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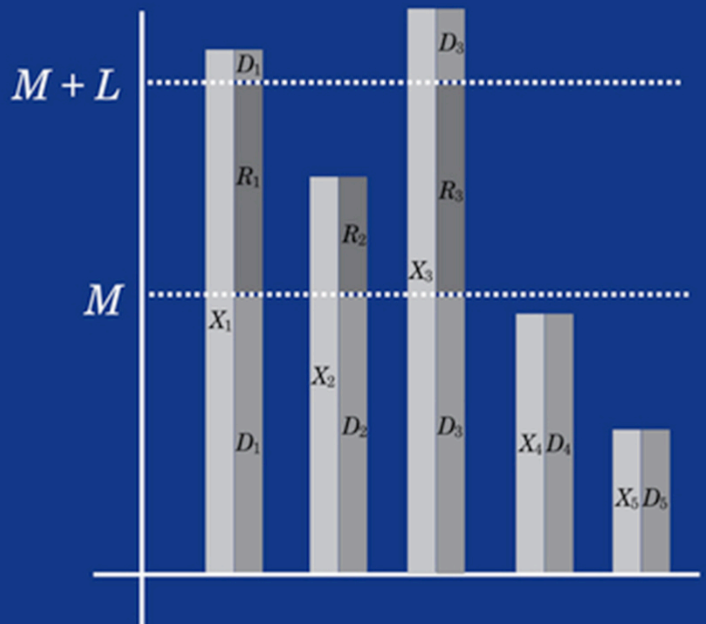

Hansjörg Albrecher
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Reinsurance: Actuarial and Statistical Aspects

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Preface

Reinsurance is a fascinating field. Several of the challenges of classical insurance are amplified for reinsurance, particularly when it comes to dealing with extreme situations like large claims and rare events. This poses particular challenges for the modelling of claims and their occurrence, which often needs to be based on only few data points. In addition, in terms of better diversification of usual-scale risk on the local and global level as well as in terms of the development of innovative and sustainable techniques to deal with risks of an unusual kind, reinsurers play a crucial role in the insurance process.

This also reflects on practitioners and researchers involved in such topics, as they have to rethink classical models in order to cope successfully with the respective challenges. Over the years, there has been enormous research activity on problems connected to reinsurance. Close to 40% of the references in our literature list have appeared over the last 10 years, with a steep upward gradient over the last 3–4 years. While there exist some excellent classical textbooks on reinsurance either from the academic or the practitioner's side, our impression was that there was no modern reference book available that gave an overview of the academic research landscape in this field and also puts it in perspective with the practical viewpoint. The main reason for writing this book was to try to address this gap, at least for actuarial and statistical matters. As all the authors are from academia, there naturally remains a bias towards the academic angle. However, numerous and enlightening discussions with insurance and reinsurance practitioners over the last few years have motivated us to produce the current account, hoping and trying to further bridge the two worlds. The focus of the book is on modelling together with the statistical challenges that go along with it. We illustrate the discussed statistical approaches alongside six case studies of insurance loss data sets, ranging from MTPL over fire to storm and flood loss data. Some of the presented material also contains new results that have not yet been published in the research literature. We hope that the material presented can trigger new research questions and foster the communication between (re)insurance practitioners and academics working in these fields. One of our main goals was to give an up-to-date overview of the relevant research literature and to frame it to questions that matter in reinsurance practice. Since this a vast topic, we naturally had to take various compromises and we apologize for possible omissions on either side.

The book is written for researchers with an interest in reinsurance problems, for graduate students with a basic knowledge of probability and statistics as well as for practitioners in the field.

We start with a general introduction to the field in Chapter 1, presenting some basic facts and motivations for reinsurance activities. We also introduce the six real-life case studies that will accompany the considerations throughout the book. In Chapter 2, we discuss the most common reinsurance forms and their properties, together with some practical aspects of their implementation. Chapter 3 is dedicated to motivating and developing models for claim size distributions that are commonly used. Here we emphasize those aspects from actuarial mathematics that are relevant for reinsurance. Reinsurance is often invoked in the presence of large claims, therefore we need a thorough discussion of models capable of catching the essentials of what actuaries would call *large*. Chapter 4 contains detailed guidelines on how to proceed in the model choice when actually facing data. Throughout the text, we illustrate the presented procedures for our case studies. Chapter 5 proceeds with models for claim numbers, both from a conceptual and a practical viewpoint. We also provide guidelines for a statistical analysis of data sets in this context. The two ingredients (claim numbers and claim sizes) are then used in Chapter 6 for the aggregation of the claims. Emphasis is put on the aggregation of independent risks, and we describe both numerical and asymptotic methods in detail. The case of dependence in the aggregation process is also discussed briefly, although not in detail, as the results typically are very sensitive to the particular dependence structure used in the modeling process, and often the number of data points does not allow one such model to be decisively favored over another. It is beyond the scope of this book to discuss all such approaches. Chapter 7 treats important actuarial aspects of reinsurance pricing, once a distribution for the individual (or aggregate) risk is available (or, rather, decided upon). In Chapter 8 we discuss some guidelines on possible criteria for the choice of reinsurance forms and the respective consequences on the optimal choice of contracts. The identification of optimal reinsurance forms has been a very active research field recently and it is impossible to reflect all these contributions in one book chapter in an exhaustive way. We instead provide an overview of some of the main approaches and contributions alongside a structure in terms of decision criteria, with an emphasis on the intuition behind the results. Since stochastic simulation is an essential tool in many models relevant in reinsurance, we cover this topic in Chapter 9 and discuss some variance reduction techniques that can help to considerably speed up calculations. Chapter 10 then examines some further topics. We first provide more information on large claim analysis, and continue with an overview of alternative risk transfer products, which can serve as a complement to traditional reinsurance. We also highlight the role of finance in reinsurance and finish with a section on catastrophe insurance. Within the chapters and in particular at the end of the chapters we provide links for further reading.

Many of the topics dealt with in the book apply to both non-life and life insurance. Even when there is a clear emphasis on non-life insurance throughout, we hope that some of our attempts may help to also be of service to life insurance. As the title suggests, this book is about (traditional) actuarial as well as statistical aspects arising in reinsurance. As is outlined in Chapter 1, reinsurance also serves financial and management purposes in practice. Correspondingly, the role of capital is nowadays an important ingredient in managing and steering reinsurance companies, and financial pricing techniques for reinsurance contracts as well as general capital management tools eventually have to complement the actuarial approach. While we do consider such aspects when discussing the pricing and the possible choice of contracts in

Chapters 7, 8 and 10, it is beyond the scope of this book to treat and reflect the merging of actuarial and financial principles in the amount of detail this may deserve from a general perspective.

The idea for writing this book was born in the legendary and productive environment of EURANDOM, Eindhoven. We would like to thank this institution for its continuing support over the years as well as the University of Lausanne and KU Leuven for generous support for extended research visits that enabled the book to progress. We also thank Sophie Ladoucette, MunichRe, and the Versicherungsverband Österreich for providing data for our case studies.

We would like to thank all the people with whom we had interesting discussions about the topic over the recent years, including the participants of the Summer School of the Swiss Actuarial Association in Lausanne in 2015, as well as short course participants in Paris, Johannesburg, Lisbon, Lyon, Luminy, Yerevan, Warsaw, and Hong Kong.

Particular thanks for stimulating discussions or advice in earlier and later stages of the book writing go to Jose Carlos Araujo Acuna, Katrien Antonio, Peiman Asadi, Alexandru Asimit, Anastasios Bardoutsos, Arian Cani, Michel Dacorogna, Dalit Daily-Amir, Michel Denuit, François Dufresne, John Einmahl, Karl-Theodor Eisele, Michael Fackler, Damir Filipović, Hans U. Gerber, Alois Gisler, William Guevara-Alarcon, Jürgen Hartinger, Christian Hipp, Frans Koning, Yuriy Krvavych, Sandra Kurmann, Sophie Ladoucette, Stéphane Loisel, Franz Prettenthaler, Christian Y. Robert, Robert Schall, Matthias Scherer, Thorsten Schmidt, Wim Schoutens, Johan Segers, Wim Senden, Stefan Thonhauser, Joël Wagner, Roel Verbelen, Robert Verlaak, Leonard Vincent, Jean-François Walhin, Gord Willmot, and Mario Wüthrich. Special thanks go to Tom Reynkens for his tremendous effort writing an R package with this book and producing the plots linked with the statistical procedures. Further thanks go to William Guevara-Alarcon and Dominik Kortschak for help with the R codes underlying the illustrations in Chapter 9, and to Roel Verbelen and Tom Reynkens for their significant contribution to the splicing methods. We will maintain a webpage connected to the book at

`http://www.hec.unil.ch/halbrech_files/reinsurance.html`

where we also intend to keep a list of misprints and remarks. We are grateful to receive relevant material sent to us by email. The R package ReIns can be found at the CRAN page

`cran.r-project.org/package=ReIns`

Hansjörg Albrecher, Jan Beirlant, and Jozef L. Teugels
Lausanne and Leuven,
December 2016

1 Introduction

1.1 What is Reinsurance?

A *reinsurance contract* is an agreement in which one party (the *reinsurer*) agrees to indemnify another party (the *reinsured*, the *first-line insurer* or also the *ceding company, cedent*) for specified parts of its underwritten insurance risk. In turn, the cedent pays to the reinsurer a *reinsurance premium* for this service. That is, in reinsurance the principle of insurance is lifted up one level, so an insurance company seeks itself the possibility of replacing parts of its future loss by a fixed premium payment (much like a policyholder does when entering an insurance contract). There are many reasons why such a risk transfer from the insurer to the reinsurer can be desirable for both parties, as well as for the economy in general, and we will outline a number of them in Section 1.2.

