Aviation Regulation Parts 91 [17] and 121 [18] and European EASA OPS Part-CAT [19].

The certification of a new airplane involves an extensive series of tests and compliance checks designed to ensure that it meets a minimum set of safety standards. FAR/CS 25 [15, 16] contain specific requirements that pertain to an airplane's performance, which must be demonstrated during the certification process. For example, FAR/CS 25.121(a) deals with an airplane's ability to climb following takeoff with one engine inoperative and with the landing gear extended. It is stipulated that the steady gradient of climb must be positive for two-engine airplanes, not less than 0.3% for three-engine airplanes, and 0.5% for four-engine airplanes, in the critical takeoff configuration. Compliance has to be demonstrated at the appropriate airplane weight, without the benefit of ground effect, with the critical engine (i.e., the engine that most adversely affects the airplane's climb performance) inoperative at a specified speed and thrust setting. This is one of many such performance requirements defined in FAR/CS 25. Airplanes that meet these requirements will have demonstrated a minimum performance capability that is considered appropriate for safe flight operations.[4]

An important output of the certification process is a formal record of the key safety-related performance capabilities of the airplane—this is the Airplane/Aeroplane Flight Manual (AFM). As the AFM is not designed to be used by flight crews, another document, known as the Flight Crew Operations/Operating Manual (FCOM), is produced based on the same performance data, but supplemented by approved manufacturers' data concerning non-safety-critical

3 The global accident rate for civil jet airplanes for the five-year period 2010–2014, measured in hull losses per 1 million flights, was 0.45; this is the equivalent of one major accident for every 2.2 million flights [14].

4 Acceptable methods to demonstrate compliance with the primary certification requirements/standards are given in FAA Advisory Circular (AC) 25-7 [20] and EASA CS-25 (Book 2) [16].

topics, such as optimum cruise speeds and all-engine climb performance. The FCOM is the primary source of performance information used by flight crews, who must operate their airplanes within a highly regulated environment, which is a feature of commercial aviation.

The rules that apply to commercial flight operations are many and varied, and these impose yet another set of constraints or limits on the performance that an airplane might achieve in routine flight operations. The International Civil Aviation Organization (ICAO) is responsible for coordinating and regulating international air travel. Central to this role is the Convention on International Civil Aviation [21], which, together with the many standards, policies, and procedures issued by ICAO, provides an internationally agreed framework for the safe operation of aircraft. This includes a set of procedures for operating within controlled airspace. It is often necessary to consider these procedures when analyzing an airplane's performance. Flight altitudes can be restricted and speed restrictions are imposed below 10 000 ft in much of the airspace used for commercial operations worldwide. These factors impact an airplane's achievable performance in service—for example, the previously mentioned restrictions can influence the time that it would take an airliner to reach its initial cruise altitude.

## 1.5 Performance-Related Activities

### 1.5.1 Performance Activities Related to the Airplane Life Cycle

Performance analyses are conducted for a variety of reasons, and the techniques that are used vary depending on the nature of the problem—for example, there is the prediction of the performance of a new airplane at the design stage; the reduction of flight-test data from a test airplane; the generation of performance data for the AFM and other key documents; the planning of flight operations taking into account real operational conditions; and the *in situ* calculations and performance monitoring undertaken by the flight crew during flight. Four distinct sets of activities that involve elements of airplane performance are identified in Figure 1.1. The activities have been arranged in the sequence in which they would first be conducted for a new airplane—that is, through the airplane's life cycle. For each activity, the physics does not change as the underlying principles of flight dynamics are the same, but the nature of the work undertaken is different as the purpose—and available data—of each activity is different. Although different calculations are carried out during the different activities, the theoretical basis for the various analyses that are conducted is largely the same.

During the design of a new airplane, engineering analyses are conducted where the performance targets—such as payload–range capability, cruise speed, fuel efficiency, and takeoff and landing capability—have been established, and the airplane's design features and aerodynamic characteristics are to be determined. At the conceptual design stage, it is usual to make a number of simplifying assumptions concerning the behavior of the airplane and regarding the operating environment, which will facilitate analytical solutions to be obtained for many performance problems. The techniques would only need to be accurate to within a few percent. In later stages of the design process, where more accurate performance predictions are required—and more data on the new design are available—more sophisticated techniques, which often involve numerical computation, are likely to be used.

The flight testing and subsequent data analyses of a new airplane type—which establishes the airplane's validated, or demonstrated, performance characteristics—involve a different set of analysis techniques and methodologies. Performance characteristics of the airplane are measured and compared to predicted values. As a range of air temperature and pressure conditions are typically encountered during flight testing, measured performance data are adjusted to

**Airplane design (by the manufacturer)**

Activities include performance analyses conducted using projected airplane data (e.g., airplane geometry, weight, aerodynamic and engine characteristics) using idealized mathematical models and historical data in support of design activities associated with the development of a new airplane (or a derivative). Primary considerations include: (1) airworthiness regulations (e.g., FAR/CS 25); (2) operational regulations (e.g., FAR 121, EASA OPS Part-CAT); and (3) customer requirements (e.g., payload–range capability, takeoff and landing distances, fuel economy).

**Flight testing and generation of performance documentation (by the manufacturer)**

Activities include the measurement of airplane performance characteristics during a series of standardized tests (described in FAA AC 25-7 and EASA CS-25 Book 2, for example), conducted in actual conditions (not idealized or model conditions) using test airplanes with extensive airborne and ground instrumentation.

Analysis of flight test and supporting performance data is conducted (1) to demonstrate compliance with the regulations (e.g., FAR/CS 25); (2) to produce the Airplane Flight Manual (AFM); and (3) to validate design characteristics and produce data that can be used by the manufacturer and operator.

Data are corrected to standard conditions (e.g., ISA). Correction factors are introduced for flight operations (e.g., to account for anticipated differences in reaction times between test pilots and line pilots in emergency situations, such as engine failure).

Operational documentation (e.g., FCOM) is generated, which must (1) consider likely operational conditions (e.g., off-ISA conditions, airfield limits); (2) introduce conservative correction factors and allowances (e.g., credit for headwind on landing); and (3) establish operational limit speeds (e.g., minimum control speed in the air).

**Performance engineering (by the operator)**

Operational flight planning activities are conducted in accordance with (1) the manufacturer's documentation (e.g., AFM); (2) the requirements of the regulatory authorities (e.g., FAR 121 and EASA OPS Part-CAT); and (3) local and international restrictions (e.g., noise limits). Activities include route planning (for standard and emergency conditions), the analysis of the airplane's performance for all critical phases of the flight, the determination of fuel requirements, weight and balance calculations, and takeoff and landing performance estimations (addressing such issues as noise abatement and reduced thrust, if applicable).

**Flight operations (by the flight crew)**

Flight operations are conducted in actual (non-idealized) weather conditions; hence corrections to published performance data may be required.

Preflight activities include the determination of the fuel required for the flight; checking of weight and center of gravity position against airplane limits; route planning (which considers forecast winds, weather, and anticipated delays); and airplane condition (e.g., restrictions due to unserviceable items).

In-flight activities include operating the airplane within the manufacturer's performance limits (e.g., speed, load factor, angle of attack), fuel status and systems monitoring, and the management of routine and emergency situations.

**Figure 1.1** Airplane performance-related activities.

represent the data as a function of pressure altitude, which is based on the idealized conditions defined in the International Standard Atmosphere (ISA). Airspeed and altitude instrumentation is calibrated to sea level (datum) conditions of the ISA. The database that is produced through the flight-test program is used to generate the performance values that are recorded in the AFM.

The determination of safety-related performance data during flight testing is a key part of the certification process in which compliance with the relevant airworthiness regulations must be demonstrated for the issue of a *type certificate*. Individual aircraft manufactured to an approved design—that is, a design for which a type certificate has been granted—may be issued a *certificate of airworthiness* by the national aviation authority of the country in which the airplane is registered. Airplanes with a valid certificate of airworthiness (which requires annual renewal) may be legally operated within the regulatory conditions of its issue.

The safe operation of any aircraft depends critically on the relevance and accuracy of the performance data that are available to the operator and flight crew. Performance calculations conducted in support of flight operations are based on manufacturers' performance data—taking account of the forecast weather, prevailing winds, runway conditions and limits, obstacle heights, and so forth. In some respects, the nature of these calculations is the reverse of that conducted during the design of the airplane: here, the airplane's performance attributes have to be determined based on known airplane characteristics; whereas during the design phase, it is the airplane characteristics that have to be determined to meet performance targets.

### 1.5.2 Performance Engineering and Flight Operations

There is a diverse set of engineering tasks and activities associated with the in-service operation of commercial jet transport airplanes that is customarily addressed under the heading of *airplane performance engineering*. The list of activities undertaken by performance engineers in support of flight operations can, for the sake of convenience, be grouped under five headings, although in reality there are many overlapping aspects. The description given below is not intended to be exhaustive, but rather illustrative of the nature of this work—these descriptions serve to establish a backdrop to the discussions presented in this book.

#### Performance Activities Associated with Specific Phases of a Flight

These activities include the determination and monitoring of the performance of an airplane during specific phases of the flight, which include takeoff (and rejected takeoff), initial climb after takeoff, takeoff flight path, *en route* climb to cruise altitude, cruise (including step climb), descent, approach (and missed approach), and landing. Key tasks are associated with establishing the airplane's performance during emergencies, such as engine failures at any stage of the flight and the determination of associated operational procedures.

#### Trip/Mission Performance

These activities consider the trip, or mission, performance of an airplane, which include payload–range assessments, determination of cost index parameters, policies for fuel tankering (fuel transportation), fuel conservation (including policy development and implementation), and considerations associated with exhaust emissions.

#### Route and Operational Flight Planning

These activities are associated with route analysis and consider such issues as foreign and domestic airspace restrictions, available air traffic tracks and flight levels, air traffic control (ATC) restrictions, suitability of destination and alternate airports, fuel considerations (including fuel planning and fuel usage monitoring), *en route* terrain considerations (including the determination of minimum safe altitudes), in-flight emergency considerations (e.g., oxygen requirements for passengers and crew), and extended-range twin-engine operations.

**Weight and Balance**

These activities are associated with establishing the airplane's operating empty weight (OEW) and center of gravity (CG) location, developing load sheets and validating payload weight procedures, assessing the CG location with respect to the manufacturer's fore and aft limits for each phase of the flight, and establishing procedures for the correct setting of the stabilizer trim for takeoff.

**Operational Support and Organization-Specific Tasks**

These activities include the preparation and upkeep of documentation for flight crews and dispatchers; implementation of applicable service bulletins, airworthiness directives and regulations; maintaining dispatch deviation documents (e.g., Master Minimum Equipment List); and a wide range of performance tasks applicable to the individual organization (e.g., monitoring reduced thrust usage in takeoff or climb, and providing input to fleet retirement/renewal decisions).

## 1.6 Analysis Techniques and Idealizations

The analysis techniques that are used to compute the performance characteristics of an airplane vary considerably depending on the purpose and the availability of data. There are many cases where a quick, simple "back of the envelope" calculation will suffice in order to obtain an approximate answer—for example, to provide a cross check against a result produced by a computer program or data contained in a reference manual. Simple, approximate methods also get used in student exercises, where real airplane data are seldom available. More sophisticated methods are, of course, needed for the computation of performance values that would be used in planning actual flight operations. In this book, a variety of methods of varying degrees of complexity are described (note that certain of these methods are only suitable for rough estimations).

Airplane performance analyses, in most cases, rely on three sets of data—these correspond to descriptions of (1) the characteristic parameters of the airplane (e.g., geometric parameters such as the wing reference area, aerodynamic relationships such as the drag polar, and the airplane's gross weight[5] and center of gravity position); (2) the characteristic parameters of the engine (e.g., net thrust and rate of fuel consumption); and (3) the environment in which the airplane is operating (e.g., ambient air temperature and pressure, wind, and runway features for takeoff and landing calculations). For example, the determination of an airplane's instantaneous rate of climb corresponding to a set of conditions (e.g., airplane gross weight, altitude, air temperature, speed, and acceleration) will require knowledge of a subset of data from the three groups, and—as will be shown later—the rate of climb depends principally on the airplane's thrust and drag.

The task of analyzing the airplane's performance is greatly simplified by the establishment of analytical models. Note that these models are not absolute laws, although many have a theoretical basis. These mathematical models are approximations of measurable parameters, which are usually only valid within specified limits. Some of these relationships can be described by surprisingly simple polynomial functions (such as the parabolic drag polar), which are adequate for approximate calculations. Certain functions, however, do not lend themselves to simple mathematical idealization. The variation of thrust lapse rate with altitude and speed, for example, is far too complex to be modeled by simple functions. Mathematical models can often be extremely useful, as they permit "exact" solutions to be found for complex scenarios, thus enabling

5 *Gross weight* is the total airplane weight at any moment during the flight or ground operation (i.e., instantaneous weight).

sensitivity analyses to be completed, thereby identifying the relationship between changes in airplane characteristic parameters and the predicted performance. Quite often, simple mathematical expressions can be used to initially study an idealized problem, and, thereafter, the effect of discrepancies between the model and reality can be addressed by adding smaller, second-order terms or correction factors to refine the calculation.

Idealizations are widely used in airplane performance studies. For most applications, the airplane can be considered to be a rigid body, permitting a classical Newtonian mechanics approach to be used to determine the equations of motion of the airplane. The application of static air loads on the wing will alter the wing's shape as it bends upwards and twists, resulting in a change to the chordwise and spanwise lift distributions, which will change the lift-induced drag. This can, in part, be accounted for in the determination of the airplane's drag polar. Dynamic air loads due to gusts (which tend to be oscillatory) momentarily influence the airplane's trajectory, but have a negligible influence on such performance parameters as climb gradient, range, or endurance.

