THE MONEY FORMULA

DODGY FINANCE PSEUDO SCIENCE AND HOW MATHEMATICIANS TOOK OVER THE MARKETS

PAUL WILMOTT

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WILEY

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To Oscar, Zachary, Genevieve, and Horatio
—Paul Wilmott

To Wendy and Katherine
—David Orrell

Contents

Acknowledgements ix

About the Authors xi

Introduction xiii

Chapter 1 **Early Models** 1

Chapter 2 **Going Random** 15

Chapter 3 **Risk Management** 35

Chapter 4 **Market Makers** 61

Chapter 5 **Deriving Derivatives** 83

Chapter 6 **What Quants Do** 109

Chapter 7 **The Rewrite** 135

Chapter 8 **No Laws, Only Toys** 149

Chapter 9 **How to Abuse the System** 169

Chapter 10 **Systemic Threat** 189

Epilogue: Keep it Simple 207

Bibliography 227

Index 237

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About the Authors

Paul Wilmott is a mathematician and serial entrepreneur. His textbooks and educational programs provide the definitive training for quants; his website wilmott.com is the center of the quant community; his eponymous bi-monthly – and according to *Esquire* the world's most expensive – magazine is a quant must-have. As a practitioner he has been a consultant to leading financial institutions and managed his own hedge fund. As a commentator he has appeared on many TV and radio programs and written OpEds for the *New York Times*. Nassim Nicholas Taleb calls him the smartest quant in the world: "He's the only one who truly understands what's going on… the only quant who uses his own head and has any sense of ethics." Paul divides his time between London, the Cotswolds, and New York.

David Orrell is an applied mathematician and writer. Founder of the scientific consultancy Systems Forecasting, his scientific work has encompassed diverse areas such as particle accelerator design, weather prediction, cancer biology, and economics. His books on subjects including prediction, economics, and science have been national bestsellers and have been translated into over ten languages. A revised and expanded version of his book *Economyths: ~~10~~ 11 Ways That Economics Gets it Wrong* is also published this year. He lives in Toronto.

Introduction

"How about the scandalous stories of thousands of families with small and medium investments who have been ruined because of the greed of financial institutions in the United States and Europe. Look at the evictions, ruined families, and suicide attempts caused by the financial crisis of those who have failed to control the capital markets or the prices of raw materials. ¡Vaya mierda! "[1]

—Response to the survey question: "Do you have any outrageous or hilarious stories that you think ought to be in Paul and David's new book? Share some details, please!" at wilmott.com

"The truth about their motivation in writing."

—Response to the survey question: "What topics should definitely feature in the book?" at wilmott.com

The global financial crisis that peaked in late 2008, and whose aftershocks have yet to fully dissipate, was the culmination of many years of dubious financial practices. If carried out alone they might have caused only localized harm, but they became aligned in the way that only the most dramatic of astrologers can dream of: a quadrillion dollars in complex financial products that no one understands; risk-management techniques that hide risk rather than decrease it; moral hazard and dangerous incentives; lack of diversification; regulators that are oblivious; mathematicians acting as psychological enablers. It was a story where the naïve, the negligent, and the downright nasty all pulled together in seizing as much as possible for themselves while almost destroying the financial foundations of the planet.

[1] We're translating from the Spanish. We think that "*¡Vaya mierda!*" is slang for "Have a great day!" but we're not sure.

Of course, things have moved on since then. The banking system has become even more concentrated. Global debt – the engine fuel of finance – has grown to unprecedented levels. Markets, in which activity is increasingly dominated by high-frequency-trading robots, experience constant "flash" events where prices suddenly go wild before returning to more normal levels. The world financial system is once again rattling at its cage, ready to blow. And quantitative finance – the use of mathematical models to assist or dictate investment decisions – has become more powerful and influential than ever.

The story, in other words, isn't over – not by a long shot. Indeed, the stakes have never been higher, which is why previously arcane topics such as hedge funds, high-frequency trading, and too-big-to-fail banks have become a major topic of often-confusing debate for everyone from TV pundits to politicians. And why the confusion is often deliberate.

It has been estimated that in 2010 the notional value of all the financial derivatives in existence was $1.2 quadrillion.[2] That's $1,200,000,000,000,000. For comparison, it's about 17 times the market capitalization of all the world's stock markets, or 150 times the value of the above-ground gold supply, or $170,000 for every living human on the planet. Actually, it's larger than the entire global economy. We'll explain this number, and how it could be interpreted, later. For the moment, let's just say that whatever it means in terms of risk, it seems like a dangerously big number for what is, let's be honest, just a service industry.