While reinsurance can be seen as a particular form of insurance, and naturally shares various common features with it, reinsurance is also quite distinct from primary insurance in a number of aspects. These include the type and magnitude of risks under consideration, the type of data available for the risk analysis, the diversification possibilities, demand/supply peculiarities of contracts quite different from the primary insurance market, and also the fact that reinsurance is a form of risk sharing among two "professional" insurance entities, so that the necessary guidelines for regulation can be quite different.

(Non-life) reinsurance contracts are typically written for one year, and one distinguishes between *obligatory treaties*, where a binding agreement is specified that applies to all risks of a specified risk class, and *facultative* arrangements, which are negotiated on each individual risk. A *facultative treaty* is then a contract where the cedent has the option to cede and the reinsurer has the option to decline or accept classified risks of a particular business line. In practice many contracts actually involve several reinsurers (e.g., the contract is negotiated with a primary reinsurer, and other reinsurers then participate proportionally in the reinsurance coverage, or a second reinsurance contract with another reinsurer is written for parts of the remaining risk of the cedent after a first contract). The relationship between insurer and reinsurer is often of a long-term nature, which also has an effect on the way reinsurance premiums are negotiated. Finally, there is no relation between a reinsurer and the individual policyholders of the

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Table 1.1 Global premium volume 2015 (in US$ billions).

	Primary insurance	Reinsurance
Life and health	2500	65
Non-life	2000	170

Source: SwissRe.

underlying risks. A reinsurer may itself enter a reinsurance contract with another insurance company on parts of the reinsured risk, and such a procedure is called *retrocession*.

Table 1.1 gives a feeling for the size of the global reinsurance market in comparison to the primary insurance market. One sees that in terms of premium volume, reinsurance is only employed for a small fraction of the primary insurance risk. However, typically the reinsured risk is the one that is complicated to assess and handle (this is one of the main reasons why it is reinsured!), which makes this type of risk particularly challenging for actuaries, statisticians, and other risk professionals. Worldwide, there are about 200 reinsurance companies today, and many of these are also acting as primary insurers in the market.

1.2 Why Reinsurance?

Let us look at why an insurance company is interested in buying reinsurance. The main function of insurance companies is to take risk. This is similar to the business model of other financial organizations, and both types leverage the capital provided by shareholders through raising debt. However, insurers raise debt by selling policies to insureds, which makes the debt very risky (due to uncertainty around the timing and severity of claims), whereas financial debt would typically rather have pre-determined expiry and face value (severity). This leveraging activity is a competitive advantage, but also makes the companies vulnerable to distress and insolvency, creating the demand for risk management. Among the available risk management tools, *risk transfer* through reinsurance then plays an important role in improving the company's overall risk profile. Let us look at some of the main motivations for the insurer to buy reinsurance as a means of risk transfer (several of which are not independent of each other):

- **Reducing the probability to suffer losses that are hard to digest**
  This is a rather general statement and many of the items below are in fact refinements of this criterion. It should be kept in mind that for an insurance company buying reinsurance means passing on some of its insurance business (i.e., its core activity), and hence typically the goal is to keep the reinsured part small. However, reduction of risk exposure can be desirable or necessary for the reasons outlined below.
- **Stabilizing business results**
  Entering a reinsurance contract reduces the volatility of the cedent's financial result, as random losses are replaced by a (typically deterministic) premium payment. That is, reinsurance can be a means to steer the volatility of an insurance company towards a

desired level, and the latter can have particular advantages (e.g., with respect to taxes, capital requirements and market expectations).

- **Reducing required capital**
  Reducing the aggregate risk will reduce the required capital to bear such risks, and in view of capital costs this may be desirable. Concretely, if the reinsurance premium (together with the administration costs) is smaller than the gain resulting from the corresponding reduction of capital, the reinsurance contract is desirable. In fact, due to the ongoing shift towards risk-based regulation, the notion of capital and its management becomes a central issue for insurance companies, and reinsurance then should be understood as a tool in this context. This corresponds to an important *finance function* of reinsurance as a substitute for capital, freeing up capacity.
- **Increasing underwriting capacity**
  In the presence of a reinsurance contract, only a certain part of the risk is assumed by the insurer, and hence under otherwise identical conditions an insurance company can afford to underwrite more and larger policies (see Chapter 2 for details), which may be desirable for various reasons, including market share targets, testing and entering of new markets, gaining (data) experience in certain business lines or regions etc. It also can lead to enhanced liquidity.
- **Accessing benefits from larger diversification pools**
  Often the primary insurers' business model is restricted to a local area, in which case attempts to look on their own for diversification possibilities outside of that market for the more dangerous part of the risks would be very costly and inefficient. Reinsurers, on the other hand, typically act on an international level and therefore have more possibilities for diversifying such risks. Consequently the amount of capital needed to safeguard these risks in the portfolio can be considerably lower for a reinsurer and so the risk transfer produces economic gain through attractive reinsurance premiums.

We mention a few further motivations:

- **Reducing tax payments**
  Equalization reserves (i.e., reserves for volatility of claims and their arrivals over longer time periods, which is, for instance, particularly important for catastrophe risks) of insurance companies are taxed in most legislations. If such reserves are paid to a reinsurance company in the form of a reinsurance premium (or, alternatively, into a respectively created captive structure, cf. Section 10.2), then the taxation pattern becomes more favorable, as for reinsurers and captives (often located in tax-favorable countries) different tax rules may apply.
- **Other legal issues**
  Reinsurance can be a helpful tool to resolve legal constraints such as regulatory compliance. For instance, if an insurance company does not have a formal license to write business in a certain country, a solution can be to find a local insurer with such a license and act as a reinsurer for this local company.
- **Financial solutions**
  The reinsurer can serve as a facilitator for financial solutions. Examples include reducing (expected) financial distress costs by providing run-off solutions (cf. Section 10.2) and portfolio transfers to other companies or the capital markets as well as setting up securitization transactions like issuing bonds.

- **Protection against model risk**
  Insurance activities are designed on the basis of stochastic models for the underlying risks. For the aggregate performance, both the understanding of the marginal risks as well as of the dependence between them is important.[1] However, every model is an imperfect description of reality, and the less experience and data one has, the higher the uncertainty about whether the model underlying the business plan is appropriate. Reinsurance is a way to mitigate model inadequacy (e.g., concerning the tails of the risks or their dependence).
- **Support in risk assessment, pricing, and management**
  In certain situations an insurance company does not have enough data points or manpower available to analyze the risks (in particular their tails), and passing on those risks to an entity with respective experience is a natural procedure, which is often much cheaper than dealing with such risks by other means. This also includes business expansions to new regions or business lines, in which the reinsurer may already have experience from earlier activities. In fact, reinsurance contracts often have a certain consultancy component, as the reinsurer may share its expertise and data on the respective risks with the cedent.

On the society level, reinsurance allows insurers to write more business, which makes insurance more broadly available and affordable. This can foster economic growth and increase stability at large. Reinsurance enables risks to be insured that otherwise would not be insurable, and assigning premiums to (i.e., quantifying) risks can also provide incentives for more risk-adequate behavior and possibly risk prevention.

For all these reasons, reinsurance serves as a tool to increase the efficiency of the marketplace. When designing reinsurance contracts, all these aspects will play some role. The goal of this book is to focus on the actuarial elements involved in the process as well as the statistical challenges that appear in this context.

1.3 Reinsurance Data

As for primary insurance, in the reinsurance business one will be interested in the statistical analysis of claim information for different types of business lines (such as car liability insurance or fire insurance), where one can obtain claim information on the individual claim level. Due to the nature of the reinsurance contract, there are, however, additional challenges with respect to the type of claim data.

Consider, for instance, the case of non-proportional reinsurance where the reinsurer will pay (parts of) the excesses over some threshold, say M. The ceding company then does not need to provide all claim information to the reinsurer. For example, information may be provided on those events only for which the incurred claim amount I (i.e., the estimate of the amount of outstanding liabilities) is larger than a certain percentage of M. Then, as long as I stays below that reporting threshold during the development process, the claim will not be known to the reinsurer and hence the

1 Here, dependence can be causal (e.g., the occurrence of a claim triggers another claim) or due to common risk drivers. An appropriate modelling of dependence can be a considerable challenge, particularly when only few data points are available and the number of dependent risks is high.