For certain performance analyses that are associated with point calculations, it is often convenient to assume that a quasi-steady-state condition exists. The airplane's velocity vector is thus assumed not to change in magnitude or direction. This implies a state of equilibrium and a balance between the forces of lift, drag, thrust, and weight, which greatly simplifies the mathematics. Considering, once again, the example of instantaneous rate of climb, such an idealization would ignore the influence of the rate of change of true airspeed with height as well as the influence of wind gradients. In this case, the assumption of quasi-steady state permits the determination of a reasonable approximation. The inclusion of the first acceleration term will provide a small refinement to the calculation; the second acceleration term (i.e., due to a wind gradient) has an even smaller influence on the computed answer for a typical *en route* climb.

Another idealization that is frequently adopted when computing a performance parameter that relies on a time-based integral assumes that the airplane's mass is constant. This is obviously not true for a jet airplane due to the continuous consumption of fuel. However, the time rate of change of the airplane's mass is small, and this permits certain performance analyses to be conducted with the assumption of constant airplane mass. For example, it is commonplace to assume that the airplane's mass does not change during the takeoff run. This assumption simplifies the mathematics and facilitates the development of a closed-form mathematical solution for the takeoff run. Nonetheless, the inclusion of mass change is possible within a sophisticated numerical routine that involves an iterative approach to the determination of the takeoff distance, and this provides a means to refine the computed distance—albeit by a very small amount.

The Earth's curvature is an important consideration for long distance navigation, but, for the most part, the performance of an airplane can be satisfactorily assessed by assuming that the Earth is flat. Additionally, the Earth's rotation does not have a significant influence on most performance parameters and is generally ignored. There are, however, certain times when a high degree of precision is warranted and it is justified to take into account small correction factors. For example, in the analysis of specific fuel consumption data recorded during flight tests, it is possible to take into account the centrifugal influence of the Earth's rotational velocity on the airplane's weight—this correction depends on the airplane's ground speed and direction of flight (or, more precisely, its true track).

A common engineering approach, when assessing the validity of such idealizations, considers the impact that the idealization has on the computed result. For example, when credit is not taken for the reducing airplane mass during takeoff, a conservative result is obtained—that is, a marginally longer takeoff distance is predicted—and this may justify the use of the idealization. Another factor that must always be considered when assessing the merits of including such refinements is the accuracy to which the other parameters in the equation can be established. There is little value in going to considerable computational effort (e.g., by accounting for aircraft mass change in a takeoff analysis) when there is uncertainty associated with another parameter in the equation that has a significantly greater influence on the final result.

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# 2

# Engineering Fundamentals

## 2.1 Introduction

This chapter introduces the fundamental engineering parameters, principles, and concepts that underpin the study of the mechanics of flight. An accessible summary of these topics is presented using language and notation that is consistent with subsequent chapters. In essence, this is the foundation upon which the analytical and numerical models that describe the performance of an airplane are constructed.

The topics of notation (representing engineering terms), abbreviations, symbols, and units—applicable to the study of airplane performance—are discussed in Section 2.2. Increasingly, SI units[1] are used in engineering the world over. Nonetheless, the foot–pound–second (FPS) unit system[2] remains widely used in certain English-speaking countries, and many practicing engineers (in the United States, for example) do not have a "feel" for SI units. Furthermore, by international agreement, altitude is measured in feet in most of the world's airspace. For these reasons, equations in this book are presented in a consistent form that will permit either SI units or FPS units to be used (Appendix B contains a comprehensive table of conversion factors).

Converting between units poses little difficulty for most applications; however, in the case of airplane performance analyses, alternative definitions for what could appear to be the same parameter can lead to errors—for example, by the inclusion or exclusion of gravitational acceleration in equations involving fuel flow. Precise definitions of mass, weight, and gravity (see Section 2.3) facilitate a better understanding of affected units and engineering terms.

The equations that describe the motion of an airplane on the ground and in the air rely on the underlying principles of rigid body dynamics and fluid dynamics—a summary of the basic concepts is presented in Sections 2.4 and 2.5, respectively.

1 The International System of Units, universally abbreviated as SI from the French *Le Système International d'Unités*, is the modern form of the metric system of measurement [1].
2 United States Customary (USC) units (known also as Standard units) of measurement are very similar—and in many cases identical—to British Imperial units of measurement, as both were historically derived from a common system of measurement, usually called English units, which were used throughout the British Empire and its region of influence. The term foot–pound–second (FPS) is used herein to describe a coherent system of measurement, consistent in all respects with USC units, that considers the pound (lb) as the fundamental unit of force and the slug as the derived unit of mass.

*Performance of the Jet Transport Airplane: Analysis Methods, Flight Operations, and Regulations*, First Edition.
Trevor M. Young.

## 2.2 Notation, Units, and Conversion Factors

### 2.2.1 Notation

The subject of flight mechanics is fraught with confusion regarding notation. Various systems have been used over the years by authors, researchers, and practicing engineers around the world. Some of these have been superseded by newer systems, but old texts are still found in libraries (and on the shelves of professors)—a potential source of confusion. The problem is not only restricted to the use of different symbols for the same parameter, but different parameters and non-dimensional groups are sometimes given the same name. This could mean that values for a particular non-dimensional parameter for an airplane could differ by a factor of two (depending on whether an American or British system has been used, for example).

Words used to describe aerodynamic characteristics, such as the breakdown of drag (see Section 7.3), are not universally consistent, and the same word may have a slightly different meaning, depending on the preference of the author or the country of origin of the book or report. A mix of American, British, old and new terms can be found in academic and commercial literature.

Yet another problem encountered in this subject is the inconsistency of what defines the positive sense of a parameter. One example is the stick/yoke force—a positive force will imply that the pilot is pulling back on the stick/yoke in one convention, while another author will define a pushing force as positive.

In this book, an effort has been made to be clear and unambiguous in defining all terms; nonetheless, certain nomenclature will have more than one meaning (an unavoidable situation in this subject).

### 2.2.2 Abbreviations and Symbols

In any scientific or technical field—and aeronautical engineering is no exception—the use of abbreviations or acronyms as a substitute for cumbersome technical terms is commonplace (e.g., TSFC for thrust specific fuel consumption). Abbreviations facilitate a conciseness in writing and are generally advantageous—readers are more easily able to comprehend the subject matter, as the information is condensed. In this book, abbreviations have been extensively used, and in all cases the abbreviations are defined when they are first mentioned; there is also a list of abbreviations in Appendix G. The abbreviations comprise multiple letters in upper and/or lower case (e.g., SAR, ppm) and are sometimes separated by punctuation marks (e.g., R/C).

The prime purpose of symbols (e.g., $V$, $C_D$, $\alpha$), on the other hand, is to represent technical terms, or parameters, in mathematical formulae. A symbol is best written as a single upper or lower case letter, augmented where necessary by subscripts and/or superscripts (e.g., $V_v$, $\delta_e$, $H^*$). The use of abbreviations in mathematical formulae, however, can lead to misunderstanding, and for this reason the practice is discouraged. For example, KE (which can represent kinetic energy) could be understood as the product of K and E. Technical parameters (i.e., variables and constants) that appear in equations are herein assigned symbols, irrespective of whether an abbreviation also exists—for example: TSFC is assigned the symbol $c$, and this is consistently used in equations. The reverse situation, however, whereby a symbol is used in text, seldom creates a problem (provided that the symbol has been properly defined, of course)—for example, $C_L$, the widely accepted symbol for lift coefficient, is frequently used both in equations and in text (in academic literature and other technical documentation).

Another convention that has been adopted herein is to set the symbols (representing variables and constants) in italics—this is done in equations and also in the text. Where it is important to identify a vector parameter, this has been done in the customary manner, using either bold font or by placing a small arrow over the symbol.

### 2.2.3 Base Quantities and Dimensions

All physical quantities used in engineering (e.g., force, entropy) can be expressed in terms of seven mutually independent base quantities—these are length, mass, time, thermodynamic temperature, amount of substance, electric current, and luminous intensity. (Interestingly, it is possible to express almost all quantities that are encountered in mechanics in terms of just three of these quantities: length, mass, and time.) Other quantities—which are called derived quantities—are obtained from equations expressed in the form of a product of powers of the base quantities.

The dimensions of the seven base quantities are usually denoted by the symbols L, M, T, Θ, N, I, and J, respectively (Table 2.1). Dimensions of other quantities are obtained by equation—for example, for an arbitrary quantity $Q$, the dimension (dim) will be

$$\dim[Q] = \mathrm{L}^{\alpha}\ \mathrm{M}^{\beta}\ \mathrm{T}^{\gamma}\ \Theta^{\delta}\ \mathrm{N}^{\varepsilon}\ \mathrm{I}^{\zeta}\ \mathrm{J}^{\eta} \tag{2.1}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$, and $\eta$ are the dimensional exponents.

Using this convention, the dimension of energy, for example, is $\mathrm{L}^2\mathrm{MT}^{-2}$, which is consistent with the equations for kinetic energy (see Equation 2.42) and potential energy (see Equation 2.44).

**Table 2.1** Base quantities

| Quantity | Dimension | SI unit | Symbol |
|---|---|---|---|
| length | L | metre *or* meter | m |
| mass | M | kilogram | kg |
| time | T | second | s |
| thermodynamic temperature | Θ | kelvin | K |
| amount of substance | N | mole | mol |
| electric current | I | ampere | A |
| luminous intensity | J | candela | cd |

### 2.2.4 Units and Conversion Factors

Various unit systems have been adopted to express the measurement of engineering quantities. Unit systems that have been defined using the fundamental equations that link quantities are called *coherent*. It is a property of coherent unit systems that the equations of mechanics that describe force, energy (or work), and power in terms of length, mass, and time hold without the introduction of constant factors (see Sections 2.3 and 2.4). The Système International [1, 2], which is widely known as SI, is such a system. Each base quantity has a base unit (see Table 2.1). Each derived quantity has a derived unit, which is obtained from the appropriate equation by replacing the dimension symbols with units. Derived units can thus be expressed in terms of the base units—for example, energy can be expressed in units of $\mathrm{m}^2\ \mathrm{kg\ s}^{-2}$, which, by definition, is equivalent to a joule (J).

SI units are widely used in aeronautical engineering, but, even then, there are a few exceptions that arise due to customary practices or regulations associated with flight operations. For example, in almost all countries in the world (see Section 5.4 for exceptions), altitude

**Table 2.2** Frequently used conversion factors

| | US Customary unit | SI unit | Conversion factor |
|---|---|---|---|
| length, height, *or* distance | ft | m | 0.3048 |
| distance[(a)] | nm | m | 1852 |
| speed[(b)] | kt | m/s | 0.5144 |
| area | $ft^2$ | $m^2$ | 0.09290 |
| volume | US gal *or* USG | $dm^3$ *or* L | 3.785 |
| force[(c)] | lb | N | 4.448 |
| pressure | $lb/in^2$ | Pa | 6895 |
| mass | slug | kg | 14.59 |
| mass[(d)] | lbm | kg | 0.4536 |
| density | $slug/ft^3$ | $kg/m^3$ | 515.4 |
| specific energy[(e)] | Btu/lbm | J/kg | 2326 |
| thrust specific fuel consumption[(f)] | $lb\ lb^{-1}\ h^{-1}$ | $mg\ N^{-1}\ s^{-1}$ | 28.33 |

**Notes:**

(a) The unit nm, when taken out of context, can be misinterpreted as the SI unit for nanometers. If confusion is likely, an alternative abbreviation (e.g., NM, nmi, naut.mi.) should be used.
(b) One knot (abbreviated as kt or, alternatively, as kn) is equal to one nautical mile per hour.
(c) The unit abbreviation lb is customarily preferred to the abbreviation lbf for pound-force.
(d) The unit abbreviation lbm is used herein for pound-mass (avoirdupois pound).
(e) Specific energy is energy per unit mass.
(f) The conversion of thrust specific fuel consumption (TSFC) is not dimensionally consistent. The TSFC can be defined as either the *mass* of fuel burned per unit of time divided by the thrust or the *weight* of fuel burned per unit of time divided by the thrust (see Section 8.4.2).

is measured in feet for airplane operations. Furthermore, distance is frequently measured in nautical miles and speed is measured in knots (see Sections 6.2 and 6.3). The acceptance of these non-SI units (i.e., feet, nautical miles, and knots) in flight operations is specifically addressed in ICAO Annex 5 (International Standards and Recommended Practices concerning units of measurement for air and ground operations) [3]—these units have been adopted as the primary measures of height, trip distance, and speed in this book.

The most frequently used conversion factors for this subject (given to four significant figures) are listed in Table 2.2; a more complete listing of conversion factors (with greater precision) is provided in Appendix B.

### 2.2.5 Temperature Scales

The four commonly used temperature scales are Celsius (unit: °C), kelvin (unit: K), Fahrenheit (unit: °F), and Rankine (unit: °R). Reference temperatures for these temperature scales are given in Table 2.3. Temperature values can be converted between Celsius and Fahrenheit as follows:

$$T_C = \left(\frac{5}{9}\right)(T_F - 32) \tag{2.2a}$$

$$T_F = \left(\frac{9}{5}\right)T_C + 32 \tag{2.2b}$$

where $T_C$ and $T_F$ are temperature values in °C and °F, respectively.

**Table 2.3** Reference temperatures for commonly used temperature scales

| Temperature scale | Absolute zero | Freezing point of pure water* | Boiling point of pure water* |
|---|---|---|---|
| Celsius | −273.15 °C | 0 °C | 100 °C |
| Fahrenheit | −459.67 °F | 32 °F | 212 °F |
| Kelvin | 0 K | 273.15 K | 373.15 K |
| Rankine | 0 °R | 491.67 °R | 671.67 °R |

***Note:** Measured under standard pressure conditions (see also Table B.4).

The conversion to absolute values can be undertaken using the following expressions:

$$T_K = T_C + 273.15 \tag{2.3a}$$

$$T_R = T_F + 459.67 \tag{2.3b}$$

where $T_K$ and $T_R$ are absolute temperature values in K and °R, respectively.

