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QUANTUM INSPIRED META-HEURISTICS FOR IMAGE ANALYSIS

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Sandip Dey would like to dedicate this book to the loving memory of his father, the late Dhananjoy Dey, his mother Smt. Gita Dey, his wife Swagata Dey Sarkar, his children Sunishka and Shriaan, his siblings Kakali, Tanusree, Sanjoy and his nephews Shreyash and Adrishan.

Siddhartha Bhattacharyya would like to dedicate this book to his father, the late Ajit Kumar Bhattacharyya, his mother, the late Hashi Bhattacharyya, his beloved wife Rashni Bhattacharyya, and Padmashree Professor Ajoy Kumar Ray, Honorable Chairman, RCC Institute of Information Technology, Kolkata, India.

Ujjwal Maulik would like to dedicate this book to his son Utsav, his wife Sanghamitra and all his students and friends.

Contents

Preface

In the present information era, the processing and retrieval of useful image information and multimedia-based data, for the purpose of faithful and realistic analysis, are supposed to be of the highest importance. One significant image processing chore is to separate objects or other important information in digital images through thresholding of the image under consideration. Efficient techniques are required in order to develop an appropriate analysis of noisy and noise-free image data to obtain suitable object-specific information.

The *soft computing approaches* have certain tools and techniques among various other approaches, which integrate intelligent thinking and principles. Fuzzy logic, Neural networks, Fuzzy sets, and Evolutionary Computation are used as the computing framework, which successfully combines these intelligent principles.

This book attempts to address the problem of image thresholding using classical algorithms. Attempts have also been made to take out the intrinsic limitations in the present soft computing methods as initiated by theoretical investigations. New versions of quantum inspired meta-heuristics algorithms have also been introduced, taking into cognizance the time and space complexity of present approaches.

The introductory chapter of the book presents a brief summary on image analysis, quantum computing, and optimization. The chapter highlights quantum solutions of NP-complete problems in brief. This introductory chapter also presents a related literature survey using different approaches, such as *quantum-based approaches, meta-heuristic-based approaches* and *multi-objective-based approaches*. The chapter also discusses the scope and organization of the book.

Chapter 2 focuses on the review of image analysis. This chapter discusses the mathematical formalism of image segmentation technique. It also highlights different digital image analysis approaches. It also throws light on popular image thresholding techniques in the binary, multi-level and gray-scale domains. A short summary of the applications of image analysis is also presented. Finally, the chapter ends with a relevant conclusion; a chapter summary and a set of exercise questions are provided.

Chapter 3 focuses on the overview of some popular meta-heuristics. The fundamentals of each of them are briefly discussed in this chapter. Pseudo-code of the corresponding meta-heuristic is also presented after each section. The summary of the chapter and a set of exercise questions are provided at the end of this chapter.

Chapter 4 addresses the intrinsic limitations of classical algorithms to deal with image data for binary thresholding. The chapter develops two different quantum-based standard conventional algorithms for binary image thresholding. The basic quantum computing principles have been recognized to develop the proposed approaches. The features of quantum computing have been properly linked with the framework of popular classical algorithms for the formation of the proposed algorithms. Experiments have been conducted with different combinations of the parameter settings. The proposed algorithms are compared with several other algorithms. The implementation results are presented for the proposed algorithms and other comparable algorithms.

In line with the objectives of Chapter 4, several novel versions of quantum inspired classical algorithms are proposed in Chapter 5. This chapter concentrates on the functional modification of the quantum inspired meta-heuristic algorithms as an attempt to extend them to multi-level and gray-scale domain. Application of the proposed algorithms is demonstrated on a set of synthetic/real-life gray-scale images. As a sequel to these algorithms, experiments were conducted with several other algorithms for comparative purposes. The experiments were conducted with different parameter values. Implementation results are reported for all the participating algorithms.

Parallel extensions to these quantum inspired classical algorithms are presented in Chapter 6. This approach introduces thresholding of color image information. The parallel operation of the proposed framework reduces the time complexity of the color image thresholding. Application of the proposed versions of quantum inspired algorithms is exhibited using thresholding of multi-level and color images. As a result of the comparative study, the proposed algorithms are compared with other popular algorithms. Implementation results with several parameter adjustments are reported.

Chapter 7 introduces several quantum inspired multi-objective algorithms using different approaches. First, an NSGA-II-based quantum inspired algorithm is proposed in a multi-objective framework. Later, several quantum inspired classical algorithms are developed in a multi-objective flavor for bi-level, multi-level and gray-scale image thresholding.

Different parameters settings are used for the clarification of proposed algorithms. Application of these algorithms is demonstrated on several real-life grayscale images. A number of other popular algorithms are used for comparative purposes. The test results are reported for all of the participating algorithms.

Finally, the concluding chapter ends the book. This chapter presents an outlook of future directions of research in this area.

Sodepur
2 November 2018

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Acronyms

ACO	Ant Colony Optimization
AI	Artificial Intelligence
BSA	Backtracking Search Optimization Algorithm
CNOT	Controlled NOT Gate
CoDE	Composite DE
DE	Differential Evolution
EA	Evolutionary Algorithm
EC	Evolutionary Computation
EP	Evolutionary Programming
ES	Evolutionary Strategies
GA	Genetic Algorithm
GP	Genetic Programming
HDQ	Heuristic Search Designed Quantum Algorithm
HV	Hypervolume
IGD	Inverted Generational Distance
MA	Metropolis Algorithm
MADS	Mesh Adaptive Direct Search
MBF	Modified Bacterial Foraging
MDS	Multidirectional Search
MODE	Multi-Objective Differential Evolutionary Algorithm
MOEA	Multi-Objective Evolutionary Algorithm
MOGA	Multi-Objective Genetic Algorithm
MOO	Multi-Objective Optimization
MOSA	Multi-Objective Simulated Annealing
MRI	Magnetic Resonance Imaging
MTT	Maximum Tsallis entropy Thresholding
NM	Nelder-Mead
NPGA	Niched-Pareto Genetic Algorithm
NSGA-II	Non-dominated Sorting Genetic Algorithm II
NSGA	Non-dominated Sorting Genetic Algorithm
OCEC	Organizational Coevolutionary algorithm for Classification
PAES	Pareto Archived Evolution Strategy
PESA	Pareto Envelope-based Selection Algorithm
PET	Positron Emission Tomography
PO	Pareto Optimal