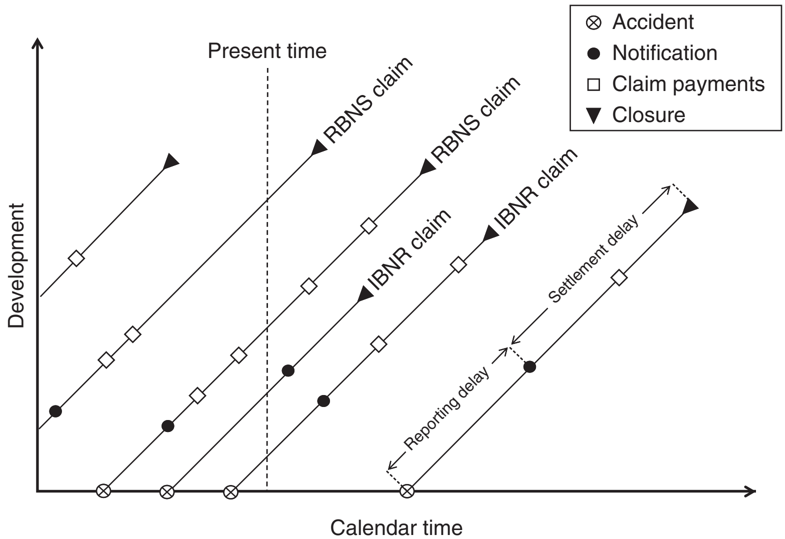

Figure 1.1 Claim development scheme.

incurred value is *left truncated* in such a case. For some lines of business, development times can be quite large (up to several decades) so that, at the time of evaluation, the cumulative payments are still a lower bound for the ultimate claim amount. The data then are *censored*. In practice, companies use claim development methods to forecast the ultimate claim amounts. Of course these also yield uncertain information, which hampers the statistical analysis. Hence, in reinsurance we face incomplete information, due to incurred but not reported (IBNR) and reported but not settled (RBNS) claims (the latter are also frequently called *open case estimates*). This is illustrated in Figure 1.1. The development of claims progresses with calendar time, and when the notification does not arrive before the present evaluation time (e.g., because the incurred value is too low), the data are left truncated (IBNR). If the claim is notified to the reinsurer but not completely settled before the evaluation time, the information is censored (RBNS).

Throughout the development of the book we will make use of the real data examples described in the following sections to illustrate the practical statistical side of implementing reinsurance treaties.

1.3.1 Case Study I: Motor Liability Data

We here present a data set on motor third-party liability (MTPL) data, gathering information about two direct insurance companies operating in the EU, named A and B hereafter. The data come from an observation period between 1991 and 2010, with evaluation date at January 1, 2011. All amounts are corrected in order to reflect costs in calendar year 2011, with inflation and super-inflation taken into account. For every claim, the payments in a given year were aggregated in a single observation. For Company A 16 years and for Company B 20 years of data are available. In the subsequent chapters we will analyze the two data sets separately: the statistical analysis of the losses will show different characteristics for these companies. For Company A, the exact

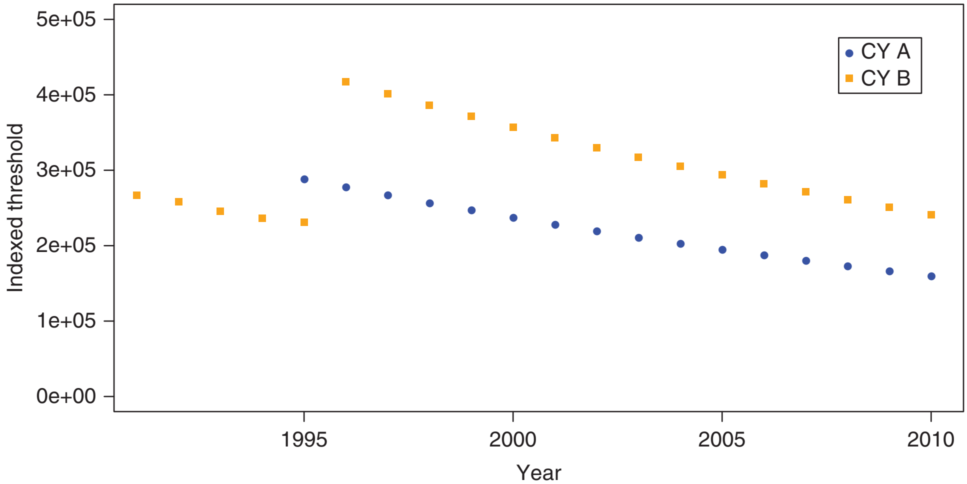

Figure 1.2 Indexed reporting thresholds of Companies A and B.

occurrence dates of the claims are also available, so the analysis of the counting process can be performed more accurately for that claim data set.

Per development year and per claim the *aggregate payment* and *incurred loss* are given. The incurred loss at a given moment in time is the sum of the already paid amount and a reserve for further payments, proposed by company experts at that moment. A claim enters the database from the moment the incurred value exceeds the reporting threshold as given in Figure 1.2. Once a claim has been reported, it stays in the data set even if the associated incurred loss falls below the reporting threshold at some point later. When estimating the loss experienced by the reinsurer, one needs to model the *reporting delay* between the accident time and the year where the claim was first reported to the reinsurer, that is, when the incurred loss I_1 first exceeds the reporting threshold. Indeed, claims that have occurred close to the evaluation time at the end of 2010 can still be IBNR to the reinsurer. In Figure 1.3 the histogram of the reporting delays is given. Given that the accident dates were only reported for Company A, we restrict the plot to this data set. The delay time is then obtained from the difference between the reporting year and accident date, rounded off in years, using the reporting threshold of the particular accident year (see Figure 1.2).

For Company A one has 849 claims of which 340 are completely developed, while the sample size for Company B is 560 of which 225 are fully developed. In Figure 1.4 we show the development of four selected claims. The cumulative payments (aggregated on a yearly basis) are indicated by a full line, while the incurred values are given by dashed lines. When payments and incurred meet, the claim is closed. The characteristics of the four depicted claims are given in Table 1.2.

Note that the information concerning the loss values and development periods is right censored since for the claims which are not fully developed at the end of 2010, the loss as well as the development time at the end of 2010 are only lower bounds for their final value. In Table 1.3 the observed numbers of claims per accident year and per

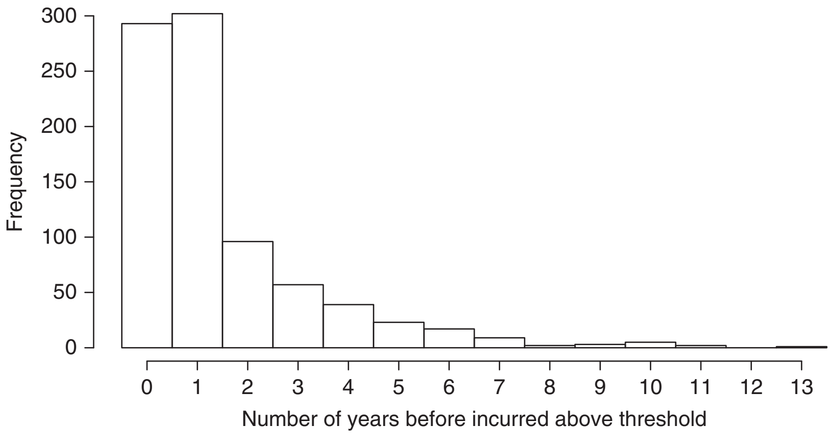

Figure 1.3 Company A: histogram of reporting delays.

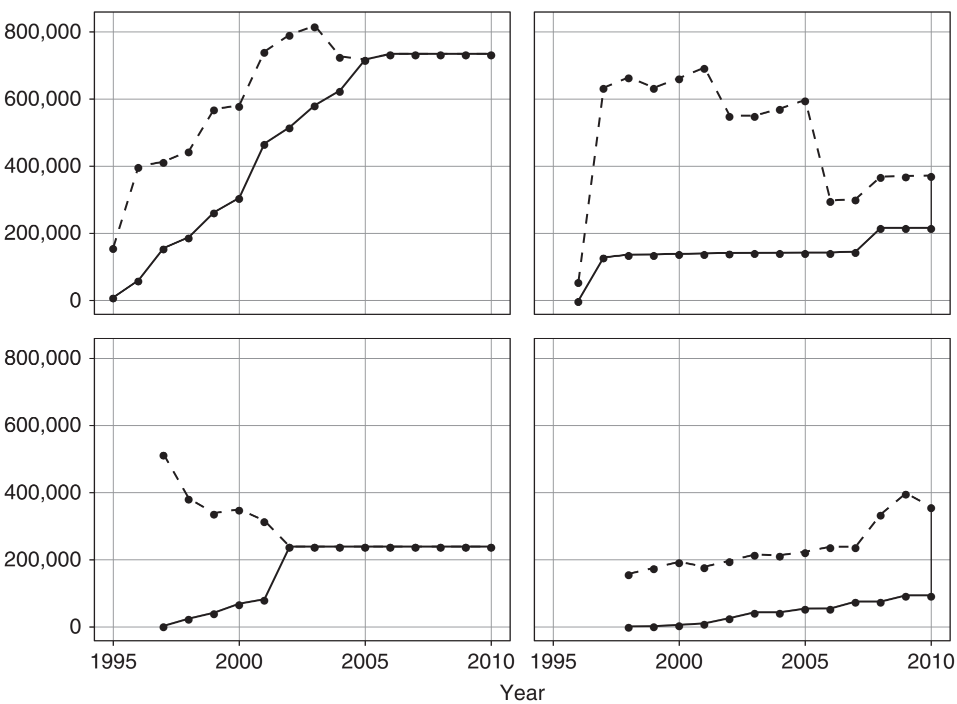

Figure 1.4 MTPL data: development pattern of four particular claims.

development time up to 2010 (in years, DY_{2010}) are given for Company A. Clearly the amount of censoring increases with increasing accident year.