### 2.2.6 Scalar and Vector Quantities

Scalar quantities are described by magnitude only, whereas vector quantities are defined by both magnitude and direction. Scalar quantities are traditionally printed in italic typeface (a convention that has been adopted herein). Vectors are printed in bold roman (i.e., upright) typeface; alternatively, they can be identified by an arrow drawn over the italicized symbol. For example, a velocity vector would be written as $\mathbf{V}$ or $\vec{V}$ and its magnitude as $V$.

## 2.3 Mass, Momentum, Weight, and Gravity

### 2.3.1 Mass

Mass is a measure of the amount of matter in a solid or fluid body. Mass is also a measure of a body's resistance to acceleration—a relationship expressed by Newton's second law:

$$F = ma \tag{2.4}$$

where $F$ is the force (typical units: N, lb);
$m$ is the mass (typical units: kg, slug); and
$a$ is the acceleration measured in the direction of the force (typical units: m/s$^2$, ft/s$^2$).

The standard units of mass are kilogram (SI unit) and slug (FPS unit). As SI and FPS are coherent unit systems, the following relationships hold:

- 1 kg is the mass of a body that will accelerate at 1 m/s$^2$ when subjected to a force of 1 N.
- 1 slug is the mass of a body that will accelerate at 1 ft/s$^2$ when subjected to a force of 1 lb.

### 2.3.2 Momentum

Momentum, which is a vector quantity, is defined as the product of a body's mass and its velocity. Standard units are kg m s$^{-1}$ and slug ft s$^{-1}$.

### 2.3.3 Weight

Weight is the gravitational force exerted on a body. When an object is allowed to fall freely in a vacuum under the influence of its own weight, it will accelerate at a rate known as the *gravitational acceleration* ($g$)—which in aviation literature is sometimes written as *gee*. Gravitational acceleration can be considered as the constant of proportionality linking weight and mass:

$$W = mg \tag{2.5}$$

where $W$ is the weight (typical units: N, lb); and
$g$ is the gravitational acceleration, measured in the direction of the weight force (typical units: m/s$^2$, ft/s$^2$).

The acceleration due to gravity varies with geographical location: it increases in magnitude with increasing latitude (i.e., it is greater at the poles than at the equator), and it reduces with increasing height above the Earth's surface (see Section 2.3.4). To simplify the analysis of air vehicles in the Earth's atmosphere, the standard value of gravitational acceleration[3] ($g_0$) is widely used, where

$g_0 = 9.80665$ m/s$^2$ (exactly) in SI units; or
$g_0 = 32.174049$ ft/s$^2$ (correct to eight significant figures) in FPS units.

This standard value of $g$ links the *units* of mass and weight, as follows:

- A body of mass 1 kg weighs 9.80665 N under standard conditions.
- A body of mass 1 slug weighs 32.174049 lb under standard conditions.

Weight-equivalent and mass-equivalent terms, based on $g_0$, can be defined as follows:

- 1 kgf (1 kilogram-force[4]) is the weight of a body of mass 1 kg under standard gravitational conditions (thus 1 kgf = 9.80665 N).
- 1 lbm (1 pound-mass) is the mass of a body that weighs 1 lb under standard gravitational conditions (thus 1 lbm = 0.03108095 slug).

Note that kilogram-force and pound-mass are non-coherent units; neither are recommended for scientific applications.

### 2.3.4 Gravitational Acceleration

The weight of a body of fixed mass will depend on its physical location. The law of universal gravitation, formulated by Isaac Newton,[5] states that every particle of matter in the universe attracts

3 The standard value of $g$, which is usually identified by the subscript zero, was agreed by the International Bureau of Weights and Measures in 1901 [1]. It is officially defined in SI units; the equivalent value in US Customary (USC) units is obtained by unit conversion.
4 The kilogram-force unit is also known as the kilopond (kp), which, note, is not part of the SI system. Although once widely used as a unit of force, the kilopond is considered an obsolete unit and is today seldom encountered in general engineering work.
5 Isaac Newton (1643–1727), an English physicist and mathematician, published the first mathematical formula of gravity and the basic laws governing the motion of bodies (see Section 2.4.1) in his three-volume work *Principia Mathematica* in 1687.

every other particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them, that is,

$$F = G\frac{m_1 m_2}{d^2} \tag{2.6}$$

where $F$ is the mutual force of attraction between two bodies (typical units: N, lb);
$G$ is the universal (or Newtonian) gravitational constant (see below);
$m_1$ and $m_2$ are the masses of the two bodies (typical units: kg, slug); and
$d$ is the distance between the centers of the two bodies (typical units: m, ft).

The CODATA[6] (2006) recommended value of $G$ in SI units is $6.67428 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ with an uncertainty of $0.001 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$. The converted value of $G$ in FPS units is $3.43978 \times 10^{-8}$ ft$^3$ slug$^{-1}$ s$^{-2}$.

An expression for the gravitational acceleration at the Earth's surface can be deduced by considering the gravitational force that acts on a body of mass $m$ if the Earth is represented by an idealized, perfectly spherical body. By combining Equations 2.5 and 2.6, it follows that

$$F = mg_{SL} = G\frac{m_E m}{r_E^2}$$

thus $$g_{SL} = G\frac{m_E}{r_E^2} \tag{2.7}$$

where $g_{SL}$ is the gravitational acceleration at sea level (typical units: m/s$^2$, ft/s$^2$);
$m_E$ is the mass of the Earth (typical units: kg, slug); and
$r_E$ is an idealized, equivalent radius of the Earth[7] (typical units: m, ft).

The gravitational pull exerted by the Earth on another object (e.g., an airplane) reduces as the object moves away from the Earth's surface. An expression for the gravitational acceleration as a function of height above sea level can be deduced from Equation 2.7, that is,

$$g_z = G\frac{m_E}{(r_E + z)^2} \tag{2.8}$$

where $g_z$ is the gravitational acceleration at height $z$ (typical units: m/s$^2$, ft/s$^2$); and
$z$ is the height of the object above sea level (typical units: m, ft).

It is convenient for certain applications to express $g_z$ in terms of the gravitational acceleration at sea level ($g_{SL}$). By dividing Equation 2.8 by Equation 2.7, it can be shown that

$$g_z = g_{SL}\left(\frac{r_E}{r_E + z}\right)^2 \tag{2.9}$$

Equations 2.7 to 2.9 describe an idealized situation, where the Earth is considered to be perfectly spherical; furthermore, the influence of the Earth's rotation about its axis has been

6 The Committee on Data for Science and Technology (CODATA), an interdisciplinary committee of the International Council for Science (ICSU), periodically publishes internationally accepted sets of values of fundamental physical constants [4].

7 For the purpose of defining standard atmospheres (see Section 4.2), an equivalent (nominal) radius of the Earth of $6.356766 \times 10^6$ m ($2.0855531 \times 10^7$ ft) is used [5]. This yields the standard value of gravitational acceleration ($g_0$) at latitude 45.5425°.

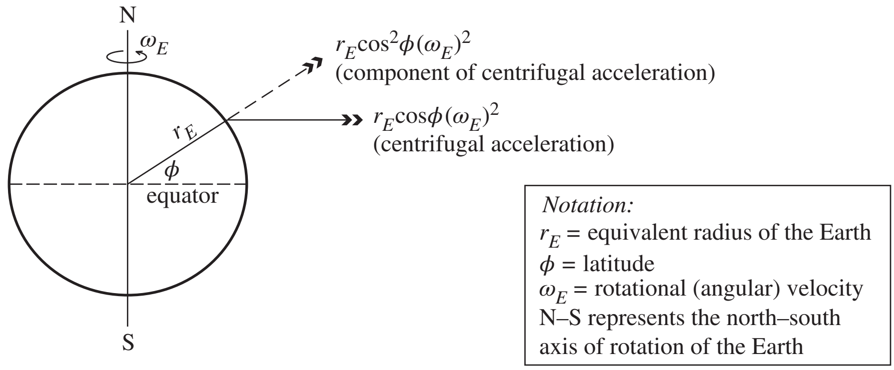



**Figure 2.1** Centrifugal acceleration of an object on the Earth's surface due to the Earth's rotation.

ignored. Both of these factors require small corrections to be included in the mathematical description of the Earth's gravitational field if precise calculations involving an object's *weight* are to be conducted. It is known that the gravitational force exerted on an object of fixed mass, at sea level, depends on latitude—it is weaker at the equator than at the poles. There are two reasons for this observation. First, the Earth is not a perfect sphere: it bulges out a little at the equator, effectively increasing the distance from the Earth's center to its surface. Secondly, the Earth's rotational (angular) velocity imparts a centrifugal force on the object, which is perpendicular to the Earth's rotational axis (see Figure 2.1). The upward component (i.e., normal to the Earth's surface) of this centrifugal acceleration opposes gravity, diminishing its apparent effect. The gravitational acceleration at sea level taking into account the Earth's rotation ($g_{SL,\phi}$) is thus

$$g_{SL,\phi} = g_{SL} - r_E \cos^2 \phi \, \omega_E^2 \tag{2.10}$$

where $\phi$ is the geographic latitude; and
$\omega_E$ is the rotational velocity of the Earth (equal to $7.292115 \times 10^{-5}$ rad/s).

Even though this centrifugal force is not due to the Newtonian gravitational attraction between the Earth and the other body, it is always present, and it is thus customary to include this correction in published data of the Earth's gravitational field.

The Earth, of course, is not a perfect sphere, and, for some applications, Equation 2.10 does not provide sufficiently accurate answers. The Earth's gravitational acceleration is well represented by the international gravity formula, which is used by the World Geodetic System WGS 84 [6]. The formula, which is based on an ellipsoidal representation of the Earth and a rotational velocity of $7.292115 \times 10^{-5}$ rad/s, expresses gravitational acceleration at sea level ($g_{SL,\phi}$) as a function of latitude ($\phi$):

$$g_{SL,\phi} = g_{eq} \left[1 + \left(\frac{b}{a}\frac{g_{po}}{g_{eq}} - 1\right) \sin^2 \phi\right] \left(\frac{1}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}}\right) \tag{2.11}$$

where $g_{eq}$ is the theoretical gravity at the equator, equal to 9.7803253359 m/s$^2$;
$g_{po}$ is the theoretical gravity at the poles, equal to 9.8321849378 m/s$^2$;
$a$ is the semi-major ellipsoidal axis, equal to $6.378137 \times 10^6$ m;
$b$ is the semi-minor ellipsoidal axis, equal to $6.3567523142 \times 10^6$ m; and
$\varepsilon$ is the first ellipsoidal eccentricity, equal to $8.1819190842622 \times 10^{-2}$.

An alternative formulation, which is used for many applications (e.g., to establish standard atmospheres—see Section 4.2), is known as Lambert's equation [5]:

$$g_{SL,\phi} = g_{ref}(1 - 2.6373 \times 10^{-3} \cos 2\phi + 5.9 \times 10^{-6} \cos^2 2\phi) \tag{2.12}$$

where $g_{ref} = 9.80616$ m/s$^2$ (32.17244 ft/s$^2$).

The gravitational acceleration corresponding to any given height and latitude ($g_{z,\phi}$) can be obtained by applying a height correction to the corresponding sea level value (given by Equation 2.11 or 2.12). This can be done using Equation 2.9, or, alternatively, using Equation 2.13 (which is a more precise method that accounts for latitude). Equation 2.13 is obtained by including the centrifugal correction in Equation 2.9, and then substituting $g_{SL}$ from Equation 2.10; thus

$$g_{z,\phi} = \left(g_{SL,\phi} + r_E \cos^2 \phi\, \omega_E^2\right) \left(\frac{r_E}{r_E + z}\right)^2 - (r_E + z) \cos^2 \phi\, \omega_E^2 \tag{2.13}$$

### 2.3.5 Influence of an Aircraft's Motion on Its Apparent Weight

The apparent weight of fast-moving objects, such as aircraft, in the Earth's atmosphere is influenced, albeit by a small amount, by the speed and direction of motion of the object.[8] If an aircraft flies at constant height above the Earth's surface on a fixed heading, it will follow a circular flight path (i.e., curved towards the Earth). Consequently, a centrifugal force will act on the aircraft, which will oppose the gravitation attraction. This is very similar in principle to the centrifugal correction (described in Section 2.3.4) due to the Earth's rotation; however, in this case, the correction depends not only on latitude but also on the aircraft's height (above sea level), ground speed, and true track (defined in Section 6.3.2).

The following equation (which is derived in reference [8]) can be used to estimate the centrifugal correction due to the aircraft's motion ($\Delta g_{cm}$):

$$\Delta g_{cm} = -\frac{V_G^2}{r_E + z} - 2\omega_E V_G \cos \phi \sin \chi \tag{2.14}$$

where $V_G$ is the ground speed of the aircraft (typical units: m/s, ft/s); and
$\chi$ is the true track angle of the flight path (unit: degrees, measured clockwise with respect to true north).

The output of Equation 2.14 is added to $g_{z,\phi}$ to give a corrected value of gravitational acceleration ($g_{z,\phi,cm}$):

$$g_{z,\phi,cm} = g_{z,\phi} + \Delta g_{cm} \tag{2.15}$$

### 2.3.6 Actual Versus Standard Gravitational Acceleration

The use of a single, standard gravitational acceleration ($g_0$) enables simplified mathematical models involving an object's weight to be used in many engineering tasks, and is sufficiently accurate for most airplane performance applications. It is common practice to use $g_0$ for flight operations work. For tasks where a high degree of precision regarding the airplane's weight is

8 On eastbound flights at the equator, the Concorde cruising at Mach 2 at 60 000 ft would have experienced a 1.5% reduction in apparent weight due to this centrifugal effect [7].

needed, such as cruise flight testing, corrections due to (1) latitude, (2) height, and (3) airplane motion need to be taken into account.