This book is not about the fallout from the crisis – plenty of books and column inches have been written about that – but about helping to prevent the next one (which won't look like the last one). To do that, it is necessary to go into the engine room of this massive shadow economy and understand how quantitative analysis works. How do you create a quadrillion dollars out of nothing, blow it away,

[2] This was estimated by the economist Tim Harford and Paul for the BBC Radio 4 program *More or Less* based on data from the website of the Bank for International Settlements. This "headline" figure, which is open to interpretation, includes both the contracts traded through an exchange and the over-the-counter market in which two parties trade directly. It is also what is called the "notional" value. If a contract specifies that it will pay you 1% of $1 million in a year's time then that would be recorded as a notional of $1 million, whereas it's really just worth about $10,000. So it's tricky to say what amount really is at risk in that $1.2 quadrillion.

and leave a hole so large that even years of the deliberately misnamed "quantitative easing" can't fill it – and then go back to doing the same thing, only faster? Part of a quant's job, as we'll see, is science, and another part (the one where mathematics is used to obfuscate reality) is the opposite of science. We will discuss both, starting with the science.

The book is divided into two main parts. The first five chapters dip into the history of quantitative finance and explain its key principles, such as risk analysis, bond pricing, portfolio insurance – all those gold-standard techniques, in short, which completely failed during the crisis, but have yet to be properly reinvented. We explore the elegant equations used in financial mathematics, and show how the deadly allure of their ice-cold beauty has misled generations of economists and investors. We trace the development of financial derivatives from bonds to credit default swaps, and show how mathematical formulas helped not just to price them, but also to greatly expand their use to the point where they dwarfed the real economy. And we show how risk-management and insurance schemes have led to more risk and less insurance than arguably at any time in history.

The second part is about the quantitative finance industry today, and how it is evolving. We will show what quants do, the techniques they use, and how they continue to put the financial system at peril. Part of the problem, we'll see, is that quants treat the economy as if it obeys mechanistic Newtonian laws, and – by nature and by training – have no feel for the chaos, irrationality, and violent disequilibrium to which markets often seem prone. The same can also be said of the regulators watching the system. We'll lower ourselves into the hidden caves of finance, with their "dark pools" navigated by swarms of high-frequency traders, and show how new ideas from areas such as complexity science and machine learning are providing analytic tools for visualizing and understanding the turbulent eddies of financial flows. Along the way, we will grapple with some of the philosophical and practical difficulties in modeling the financial system – and show how models are often used less for predicting the future than for telling a story about the present.

The authors are both Oxford-trained applied mathematicians, who have worked in a variety of industries but otherwise come to this project from different angles. Paul is a quintessential insider – named "arguably the most influential quant today" by *Newsweek* – but he is

also (as visitors to quant forum wilmott.com will know) a longstanding critic of standard practices. David works primarily in the areas of mathematical forecasting and computational biology (he invented a program called "Virtual Tumour," which gives you an idea). He has argued in a number of books that economics needs to take a similarly biological approach – and that our out-of-control financial sector is in serious need of a health check.

The Money Formula provides new insights into one of the largest, best-paid, but least-understood industries in the world – and the one with the most capacity to either help our future economic development or give it the financial equivalent of a cardiac arrest.

We begin by turning to the early 18th century, when France was seeking financial advice from a mathematician.

CHAPTER 1

Early Models

"Nature, and Nature's Laws lay hid in Night.
God said, Let Newton be! And All was Light."

—Alexander Pope

"Beelzebub begat Law
Law begat the Mississippi
The Mississippi begat the System (etc.)"

—Het Groote Tafereel der Dwaasheid
(The Great Mirror of Folly)

The mathematical models used by quants are based on ideas and concepts developed by generations of economists. They in turn were heavily influenced by physics. But is it really possible to model the markets as a kind of physical system, or is quantitative finance more like a set of mathematical tricks for betting on markets? This chapter traces the development of economics; looks at the basic assumptions such as equilibrium and rationality that have shaped both economics and finance; and considers the dual nature of quantitative finance, as exemplified by two men – John Law and Isaac Newton.

In 1705, Scotland was contemplating union with its neighbor England. The English economy was riding high, and Scotland's leaders thought this might be an opportune moment for a merger. However, not everyone thought hooking up was a good idea. One

person who argued against it was the banker, gambler, and social climber John Law. He went so far as to propose an entirely new monetary system for Scotland, which he claimed would go beyond the English system and in a stroke solve his country's monetary problems while boosting trade.