In Figures 1.5 and 1.6, time plots of the incurred loss data of Company A and Company B, respectively, are given as a function of accident year.

Table 1.2 Company A: characteristics of the claims from Figure 1.4

Claim	Reporting year	Closing year	Development time (years)
Top left	1995	2005	10
Top right	1996	-	≥ 15
Bottom left	1997	2002	5
Bottom right	1998	-	≥ 13

Table 1.3 Company A: observed number of claims per accident year and per number of development years in 2010 (DY_{2010})

	DY_{2010}																Nr.		Prop.
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	censored	Total	non-censored
1995	0	0	2	3	4	8	1	1	2	2	2	2	1	1	2	2	11	44	0.75
1996	0	1	0	0	4	1	6	2	3	4	3	4	2	3	0		14	47	0.7
1997	0	1	3	3	1	3	6	6	4	3	2	2	5	1			10	50	0.8
1998	0	0	0	6	3	8	8	7	5	4	4	4	0				21	70	0.7
1999	0	0	0	3	2	4	3	2	1	4	4	3					17	43	0.6
2000	0	1	1	1	4	6	8	6	2	7	2						19	57	0.67
2001	0	0	1	2	5	4	6	4	9	3							23	57	0.6
2002	0	0	1	2	5	1	3	2	1								27	42	0.36
2003	0	2	2	5	7	6	5	1									38	66	0.42
2004	0	0	1	6	8	5	4										41	65	0.37
2005	0	0	1	2	6	1											46	56	0.18
2006	0	0	2	2	2												43	49	0.12
2007	0	0	0	4													65	69	0.06
2008	0	0	0														69	69	0.00
2009	0	0															55	55	0.00
2010	0																10	10	0.00
Censored	10	55	69	65	43	46	41	38	27	23	19	17	21	10	14	11	509		
Total	10	60	83	104	94	93	91	69	54	50	36	32	29	15	16	13		849	0.60

The classical statistical procedure to estimate the distribution of right censored random variables is given by the Kaplan–Meier estimator of the distribution function. This estimator is discussed in more detail in Chapter 4. Note from these plots that about half of the claims are expected to demand a development period of at least 10 years.

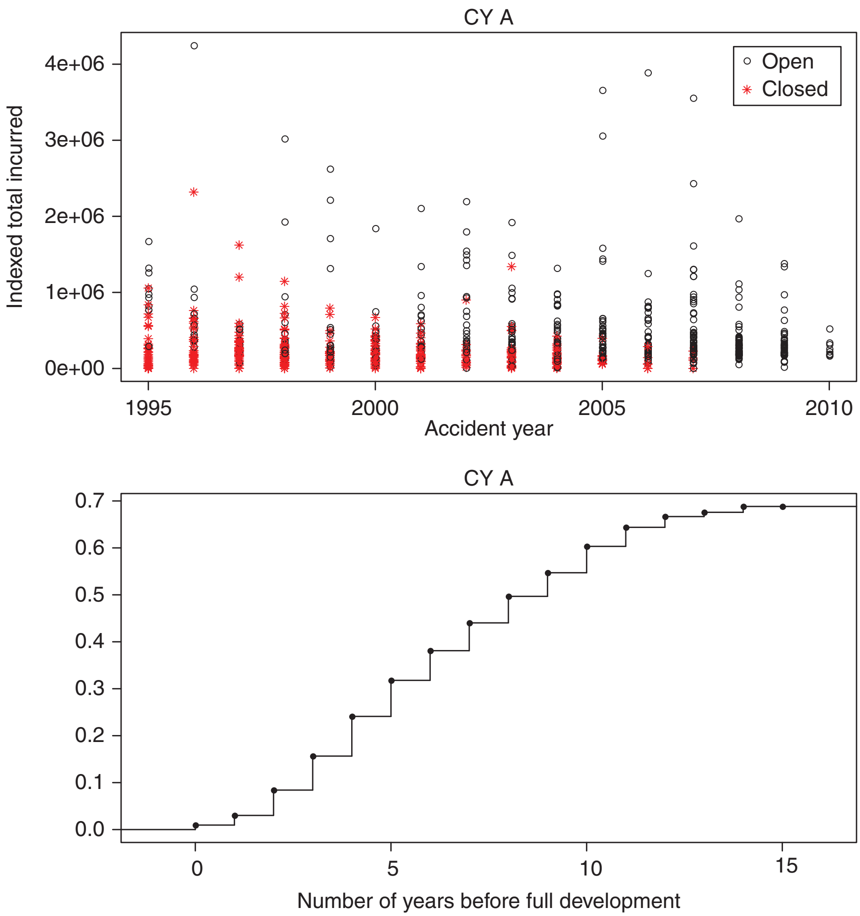

Figure 1.5 Company A: incurred losses (top); Kaplan–Meier estimator for the distribution function of the number of development years (bottom).

Alongside the aggregate payment and the incurred loss, when analysing the risk many companies compute *ultimate loss amounts* for claims that are still under development. These ultimates are statistical estimates of the final loss. The ultimate value of course equals the final aggregate payment in case the claim is closed during the period of study. In practice, the ultimate estimates for non-closed claims are often primarily based on chain ladder development factors based on paid and incurred loss triangles (e.g., see Wüthrich and Merz [797] and Radtke et al. [638]), but then applied on the individual loss data, see also Drieskens et al. [308]. In Figure 1.7 scatterplots of the ultimate against the incurred losses for the data of the two companies are given. Note that the regression fits on these scatterplots for the claims that are still open at the end of the observation period indicate a linear relation between ultimate and incurred values with a negligible intercept: ultimate $= a \times$ incurred, for some $a > 1$.

Finally, in Figure 1.8 we plot the daily cluster sizes for the claims of Company A. Up to three claims per day were observed.

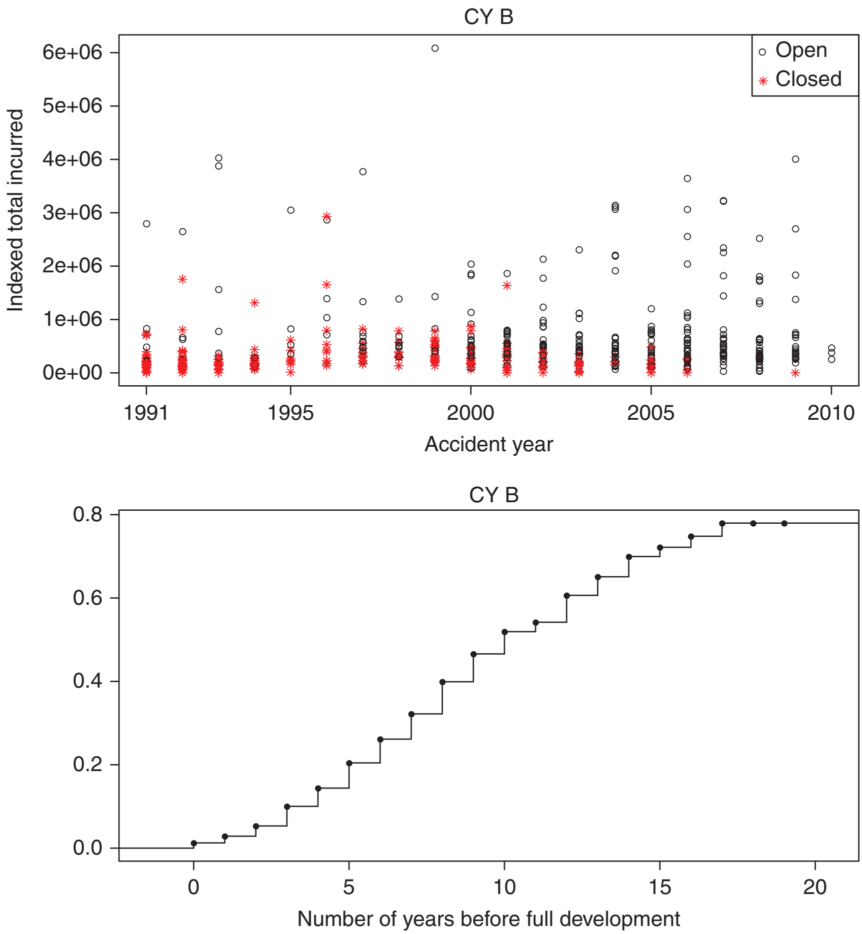

Figure 1.6 Company B: incurred losses (top); Kaplan–Meier estimator for the distribution function of the number of development years (bottom).

1.3.2 Case Study II: Dutch Fire Insurance Data

We will use claim data from the Dutch fire insurance market between 2000 and 2015, provided by a reinsurance company. The date for every fire is known, together with the type of building and regional information. Here the development times are short. Figure 1.9 depicts the logarithm of the claim sizes as a function of time as well as the daily cluster sizes (one sees up to five claims per day). The loss data are indexed to 2015. The reporting threshold equals a value equivalent to 2 million Dutch guilders up to 2002, after which 1 million Euros is used.