(1) The standard gravitational acceleration (see Section 2.3.3) matches the actual sea level value at about 45° latitude. The discrepancy between $g_0$ and the actual value of $g$ (at sea level) is thus negligible in mid-latitude regions, with the error increasing to a maximum of 0.27% as the observer moves towards the equator or the poles (as described by Equation 2.11).
(2) The variation of $g$ with height, at a given latitude, within the height range used for commercial airplane operations is small—but not always negligible. It can be shown using Equation 2.13 that $g$ reduces by 0.43% from sea level to a height of 45 000 ft (13 716 m), for example. Precise values of $g$ can be obtained by applying the height correction, given by Equation 2.13 (or by Equation 2.9—which yields almost the same result) to the sea level value predicted by Equation 2.11.
(3) The centrifugal influence on an airplane's weight due to its motion can be estimated using Equation 2.14. The correction depends on the airplane's location (specifically its latitude and height), ground speed, and true track. The influence is greatest for eastbound flights at the equator—for example, at a cruise speed of Mach 0.86 at 40 000 ft, the apparent weight of an aircraft due to its motion is reduced by 0.48%.

## 2.4 Basics of Rigid Body Dynamics

### 2.4.1 Newton's Laws of Motion

Before the forces acting on an airplane in flight are considered, it will be useful to review the basic theory of the dynamics of bodies in motion. The starting point of any such discussion is Newton's laws of motion, which, in modern terminology, can be stated as follows:

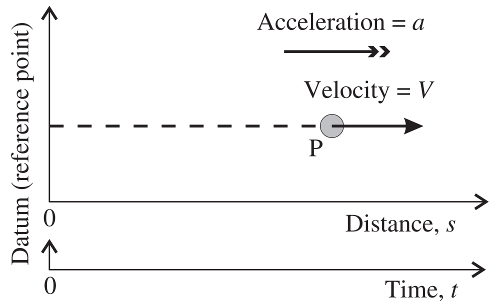
**Law 1:** A body remains at rest or continues to move in a straight line at a uniform velocity unless acted upon by an unbalanced force.
**Law 2:** The acceleration of a body is proportional to the net force applied and acts in the same direction as the force. Alternatively, it can be stated that the time rate of change of a body's momentum vector is equal to the applied force vector.
**Law 3:** The forces that exist between two interacting bodies are equal in magnitude, collinear, and directed in the opposite sense on each body. Alternatively, it can be stated that forces of action and reaction are equal and opposite.

Newton's laws essentially refer to the motions of *particles*. When applied to the motion of a large body, such as an airplane, a point mass at the airplane's center of gravity is taken to represent the airplane; furthermore, the effects of elastic distortions of the body are ignored (i.e., the airplane is assumed to be a rigid body).

### 2.4.2 Rectilinear Motion

Rectilinear motion describes the motion of a body in a straight line. Under such a condition, the position of the body (P), at any instant in time $t$, is given by the coordinate $s$, which is defined as its distance from a fixed datum (Figure 2.2). For relatively short distances (e.g., runway length), the preferred units of measurement are meters or feet, but for longer distances (e.g., trip length) the preferred units are kilometers (km) or nautical miles (nm), although statute miles (mi) are also used occasionally.

Figure 2.2 Rectilinear motion.




The instantaneous velocity of a body is the rate of change of distance with respect to time at the point of interest (specified by either $s$ or $t$):

$$V = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \text{or} \quad V\,\mathrm{d}t = \mathrm{d}s \tag{2.16}$$

where $V$ is the velocity (typical units: m/s, ft/s, km/h, kt, mi/h);
$s$ is the distance (typical units: m, ft, km, nm, mi); and
$t$ is the time (typical units: s, h).

The instantaneous acceleration is the rate of change of velocity with respect to time at the point of interest (specified by $s$, $t$, or $V$):

$$a = \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} \quad \text{or} \quad a\,\mathrm{d}t = \mathrm{d}V \tag{2.17}$$

where $a$ is the acceleration (typical units: m/s$^2$, ft/s$^2$).

A differential equation relating velocity, acceleration, and distance can be obtained from Equations 2.16 and 2.17:

$$a = \frac{\mathrm{d}V}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}s}V \quad \text{or alternatively} \quad a = \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{2}V^2\right)$$

Thus $a\,\mathrm{d}s = V\,\mathrm{d}V$ (2.18)

Note that velocity $V$ and acceleration $a$ are algebraic quantities that are defined as positive in the positive direction of $s$.

When the acceleration is constant, the differential Equations 2.17 and 2.18 can be evaluated directly. The initial condition is defined as $s(0) = 0$ and $V(0) = V_0$.

$$a\int_0^t \mathrm{d}t = \int_{V_0}^{V} \mathrm{d}V \qquad \text{or} \quad V = V_0 + at \tag{2.19}$$

$$a\int_0^s \mathrm{d}s = \int_{V_0}^{V} V\,\mathrm{d}V \quad \text{or} \quad V^2 = V_0^2 + 2as \tag{2.20}$$

Substituting $V$ from Equation 2.19 into Equation 2.16 gives

$$\int_0^s \mathrm{d}s = \int_0^t (V_0 + at)\,\mathrm{d}t \quad \text{or} \quad s = V_0 t + \frac{1}{2}at^2 \tag{2.21}$$

### 2.4.3 Motion With Respect to a Moving Datum (Reference System)

In Section 2.4.2, the rectilinear motion of bodies is considered with respect to a datum (reference system) that is fixed in space. This is a special case of the more general condition where the

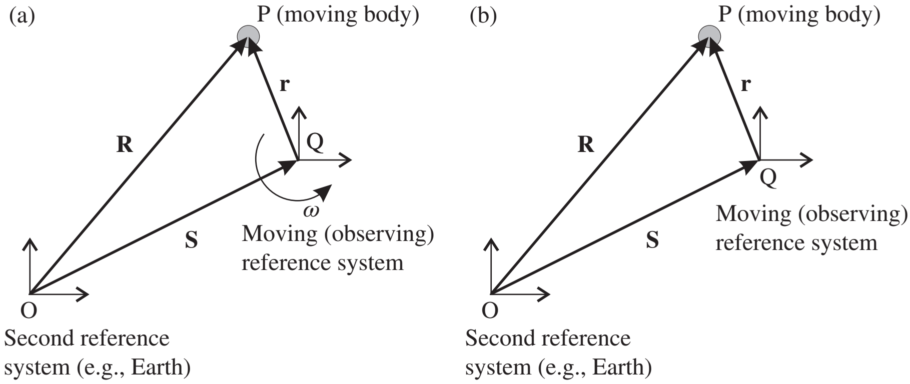



**Figure 2.3** Relative motion: (a) Translating and rotating reference system, and (b) Translating reference system.

coordinate system is itself moving with respect to a second reference system (e.g., the Earth). Analyzing the motions of bodies with respect to axes fixed to the Earth is satisfactory for most applications; however, for certain problems, a non-rotating coordinating system (e.g., based on the sun) must be used. There is one particular type of reference system that is convenient for the analysis of the dynamics of bodies: it is called an *inertial frame of reference*. By definition, such a system is either stationary or moving at a constant velocity.

Three cases—representing progressively simpler situations—are described in this section: (1) a moving reference system that translates and rotates; (2) a moving reference system that translates, but does not rotate; and (3) rectilinear motion.

#### (1) Translating and Rotating Reference System

The origin of the fixed set of axes is designated as O and that of the moving axes as Q (see Figure 2.3a). The vector **r** represents the relative motion of the body (P) with respect to the moving (or observing) axes and the vector **S** represents the motion of Q with respect to O. Furthermore, the moving axes are also turning at a rate $\omega$. The vector **R** describes the position of P with respect to O:

$$\mathbf{R} = \mathbf{S} + \mathbf{r} \tag{2.22}$$

The absolute velocity and acceleration of P (i.e., with respect to O) is given by

$$\dot{\mathbf{R}} = \dot{\mathbf{S}} + \dot{\mathbf{r}} + (\omega \times \mathbf{r}) \tag{2.23}$$

and $$\ddot{\mathbf{R}} = \ddot{\mathbf{S}} + \ddot{\mathbf{r}} + (\dot{\omega} \times \mathbf{r}) + \omega \times (\omega \times \mathbf{r}) + 2(\omega \times \dot{\mathbf{r}}) \tag{2.24}$$

The last term in Equation 2.24 is known as the Coriolis acceleration.

#### (2) Translating Reference System

A simpler solution arises when the reference system translates, but does not rotate (Figure 2.3b). The velocity and acceleration of P are obtained by vector summation:

$$\dot{\mathbf{R}} = \dot{\mathbf{S}} + \dot{\mathbf{r}} \tag{2.25}$$

$$\ddot{\mathbf{R}} = \ddot{\mathbf{S}} + \ddot{\mathbf{r}} \tag{2.26}$$

Note that if Q is moving at a constant velocity (as would be the case for an inertial reference system), then $\ddot{\mathbf{S}} = 0$ and the absolute acceleration of P is the same as the observed acceleration (this is a useful property for the determination of inertia forces, for example).

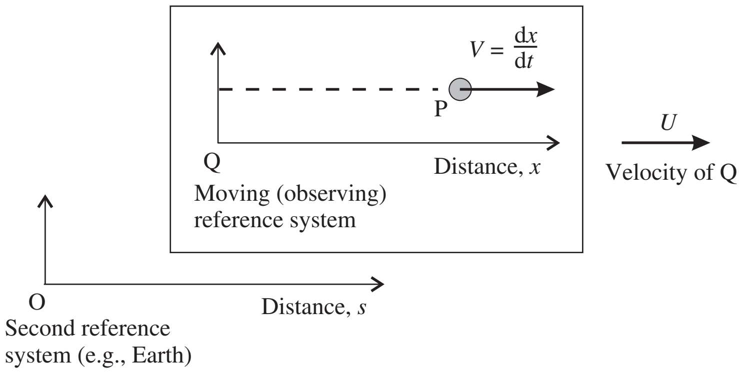



**Figure 2.4** Relative motion: rectilinear movement of reference system.

**(3) Rectilinear Motion**

A further simplification can be introduced when considering rectilinear motion (Figure 2.4). In this case, vector notation is not required as the moving reference system is translating in the same direction as the motion of the body, but not necessarily in the same sense. The velocity and acceleration of the body (P) with respect to O are given by

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} + U = V + U \tag{2.27}$$

and
$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} + \frac{\mathrm{d}U}{\mathrm{d}t} \tag{2.28}$$

where $x$ is the distance (abscissa) of P with respect to the moving axes;
$V$ is the velocity of P with respect to the moving axes;
$s$ is the distance (abscissa) of P with respect to the second axes (e.g., Earth); and
$U$ is the velocity of Q with respect to the second axes.

### 2.4.4 Curvilinear Motion

Curvilinear motion—which is a special case of the more general three-dimensional motion of a body—describes motion along a curved path that lies on a single plane. Normal ($n$) and tangential ($t$) coordinates are usually selected to describe such motions. The $n$ and $t$ coordinates move along the path with the body, such that the positive direction for $n$ is always towards the center of curvature of the path (Figure 2.5). Circular motion is a special case of curvilinear motion where the radius of curvature is constant (Figure 2.6).

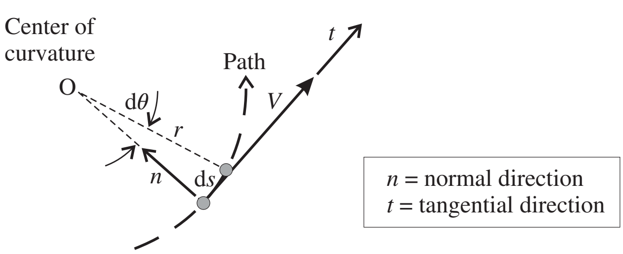



**Figure 2.5** Curvilinear motion.

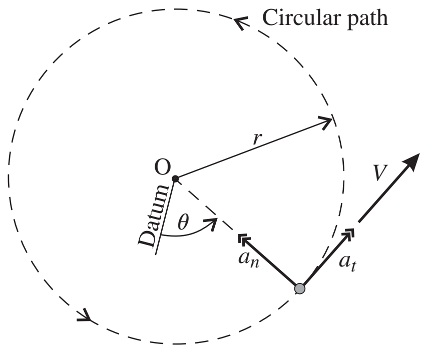



**Figure 2.6** Circular motion.

In Figure 2.5 the change in direction is shown as $d\theta$, where $\theta$ is measured in radians. The magnitude of the velocity, $V$, can be expressed as follows:

$$V = \frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt} = r\frac{d\theta}{dt} \tag{2.29}$$

as the differential displacement $ds$ is given by $ds = r\,d\theta$.

Acceleration, by definition, is the time rate of change of velocity. As velocity is a vector, the net acceleration includes both the change in magnitude and the change in direction. The acceleration due to the change in magnitude is given by

$$a_t = \frac{dV}{dt} \tag{2.30}$$

and the acceleration due to the change in direction is given by

$$a_n = \frac{V^2}{r} = V\frac{d\theta}{dt} = r\left(\frac{d\theta}{dt}\right)^2 \qquad \text{(using Equation 2.29)} \tag{2.31}$$

### 2.4.5 Newton's Second Law

Newton's second law (see Section 2.4.1) can be used to analyze the motion of a body when subjected to forces applied through the mass center. When Cartesian coordinates (see Section 3.3) are used, the forces are resolved into X, Y, and Z components. The acceleration can be calculated from the following equations:

$$\left.\begin{aligned} \sum F_x &= m a_x \\ \sum F_y &= m a_y \\ \sum F_z &= m a_z \end{aligned}\right\} \tag{2.32}$$

Similarly, when normal and tangential coordinates are used, the forces are resolved into $n$ and $t$ components and the accelerations calculated:

$$\left.\begin{aligned} \sum F_n &= m a_n \\ \sum F_t &= m a_t \end{aligned}\right\} \tag{2.33}$$

The net acceleration is given by

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{2.34}$$

or $$a = \sqrt{a_n^2 + a_t^2} \tag{2.35}$$

### 2.4.6 Work

Work, which is given the symbol $U$, is done when the point of application of a force is moved through a finite distance. Work is a scalar quantity; work is positive if the "working" component of the force is in the direction of the displacement and negative if it is in the opposite direction. The work done by force **F** during a differential displacement d**s** is given by the dot product of the vector **F** and the vector displacement d**s** (see Figure 2.7).

$$\mathrm{d}U = \mathbf{F} \bullet \mathrm{d}\mathbf{s} \qquad \text{or} \qquad U = \int \mathbf{F} \bullet \mathrm{d}\mathbf{s} \tag{2.36}$$

Now, as $\varepsilon$ is the angle between **F** and d**s**, Equation 2.36 can be written as

$$\mathrm{d}U = F \cos\varepsilon \, \mathrm{d}s \qquad \text{or} \qquad U = \int F \cos\varepsilon \, \mathrm{d}s \tag{2.37}$$

Equation 2.37 can be interpreted as either the component of force acting in the direction of the displacement multiplied by the displacement, or the force multiplied by the component of displacement measured in the direction of the force. The former interpretation leads to the following expression for work:

$$U = \int \mathbf{F} \bullet \mathrm{d}\mathbf{s} = \int F_x \, \mathrm{d}x + \int F_y \, \mathrm{d}y + \int F_z \, \mathrm{d}z \tag{2.38}$$

where $F_x$, $F_y$, and $F_z$ are the components of the force **F** in the directions X, Y, and Z, respectively.