PSNR	Peak Signal-to-Noise Ratio
PSO	Particle Swarm Optimization
QC	Quantum Computer
QEA	Quantum Evolutionary Algorithm
QFT	Quantum Fourier Transform
QIACO	Quantum Inspired Ant Colony Optimization
QIACOMLTCI	Quantum Inspired Ant Colony Optimization for Color Image Thresholding
QIDE	Quantum Inspired Deferential Evolution
QIDEMLTCI	Quantum Inspired Deferential Evolution for Color Image Thresholding
QIEA	Quantum Inspired Evolutionary Algorithms
QIGA	Quantum Inspired Genetic Algorithm
QIGAMLTCI	Quantum Inspired Genetic Algorithm for Color Image Thresholding
QIMOACO	Quantum Inspired Multi-objective Ant Colony Optimization
QIMOPSO	Quantum Inspired Multi-objective Particle Swarm Optimization
QINSGA-II	Quantum Inspired NSGA-II
QIPSO	Quantum Inspired Particle Swarm Optimization
QIPSOMLTCI	Quantum Inspired Particle Swarm Optimization for Color Image Thresholding
QISA	Quantum Inspired Simulated Annealing
QISAMLTCI	Quantum Inspired Simulated Annealing for Color Image Thresholding
QISAMO	Quantum Inspired Simulated Annealing for Multi-objective algorithms
QITS	Quantum Inspired Tabu Search
QITSMLTCI	Quantum Inspired Tabu Search for Color Image Thresholding
RMSE	Root mean-squared error
SA	Simulated Annealing
SAGA	Simulated Annealing and Genetic Algorithm
SC	Soft Computing
SOO	Single-Objective Optimization
SPEA2	Strength Pareto Evolutionary Algorithm 2
SPEA	Strength Pareto Evolutionary Algorithm
SVM	Support Vector Machines
TS	Tabu Search
TSMO	Two-Stage Multithreshold Otsu
TSP	Travelling Salesman Problems
VEGA	Vector-Evaluated Genetic Algorithm
X-ray	X-radiation

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Introduction

A quantum computer, as the name suggests, fundamentally works on several quantum physical characteristics. It is also considered as the field of study, primarily focused on evolving computer technology using the features of quantum theory, which expounds the nature of energy and substance and its behavior on the quantum level, i.e., at the atomic and subatomic level. Developing a quantum computer would mark an advance in computing competency far superior to any current computers. Thus, the use of quantum computers could be an immense improvement on current computers because they have enormous processing capability, even exponentially, compared to classical computers. The supremacy of processing is gained through the capacity of handling multiple states at once, and performing tasks exploiting all the promising permutations in chorus. The term *quantum computing* is fundamentally a synergistic combination of thoughts from quantum physics, classical information theory, and computer science.

Soft computing (SC), introduced by Lotfi A. Zadeh [282], manages the soft meaning of thoughts. SC, comprising a variety of thoughts and practices, is fundamentally used to solve the difficulties stumbled upon in real-life problems. This can be used to exploit the uncertainty problem almost with zero difficulty. This can also handle real-world state of affairs and afford lower solution costs [29]. The advantageous features of SC can best be described as leniency of approximation, vagueness, robustness, and partial truth [103, 215]. This is a comparatively novel computing paradigm which involves a synergistic amalgamation of essentially several additional computing paradigms, which may include fuzzy logic, evolutionary computation, neural networks, machine learning, support vector machines, and also probabilistic reasoning. SC can combine the aforementioned computing paradigms to offer a framework for designing many information processing applications that can function in the real world. This synergism was called computational intelligence by Bezdek [24]. These SC components are different from each other in more than one way. These can be used to operate either autonomously or conjointly, depending on the application domain.

Evolutionary computation (EC) is a search and optimization procedure which uses biological evolution inspired by Darwinian principles [14, 83, 136]. It is stochastic and delivers robust search and optimization methods. It starts with a pool of trial solutions in its search space, which is called the *population*. Numerous in-built operators are generally applied to each individual of the population, which may cause population diversity and also leads to better solutions. A metric, called the fitness function (objective function), is employed to determine the suitability of an individual in the population at any

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particular generation. As soon as the fitness of the existing individuals in the population is computed, the operators are successively applied to produce a new population for the successive generations. Distinct examples of EC may include the Differential Evolution [242], Genetic Algorithms [127, 210], Particle Swarm Optimization [144], and Ant Colony Optimization [196], to name but a few. Simulated annealing [147] is another popular example of meta-heuristic and optimization techniques in this regard. This technique exploits the features of statistical mechanics concerning the behavior of atoms at very low temperature to find minimal cost solutions of any given optimization problem. EC techniques are also useful when dealing with several conflicting objectives, called the multi-objective evolutionary techniques. These search procedures provide a set of solutions, called optimal solutions. Some typical examples of these techniques may include the multi-objective differential evolutionary algorithm (MODE) [275], the multi-objective genetic algorithm (MOGA) [172, 183], and multi-objective simulated annealing (MOSA) [237], to name but a few.

Fuzzy logic tenders more elegant alternatives to conventional (Boolean) logic. Fuzzy logic is able to handle the notion of partial truth competently [139, 141, 215, 282, 283]. A neural network is a computing framework comprising huge numbers of simple, exceedingly unified processing elements called artificial neurons, which add up to an elemental computing primitive [82, 102, 150]. Machine learning is a kind of intelligent program which works on example data. It learns from previous experiences and is used to enhance the performances by optimizing a given criterion [5, 156, 178]. Support vector machines (SVM) are known to be the collection of supervised learning techniques. SVMs are very useful in regression and classification analysis [38, 50]. SVMs are fit to handle a number of real-life applications, including text and image classification, or biosequences analysis, to name but a few [38, 50]. Nowadays SVMs are often used as the standard and effective tool for data mining and machine learning activities. Probabilistic reasoning can be defined as the computational method which uses certain logic and probability theory to handle uncertain circumstances [201, 202].

Many researchers utilize the basic features of quantum computing in various evolutionary algorithmic frameworks in the soft computing discipline. The underlying principles of quantum computing are injected into different meta-heuristic structures to develop different quantum inspired techniques. In the context of image analysis, the features are extracted both from pictographic and non-numeric data and are used in these algorithms in different ways [27]. This chapter provides an insight into the various facets of the *quantum computing, image segmentation, image thresholding, and optimization*. This chapter is arranged into a number of relevant sections. Section 1.1 presents an overview of the underlying concepts of image analysis. A brief overview of image segmentation and image thresholding is discussed in this section. Section 1.2 throws light on the basics of quantum computing in detail. Section 1.3 discusses the necessity of optimization in the real world. This section presents different types of optimization procedures with their application in the real world. Apart from the above issues, this chapter also presents a short description of the literature survey on related topics. Different types of approaches in this regard are in detail presented in Section 1.4. The organization of the book is presented in Section 1.5. The chapter concludes in Section 1.6. It also shows the direction of research that can be used as future reference. A brief summary of the chapter is given in Section 1.7. In Section 1.8, a set of questions related to the theme of the chapter is presented.