1.3.3 Case Study III: Austrian Storm Claim Data

Sometimes individual claim data are not available, and instead claims aggregated over time or regions have to be used. As an illustration, we will use data from historical storm losses of residential buildings in Austria in the period 1998–2009, aggregated over two-digit postcode regions. This data set contains 36 storm events and was provided by the

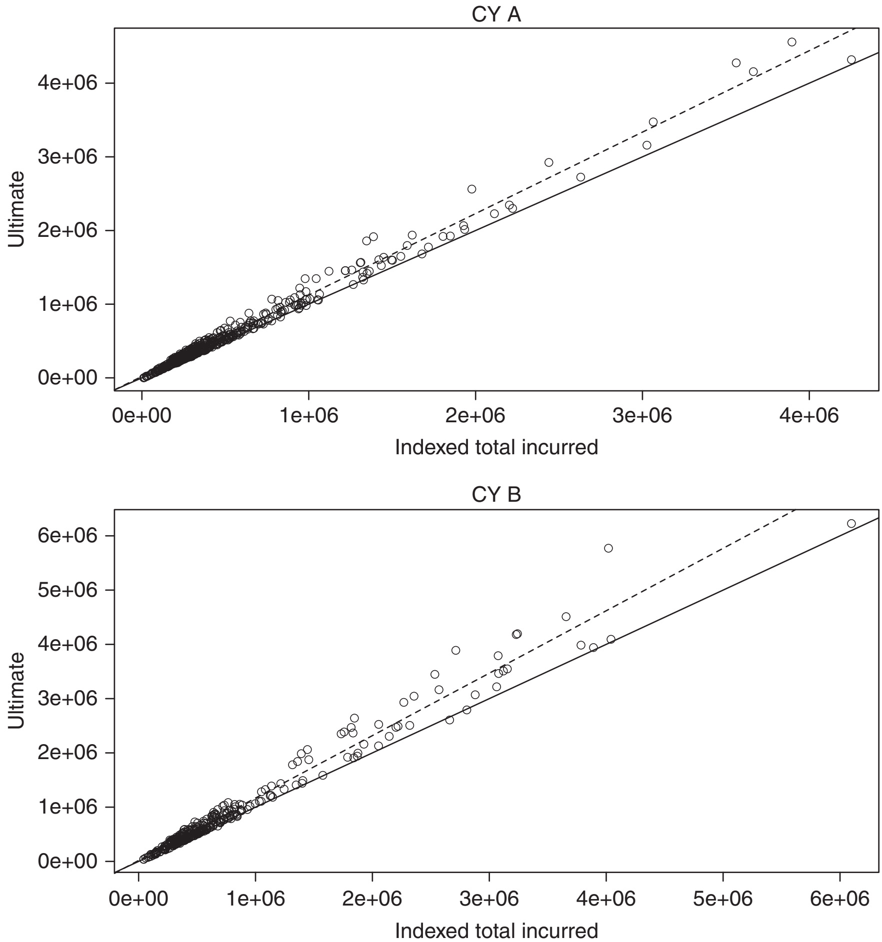

Figure 1.7 Ultimate versus incurred losses with least squares regression fit for the open claims of Company A (top) and Company B (bottom).

Austrian Association of Insurance Companies (VVÖ). The data are indexed according to the building value index and normalized with respect to the overall building stock value in the respective year. Using actual wind fields of each storm on a fine grid, Prettenthaler et al. [633] formulated a building-stock-value-weighted wind index W for each region and storm, and then developed a stochastic model relating wind speed and actual losses (expressed per million of the building stock value). Figure 1.10 depicts the losses of the 36 storms in the data set as a function of this wind index W for Vienna and the province of Upper Austria. Here one studies the distribution of the loss data as a function of W in a regression setting.

1.3.4 Case Study IV: European Flood Risk Data

Floods rank amongst the most wide-reaching natural hazards. Losses from floods show an increasing trend which (to a considerable extent) is attributable to socio-economic factors, including population growth, economic development and construction

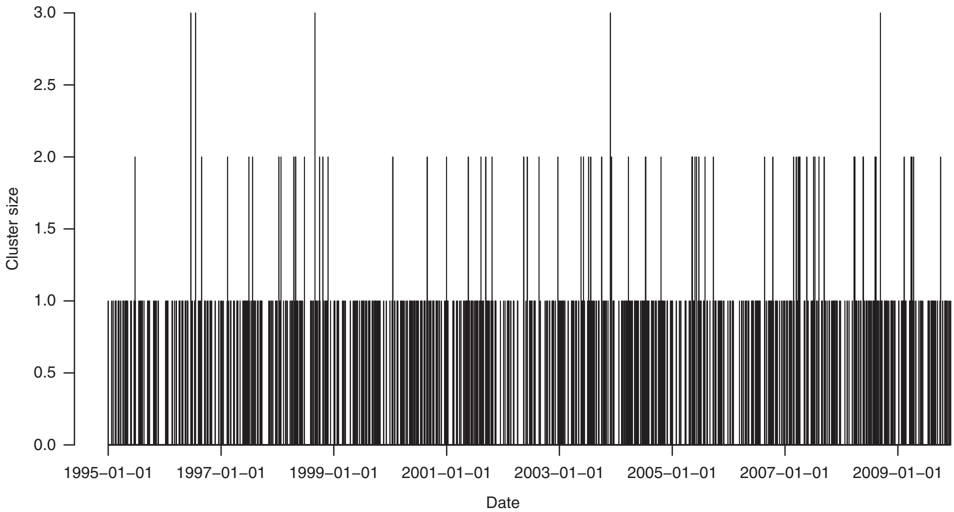

Figure 1.8 Company A: observed cluster sizes of the claim number process.

activities in vulnerable areas. In Prettenthaler et al. [632] (indexed) flood loss data across Europe (provided by Munich Re NatCatSERVICE, 2014) were transformed into losses expressed as a percentage of building stock value, and then used to determine loss quantiles as required for flood risk management. Figure 1.11 depicts the respective aggregate annual losses for the period 1980–2013 for Germany and the UK.

1.3.5 Case Study V: Groningen Earthquakes

Next to loss amount data, reinsurers also need to analyze the physical phenomena causing damage. A classical example is earthquake risk. We discuss the Groningen earthquakes caused by gas extraction. The Groningen field is the largest gas field of Western Europe, with 2800 billion cubic metres available and 800 billion cubic metres left. The pressure inside the gas layers decreases due to the extraction, and the layers on top collapse. This collapse does not happen homogeneously, which causes the earthquakes. Hundreds of earthquakes have been detected since 1986 with magnitudes between 2 and 3 on the Richter scale, and 14 larger than 3 (Figure 1.12). The damage to houses and public buildings was substantial, with many buildings needing reinforcements. The largest observed magnitude was 3.6 (Huizinge, August 2012). In this context, the estimation of the maximum possible magnitude is the main goal. Depending on the research team, maximum magnitudes between 3.9 and 5 were predicted, see for instance Bourne et al. [157].

1.3.6 Case Study VI: Danish Fire Insurance Data

It is quite common to combine reinsurance forms across various lines of business (LoB), so modelling the dependence of the different LoB is important. To illustrate the appropriate multivariate models and statistical methodology, we will use the Danish fire insurance data set containing information on 2167 fire losses over the period

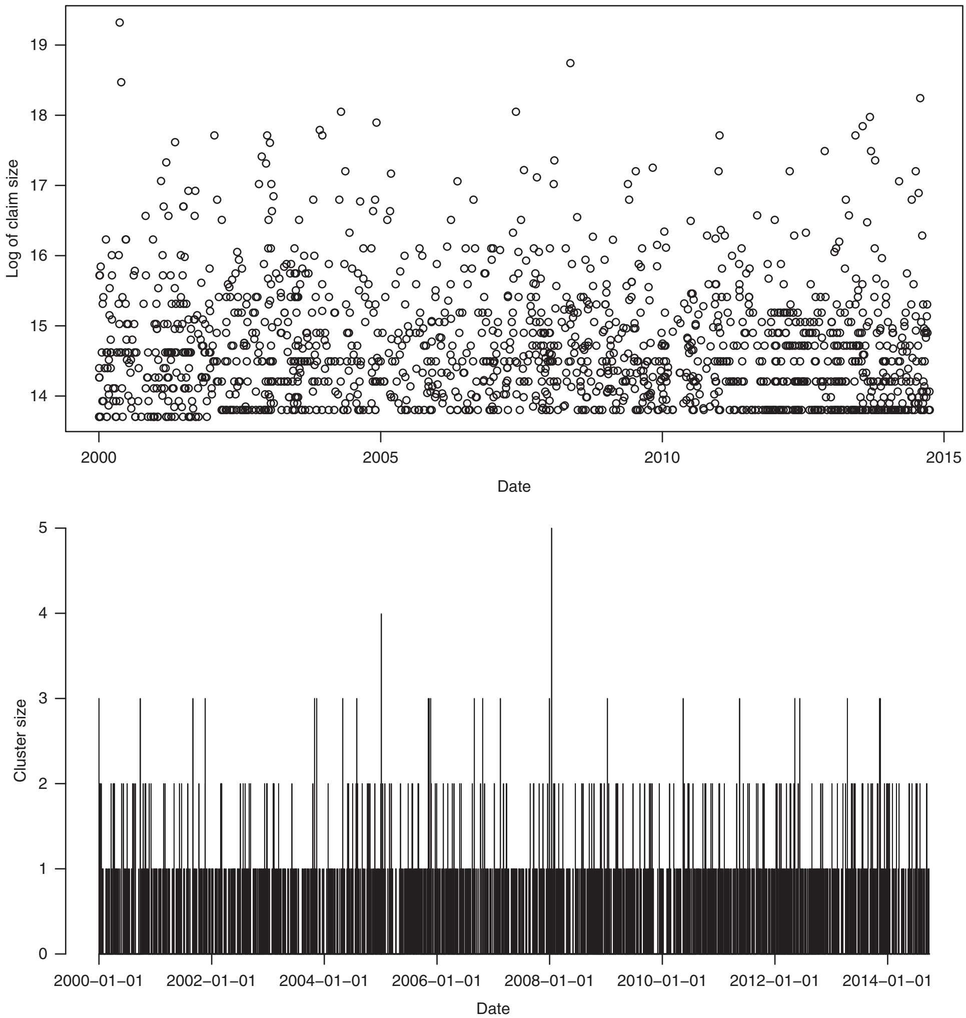

Figure 1.9 Dutch fire insurance claims: log-claims as a function of time for Dutch fire insurance (top); observed cluster sizes of the claim number process (bottom).