The standard SI unit of work is the joule (J), where 1 J = 1 N m. In FPS units, work is usually measured in ft lb.

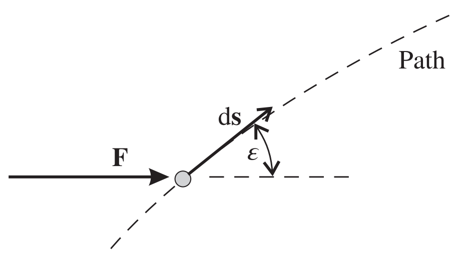



**Figure 2.7** Work done by a force that is not collinear with the path.

### 2.4.7 Power

Power ($P$) is defined as the time rate of doing work:

$$P = \frac{\mathrm{d}U}{\mathrm{d}t} \tag{2.39}$$

When a force **F** moves through the distance d**s** in the time d$t$, the power developed is given by

$$P = \frac{\mathrm{d}U}{\mathrm{d}t} = \mathbf{F} \bullet \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = \mathbf{F} \bullet \mathbf{V} \tag{2.40}$$

where $P$ is the power (typical units: W, ft lb s$^{-1}$);
$U$ is the work done (typical units: J, ft lb);
$t$ is time (typical unit: s);
d**s** is the differential displacement (typical units: m, ft);
**F** is the force vector (typical units: N, lb); and
**V** is the velocity vector (typical units: m/s, ft/s).

Power is a scalar quantity. The standard SI unit of power is the watt (W), where 1 W = 1 J/s = 1 N m/s. The FPS unit of power is ft lb/s; although in USC units, power is frequently measured in horsepower (hP), where 1 hP = 550 ft lb/s.

### 2.4.8 Kinetic Energy

Kinetic energy, within a conservative system, can be defined as the work that must be done on a body to accelerate it from a state of rest to a given velocity. If the force $F$ results in the mass $m$ being accelerated from zero velocity to velocity $V$ over the distance $s$, then the work done ($U$) can be determined from Equations 2.4 and 2.20:

$$U = F\,s = m\,a\,s = \frac{1}{2} m V^2 \tag{2.41}$$

The kinetic energy of the body moving at the velocity $V$ is thus

$$E_k = \frac{1}{2} m V^2 \tag{2.42}$$

where $E_k$ is the kinetic energy (typical units: J, ft lb);
$m$ is the mass (typical units: kg, slug); and
$V$ is the velocity (typical units: m/s, ft/s).

Kinetic energy, by definition, is a scalar quantity and is always positive.

### 2.4.9 Potential Energy

When work is done by moving a body within a force field (such as a gravitational field) or by deforming an elastic member (such as a spring), the body is said to have gained potential energy by virtue of its "elevated" position with respect to an energy baseline or datum.

The gravitational potential energy associated with a body of mass $m$, when it is raised from a datum elevation $h_0$ to the height $h$, is defined as follows:

$$E_p = m \int_{h_0}^{h} g \,\mathrm{d}h \tag{2.43}$$

where $E_p$ is the potential energy (typical units: J, ft lb);
$g$ is the gravitational acceleration (typical units: m/s$^2$, ft/s$^2$);
$h$ is height (typical units: m, ft); and
$h_0$ is the datum height (typical units: m, ft).

If $g$ is considered to be constant, for example if $g = g_0$, then

$$E_p = m g_0 (h - h_0) \tag{2.44}$$

## 2.5 Basics of Fluid Dynamics

### 2.5.1 Density, Specific Weight, and Specific Gravity

*Density*, which is it is usually assigned the Greek letter rho ($\rho$), is defined as mass per unit of volume:

$$\rho = \frac{m}{v} \tag{2.45}$$

where $m$ is the mass (typical units: kg, slug); and
$v$ is the volume (typical units: $m^3$, $ft^3$).

The standard units of density are $kg/m^3$ and $slug/ft^3$ (where $1\ slug\ ft^{-3} = 1\ lb\ ft^{-4}\ s^2$).

*Specific weight* is defined as weight per unit of volume, that is,

$$w = \frac{W}{v} \tag{2.46}$$

where $w$ is the specific weight (typical units: $N/m^3$, $lb/ft^3$); and
$W$ is the weight (typical units: N, lb).

The specific weight of a substance relates to its density:

$$w = \rho g \tag{2.47}$$

*Specific gravity* (relative density[9]) is defined as follows:

$$\sigma_{SG} = \frac{\rho}{\rho_{ref}} \tag{2.48}$$

where $\sigma_{SG}$ is the specific gravity (dimensionless);
$\rho$ is density (typical units: $kg/m^3$, $slug/ft^3$); and
$\rho_{ref}$ is a reference density (typical units: $kg/m^3$, $slug/ft^3$).

The reference density ($\rho_{ref}$) is the density of pure water at a specified reference temperature, which is normally 4 °C (39.2 °F), although other temperatures, such as 60 °F (15.6 °C) and 68 °F (20 °C), are also used. The reference density of pure water at 4 °C is 1000 $kg/m^3$ (1.9403 $slug/ft^3$).

### 2.5.2 Pressure in Fluids At Rest

Pressure ($p$), by definition, is force per unit of area:

$$p = \frac{F}{A} \tag{2.49}$$

where $p$ is the pressure (typical units: Pa, $lb/ft^2$, $lb/in^2$);
$F$ is the force (typical units: N, lb); and
$A$ is the area (typical units: $m^2$, $ft^2$).

The standard SI unit of pressure is the pascal (Pa), where $1\ Pa = 1\ N/m^2$. The FPS unit of pressure is $lb/ft^2$; although in USC units, pressure is often measured in $lb/in^2$ (psi).

9 The usage of the term *specific gravity* in scientific literature has declined in recent decades; many authorities prefer the term *relative density* [9].

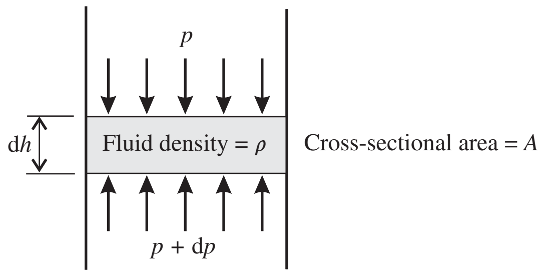



**Figure 2.8** Pressure due to the mass of a fluid.

In Figure 2.8 the fluid is stationary under the influence of gravity. The mass ($m$) of the fluid of the defined element, which has a height d$h$ and an area $A$, is the product of the density ($\rho$) and the volume ($A$d$h$). The weight of the element of fluid is $mg$. As the fluid is in a state of equilibrium, the sum of the forces ($F$) acting on the element is zero.

$$\sum F = (p + \mathrm{d}p)A - pA - mg = 0$$

Thus $(p + \mathrm{d}p)A - pA - (\rho A\,\mathrm{d}h)g = 0$

and $\mathrm{d}p = \rho g\,\mathrm{d}h$ (2.50)

Equation 2.50 is an expression of Pascal's principle, or Pascal's law,[10] and is often referred to as the *hydrostatic equation.*

### 2.5.3 Mass Flow Rate and Continuity of Flow

The mass flow rate ($Q$) of a fluid moving through a pipe (or stream tube) is equal to the fluid mass that passes a particular station (or location) per unit of time.

$$Q = \frac{\Delta m}{\Delta t} = \rho A V \tag{2.51}$$

where $\Delta m$ is the change in mass (typical units: kg, slug);
$\Delta t$ is the change in time (typical unit: s);
$\rho$ is the density (typical units: kg/m$^3$, slug/ft$^3$);
$A$ is the cross-sectional area of the pipe or stream tube (typical units: m$^2$, ft$^2$); and
$V$ is the velocity (typical units: m/s, ft/s).

An alternative—and widely used—designation for mass flow rate is $\dot{m}$, thus

$$Q = \dot{m} = \frac{\mathrm{d}m}{\mathrm{d}t} \tag{2.52}$$

The standard units of mass flow rate are kg/s and slug/s (where 1 slug s$^{-1}$ = 1 lb ft$^{-1}$ s).

The continuity equation can be deduced by considering fluid moving uniformly through a horizontal pipe (or stream tube) of changing cross section, as shown in Figure 2.9; it is apparent that the mass of fluid that passes Station 1 every second is equal to the mass of fluid that

10 Blaise Pascal (1623–1662), the French mathematician, physicist, and philosopher, defined a set of principles in 1647 that describe the pressure in fluids [10].

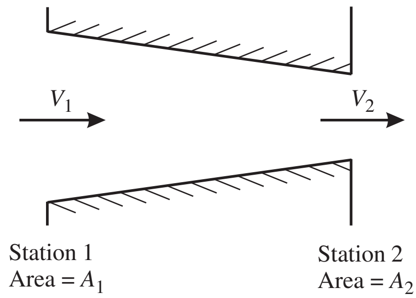



**Figure 2.9** Flow in a converging pipe (or stream tube).

passes Station 2 every second. This principle is known as the *continuity of flow* and is expressed mathematically as

$$\rho_1 A_1 V_1 = Q_1 = \rho_2 A_2 V_2 = Q_2 \tag{2.53}$$

where subscript 1 denotes Station 1 and subscript 2 denotes Station 2.

or $$\rho A V = \text{constant} \tag{2.54}$$

The continuity equation can also be written in differential form—from Equation 2.54 it follows that

$$\rho A \, \mathrm{d}V + \rho V \, \mathrm{d}A + AV \, \mathrm{d}\rho = 0 \tag{2.55}$$

Dividing through by $\rho AV$ gives the required equation:

$$\frac{\mathrm{d}V}{V} + \frac{\mathrm{d}A}{A} + \frac{\mathrm{d}\rho}{\rho} = 0 \tag{2.56}$$

Equation 2.56 applies to both liquids and gases. Whereas liquids are considered to be incompressible, gases can easily be compressed. However, when the velocity of the gas is relatively low, the effects of compressibility are small. It is convenient for many low-speed aerodynamic applications to idealize the properties of air and to consider it to be incompressible. In such cases, the density will not change and Equation 2.56 can be simplified as follows:

$$\frac{\mathrm{d}V}{V} + \frac{\mathrm{d}A}{A} = 0 \quad \text{(applicable to incompressible flow)} \tag{2.57}$$

And, it is evident from Equation 2.54 that

$$AV = \text{constant} \tag{2.58}$$

### 2.5.4 Newton's Second Law Applied to Fluid Flow

According to Newton's second law of motion (see Section 2.4.1), the change in momentum of the object with respect to time is equal to the applied force. Considering unidirectional flow, this can be expressed as

$$F = \frac{\Delta(mV)}{\Delta t} \tag{2.59}$$

where $F$ is the force (typical units: N, lb);
$\Delta(mV)$ is the change in momentum or impulse (typical units: kg m s$^{-1}$, slug ft s$^{-1}$); and
$\Delta t$ is the change in time (typical unit: s).

When considering fluid flow, Equation 2.59 is best written in terms of mass flow rates:

$$F = \dot{m}_2 V_2 - \dot{m}_1 V_1 \tag{2.60}$$

where $\dot{m}$ is the mass flow rate (typical units: kg/s, slug/s);
$V$ is the velocity (typical units: m/s, ft/s);
and where subscript 1 denotes the entry condition and subscript 2 denotes the exit condition.

In a propulsion system, however, it is the reaction of accelerating the fluid—as expressed by Newton's third law of motion—that is of interest. The reaction force, which acts on the powerplant, is equal in magnitude to the accelerative force that acts on the fluid, but opposite in direction.

### 2.5.5 Bernoulli Equation for Incompressible Flow and Dynamic Pressure

The cross-sectional area of the horizontal frictionless pipe (or stream tube) in Figure 2.10 is shown to reduce in the direction of the flow ($x$). If the fluid is considered to be incompressible, then Equation 2.58 applies, and it can be concluded that the velocity increases in the direction of the flow; that is, the flow accelerates. By definition, an incompressible flow regime is one in which appreciable changes in fluid density along streamlines do not occur (see Section 3.5.1). By considering the net force that causes this acceleration, a most useful relationship—linking pressure, density, and velocity—can be derived. This is the Bernoulli equation[11] for incompressible flow.