1.1 Image Analysis

Image analysis has a vital role in extracting relevant and meaningful information from images. There are few automatic or semi-automatic techniques, called *computer/machine vision, pattern recognition, image description, image understanding* to name but a few, used for this purpose. Image segmentation can be thought of as the most fundamental and significant step in several image analysis techniques. A good example of image analysis may involve the organized activities of the human eye with the brain. Computer-based image analysis can be thought of as the best alternative which may reduce human effort in order to make this process faster, more efficient, and automatic. Image analysis has numerous applications in a variety of fields such as medicine, biology, robotics, remote sensing, and manufacturing. It also makes a significant contribution in different industrial activities such as process control, quality control, etc. For example, in the food industry, image analysis plays a significant role to ensure the uniform shape, size and texture of the final food products.

In medical image analysis, clinical images of different views are captured to diagnose and detect diseases in relation to body organs, and study standard physiological procedures for future references. These investigations can be accomplished through images attained from various imaging technologies, such as magnetic resonance imaging (MRI), radiology, ultrasound, etc. For example, image analysis methodology is of the utmost importance in cancer detection and diagnosis [44], thus it helps the physician to ensure accurate treatment for their patient. In the context of cancer treatment, several features like shape, size, and homogeneity of a tumor are taken into consideration when classifying and diagnosing cancer images. Different image analysis algorithms can be introduced that can help radiologists to classify tumor images.

The steps involved in image analysis are presented in Figure 1.1 [112]. Each step is discussed in the following in brief.

1. *Image acquisition*: This is the first step of every vision system. Image acquisition means acquiring a digital image. After obtaining the image successfully, several processing approaches can be used on the image in order to fulfill the different vision

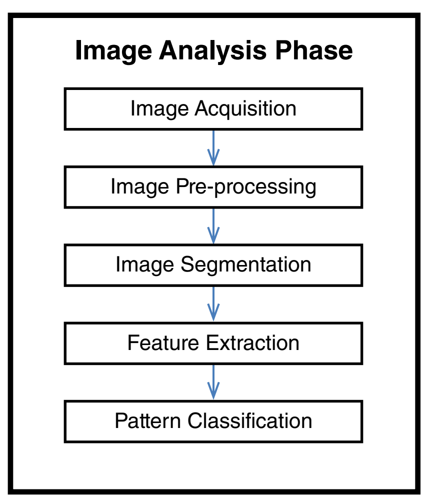


Figure 1.1 Steps in image analysis.

tasks required nowadays. However, if the image cannot be acquired competently, the anticipated tasks may perhaps not be completed successfully by any means.
2. *Image pre-processing*: This step involves improving the image to be suitable for analysis. In this phase, the quality of image is improved by introducing different techniques, such as contrast enhancement, noise contraction, and sharpening of image. The output image can be sent to perform the next step.
3. *Image segmentation*: In this step, the image is partitioned into several regions and the regions of interest are taken out from the input image.
4. *Feature extraction*: This step converts the input image to a number of features on the basis of the traits of the segmented image. Resulting from the discovery of certain facts some data are obtained as the output of this step.
5. *Pattern classification*: This is the final step of image analysis. The extracted features obtained in the last phase are used to classify the given image.

Various techniques can be applied to execute the steps in image analysis depending on the intended application. So, the selected technique for performing each step is of the utmost importance to achieve the desired results from the proposed algorithm.

1.1.1 Image Segmentation

Segmentation separates the patterns into a number of uniform, non-overlapping and homogeneous regions or segments (classes), in which the members of any particular segment are similar to each other and the members in the different segments possess dissimilarity among themselves [9, 22, 87, 133, 166, 194, 203, 252, 258]. The patterns carrying the similar features are known to be clusters/segments. Segmentation is equally effective for localizing and identifying object-specific features both from nonnumeric and pictorial datasets. The challenges lie in the attempt to emulate human perception and intelligence to extract underlying objects accurately. The foremost objective of the segmentation process is to detect the pertinent and meaningful data by taking out the redundant part embedded within. The foundation of a segmentation technique is basically contingent on the assortment of the representation of data elements, the proximity measurement between them and also their grouping. Thus, certain metrics are commonly used for measuring the similarity or difference between the patterns. So far, segmentation has been successfully applied in diverse fields, which may include different engineering disciplines like electrical engineering, mechanical engineering, and others. Apart from that, it has also been widely used in various other fields, such as remote sensing, machine learning, robotic vision, pattern recognition, artificial intelligence, economics, medical sciences, and many others.

Formally, the term "Image segmentation" is defined as follows:

$$1)\ \mathcal{I} = \bigcup_{i=1}^{p} R_i \tag{1.1}$$

$$2)\ R_m \cap R_n = \phi, m \neq n \tag{1.2}$$

where it is assumed that an image, $\mathcal{I}$ is segmented into p number of regions, namely, $R_1, R_2, \ldots, R_p$ [153, 162, 193]. A comprehensive exploration of diverse segmentation methods is available in the literature [27, 97, 231]. Of late, color image segmentation has become a trusted addition in many areas of application [112, 152, 229, 248, 264]. A color pixel is typically manifested as a mixture of different color constituents. The synthesis of different color constituents in color images usually enhances an enormous amount of intrinsic computational complexities in respect of color image processing. As a result, it becomes more challenging to take them up in real-life situations. Some typical examples of color image segmentation may include robotics, object recognition, and data compression to name but a few [27]. A pixel in a color image is recognized in multidimensional color space, which basically enhances the processing complexity in real-life applications. Compared to image segmentation in monochrome image, a higher number of parameters is required to be tuned for optimality in color image segmentation [27].

1.1.2 Image Thresholding

Thresholding is well recognized and probably the most effective tool in the context of image processing (image segmentation) and pattern recognition. From an implementation point of view, it can be considered as the simplest technique among others, which generally provides the most accurate results. Thresholding is basically used in segregating the background and foreground information of the image. This technique is very effective in ascertaining the dissimilar homogeneous components (gray value, color) of the image [97]. Thresholding is equally effective for the images possessing nonhomogeneous components.(textured images).

The threshold can be found by using two approaches, namely, parametric and nonparametric [3, 261, 277]. In the first approach, the distribution of dissimilar gray levels of an object class guides the location of the thresholds. For example, Wang and Haralick [268] used a parametric approach where they divided the pixels of an image into two categories, namely, edge and non-edge pixels. Thereafter, in consonance with the local neighborhoods, the edge pixels are re-classified into two groups, referred to as relatively dark and relatively bright pixels. Afterwards, two histograms are individually and successively drawn from the pixels of each group. The highest peaks are selected from these histograms as the thresholds. Another popular parametric approach is popularly known as moment preserving thresholding, in which the image is segmented on the basis of the condition that the original and thresholded image must have the identical moments [261]. In the latter approach, the concept of optimality is used to find the threshold values, where the threshold divides the gray-level regions of an image on the basis of certain discerning criteria, such as the entropy, cross-entropy, within or between class variance, so on. Typical examples of nonparametric approaches may include Otsu's method [192], Pun's method [206], Kapur's method [140], to name but a few. Otsu's method [192] is clustering-based, where the optimal threshold values are selected by maximizing the between-class variance with a comprehensive search mechanism. Pun's method [206] and Kapur's method [140] are two entropy-based methods, where the gray levels are classified into different classes, and the threshold value is obtained by maximizing the entropy of the histograms of the members of that class. As the consequence of the extensive research over the last few years, a plethora of robust thresholding methods of the parametric or nonparametric type are now available in the literature [220].