1980–1990. The data have been adjusted for inflation to reflect 1985 values and are expressed in millions of Danish kroner. The total loss amount X_i of the ith claim is subdivided into damage to building ($X_{i,1}$), damage to content ($X_{i,2}$) (e.g., furniture and personal property) and loss of profits ($X_{i,3}$). A claim is only registered if the total loss exceeds 1 million kroner, that is, $X_{i,1} + X_{i,2} + X_{i,3} \geq 1$. This data set was collected at the Copenhagen Reinsurance Company and can nowadays be seen as a folklore example as it has been studied extensively over the years in the academic literature (e.g., see Embrechts et al. [329]). In Figure 1.13 a scatterplot matrix is given for the log-transformed data. On the diagonal, histograms of the logarithm of the marginal losses are given. Note that several claims exhibit losses in only one or two of the components (for only 517 claims there is a loss in each of the three components).

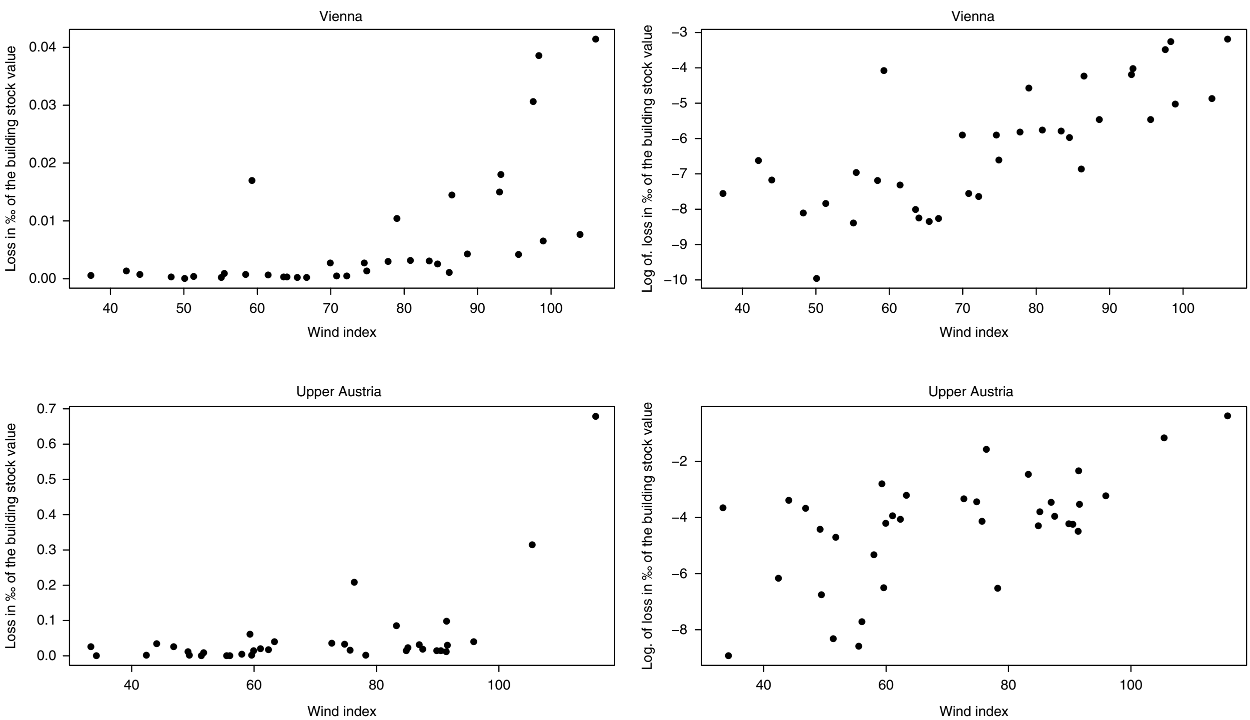

Figure 1.10 Normalized loss data against wind index W for Vienna (top) and Upper Austria (bottom); original scale (left) and log-scale (right).

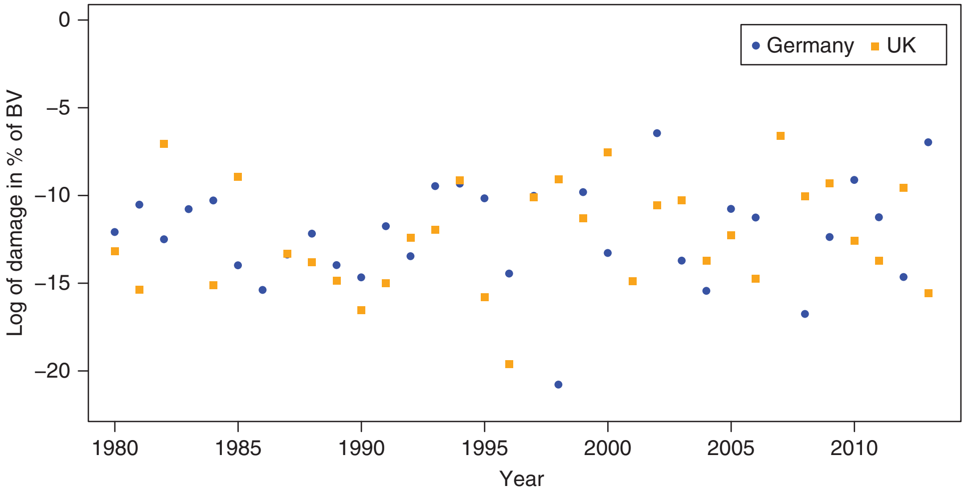

Figure 1.11 Flood risk: aggregate annual losses (in log scale) by percentage of building value for Germany and the UK.

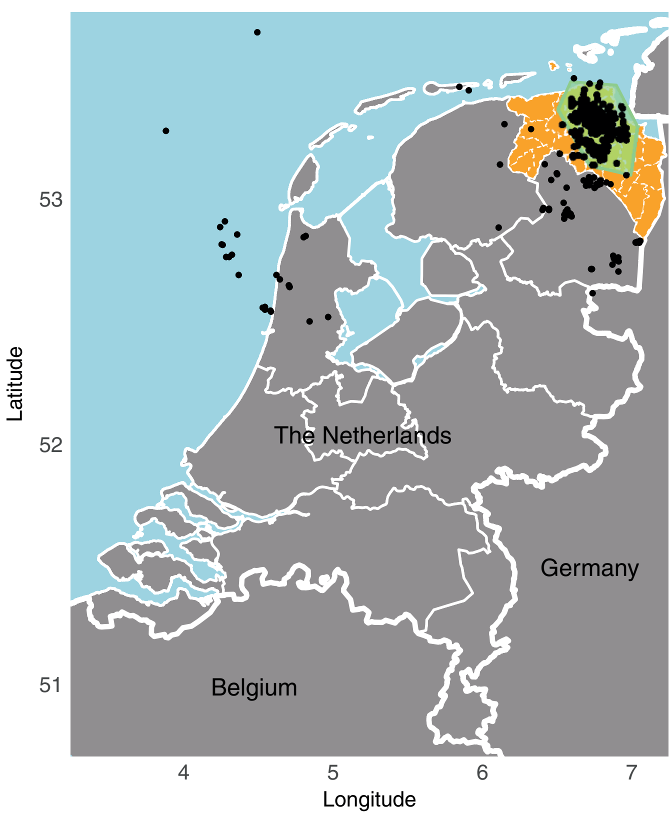

Figure 1.12 Induced (dark points) earthquakes in the northern part of the Netherlands with magnitudes larger than 1.5.