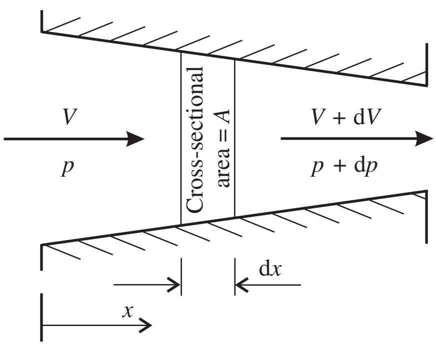



**Figure 2.10** Flow velocity and pressure in a converging pipe (or stream tube).

Consider the sum of the forces in direction $x$ acting on the element of fluid, which has a width $dx$ and a mean cross-sectional area $A$. Now, apply Newton's second law to the element:

$$\sum F = pA - (p + dp)A = m\frac{dV}{dt}$$

or $$-A\,dp = (\rho A\,dx)\frac{dV}{dt}$$

hence $$dp = -\rho\frac{dV}{dt}dx = -\rho\frac{dx}{dt}dV = -\rho V\,dV \tag{2.61}$$

and $$\int dp = -\int \rho V\,dV \tag{2.62}$$

11 Named after Daniel Bernoulli (1700–1782), the Swiss mathematician and physicist, who, in 1738, in his acclaimed publication *Hydrodynamica*, established the fundamental relationship between pressure, density, and velocity of fluids.

With the assumption that the flow is incompressible, the density is constant and $\rho$ can be taken out of the integral expression. Evaluation of the integral yields the desired result:

$$p + \frac{1}{2}\rho V^2 = \text{constant} \tag{2.63}$$

Equation 2.63 is the well-known Bernoulli equation for incompressible flow.

The term $(1/2)\rho\, V^2$, which appears in Equation 2.63, has units of pressure. This term appears frequently in the study of fluid dynamics; it is called the *dynamic pressure* and is often given the symbol $q$. By definition,

$$q = \frac{1}{2}\,\rho\, V^2 \tag{2.64}$$

It is seen from Equation 2.63 that the sum of the pressure $p$—which in this context is called the *static pressure*—and the dynamic pressure $q$ of the fluid in the pipe (or stream tube) is constant. The sum of these two terms is known as the *total pressure*; note that total pressure will remain constant in the absence of work done on the fluid or losses incurred.

$$p_t = p + q \tag{2.65}$$

where $p_t$ is the total pressure (typical units: Pa, lb/ft$^2$);
$p$ is the static pressure (typical units: Pa, lb/ft$^2$); and
$q$ is the dynamic pressure (typical units: Pa, lb/ft$^2$).

### 2.5.6 Ideal Gas Law

The ideal (or perfect) gas law is the equation of state of an ideal (or perfect) gas.[12] It describes the state of a gas in terms of the pressure, temperature, volume, and amount of matter. The most useful form of this equation for aerodynamic analyses is

$$p = \rho R T \tag{2.66}$$

where $p$ is the pressure (typical units: Pa, lb/ft$^2$);
$\rho$ is the density (typical units: kg/m$^3$, slug/ft$^3$);
$R$ is the specific gas constant, usually just called the *gas constant*; and
$T$ is the absolute temperature (typical units: K, °R).

The standard values of the gas constant [11] applicable to the International Standard Atmosphere (ISA)[13,14] are

$R = 287.05287$ m$^2$ s$^{-2}$ K$^{-1}$ (or N m kg$^{-1}$ K$^{-1}$) for SI units;
$R = 3089.81138$ ft$^2$ s$^{-2}$ K$^{-1}$ (or ft lb slug$^{-1}$ K$^{-1}$) for FPS units, with temperature in K; or
$R = 1716.56188$ ft$^2$ s$^{-2}$ °R$^{-1}$ (or ft lb slug$^{-1}$ °R$^{-1}$) for FPS units, with temperature in °R.

12 The main constituent gases of dry air, at the temperatures and pressures encountered in the region of the atmosphere used for commercial flight operations, can be treated as ideal gases with reasonable accuracy.
13 The International Standard Atmosphere (ISA) is an idealized model of the properties of the atmosphere; it is widely used in aviation (see Section 4.2).
14 It is common practice, when determining standard atmospheric models, for authorities to define the fundamental constants in SI units and then to convert the computed atmospheric properties to FPS units. If FPS units are used to calculate atmospheric properties, the constants (see Table 4.1) should be expressed with a large number of significant figures (so as to ensure that the computed atmospheric properties are unchanged).

The gas constant for a particular gas is defined as the ratio of the universal gas constant (which is applicable to all gases) to the molar mass of that gas:

$$R = \frac{\bar{R}}{m_0} \tag{2.67}$$

where $\bar{R}$ is the universal gas constant (typical units: N m K$^{-1}$ kmol$^{-1}$); and
$m_0$ is the molar mass of the gas (typical unit: kg/kmol).

The ISO (International Organization for Standardization) value of $\bar{R}$, as derived from the Boltzmann constant, is 8314.4621 N m K$^{-1}$ kmol$^{-1}$ [4]. The determination of atmospheric properties (e.g., pressure, density) in the model atmospheres used in aviation (see Section 4.2), however, is based on a gas constant of 8314.32 N m K$^{-1}$ kmol$^{-1}$ (defined in SI units). Note that this minor disparity from the accepted ISO value, given above, does not represent a reduction in accuracy of the computed atmosphere models.

In a real atmosphere, the composition of the air is not entirely uniform (e.g., it changes due to the presence of water vapor or contaminants), and this influences the molar mass. For the purpose of defining a standard atmosphere, the molar mass of air is based on the properties of a mixture of gases typical of the composition of clean, dry air (see Dubin, *et al.* [12], ISO [13], or ICAO [5] for details). As there is little change in the composition of dry air up to a height of 90 km (295 276 ft), a single value for $m_0$ is used. In the ISA, the molar mass is taken as

$$m_0 = 28.964420 \text{ kg/kmol}$$

Note that there exists a second, very similar, parameter, which is also known as the gas constant—and this can be confusing as it is sometimes also given the symbol $R$. Herein, this alternative form of the gas constant is designated as $R'$ and it is defined as

$$R' = \frac{R}{g} \tag{2.68}$$

It follows that the ideal gas law can thus be written as

$$p = \rho g R' T \tag{2.69}$$

Equation 2.66, however, is simpler to use as $R$ is valid for all altitudes (the alternative form of the gas constant, $R'$, is not used herein).

### 2.5.7 Specific Heats of a Gas

The *specific heat* (which is also known as the *specific heat capacity*) of a gas, by definition, is the amount of heat energy needed to raise the temperature of a unit mass of the gas by 1 K (or alternatively by 1 °R). Commonly used SI units are J kg$^{-1}$ K$^{-1}$ (which is equivalent to m$^2$ s$^{-2}$ K$^{-1}$). The specific heat of a particular gas, or mixture of gases, depends on the manner in which this operation takes place. It is evident from the ideal gas law (Equation 2.66) that the state of a fixed mass of gas depends on its pressure, volume, and temperature. By holding, in turn, each of the variables constant, three fundamental processes (or operations) can be defined that describe the relationship between the two remaining variables:

(1) constant pressure (isobaric) process;
(2) constant volume (isochoric) process; or
(3) constant temperature (isothermal) process.

The specific heat at constant pressure ($C_p$) and the specific heat at constant volume ($C_v$) are parameters of fundamental importance in the study of gas dynamics. The difference between the two values can be shown [14], for an ideal gas, to be equal to the gas constant, that is,

$$C_p - C_v = R \tag{2.70}$$

Moreover, it is the ratio of the two specific heats of air that frequently appears in aerodynamic analyses. This ratio is assigned the Greek letter gamma ($\gamma$). As air is assumed to be a mixture of two ideal diatomic gases in the ISA, it has a value of 1.4 (exactly). Thus

$$\gamma = \frac{C_p}{C_v} = 1.4 \qquad \text{(for air in the ISA)} \tag{2.71}$$

### 2.5.8 Adiabatic Processes

An adiabatic process is one in which no heat is added or removed. The properties of an ideal gas undergoing a reversible adiabatic process can be expressed mathematically by the following equations [14], which can be derived from the first and second laws of thermodynamics:

$$\frac{p}{T^{\left(\frac{\gamma}{\gamma-1}\right)}} = \text{constant} \tag{2.72}$$

$$pv^{\gamma} = \text{constant} \tag{2.73}$$

and $$vT^{\left(\frac{1}{\gamma-1}\right)} = \text{constant} \tag{2.74}$$

where $p$ is the pressure (typical units: $N/m^2$, $lb/ft^2$);
$T$ is the absolute temperature (typical units: K, °R); and
$v$ is the volume (typical units: $m^3$, $ft^3$).

It is convenient to rewrite these fundamental expressions in terms of density rather than volume. If the fluid mass is constant, Equations 2.73 and 2.74 can be expressed as follows:

$$\frac{p}{\rho^{\gamma}} = \text{constant} \qquad \text{(using Equation 2.45)} \tag{2.75}$$

and $$\frac{T^{\left(\frac{1}{\gamma-1}\right)}}{\rho} = \text{constant} \qquad \text{(using Equation 2.45)} \tag{2.76}$$

### 2.5.9 Speed of Sound and Mach Number

Gases are highly compressible, and, when disturbed, a pressure wave will result that propagates through the gas; the greater the compressibility of the gas, the lower will be the speed of wave propagation. Sound propagates as a pressure wave in air, and the speed of sound thus depends on the compressibility of the air, which is not constant. The speed of sound in air ($a$) is given by

$$a = \sqrt{\gamma R T} \tag{2.77}$$

where $a$ is the speed of sound (typical units: m/s, ft/s);
$\gamma$ is the ratio of specific heats of air (dimensionless);
$R$ is the gas constant (defined in Section 2.5.6); and
$T$ is the absolute temperature (typical units: K, °R).

Since $\gamma$ and $R$ are both constants in the ISA, the speed of sound (i.e., sonic speed) in air is seen to depend solely on the square root of the absolute temperature. As temperature changes, due to varying atmospheric conditions, for example, so the speed of sound will change. Ambient air temperature usually decreases monotonically with increasing altitude up to the tropopause (see Chapter 4); hence the speed of sound will also decrease with increasing altitude.

The Mach number[15] ($M$) of a gas, by definition, is the ratio of the speed of the gas ($V$) to the speed of sound in the gas at the point of interest ($a$), that is,

$$M = \frac{V}{a} \tag{2.78}$$

When $M < 1$, the flow is subsonic; when $M = 1$, the flow is sonic; and when $M > 1$, the flow is supersonic. Transonic describes a flow field in which there is a mix of subsonic and supersonic flow.

#### 2.5.10 Total Temperature and Total Pressure

When a gas is brought to rest in an adiabatic process, the gas temperature will rise. The total, or stagnation, temperature is given by

$$T_t = T\left(1 + \frac{\gamma - 1}{2}M^2\right) \tag{2.79}$$

where $T_t$ is the total, or stagnation, temperature (absolute); and
$T$ is the static temperature (absolute).

By substituting $\gamma = 1.4$, the total temperature for air, in this idealized condition, is

$$T_t = T(1 + 0.2M^2) \tag{2.80}$$

Instruments designed to measure total temperature (such as the temperature probes used on aircraft) are usually not able to recover the full temperature rise and a recovery factor has to be introduced to account for this.

$$T_{t,i} = T(1 + 0.2k\,M^2) \tag{2.81}$$

where $T_{t,i}$ is the indicated total temperature (absolute); and
$k$ is the recovery factor (typically 0.9 to 1.0, depending on the instrument).

Similarly, the total pressure ($p_t$) is given by[16]

$$p_t = p\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\left(\frac{\gamma}{\gamma-1}\right)} \tag{2.82}$$

By substituting $\gamma = 1.4$, Equation 2.82 can be written as

$$p_t = p(1 + 0.2\,M^2)^{3.5} \tag{2.83}$$

15 Named after Ernst Mach (1838–1916), Austrian-born physicist and philosopher. From *ca.* 1881, Mach conducted pioneering research into supersonic flow, describing and successfully recording the existence of shock waves in 1886 [15]. In honor of his achievements, Jacob Ackeret, the eminent Swiss aeronautical engineer, in 1929 suggested calling the ratio of flow speed to the local speed of sound the *Mach number* [16].
16 The derivation of Equation 2.82 is given in Appendix D, Section D.2.1.

### 2.5.11 Bernoulli Equation for Compressible Flow

For speeds greater than about Mach 0.3, appreciable changes in density occur due to the compression of the air. Equation 2.63, the incompressible-flow Bernoulli equation, was derived by assuming that the density of the air in the stream tube (illustrated in Figure 2.10) was constant. For higher speeds, the following equation, which takes into account the change in air density, is appropriate:[17]

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho}\right)+\frac{1}{2}V^2=\text{constant} \tag{2.84}$$

Equation 2.84 is an expression of the Bernoulli equation for compressible flow. The equation can be rearranged and written in several different ways—for example, Mach number can be substituted for velocity, as shown below.

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho}\right)+\frac{1}{2}M^2a^2=\text{constant} \qquad \text{(using Equation 2.78)} \tag{2.85}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho}\right)+\frac{1}{2}M^2\gamma RT=\text{constant} \qquad \text{(using Equation 2.77)} \tag{2.86}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho}\right)+\frac{1}{2}M^2\gamma\left(\frac{p}{\rho}\right)=\text{constant} \qquad \text{(using Equation 2.66)} \tag{2.87}$$

or

$$\left(\frac{\gamma}{\gamma-1}+\frac{\gamma}{2}M^2\right)\left(\frac{p}{\rho}\right)=\text{constant} \tag{2.88}$$

### 2.5.12 Viscosity

The viscous nature of air is evident when observing flow parallel to a stationary body within the boundary layer. The boundary layer (discussed later in Section 3.6) can be described as the layer of air that is slowed down due to the presence of the body. The *no slip* boundary condition implies that the flow has zero velocity at the surface of the body. The flow velocity ($u$) increases with increasing distance from the surface ($y$).