Sezgin et al. [231] have presented a comprehensive survey of various thresholding methods.

Any thresholding method is usually of two types: bi-level thresholding and multi-level thresholding. In bi-level thresholding, the image is basically divided into two components, foreground (object) and background, containing different gray-level distribution. Hence, pixels of an image are grouped into two classes in this thresholding method. The pixels with gray levels above a certain threshold are kept in one group, and the other group is formed with the rest of the pixels of that image. The bi-level image thresholding can be computationally extended to its multi-level version, where pixels of an image are segregated into several classes with specific ranges defined by several thresholds.

In both classical and intelligent approaches, the purpose of thresholding also includes reducing the image information complexity by transforming it into monochrome versions, thereby allowing a faithful analysis of the image scene [112, 132, 204, 225, 229]. Thresholding is basically applied to discriminate objects from the background image in an efficient manner. In addition, it can also be used to separate objects from images which comprise of different gray-levels [230, 284]. On the basis of the number of threshold values selected, the thresholding method can be categorized as follows.

1. *Bi-level thresholding*: In this category, the pixel intensity values of the image are grouped into two classes. This kind of thresholding method accepts the gray level/color image as the input and converts it to its corresponding binary image output. The conversion is accomplished on the basis of a predefined pixel intensity value, called the threshold. Based on some criteria, one threshold value as the pixel is chosen from the image, which in turn divides the pixels of the image into two groups. These groups are referred to as object (O) (sometimes called foreground) and background (B). Generally, the pixel intensity values in group (O) are greater than the threshold while the group (B) contains smaller pixels than the threshold or vice versa [220]. To conclude, each element in (O) and (B) is set to be 1 (white) and 0 (black), respectively [278]. Theoretically, for an image (I) and its corresponding threshold value ( ), the subsequent features must be satisfied [25, 41, 251, 261, 278]:

   $$1)\ \ \mathcal{I} \in \{0, 1, 2, \dots, L-1\} \tag{1.3}$$

   $$2)\ \ O = \{\mathcal{I} | \mathcal{I} > \theta\} \text{ and } B = \{\mathcal{I} | \mathcal{I} \leq \theta\} \tag{1.4}$$

2. *Multi-level thresholding*: When the number of classes of pixels exceeds two, it is called multi-level image thresholding. In this kind of thresholding, multiple number of threshold values as pixels are selected. In this category, the number of groups yielded is one more than the number of threshold selected for image thresholding. As a higher level of thresholding may necessitate more calculations, the time complexity of algorithms increases proportionally with the increase of level of thresholding in multi-level thresholding [10, 39, 119, 120, 279]. This could cause significant difficulties especially when higher level threshold values are required to be evaluated. Hence, multi-level image thresholding possesses more complexity compared to the other one. There exists dissimilar algorithms for bi-level image thresholding in the literature, which can be extended to their respective multi-level versions, if required [120, 185]. Although image thresholding, as stated above, results in a monochrome image, of late, researchers have resorted to multi-level

thresholding [161] to generate multi-point thresholding for the faithful segmentation of gray-level images.

Both of these image thresholding versions can be identified by acclimatizing parametric or nonparametric approaches [161, 186]. So far, different algorithms have been designed to fulfill different purposes. Some distinctive applications of thresholding comprise document image analysis [2], image segmentation [274], or nondestructive quality inspection of materials [230].

Several classical methods [112, 231, 258] have been introduced to attain an apposite thresholding criterion for gray-level images. A score of approaches have been developed to address the problem of image thresholding [231]. A few distinguished methods among them are as follows:

1. *Shape-based methods*: In this category, the peaks, valleys, and curvatures of the smoothed histogram are analyzed [213].
2. *Clustering-based methods*: Here, the gray-level samples are clustered in two sections as background and foreground [192].
3. *Entropy-based methods*: In entropy-based methods, the entropy of the foreground and background regions are used to determine thresholds [158]
4. *Object attribute-based methods*: These methods aim to discover a similarity measure between the gray-level and its binary version. This similarity measure may include edge coincidence, fuzzy shape resemblance, etc.
5. *Spatial methods*: These kinds of methods usually use higher-order probability distribution and/or correlation between pixels [35].
6. *Local methods*: This method acclimatizes the threshold value on every individual pixel to the local image features.

Among the different soft computing approaches in this direction, either a deterministic analysis of the intensity distribution of images or heuristic search and optimization techniques are most extensively used [36, 90, 186, 211]. A survey of classical and non-classical techniques for image thresholding and segmentation is available in [27]. But the inherent problem of these optimization techniques lies in their huge time complexities.

1.2 Prerequisites of Quantum Computing

A quantum computer (QC), as the name implies, fundamentally works on quite a few quantum physical features. In contrast to classical computers, a QC has a faster processing capability (even exponentially), hence, these can be thought of as an immense alternative to today's classical computers. The field of quantum computing [173] has developed to provide a computing speed-up of the classical algorithms by inducing physical phenomena such as, superposition, entanglement, etc. It entails these thoughts to develop a computing paradigm much faster in comparison to the conventional computing. The upsurge in the processing speed is acquired by dint of exploiting the inherent parallelism perceived in the qubits, the building blocks of a quantum computer [57]. Hence, the term *quantum computing* can be primarily considered as a synergistic amalgamation of concepts from quantum physics, computer science, and classical information theory.

Deutsch and Jozsa [58] and, later, Shor [235] exhibited some concrete problems where they proved that such speed-up is possible. The basics of a few QC properties are addressed in the following subsections.

1.2.1 Dirac's Notation

The state of a quantum system is basically described in a complex divergent Hilbert space, symbolized by the notation $\mathcal{H}$. The utility of the "bra-ket" notation is very noteworthy in quantum mechanics. Paul Dirac introduced this standard notation in 1939, and hence, this notation is at times popularly known as Dirac's Notation. The "ket" vector is symbolized as $|\psi\rangle$ and, its Hermitian conjugate (conjugate transpose), referred to as "bra", is denoted by $\langle\phi|$. The "bracket" is formed by uniting these two vectors, and is represented by $\langle\phi|\psi\rangle$[30, 77, 121].