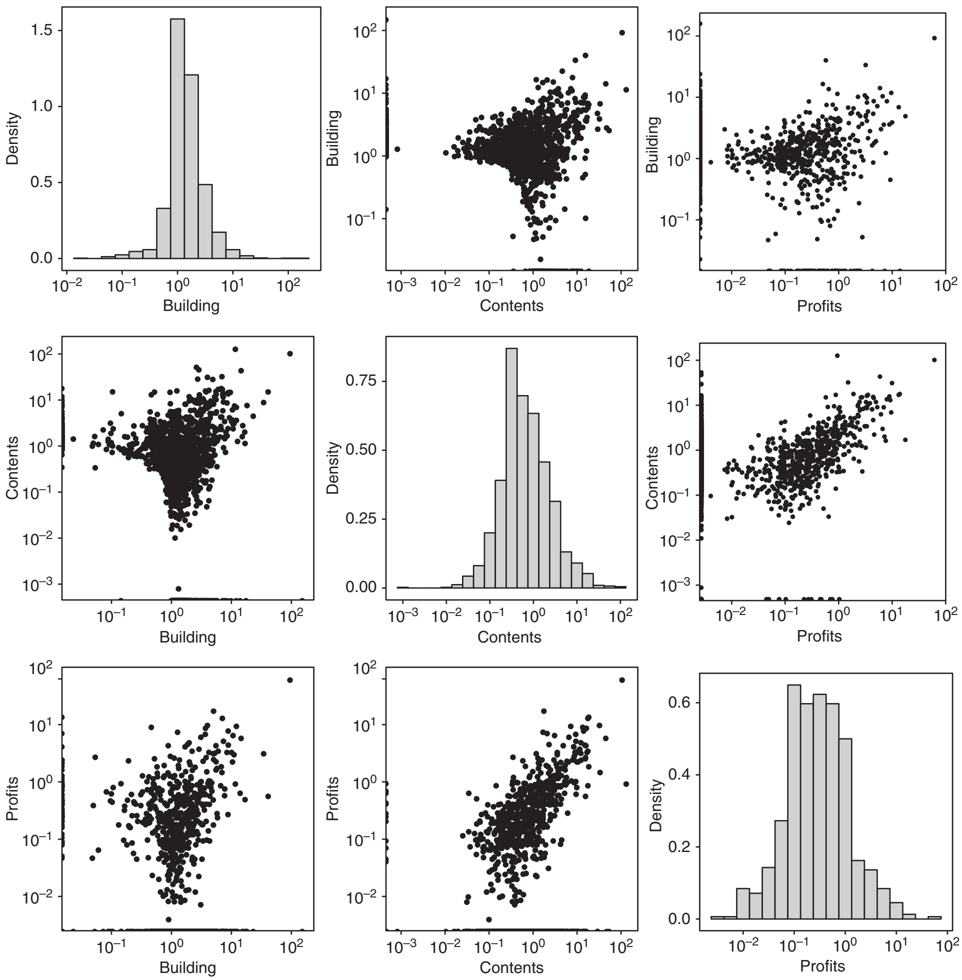

Figure 1.13 Danish fire insurance data: scatterplot matrix on the log-scale.

The occurrence dates are also given and hence simultaneous occurrences of claims for the three components can be observed. Figure 1.14 illustrates the occurrences and the cluster sizes when all portfolio components were affected.

1.4 Notes and Bibliography

There are a number of classical textbooks available which provide a general introduction to reinsurance, for example Carter [185], Gerathewohl [382], Grossmann [409], Strain [710], Gastel [375], Schwepcke [686], and Walhin [765]. A number of articles in Teugels and Sundt [741] also deal with the topic. For the role of reinsurance in risk management, see D'Outreville [598]. More recent and shorter overviews can be found in Liebwein [543], Albrecher [13], Outreville [599], Bernard [120], and

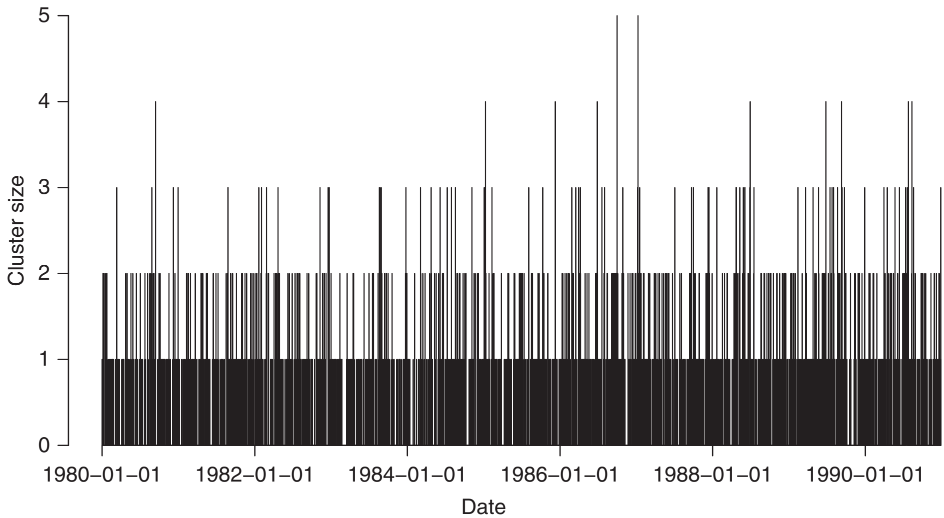

Figure 1.14 Danish fire insurance data: cluster arrivals, selecting the claim dates where each component is addressed.

Deelstra et al. [267]. Furthermore, a number of basic textbooks on risk theory contain sections on reinsurance. Examples include (in alphabetic order) Beard et al. [93], Beekman [94], Borch [149], Bowers et al. [157], Bühlmann [165], Cramér [234], Daykin et al. [254], De Vylder [263], Gerber [383], Goovaerts et al. [402], Heilmann [435], Kaas et al. [475], Klugman et al. [491], Lundberg [551], Mack [555], Rolski et al. [652], Schmidt [677], Seal [690], Straub [711], and Sundt [716]. For a discussion on the challenges and opportunities of reinsurance as an international business, see Göbel [392]. A recent overview from a practical perspective can be found in Swiss Re [725]. The increasing role of the notion of capital and capital management in running insurance and reinsurance companies, which can be seen as an ongoing change of paradigm in the insurance industry, is highlighted in Dacorogna [243], see also [689] and Krvavych [510, 512].

The number of 200 reinsurance companies can be compared with the more than 10,000 primary insurance companies in the market today (using economic arguments, Powers and Shubik [627] in fact claim that the "optimal" number of reinsurers in the market is connected to the number of primary insurers by a square-root rule).

Historically, the first documented reinsurance contract dates back to 1370, when the cargo of a ship sailing from Genoa to Sluis (near Bruges in Flanders) was reinsured by the direct insurer for the more dangerous part of the journey from Cadiz to Sluis (interestingly, the contract did not state the premium, which most likely was done to avoid usury discussions). The first reinsurance company was founded much later, in 1846, in Cologne after the big fire of Hamburg in 1842, and the first retrocession contract seems to date back to 1854, involving Le Globe Compagnie d'Assurance contre L'incendie. Soon the (nowadays) major European reinsurance companies were founded, and the American Life Reinsurers followed in the early 20th century. For a detailed account of the history of reinsurance, see Kopf [496], Holland [451], and Borscheid et al. [154].

2

Reinsurance Forms and their Properties

Let $\{X_i; i \in \mathbb{N}\}$ be random variables denoting the claim sizes that the first-line insurer experiences and let $\{N(t); t \geq 0\}$ be a counting process, where $N(t)$ represents the number of claims up to time $t > 0$ (measured in years). Then the *total* or *aggregate claim amount* at time t for the first-line insurer is given by

$$S(t) = \begin{cases} \sum_{i=1}^{N(t)} X_i & \text{if } N(t) > 0, \\ 0 & \text{if } N(t) = 0. \end{cases}$$

Recall that most (non-life) contracts are written for the duration of one year, so the static random variable $S(1)$ will be of prime interest in many applications.

In a reinsurance contract, this aggregate claim size is now sub-divided into

$$S(t) = D(t) + R(t),$$

where $D(t)$ is the deductible (retained) amount that stays with the first-line insurer after reinsurance and $R(t)$ is the amount paid by the reinsurer. For many reinsurance contracts the splitting will be defined on the individual risks X_i, and in this case we write $X_i = D_i + R_i$ (or just $X = D + R$ for short, in case they all follow the same distribution).

We will now discuss the most common obligatory reinsurance forms and their properties. We start with proportional (also called *pro-rata*) treaties.

2.1 Quota-share Reinsurance

The simplest possible reinsurance form is *quota-share* (QS) *reinsurance*, which is a fully proportional sharing of the risk, that is,

$$R = a \cdot X \text{ and } R(t) = a \cdot S(t)$$

for a *proportionality factor* $0 < a < 1$.