When the flow is laminar, the air may be considered to move in discrete layers of finite thickness (see Figure 2.11). The velocity of the flow (which is represented by the length of the arrows) is reduced, or retarded, due to the shear stresses that act between the layers of air. The *dynamic*

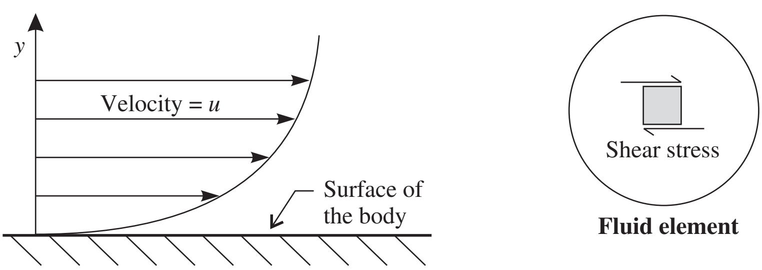



Figure 2.11 Velocity profile in the boundary layer, indicating the shear stresses parallel to the surface acting on a fluid element.

17 The derivation of Equation 2.84 is given in Appendix D, Section D.2.2.

*viscosity*, which is usually just called the *viscosity*, is assigned the Greek letter mu ($\mu$); it is equal to the constant of proportionality that relates the shear stress ($\tau$) between adjacent layers of the flow to the velocity gradient at that point:

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} \tag{2.89}$$

where $\tau$ is the shear stress in the fluid (typical units: N/m$^2$, lb/ft$^2$);
$\mu$ is the viscosity (or dynamic viscosity) of the flow;
$u$ is the flow velocity at a distance $y$ from the surface (typical units: m/s, ft/s); and
$y$ is the distance from the surface (typical units: m, ft).

The standard units of dynamic viscosity are N s m$^{-2}$ (which is equivalent to Pa s or kg s$^{-1}$ m$^{-1}$ in the SI system) and lb s ft$^{-2}$ (which is equivalent to slug s$^{-1}$ ft$^{-1}$).

*Kinematic viscosity*, which is assigned the Greek letter nu ($\nu$), is defined as the ratio of the dynamic viscosity of a fluid to its density.

$$\nu = \frac{\mu}{\rho} \tag{2.90}$$

The standard units of kinematic viscosity are m$^2$ s$^{-1}$ and ft$^2$ s$^{-1}$.

The viscosity of air is a function of temperature ($T$)—this can be fairly accurately described by the Sutherland equation,[18] where the empirical constants $\beta_s$ and $S$ are determined by experiment:

$$\mu = \frac{\beta_s T^{3/2}}{T + S} \tag{2.91}$$

where $\beta_s = 1.458 \times 10^{-6}$ N s m$^{-2}$ K$^{-1/2}$ for SI units; or
$\beta_s = 3.04509 \times 10^{-10}$ lb s ft$^{-2}$ K$^{-1/2}$ for FPS units; and
$S = 110.4$ K.

It is reported [5, 11] that Equation 2.91 is not valid for very high or very low temperatures or under the atmospheric conditions that exist at altitudes above about 90 km (295 276 ft).

The viscosity of air at the ISA sea-level datum conditions (see Section 4.2) can be determined from Equation 2.91 by setting $T = T_0 = 288.15$ K:

$\mu_0 = 1.7894 \times 10^{-5}$ N s m$^{-2}$ for SI units; or
$\mu_0 = 3.7372 \times 10^{-7}$ lb s ft$^{-2}$ for FPS units.

### 2.5.13 Reynolds Number

Reynolds number ($R_e$) is a non-dimensional characteristic number,[19] which is defined as follows:

$$R_e = \frac{\rho u_\infty l}{\mu} \tag{2.92}$$

where $u_\infty$ is the flow velocity (typical units: m/s, ft/s); and
$l$ is a characteristic length (typical units: m, ft).

18 The formulation was proposed by William Sutherland (1859–1911), an Australian physicist, in 1893 [17].
19 The characteristic number is named in honor of Osborne Reynolds (1842–1912), the British engineer and physicist, who proposed its use in 1883 for characterizing flow behavior [18].

Alternatively, Reynolds number can be expressed in terms of kinematic viscosity, that is,

$$R_e = \frac{u_\infty l}{\nu} \tag{2.93}$$

The Reynolds number, in a sense, relates inertia effects to viscous effects in the flow. The characteristic length ($l$) is selected to suit the application—for example, when the object being studied is an airfoil, the characteristic length will usually be the chord. However, when studying the flow at a particular point on an aerodynamic surface (e.g., a wing) the local Reynolds number will be based on a dimension that defines that point, such as the distance from the airfoil leading edge.

Reynolds number is a key parameter in the understanding of the behavior of boundary layers in that it influences the thickness of the boundary layer relative to the body dimensions. For example, with increasing flow velocity and the other parameters (viz., $\rho$, $\mu$, and $l$) remaining unchanged, the Reynolds number will increase; there will be reduced rate of diffusion of momentum from outer flow layers resulting in a thinner boundary layer.

## 2.6 Further Reading

There are many textbooks that contain expanded descriptions of the topics introduced in this chapter—relevant works include:

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Munson, B.R., Rothmayer, A.P., Okiishi, T.H., and Huebsch, W.W., *Fundamentals of fluid mechanics*, 7 th ed., John Wiley & Sons, New York, NY, 2013.

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2 ISO, *Quantities and units*, ISO 31, International Organization for Standardization, Geneva, Switzerland, 1992.

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5 ICAO, "Manual of the ICAO standard atmosphere: Extended to 80 kilometres (262 500 feet)," 3rd ed., Doc. 7488, International Civil Aviation Organization, Montréal, Canada, 1993.

**6** NIMA, "Department of defense world geodetic system 1984: Its definition and relationships with local geodetic systems," TR 8350.2, 3rd ed., National Imagery and Mapping Agency, St. Louis, MO, Jan. 3, 2000.

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**8** Blake, W. and Performance Training Group, "Jet transport performance methods," D6-1420, Flight Operations Engineering, Boeing Commercial Airplanes, Seattle, WA, Mar. 2009.

**9** McNaught, A.D. and Wilkinson, A., *International Union of Pure and Applied Chemistry: Compendium of chemical terminology (Gold Book)*, 2nd ed., Blackwell Science, Oxford, UK, 1997.

**10** Adamson, D., *Blaise Pascal: Mathematician, physicist and thinker about God*, Palgrave Macmillan, London, 1995.

**11** ESDU, "Equations for calculation of international standard atmosphere and associated off-standard atmospheres," Data item 77022, Amdt. B, IHS ESDU, 133 Houndsditch, London, UK, Feb. 1986.

**12** Dubin, M., Hull, A.R., and Champion, K.S.W., "US standard atmosphere 1976," NASA TM-X-74335 / NOAA S/T-76-1562, National Aeronautics and Space Administration, National Oceanic and Atmospheric Administration, United States Air Force, Washington, DC, 1976.

**13** ISO, *Standard atmosphere*, ISO 2533, International Organization for Standardization, Geneva, Switzerland, 1975.

**14** Rogers, G.F.C. and Mayhew, Y.R., *Engineering thermodynamics: Work and heat transfer*, 4th ed., Pearson Education, Harlow, UK, 1996.

**15** Reichenbach, H., "Contributions of Ernst Mach to fluid mechanics," *Annual Review of Fluid Mechanics*, Vol. 15, pp. 1–28, 1983.

**16** Rott, N., "Jacob Ackeret and the history of the Mach number," *Annual Review of Fluid Mechanics*, Vol. 17, pp. 1–9, 1985.

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# 3

# Aerodynamic Fundamentals

## 3.1 Introduction

This chapter introduces several fundamental aspects of aerodynamics and flight controls, which are key to an understanding of the performance of the jet airplane—these topics include: aerodynamic notation and definitions (Section 3.2); coordinate systems and sign conventions (Section 3.3); aerodynamic forces and moments (Section 3.4); compressibility (Section 3.5); boundary layers (Section 3.6); high lift devices (Section 3.7); and controls for pitch, roll, and yaw (Section 3.8). The purpose of the chapter is to provide background information on these topics in a style and format that is consistent with the treatment of airplane performance as presented in subsequent chapters. There are many excellent texts (as noted in Section 3.9) that provide a wealth of information on the subject of aerodynamics and flight controls—the reader is directed to these works for more information on the topics introduced in this chapter.

## 3.2 Standard Definitions and Notation

### 3.2.1 Airfoil Section Definitions

A cross-sectional cut through an airplane's wing defines an *airfoil*, which is also known as an *airfoil section* (frequently shortened to just *section*). The shape definition (profile) of the airfoil section would typically change from the wing root to the wingtip. A great many airfoil sections have been developed and tested, and a considerable amount of data are available regarding the characteristics of these different airfoils. The development of modern, efficient airfoil sections owes a great deal to the work done at NACA[1] in producing the so-called classic NACA airfoils.[2] Although these designs have been around for many years, they are still found in many sectors of the aviation industry and are often used as a benchmark against which modern designs are compared. The main geometric parameters that define an airfoil are shown in Figure 3.1.

#### Profile Definition

It is convenient to separate the profile (or shape definition) of an airfoil section into its thickness distribution (which has a major influence on the profile drag—see Section 7.3.2) and a zero

1 NACA (National Advisory Committee for Aeronautics): precursory organization to NASA (National Aeronautics and Space Administration), which was formed in 1958.
2 References [1] and [2] provide geometrical definitions and experimental results for an extensive range of NACA airfoils.

*Performance of the Jet Transport Airplane: Analysis Methods, Flight Operations, and Regulations*, First Edition.
Trevor M. Young.

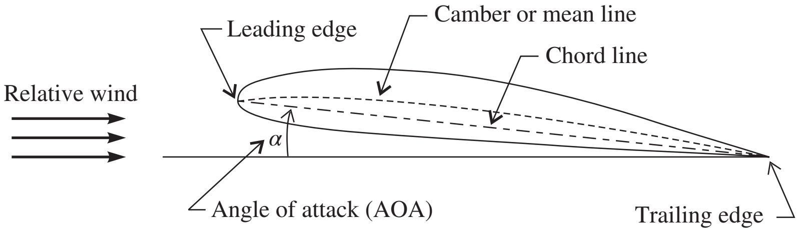


**Figure 3.1** Airfoil definitions.

thickness camber line (which has a major influence on lift, pitching moment, and lift-dependent drag). The usual method for constructing an airfoil is from coordinate data of the thickness and camber distributions. The leading edge (or nose) definition is then blended into the upper and lower profiles. For NACA airfoils, the nose section is circular, and the nose radius has its center on a line drawn tangential to the camber line at the leading edge; elliptical or near elliptical nose sections are also used.

#### Camber Line and Chord Line

The mean camber line—or camber line as it is usually called—is a locus of the midpoints between the upper and lower surfaces measured normal to the camber line itself. A straight line joining the start and end points of the camber line defines the *chord*, which is given the symbol $c$. A measure of the amount of camber of an airfoil is given by the maximum distance from the chord line to the camber line; this is usually expressed as a percentage of the chord. An airfoil with zero camber is symmetrical about the chord line.

#### Maximum Thickness-to-Chord Ratio

The maximum thickness-to-chord ratio, $(t/c)_{max}$—which is sometimes shortened to thickness ratio—is an important design parameter for an airfoil section. This parameter has a significant influence on the stall and drag characteristics of an airfoil section. Thinner sections have a lower drag coefficient and achieve a delay in the onset of drag rise associated with shock wave formation. Increased structural complexity and weight, as well as reduced wing internal volume (which is required for fuel and, in many cases, the undercarriage), however, limit the extent by which designers can reduce the wing thickness.

#### Angle of Attack

The angle of attack (AOA) is the angle of inclination of the airfoil section measured from the relative wind (or local airflow) to the chord line (Figure 3.1). The angle of attack is customarily assigned the Greek letter alpha ($\alpha$) and is defined as positive when the nose of the airfoil is rotated upward.[3]

#### Two-Dimensional Flow Field

It is convenient for the aerodynamic assessment of a wing to commence with a study of a two-dimensional flow field around an airfoil that represents a small spanwise strip (or segment) of the wing. This is obviously an idealized representation as it considers neither the out-of-plane, or spanwise, component of flow nor the complex three-dimensional effects that arise near the

3 In some reference works, principally older British texts, the angle of attack is called the angle of incidence. This term can cause confusion as the angle between the wing centerline chord and the fuselage datum is called the wing incidence angle or simply the incidence angle.

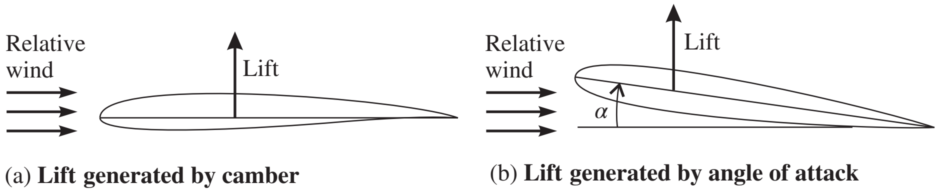


**Figure 3.2** Lift can be generated by (a) camber or (b) angle of attack.

wingtip. Such a flow field can be achieved in a wind tunnel using a zero-sweep, high aspect ratio wing model with a constant airfoil section along its span. With a stream of air flowing over the airfoil, an aerodynamic force acting on the model is generated. This force can be decomposed into *lift* and *drag* components. Lift can be generated by cambered airfoils (Figure 3.2a) or by angle of attack (Figure 3.2b) or—as is frequently the case—by a combination of both.

### 3.2.2 Wing Geometric Definitions

The wing geometric parameters that are used in aerodynamic analyses are defined on an idealized reference wing planform, which is based on the airplane's actual wing area and shape (see Figure 3.3).

#### Reference Wing Planform

The reference wing planform is an equivalent, straight-tapered planform that extends to the fuselage centerline. Each wing panel (i.e., left and right wing) is of trapezoidal shape; the leading and trailing edges are straight and the wingtip chords are parallel to the fuselage centerline. A reference wing planform is an idealization of the airplane's actual wing planform, with leading and trailing edge devices (e.g., flaps and slats) retracted. Airplane manufacturers employ their own specific methods to define a reference wing planform; a typical approach is described in ESDU 76003 [3]. A projection of the actual wing shape onto a horizontal plane is initially generated. Features such as engine nacelles, rounding of the wingtips, winglets, fillets at the wing/fuselage intersection, and so forth are then removed. Cranks on the leading and trailing edges are eliminated to generate a straight-tapered wing with the same tip chord and the chord at the wing/fuselage junction adjusted to give the same exposed wing area. The straightened leading edges are extended to the fuselage centerline—a location known as the wing apex. Similarly, the straightened trailing edges are extended to the fuselage centerline to complete the two trapezoidal shapes.