Formally, a quantum system can be described as follows:

$$|\psi\rangle = \sum_j c_j |\phi_j\rangle \tag{1.5}$$

where, $|\psi\rangle$ is a wave function in $\mathcal{H}$. $|\psi\rangle$ acts as a linear superposition encompassing the basic states ϕ_j. c_j are the complex numbers which satisfies the following equation, as given by

$$\sum_j c_j^2 = 1 \tag{1.6}$$

1.2.2 Qubit

The quantum bit, or in short, qubit can be described as the basic constituent of information in quantum computing. Basically, a qubit state can be represented as a unit vector, defined in 2D complex vector space. Theoretically, a quantum bit can possess an inestimable number of basic states as required which help to provide exponentially augmented information in QC. The basic quantum states are labeled as $\{|0\rangle, |1\rangle\}$, where,

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{1.7}$$

Occasionally, $|0\rangle$ and $|1\rangle$ are referred to as "ground state" and "excited state", respectively.

1.2.3 Quantum Superposition

The quantum superposition principle, which expresses the idea that a system can exist simultaneously in two or more mutually exclusive states, is at the heart of the mystery of quantum mechanics. Considering the two state vectors in QC, the superposition between the states is represented by the equation $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where, $(\alpha, \beta) \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$. The superposed quantum states are forced to be collapsed into a single state for quantum measurement. The probability of transforming it into the state $|1\rangle$ is $|\alpha|^2$ and that of $|0\rangle$ is $|\beta|^2$, respectively [30, 77, 173, 266].

1.2.4 Quantum Gates

The quantum gates (sometimes called quantum logic gates) are usually hardware tools which operate on preferred qubits using a fixed unitary operation over a predetermined period of time. Thus, quantum gates are reversible, which means, for n number of inputs, there must be n number of outputs. Some typical examples of quantum gates are NOT gate, C-NOT (Controlled-NOT) gate, controlled phase-shift gate, Hadamard gate, Toffoli gate, and Fredkin gate. Theoretically, for the unitary operator, U, the following equations must hold:

$$U^{+} = U^{-1} \text{ and } UU^{+} = U^{+}U = I \tag{1.8}$$

For the Hermitian operator, H

$$U = e^{iHt} \tag{1.9}$$

Quantum gates can be categorized into three categories:

1. One-qubit quantum gates.
2. Two-qubit quantum gates.
3. Three-qubit quantum gates.

A brief summary of popular quantum gates, of each category, is presented below.

1.2.4.1 Quantum NOT Gate (Matrix Representation)

In general, the matrix representation of a quantum gate can be given as

$$\sum_j |input_j\rangle\langle output_j|$$

In this gate, for input $|0\rangle$, output will be $\langle 1|$, and for input $|1\rangle$, output will be $\langle 0|$. The quantum NOT gate can be represented as

$$\begin{aligned} &= |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} \\ &= \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \end{aligned}$$

1.2.4.2 Quantum Z Gate (Matrix Representation)

In this gate, for input $|0\rangle$, output will be $\langle 0|$, and for input $|1\rangle$, output will leads to $\langle -1|$. The quantum Z gate can be represented as

$$\begin{aligned} &= |0\rangle\langle 0| + |1\rangle\langle -1| = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & -1\end{bmatrix} \\ &= \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \end{aligned}$$

1.2.4.3 Hadamard Gate

The most popular quantum gate is known as the Hadamard gate (H). This gate works on a single qubit, and performs the unitary transformation, called the Hadamard transform. It can be defined as follows.

	Gate notation	Matrix representation
Hadamard gate	$\|y\rangle \;—\boxed{H}—\; (-1)^y\,\|y\rangle + \|1-y\rangle$	$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

The matrix given here forms the computational basis $\{|0\rangle|1\rangle\}$. The schematic representation of the H gate works on a qubit in state $|y\rangle$, where $y = 0, 1$.

1.2.4.4 Phase Shift Gate

Like the Hadamard gate, the Phase shift gate works on a single qubit, and it is also represented as 2×2 matrices. The gate notation and the matrix representation of this logic gate are given below.

	Gate notation	Matrix representation
Phase shift gate	$—\boxed{\phi}—$	$R(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$

1.2.4.5 Controlled NOT Gate (CNOT)

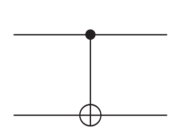
Unlike the Hadamard gate, the CNOT gate possesses two input qubits, called the control and target qubit. The first line is known as the Control qubit, while the second line signifies the target qubit. The CNOT gate works on the basis of the following condition.

1. ***Case 1:*** If control qubit = 0, the target qubit is required to be left alone.
2. ***Case 2:*** If control qubit = 1, then the target qubit is required to be flipped.

Gate notation Matrix representation

Controlled-NOT gate

$|x\rangle$ $|y\rangle$



$|x\rangle$ $|x\rangle \otimes |y\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The schematic representation of the *CNOT* gate is given here. The truth table of this gate is presented below.

Input		Output	
$\|x\rangle$	$\|y\rangle$	$\|x\rangle$	$\|x\rangle \otimes \|y\rangle$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

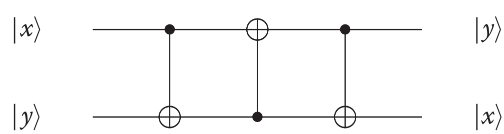
1.2.4.6 SWAP Gate

The swap gate is used to swap the states of a pair of qubits. This gate is usually made by using three CNOT gates. This gate works as follows.

First, in this gate input is represented as $|x, y\rangle$. This first CNOT gate has the output of $|x, x \otimes y\rangle$, which acts as the input of the second CNOT gate to produce $|x \otimes (x \otimes y), x \otimes y\rangle = |y, x \otimes y\rangle$. Finally, this is fed in as the input of third CNOT gate, which produces $|y, y \otimes (x \otimes y)\rangle = |y, x\rangle$.

Gate notation — Matrix representation

Swap gate



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The schematic representation of the Swap gate is presented here. The truth table of the Swap gate is presented below.

Input		Output	
$\|x\rangle$	$\|y\rangle$	$\|y\rangle$	$\|x\rangle$
0	0	0	0
0	1	1	0
1	0	0	1
1	1	1	1

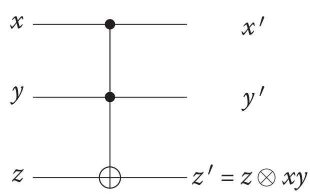
1.2.4.7 Toffoli Gate

The Toffoli gate, alias the controlled-controlled-NOT, is a popular universal reversible gate (logic gate). The Toffoli gate can be used to construct any reversible circuit. This gate is composed of 3 input bits and 3 output bits. In this logic gate, any one of the following can occur.

1. ***Case 1:*** If each of first two bits = 1, the third bit is inverted.
2. ***Case 2:*** Otherwise, all bits remains unchanged.

Gate notation | Matrix representation

Toffoli gate



$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The gate notation and matrix representation of the Toffoli gate are presented here. The truth table of this gate is given below.