This form of reinsurance is popular in almost all insurance branches, particularly due to its conceptual and administrative simplicity. In general the first-line insurer will also

Reinsurance: Actuarial and Statistical Aspects, First Edition.
Hansjörg Albrecher, Jan Beirlant and Jozef L. Teugels.

cede to the reinsurer a similarly determined proportion of the premiums (see Chapter 7 for details). If the distribution of X is available, one immediately has

$$\mathbb{P}(R \leq x) = F_X\left(\frac{x}{a}\right), \quad \mathbb{P}(D \leq x) = F_X\left(\frac{x}{1-a}\right)$$

expressed in terms of the cumulative distribution function (c.d.f.) $F_X(x) = \mathbb{P}(X \leq x)$. For the aggregate risk, correspondingly

$$\mathbb{P}(R(t) \leq x) = \mathbb{P}(S(t) \leq x/a), \quad \mathbb{P}(D(t) \leq x) = \mathbb{P}(S(t) \leq x/(1-a))$$

and for the moment-generating function

$$\mathbb{E}(e^{sR(t)}) = \mathbb{E}(e^{(as)S(t)}), \quad \mathbb{E}(e^{sD(t)}) = \mathbb{E}(e^{((1-a)s)S(t)}), \tag{2.1.1}$$

so one only needs to evaluate the moment-generating function of $S(t)$ at a different argument. As a result, the rth moments ($r \in \mathbb{N}$) are given by

$$\mathbb{E}(R^r) = a^r\, \mathbb{E}(X^r), \quad \mathbb{E}(D^r) = (1-a)^r \mathbb{E}(X^r).$$

Note that both the coefficient of variation and the skewness coefficient ν do not change under a QS treaty:

$$\mathrm{CoV}(R(t)) = \frac{\sqrt{\mathrm{Var}\,(R(t))}}{\mathbb{E}(R(t))} = \mathrm{CoV}(D(t)) = \mathrm{CoV}(S(t)),$$

$$\nu_{R(t)} = \mathbb{E}\left(\frac{R(t) - \mathbb{E}(R(t))}{\sqrt{\mathrm{Var}\,(R(t))}}\right)^3 = \nu_{D(t)} = \nu_{S(t)}.$$

2.1.1 Some Practical Considerations

QS reinsurance can be understood as (virtually) increasing the available solvency capital. To see that in a simple example, consider the probability

$$\mathbb{P}(v + P(t) - S(t) > 0)$$

that at some time $t > 0$ the capital v together with the received premiums $P(t)$ suffices to cover the claims $S(t)$. Then, after entering a QS treaty and assuming that premiums are shared with the same proportion, this probability changes to

$$\mathbb{P}(v + (1-a)P(t) - (1-a)S(t) > 0) = \mathbb{P}\left(\frac{v}{1-a} + P(t) - S(t) > 0\right).$$

In practice, a further positive effect of QS reinsurance is to improve the premium-to-surplus ratio: according to statutory accounting principles implied by the regulator, an insurer typically has to immediately include in the balance sheet all the expenses connected to issuing a policy, but the respective premium can only be entered gradually over the duration of the policy; the correspondingly needed *unearned premium reserve* considerably reduces the surplus and a QS arrangement will improve this situation, as it reduces that reserve and the expenses simultaneously (see, for example, [585]). QS contracts are often used at the initiation of smaller companies to broaden their chances for underwriting policies and to gain experience in a new market with a limited amount of risk. For reinsurers, in turn, a QS arrangement can also have the advantage of gaining claim experience in that particular market, which may be useful in other related portfolios.

QS arrangements are easy to combine, that is, an insurer can have simultaneous QS contracts on the same portfolio with different reinsurers. Also, due to the proportional share that is left with the insurer, the risk of some forms of *moral hazard* (like sloppy claim settlement procedures) is avoided.

One of the main shortcomings of QS reinsurance is that, due to its form, *all* claims are partly reinsured, not just the largest of them. This is often not ideal, as claims from small policies could have easily been borne by the insurer alone (and passing on those parts of the portfolio is a non-attractive loss of insurance business).

2.2 Surplus Reinsurance

A reinsurance form that improves on the disadvantages of QS treaties, but keeps its main advantages, is *surplus reinsurance*, which is a proportional reinsurance form for which the proportionality factor depends on the coverage limit in the underlying policy (sum insured). Let Q_i be the sum insured (policy limit) of claim X_i. For a fixed *retention line* M the reinsured amount is then given by

$$R_i = \left(1 - \frac{M}{Q_i}\right) X_i \cdot 1_{\{Q_i > M\}}, \quad D_i = X_i\, 1_{\{Q_i \le M\}} + M \frac{X_i}{Q_i} 1_{\{Q_i > M\}}, \tag{2.2.2}$$

where $1_{\{A\}}$ denotes the indicator function of event A. Altogether,

$$R(t) = \sum_{i=1}^{N(t)} R_i, \quad D(t) = \sum_{i=1}^{N(t)} D_i.$$

The ratio $V_i := X_i/Q_i$ is called the *loss degree* of claim X_i. With a surplus reinsurance each claim with an insured sum below M is fully kept by the insurer, and otherwise the relative participation of the reinsurer in the claim payment is larger the larger the underlying sum insured is (see Figure 2.1). Consequently, this reinsurance form retains the advantages of the proportionality for each claim payment, but only reinsures claims from larger policies. Due to the proportionality feature, the determination of premiums is again rather simple. In some cases Q_i is alternatively the probable maximum loss (PML) of claim X_i (see Chapter 7). From the definition, it becomes clear that the maximum retained size of each claim is M ("the line"). The surplus reinsurance contract homogenizes the portfolio of the first-line insurer, as illustrated in the following simple example.

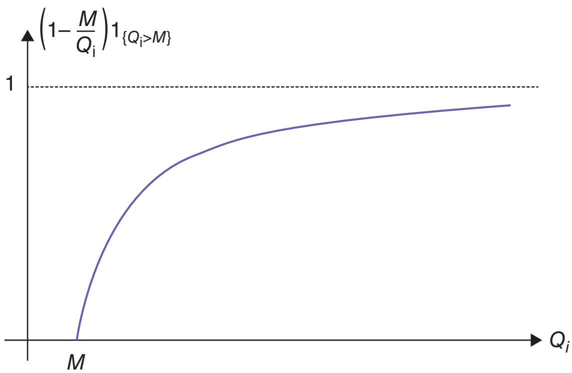

Figure 2.1 Proportionality factor of the reinsurer as a function of insured sum.

Example 2.1 Assume there are 100 independent policies in an insurance portfolio. For each policy, a claim occurs with probability 0.01 within the next year. For 70 policies, the claim size is Q_1 and for 30 policies the claim size is $Q_2 > Q_1$, given a claim occurs (i.e., for simplicity here the claim size is always equal to the policy limit). Then the insurer expects one claim during this year and an aggregate claim payment of $E(S(1)) = 0.7Q_1 + 0.3Q_2$. If the insurer charges this amount as the overall premium, this will be sufficient to cover this one expected claim if it is one with insured sum Q_1, but not if it comes from a policy with insured sum Q_2. However, if the insurer buys surplus reinsurance with $M = Q_1$ for a pure premium of $\mathbb{E}(R(1)) = 0.3(Q_2 - Q_1)$, then the remaining amount $\mathbb{E}(D(1)) = Q_1$ is sufficient to cover the retained amount of that expected claim, no matter from which type of policy it comes.

In order to determine the distributional properties of the retained and reinsured amount under surplus reinsurance, it is helpful to consider the insured amount of a claim as a random variable with (c.d.f) F_Q (based on frequencies of the sums insured specified in the policies of the portfolio and the respective claim occurrence probabilities, for example in Example 2.1 Q would have a two-point distribution with $\mathbb{P}(Q = Q_1) = 1 - \mathbb{P}(Q = Q_2) = 0.7$).[1] The distributions of the quantities R and D are then given by

$$\mathbb{P}(D \le x) = \int_0^\infty \mathbb{P}\left(X \le x \max\{1, y/M\} | Q = y\right) \, dF_Q(y), \tag{2.2.3}$$

$$\mathbb{P}(R \le x) = \int_M^\infty \mathbb{P}\left(X \le \frac{x}{1 - \frac{M}{y}} \middle| Q = y\right) \, dF_Q(y) \, . \tag{2.2.4}$$

1 Such an approach is akin to the philosophy of the collective risk model, where a heterogeneous portfolio is treated as a homogeneous one, but equipped with a mixture distribution for the claim size to take into account the heterogeneity.

For the moments of D, we have

$$\begin{aligned}
\mathbb{E}(D^r) &= \int_0^\infty \mathbb{E}(D^r|Q=y)\,dF_Q(y) \\
&= \int_0^M \mathbb{E}(X^r|Q=y)\,dF_Q(y) + \int_M^\infty M^r\mathbb{E}\left(\left(\frac{X}{Q}\right)^r |Q=y\right)\,dF_Q(y) \\
&= \int_0^\infty \min\{y^r, M^r\}\mathbb{E}(V^r|Q=y)\,dF_Q(y). \qquad (2.2.5)
\end{aligned}$$

In practice it may often be reasonable to assume that the loss degree is independent of the sum insured (particularly if the sums insured do not vary too much across policies). In that case, (2.2.5) simplifies to

$$\mathbb{E}(D^r) = \mathbb{E}(V^r)\int_0^\infty \min\{y^r, M^r\}\,dF_Q(y).$$

For the reinsured amount, the respective expression is slightly more involved, but for the first moment one easily gets

$$\mathbb{E}(R) = \int_0^\infty \max\{y-M, 0\}\mathbb{E}(V|Q=y)\,dF_Q(y)$$

and under independence of V and Q

$$\mathbb{E}(R) = E(V)\int_0^\infty \max\{y-M, 0\}\,dF_Q(y).$$

Surplus reinsurance is very popular, particularly in fire insurance, as well as property, accident, engineering and marine insurance. Typically, there is an upper limit $Q_i \le (k+1)M$ ("k lines") in the treaty, that is, the ceded share is capped by

$$R_i = \min\left\{1 - \frac{M}{Q_i}, 1 - \frac{1}{k+1}\right\} X_i \cdot 1_{\{Q_i > M\}},$$

and the remaining part for the policies with larger sums insured is then negotiated on a facultative basis. Also, for certain policies the insurer may decide to retain several, say $m < k$, lines and only reinsure the remaining $k - m$ lines (e.g., see [585]). In general, it is not uncommon to apply a *table of lines*, that is, different retention lines to various groups of similar risks. The retention line is then often chosen in a way to aim for the same maximum loss (*method of inverse claim probability*) or average loss (*method of inverse rate*) for each policy (cf. [526]).