#### Wing Reference Area

The wing reference area ($S$) is the area of the reference wing planform. Note that wing reference areas are not necessarily identical to actual wing areas. This may appear to be a problem, but in reality such discrepancies are of no real concern as the wing reference area is only a reference dimension that is primarily used to calculate aerodynamic parameters, such as lift and drag coefficients. It is used to compute the airplane's *wing loading*, which is the ratio of the airplane's mass or weight to the wing reference area. The reference area is a constant dimension for an airplane type and does not change when the actual wing area changes due to the extension of flaps, for example. It is interesting to note that airplane manufacturers will usually use the same reference area for derivative models of an airplane type (e.g., the Boeing 747-400 featured a wingtip extension, but the reference area was the same as earlier versions of the B747).

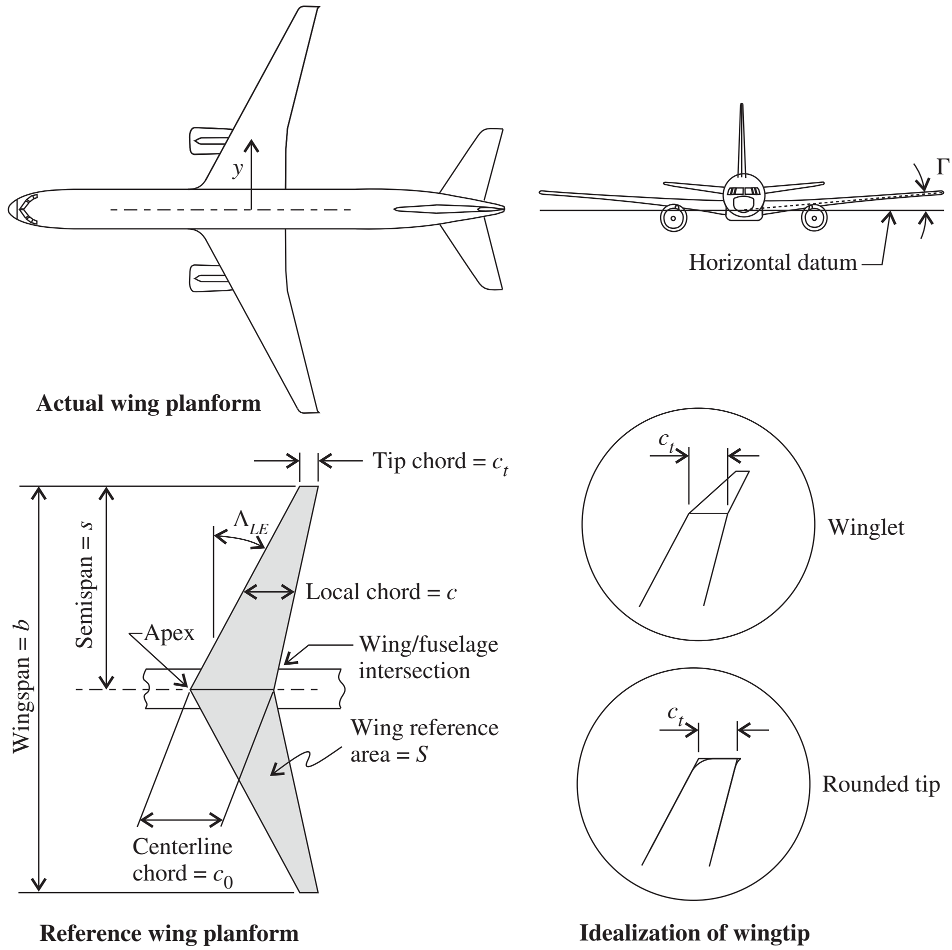


**Figure 3.3** Reference wing planform.

#### Exposed Wing Area

The exposed wing area ($S_{exp}$) is the portion of the reference area that excludes the part of the wing inside the fuselage; it is measured outwards from the wing/fuselage intersection on the reference wing planform. Typically, $S_{exp}$ is sized to match the area of the actual basic wing planform.

#### Wingspan

The wingspan ($b$) is the distance from wingtip to wingtip on the reference planform, measured normal to the airplane's centerline. The semispan distance is usually denoted as $s$ or $b/2$. In constructing a reference wing planform, any rounding of the wingtip is ignored and the leading and trailing edges are extended outwards to define the trapezoidal shape. Out-of-plane wingtip extensions (i.e., winglets) are ignored and a hypothetical wingtip defined at the winglet/wing junction.

#### Wing Chords

The wing chord ($c$) is measured from the leading edge to the trailing edge on the reference planform, parallel to the centerline of the fuselage (Figure 3.3). The *centerline chord* ($c_0$) is unambiguously defined as the chord of the reference planform at the fuselage centerline. Note that certain conventions (e.g., Boeing [4]) use the term *root chord* ($c_r$) to describe the centerline chord; other conventions (e.g., ESDU [3]) use it to describe the chord at the wing/fuselage junction. The *tip chord* ($c_t$) is measured at the semispan location on the reference planform.

### Wing Taper Ratio

The wing taper ratio, which is frequently shortened to *wing taper*, is assigned the Greek letter lambda ($\lambda$); it is defined as the ratio of the tip chord to the centerline chord:

$$\lambda = \frac{c_t}{c_0} \tag{3.1}$$

Taper affects the distribution of lift along the wingspan. The Prandtl wing theory[4] indicates that the minimum induced drag occurs when the lift is distributed in an elliptical fashion. For a wing without twist or sweep, this occurs when the wing planform is itself elliptical. An elliptical wing has superior aerodynamic characteristics than a rectangular wing. By tapering the wing, the lift distribution can be improved—for example, a taper ratio of 0.45 on an unswept wing will approximate the ideal elliptical lift distribution.

### Lateral Coordinates

The lateral coordinate ($y$) measured from the fuselage centerline is frequently non-dimensionalized using the semispan distance. It is usually denoted by the Greek letter eta ($\eta$); thus

$$\eta = \frac{y}{s} = \frac{2y}{b} \tag{3.2}$$

### Aspect Ratio

The aspect ratio is usually represented by the letter $A$ (note that other conventions exist—for example, the ratio is sometimes written as $A_R$). For a rectangular wing planform, the aspect ratio is the ratio of the wingspan to the wing chord. For all other wing planform shapes, the aspect ratio can be determined from the reference area ($S$) and the wingspan ($b$), as follows:

$$A = \frac{b^2}{S} \tag{3.3}$$

For the reference wing planform (i.e., a straight tapered trapezoidal planform), the centerline chord can be expressed in terms of the wing area and wingspan or, alternatively, the wing area and aspect ratio:

$$c_0 = \frac{2S}{(1+\lambda)b} = \frac{2}{(1+\lambda)}\sqrt{\frac{S}{A}} \tag{3.4}$$

Jet transport airplanes have moderately high aspect ratio wings when compared to other classes of aircraft. The aspect ratio has a major influence on the lift-dependent drag (see Section 7.3.4) and also on the lifting ability of the wing at a given angle of attack. A high aspect ratio wing has a steep lift-curve slope (see Section 3.4.7).

4 Named after Ludwig Prandtl (1875–1953), the eminent German scientist who conducted pioneering aerodynamics research from 1901 to 1953, initially at the Technical School Hannover (now Technical University Hannover), but mostly at the University of Göttingen [5]. So profound was his influence that he is sometimes called the *father of modern aerodynamics*.

### Sweep Angle

The wing sweep angle, or sweepback angle, is measured from a line drawn perpendicular to the fuselage centerline to a reference line on the wing planform, usually taken as the quarter chord line. It is given the upper case Greek letter lambda ($\Lambda$). It is common practice to add a subscript to identify the planform reference line to which the angle refers—for example, the leading edge sweep angle is designated as $\Lambda_{LE}$ or $\Lambda_0$. The quarter chord sweep angle (designated as $\Lambda_{1/4}$ or $\Lambda_{0.25}$ or $\Lambda_{0.25c}$) is the sweep angle of a line joining the 25% point of the centerline chord to the corresponding point on the wingtip chord. The 25% chord sweep angle is often selected to characterize a wing planform, although the 50% chord sweep angle may also be used for this purpose. The aerodynamic effect of sweeping the wing back is that it delays the subsonic drag-rise associated with shock-wave formation at transonic speeds and reduces the peak drag coefficient.[5] Wing sweep also improves the lateral (roll) stability of an airplane.

### Wing Dihedral Angle

The wing dihedral angle, which is assigned the upper case Greek letter gamma ($\Gamma$), is the angle measured from a horizontal datum plane passing through the centerline chord to a reference line (drawn from the wing centerline to the wingtip) when the airplane is viewed from the front—see Figure 3.3. If the wingtips are located above the horizontal datum line, the wing is said to have a positive dihedral angle, whereas if the wingtips are below the datum, the airplane has negative dihedral, or anhedral. The three-dimensional geometry of a modern wing design is complicated, incorporating varying twist, thickness, and camber along the wingspan—and in these cases the establishment of a wing reference line is not trivial. In many cases, the wing is twisted about a reference line (say joining the 25% chord points), and for these aircraft types this line can be used to define the dihedral angle. In flight, dihedral has a strong influence on an airplane's lateral (roll) stability.

### Wing Twist

The geometric wing twist ($\varepsilon_g$) is the angle of incidence of the wingtip relative to that of the centerline section. By convention, a positive twist angle is associated with the nose of the wingtip section being rotated upwards. The term *washout* is used to describe a wing where the tip section has been rotated downwards. Washout has a desirable influence on the stalling characteristics of a wing and is widely used in the design of aircraft of many different types. The effect of washout is to reduce locally the angle of attack of the outboard portion of the wing. This results in the wing first stalling at an inboard location, while maintaining good airflow over the outboard portion of the wing. Wing twist also affects the spanwise lift distribution and can be used to reduce the induced drag by tailoring the lift distribution to approximate an elliptical distribution. However, it is not possible to optimize this for all flight conditions. The spanwise lift distribution can also be tailored by varying the airfoil section properties, such as profile and camber, with span—this is known as aerodynamic twist.

### Wing Incidence Angle

The wing incidence (or setting) angle is the angle of the wing centerline chord with respect to the fuselage datum line. From a design perspective, it is usually selected to minimize drag at the design operating condition. At the selected flight condition, the wing will be operating at

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5 The idea of sweeping an airplane's wing back to delay the effects of compressibility was first proposed by Adolf Busemann (1901–1986), a German aerodynamicist, in 1935 [6].

the desired angle of attack for the design lift coefficient and the fuselage will be at the angle of attack for minimum drag; for a typical commercial jet transport airplane this is close to 0°.

#### Mean Aerodynamic Chord (MAC)

The mean aerodynamic chord ($\bar{\bar{c}}$)—or aerodynamic mean chord, as some authorities prefer—is the chord of a rectangular wing, of span $b$, that has essentially the same lift and pitching moment characteristics as the actual wing.[6] The MAC of a planar wing (i.e., a wing without dihedral or out-of-plane wingtip extensions) is given by

$$\bar{\bar{c}} = \frac{\int_0^{b/2} c^2 \, \mathrm{d}y}{\int_0^{b/2} c \, \mathrm{d}y} \tag{3.5}$$

If the actual wing is represented by an equivalent straight-tapered planform, the MAC is given by

$$\bar{\bar{c}} = \frac{2}{3} c_0 \left[ 1 + \frac{\lambda^2}{1 + \lambda} \right] \tag{3.6}$$

where $c_0$ is the centerline chord, which can be expressed in terms of wing area, span, and taper ratio, as indicated by Equation 3.4.

The MAC is used as a reference length for many aerodynamic calculations—for example, it is used to non-dimensionalize pitching moment coefficients (see Section 3.4.6). The position of the airplane's center of gravity (see Section 19.3.1) and aerodynamic center (defined in Section 3.4) are typically expressed relative to $\bar{\bar{c}}$. As the MAC is defined by pitching moment considerations, it follows that it only has a longitudinal position; by convention, $\bar{\bar{c}}$ is shown on the fuselage centerline. For a straight-tapered wing, the leading edge of the MAC (often abbreviated as LEMAC) is located behind the wing apex by a distance of

$$\bar{\bar{x}}_0 = c_0 \left[ 1 + \frac{2\lambda}{12} \right] A \tan \Lambda_0 \tag{3.7}$$

It is convenient, for certain applications, to determine the spanwise position where the local chord is equal to the MAC. For a straight-tapered wing, the lateral coordinate ($y$) where this occurs is given by

$$y = c_0 \left[ \frac{1 + \lambda}{12} \right] A = \frac{b}{6} \left[ \frac{1 + 2\lambda}{1 + \lambda} \right] = \frac{s}{3} \left[ \frac{1 + 2\lambda}{1 + \lambda} \right] \tag{3.8}$$

The fraction of the wing semispan at which $c = \bar{\bar{c}}$ is herein represented by the Greek letter kappa ($\kappa$)—thus for a straight-tapered wing,

$$\kappa = \frac{1}{3} \left[ \frac{1 + 2\lambda}{1 + \lambda} \right] \tag{3.9}$$

A simple graphical construction technique can be used to determine $\bar{\bar{c}}$, $\bar{\bar{x}}_0$, and $\kappa$ for a straight-tapered wing—this is illustrated in Figure 3.4.

6 Cautionary note: The mean aerodynamic chord (MAC) is sometimes given the symbol $\bar{c}$. This practice can be confusing, however, as the symbol $\bar{c}$ is also widely used to represent the geometric mean chord (see Equation 3.10).