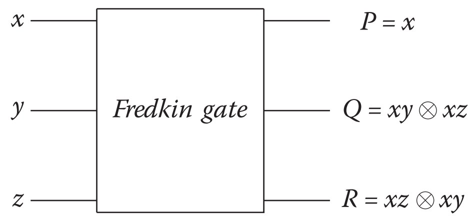
Input			**Output**		
x	y	z	x'	y'	z'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

1.2.4.8 Fredkin Gate

Like the Toffoli gate, the Fredkin gate is also a universal gate, appropriate for reversible computing. This logic gate can be effectively used to construct any arithmetic or logical operation. The Fredkin gate, alias the controlled-SWAP gate, is basically a circuit, which has 3 input bits and 3 output bits. It transmits the first bit unaffected and swaps the last pair of bits if and only if the first bit is set to be 1.

Gate notation | Matrix representation

Fredkin gate



$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The gate notation and matrix representation of the Fredkin gate are presented here. The truth table of this gate is shown below.

Input			**Output**		
x	y	z	P	Q	R
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

1.2.4.9 Quantum Rotation Gate

The quantum rotation gate is a popular quantum gate, generally employed to update any qubit by applying a rotation angle. A distinctive paradigm of this gate used to update any qubit (suppose, k^{th} qubit) (α_k, β_k), can be described as

$$\begin{bmatrix} \alpha'_k \\ \beta'_k \end{bmatrix} = \begin{bmatrix} cos(\theta_k) & -sin(\theta_k) \\ sin(\theta_k) & cos(\theta_k) \end{bmatrix} \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} \tag{1.10}$$

where, θ_k represents the rotation angle of each qubit. The value of θ_k is chosen according to the given problem.

1.2.5 Quantum Register

An anthology of qubits is generally referred to as a quantum register. The size of this register is measured by the size of qubits exercised. The qubit to number conversion can be shown using a typical example given by

$$\underbrace{|1\rangle \otimes |0\rangle \otimes \cdots \otimes |0\rangle \otimes |1\rangle}_{n\ qubits} \equiv |\underbrace{10 \cdot \cdot 01}_{n\ bits}\rangle \equiv |EDN\rangle$$

where EDN stands for equivalent decimal number and $\otimes$ signifies tensor product.

Tensor product: One can construct a new vector space using two given vector spaces. Suppose Y and Z are such two vector spaces. The third vector space is mathematically defined as $Y \otimes Z$, called the tensor product of Y and Z.

Definition: As revealed by quantum mechanics, each and every system $\mathcal{S}$ is described by dint of a Hilbert space H. Suppose the system $\mathcal{S}$ comprises two subsystems, called S_a and S_b. Symbolically, $\mathcal{S}$ can be represented as $\mathcal{S} = S_a \cup S_b$. According to the quantum theory, the Hilbert spaces of $\mathcal{S}$, S_a and S_b are correlated by a tensor product, as given by $H = H_1 \otimes H_2$.

Illustration 4: Suppose a quantum register is used to store information as a binary stream. For instance, the decimal number 7 can be represented by a quantum register in state $|1\rangle \otimes |1\rangle \otimes |1\rangle$. Let us describe it in more compact notation. Suppose $b = |b_0\rangle \otimes |b_1\rangle \otimes \ldots |b_{n-2}\rangle \otimes |b_{n-1}\rangle$, where $b_j \in \{0, 1\}$. It signifies a quantum register having the value of $b = 2^0 b_0 + 2^1 b_1 + 2^2 b_0 + \ldots + 2^{n-2} b_{n-2} + 2^{n-1} b_{n-1}$. In this representation, there may be 2^n states of this form, which represents n-length binary strings or numbers from 0 to $2^{(n-1)}$. They form a "computational basis," $b \in \{0, 1\}^n$, where b composed of n-length binary string indicates that $|b_j|$ is a part of the "computational basis."

A group of n qubits taken together is known as a quantum register of size n. Accordingly, the decimal 1 and 5 can be stored in a quantum register of size 3 as follows:

$$|0\rangle \otimes |0\rangle \otimes |1\rangle \equiv |001\rangle \equiv |1\rangle$$
$$|1\rangle \otimes |0\rangle \otimes |1\rangle \equiv |101\rangle \equiv |5\rangle$$

These two numbers can be stored in chorus as

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \otimes |1\rangle \equiv \frac{1}{\sqrt{2}}(|001\rangle + |101\rangle) = \frac{1}{\sqrt{2}}(|1\rangle + |5\rangle)$$

where, $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is called the superposed form of quantum states.

1.2.6 Quantum Entanglement

Quantum entanglement is basically a quantum mechanical phenomenon where the quantum states comprising at least two objects can be described in relation to one another, even though the distinct objects might be spatially isolated. This causes correlations among perceptible physical features in the quantum systems. The quantum entanglement ensures that any alteration made in one object will definitely affect the other one. In real life, there may subsist untold love between a boy and a girl, yet they feel unseen romance, affections and mystical connections for each other. Such connections also happen in the subatomic world, which is known as entanglement between quantum states.

Let there be a bipartite system $\mathcal{S} = S_a \cup S_b$, where, S_a and S_b are two subsystems of the system $\mathcal{S}$. A pure state of $\mathcal{S}$ is basically a vector $\phi \in H$, where H is known as Hilbert space. The state ϕ is called simple, separable, non-entangled or even factorable if ϕ can be expressed as $\phi = c \otimes d$ for some $c \in H_a$ and $d \in H_b$. Otherwise, the state is called the entangled state. For the non-entangled state, ϕ, S_a and S_b are in states c and d, respectively. It should be noted that if ϕ is entangled, it is not separable. In quantum computing, a pure state comprising two qubits is said to be in the entangled form if the state cannot be represented (not separable) as a tensor product of the participating states of these qubits as $|\vartheta_1\rangle \otimes |\vartheta_2\rangle$.

Quantum entanglement is the utmost notable feature of quantum mechanics. It is the basis of several applications of quantum mechanics, such as quantum teleportation, quantum computation, quantum cryptography, and quantum information so on [190]. The use of innumerable entangled qubits in QC, can accelerate computational capability as compared to its classical version.

Note: We consider two entangled states as follows.

1. ***Case 1:*** Suppose, in a system, the first qubit is given as $|0\rangle$ and the second qubit is given as $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Then the state of this system (two-qubit) can be written as the tensor product of these two qubits as follows:

$$\frac{1}{\sqrt{2}}|0\rangle \otimes (|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

2. ***Case 2:*** The entangled states such as $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ or $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ cannot be represented as the product of the two-qubits states.

1.2.7 Quantum Solutions of NP-complete Problems

Researchers are sometimes capable of constructing algorithms that can convey a solution for some particular given problem. These kinds of problems are solvable by a number of computational steps delimited by a polynomial m^i where m is the input size and i is a constant. This kind of problem, where a polynomial-time algorithm exists, is categorized as class P problems. It is known that class P problems are solvable with high efficacy. There is one more category where a given arrangement is validated to confirm whether it is a solution to the given problem in polynomial time or not. Such methods exist which can find a solution of this kind. These kinds of problems demand an exhaustive search by configuring all possible arrangements until there is a result of the given problems under a polynomial time. Such problems fall into the class NP. So it is clear that $P \subseteq NP$. NP-complete problems are figured with the subclass of NP. Basically, this subclass surrounds both the NP and NP-hard problems. For a NP-hard problem, there exists an NP-complete problem which can be reduced to an NP-hard problem in polynomial time. Quantum mechanics can store and influence a huge amount of data and processes information using the minimum number of quantum particles due to its rigid architecture. Its hardware is capable of dealing with all possible combinations of solutions simultaneously. It has a fabulous performance in certain kinds of problems like performing the factorial of an integer number as shown in Shor's algorithm [235]. For class P problems, QC has a very high efficacy compared to its classical counterparts, while for other NP-complete problems, it may not live up to its publicity. In [58], the authors have explained that some classically hard problems may be solved efficiently in quantum computing. Later, Grover's algorithm [117] has attracted interest in developing a more efficient search algorithm.

1.3 Role of Optimization

Optimization is referred to as a selection mechanism, used to attain the best combination of feasible solutions from an ample number of acceptable solutions of a specific problem. This selection process is guided by several predefined single/multiple criteria. This is accomplished through a systematic approach by selecting the appropriate values of integer or real variables within an allowable set. The foremost objective of optimization is to get the best possible solutions of any specific problem based on the objective functions for a defined domain. The possible solutions of any certain problem are initially found by exploring its search space, and then they are optimized within a nominal time frame. In principle, the optimization process handles those problems which require one or more of its objectives to be minimized or maximized, where the objectives are basically functions of several integer or real variables. The optimization technique can be categorized as follows.

1. Single-objective optimization
2. Multi-objective optimization

A brief overview of these two kinds of optimization technique is described in the subsequent subsections.

1.3.1 Single-objective Optimization

A single-objective optimization (SOO) problem deals with one objective function. The foremost goal of SOO is to search for the "best" possible solution corresponding to the maximization or minimization of a single objective function that combines all dissimilar objectives into one. This kind of optimization can be effective as a tool which gives decision-makers insight into the type of problem. It is usually unable to deliver the alternative solution sets that trade dissimilar objectives against one another. In this optimization technique, the attention is usually directed towards determining a solution which accomplishes global optima. Formally, the optimization technique is defined as follows.

Let $g : U \to \mathfrak{R}$ be a given function, which maps a set U to the set of real numbers, $\mathfrak{R}$. The goal of single-objective optimization is to determine an optimum element $y_0 \in U$ such that, $g(y_0) \leq g(y), \forall y \in S$ occurs in performing minimization, while $g(y_0) \geq g(y), \forall y \in U$ occurs in accomplishing maximization

Here, $U \in \mathfrak{R}^n$ comprises an assortment of entities, like equalities, inequalities or constraints and $\mathfrak{R}^n$ denotes the Euclidean space. Here, U signifies a subset of the Euclidean space R^n which is an assortment of entities such as equalities, inequalities or constraints. Each member of U ought to satisfy these said entities. The domain of the above function is known as the search space, and the members of U are referred to as feasible or candidate solutions. The function g is termed the objective function or cost function. Among all feasible solutions, a particular solution which optimizes (minimizes or maximizes) the given objective function is known as an optimal solution.

Different SOO techniques can be classified into the following categories [19, 185, 186].

- *Enumerative techniques*: Enumeration methods are very useful in solving combinatorial optimization problems. These methods encompass assessing every points of the finite, or even discretized infinite search space to attain the optimal solution [19]. Dynamic programming is a popular example of this category of search method. Enumeration methods can be broadly categorized into the following categories.
  (1) Explicit complete enumeration: In this category, all potential alternatives are completely enumerated and a comparison is made among them to get the best possible solution. It is basically impossible as well as very expensive to solve complex problems.
  (2) Implicit complete enumeration: Portions of the solution space that are certainly sub-optimal are left out in this category. This decreases complexity because only the most promising solutions are taken into consideration. In this kind of enumeration, different methods, such as Branch & Bound, dynamic optimization, etc. are generally used.
  (3) Incomplete enumeration: In this category, certain heuristics are applied to select alternatives by only observing portions of the solution space. This does not promise an optimal solution, instead, it delivers estimated solutions.
- *Calculus-based techniques*: The calculus-based methods, also referred to as numerical methods, mean the solution of any optimization problem is found on the basis of a set of necessary and sufficient criteria [19]. This kind of method can be further divided into two different groups, namely, direct and indirect methods. The direct search method is a kind of optimization method used to solve such optimization problems, which does not necessitate any information relating to the gradient of the fitness (objective) function. The traditional optimization methods generally use information about the higher derivatives or gradient for searching an optimal point, whereas a direct search method explores the search space to look for a set of points in all directions of the current point, and selects one of them, which possesses the best objective value compared to the others. The non-continuous or non-differentiable type of objective function can be used to solve problems in the direct search method. Typical applications of this category may include Mesh Adaptive Direct Search (MADS) [11], the Nelder-Mead algorithm (NM) [101, 154], Multidirectional Search (MDS) [259], to name but a few. The indirect search algorithms are based on the derivatives or gradients of the given objective function. The solution of the set of equations is obtained by equating the gradient of the objective function to zero. The calculus-based methods may be very effective in solving the trivial class of unimodal problems [19].
- *Random techniques*: Compared to enumerative methods, random search methods basically use the additional information related to the search space that in turn guides them to possible areas of the search space [24, 112]. This method can be further subdivided into two different categories, namely, single-point search and multiple-point search. In the first category, the aim is to search for a single point, whereas several points are needed to be searched at a time for the multiple-point search method. Some

popular examples of single-point search methods involve Simulated annealing [147], Tabu search [109], etc. Some population-based evolutionary algorithms, such as Particle Swarm Optimization [144], Genetic Algorithms [127], differential Evolution [76], and Ant Colony Optimization [242], are popular examples of the latter category. The guided random search methods are very effective in such problems where the search space is vast, multi-modal, and not continuous. Instead of providing the exact solution, the random search methods deliver a near-optimal solution across an extensive range of problems.

1.3.2 Multi-objective Optimization

Multi-objective optimization (MOO), also known as the multi-attribute or multi-criteria optimization technique [54] that simultaneously optimizes several objectives as one in respect of a set of specific constraints. As opposed to the SOO, the MOO deals with various conflicting objectives and provides no single solution (optimal). The collaboration among diverse objectives presents compromised solutions, which are principally referred to as the trade-off, non-inferior, non-dominated, or even Pareto-optimal solutions.

There are numerous real-life decision-making problems which have several objectives, such as minimizing risks, maximizing reliability or minimizing deviations from anticipated levels, minimizing cost, etc. In MOO, it is not always possible to find a solution in the search space, which produces the best fitness values with regards to each objective. In the given search space, there may be a group of solutions having better fitness with respect to a set of objectives and worse value for the rest as compared to other solutions. From the perspective of MOO, the term "domination" between a pair of solutions can be described as follows [54, 177]:

Suppose a MOO problem finds a set of n solutions, say, $\{y_1, y_2, \dots, y_n \in Y\}$ for m objectives, say, $\{O_1, O_2, \dots, O_m \in O\}$, Y and O are known as the solution space and objective space, respectively. A solution (say, y_i) dominates others (say, y_j), where $1 \leq i, j \leq n$ and $i \neq j$, if the following conditions are satisfied:

1. The solution y_i is not inferior to y_j $\forall$ O_k, $1 \leq k \leq m$.
2. The solution y_i must possess better value than y_j in at least one O_k, $1 \leq k \leq m$.

Since, there exists no solution in the Pareto-optimal set, which holds the top spot of all objectives, some specific problem knowledge and decision-making capabilities to select preferred solutions in MOO [54] have to be found. Over the last few years, a number of techniques have been proposed in this literature that have coped with multi-objective optimization problems in different facets [177].

1.3.3 Application of Optimization to Image Analysis

Image analysis appears to be a part of decision support systems in a variety of applications, such as medical, military, industrial and many others. Several techniques have already been introduced in the literature so far to solve different image processing activities. They usually necessitate few method-specific parameters tuning in an optimal way to accomplish the best performance. The obligation to achieve the best results changes the structure of the methods candidate into a corresponding optimization problem. Optimization is an important technique used to solve several problems in image processing. Such a reality is evident as a significant number of researchers use optimization techniques in their research work. Classical optimization techniques habitually face excessive difficulties when dealing with images or systems comprising distortions and noise. In these situations, a variety of evolutionary computation techniques have been shown to be viable alternatives that can efficiently address the challenge of different real-life image processing problems [51]. Image analysis is a dynamic and fast-growing field of research. Apart from that, each novel technique proposed by different researchers in any field is rapidly recognized and -simulated for image processing tasks. Some state-of-the-art techniques that can handle the challenges and image processing problems are available in the literature [191]. The rich amount of information is already available in the literature makes it easy to find the exact optimization technique for a certain image application.

1.4 Related Literature Survey

This section presents a write-up discussion about a brief review of various approaches to handle different optimization problems. This section tries to provide a fundamental overview of present trends to solve different optimization problems. These aforementioned approaches are broadly classified into three categories, namely, quantum-based approaches, meta-heuristic-based approaches, and multi-objective-based approaches. Most of the quantum-based approaches mainly use the basics of quantum computing to serve the optimization purpose. The meta-heuristic-based approaches use the basic anatomy of different meta-heuristics to develop a variety of techniques to handle different optimization problems. Lastly, the multi-objective-based approaches are used to develop different Pareto-based techniques to handle a number of objectives simultaneously for optimization.

This section presents a brief literature survey of the aforesaid trends in single and multi-objective optimization.

1.4.1 Quantum-based Approaches

The field of quantum computing became popular when the notion of a quantum mechanical system was anticipated in the early 1980s [23]. The aforesaid quantum mechanical machine is able to solve some particular computational problems efficiently [117]. In [89], the author has recognized that the classical computer faces a lack of ability while simulating quantum mechanical systems. The author presented a structural framework to build Quantum Computer. Alfares and Esat [4] analyzed how the notion of quantum algorithms can be applied to solve some typical engineering optimization problems. According to them some problems may arise when the features of QC have been applied. These problems can be avoided by using certain kind of algorithms. Hogg [125] presented a framework for a structured quantum search where Grover's algorithm was applied to correlate the cost with the gates, behavior. In [126], the authors extended the work and proposed a new quantum version of combinatorial optimization. Rylander et al. [216] presented a quantum version of genetic algorithm where the quantum principles like superposition and entanglement are employed on a modified genetic algorithm. In [181], Moore and Narayanan proposed a framework for general quantum-inspired algorithms. Later, Han and Kim [121] developed a quantum-inspired evolutionary algorithm which was applied to solve the knapsack problem. Here, qubit is used for the probabilistic representation. A qubit individual is represented by a string of qubits. The authors introduce the quantum gate as a variation operator in order to drive the qubit individuals toward better solutions. A new improved version of this algorithm has been presented in [122]. Here, the authors proposed a termination criterion to accelerate the convergence of qubit individuals. Another version of a quantum genetic algorithm has been proposed by Zhang et al. where the strategies to update the quantum gate by utilizing the best solution and also introducing population catastrophe have been shown. The improved version of the work presented in [121] has been proposed by Zhang et al. where they applied a different approach to get the best solution [287]. Narayan and Moore presented a genetic algorithm where quantum mechanics was used for the modification of a crossover scheme [189]. Moreover, Li and Zhuang developed a modified genetic algorithm using quantum probability representation. They adjusted the crossover and mutation processes to attain the quantum representation [157]. In [188], the authors presented a quantum-inspired neural network algorithm where also the basic quantum principles were employed to symbolize the problem variables. The instinctive compilation of information science with the quantum mechanics constructs the concept of quantum computing. The quantum evolutionary algorithm (QEA) was admired as a probability-based optimization technique. It uses qubits encoded strings for its quantum computation paradigm. The intrinsic principles of QEA help to facilitate maintaining the equilibrium between exploitation and exploration. In recent years, some researchers have presented some quantum evolutionary algorithms to solve particular combinatorial optimization problems. A typical example of this algorithm is filter design by Zhang et al. [285]. A group of heuristic search algorithms have been designed for quantum computers by different researchers. They call these algorithms heuristic search designed quantum algorithms (HDQs) [114, 115, 219, 239]. The capability of HDQs is checked by simulating a quantum computer, such that the efficiency of a quantum algorithm can be judged on classical hardware. Some authors have combined quantum computation with genetic algorithms and developed applications where fitness functions are varied between genetic steps based on the number of outer time-dependent inputs. A few distinctive examples of this category are given in [262, 272]. These papers include some schemes for quantum control processes. Here, genetic algorithms are employed for optimally shaped fields to force a few preferred physical processes [262, 272]. Aytekin et al. [12] developed a quantum-based automatic object extraction technique where quantum mechanical principles were used as the basic constituents of the proposed technique. With reference to Artificial Intelligence (AI), some authors have developed quantum behaved applications on AI. A few of them are presented in [124, 126, 137, 267]. Hogg [125] proposed a new framework for a structured quantum search. In his proposed framework, the author used Grover's algorithm [117] to associate the cost with the activities of the quantum gate. Lukac and Perkowski [164] applied a different approach where they considered each individual in the population as quantum circuits and used the elements of population for the objective quantum circuit. Two popular quantum algorithms developed so far are Quantum Fourier Transform (QFT) [235], and the Grover Search Algorithm [117].