Part of England's success was due to its newly created central bank, the Bank of England, and efficiencies created by the introduction of bank notes. However, Law thought he could do better. According to him, the problem with this new English paper money was not that it was too radical, but that it was not radical enough, since it was still exchangeable for gold. Its supply was therefore determined not by the needs of the economy, but by the quantity of precious metal that happened to be in circulation at the time. In his text *Money and Trade Consider'd with a Proposal for Supplying the Nation with Money*, he argued that Scotland needed a central bank of its own, that issued its own paper currency, but one that was backed only by the state rather than by precious metal. After all, according to this son of an Edinburgh goldsmith, money was just a "Sign of Transmission," like a casino chip, and not a store of real wealth.

The stakes for Law were greater even than the questions of Scottish independence or the meaning of monetary value. Ten years earlier, he had been charged with murder following a duel in London. After being imprisoned, he soon escaped and fled to Amsterdam. For several years he had toured around Europe, supporting himself and his young family by gambling (a trained mathematician, he claimed to have a system), before returning to Scotland. But if that country joined with England, he would have to leave or find himself back in jail.

This time, the dice did not fall in Law's favor. His radical monetary proposal was rejected by parliament, the union with England went ahead, and Law was again on the run from the law.

He set himself up in Paris, playing cards at all the fashionable salons. His system was extremely successful – so much so that he drew the attention of the Chief of Police, M. d'Argenson, who expelled him from the city. Again he hit the road, touring through Germany and Italy in a coach, amassing considerable wealth from his winnings; his prowess at gambling becoming something of a legend. When the "Sun King" Louis XIV died, leaving his country with a massive debt (incurred from wars and the construction of his palace at Versailles) and a bankrupt treasury, Law saw an opportunity and returned to

France. There was a shortage of money, and he had the answer. He quickly won over the regent, Philippe d'Orléans, who took a chance on the Scotsman and appointed him as Controller General of Finances – perhaps with the hope that his "system" would work as well for the economy as it did at cards.

Monetary Alchemy

Law's plan for the country – and he did not lack ambition – consisted of two parts. The first was to set up a state bank financed initially by himself, the Banque Générale, that would issue paper money redeemable in gold or silver. The bank was hugely successful, and its notes soon attracted a premium just for their convenience over coins. The second, which followed two years later, was to establish a company called the Mississippi Company, that would be granted a royal monopoly on trade with Louisiana – a vast region that encompassed the entire Mississippi River Valley.

Neither idea was new. The Bank of England and the Bank of Amsterdam already issued paper receipts for gold that could be traded as money. The Mississippi Company was modeled on the East India Companies of Britain and Holland. Law's brilliant idea was to connect the two, and unleash the alchemical power of paper money. Paper shares in the company could be bought using the paper money produced by the bank, in what seemed like a kind of perpetual-motion machine. In 1718 the bank was nationalized, becoming the Banque Royale; with this royal approval obtained, it was then announced that its notes would no longer be redeemable for precious metal.

Money was finally untethered from metal, its value determined instead by the authority of the French crown. A positive consequence was that the state could print as much money as it needed to satisfy the ravenous public appetite for shares, as people flocked from all over the country and abroad to take part in the economic miracle of Law's system. With all this money circulating around at a ferocious rate, the economy boomed. The word "millionaire" came into use for the first time. In 1719 alone, the Company share price vaulted from 500 livres to over 10,000 livres. The dropping of the dead-weight connection to metal also released any restraints on Law's bounding ambitions. In no time he was arranging for the Company to buy the national debt, and have the right to collect taxes. This required

issuing many more shares, and many more paper notes to buy them with. Which is when Law's system started to reveal its flaws.

While Law was certainly correct that money serves as a "Sign of Transmission," its value also depends on the confidence and trust of the community, and he had made the same mistake that he had made as a gambler in Paris, which was to fail to arrange buy-in from all the relevant players. Then it was the Chief of Police, d'Argenson, now it was the business and banking community (which included d'Argenson, who had become a prominent businessman). Rumors began to circulate that Louisiana was not quite the wealth generator it was cracked up to be, and Mississippi Company shareholders began to suspect they were being sold down the river.

The trip down was just as brief and thrilling as the way up. Suffice to say that, as the Company's share price drained away, and the value of the bank's paper notes approached zero, Law was again drummed out of Paris, and the country, and ended up near destitute in Venice. The story ought to serve as a cautionary tale for present-day central bankers. Oh, except that these days no bankers, central or otherwise, ever end up destitute.

Gold Standard

While Law was introducing the French to the benefits, perils, and general excitement of fiat currencies and financial innovation, Isaac Newton was serving as Warden of the Mint in England. Newton is of course best known for his famous contributions to physics, but he worked at the Mint from 1696 until his death in 1727. It is safe to say that his approach to finance was the opposite of Law's. At exactly the same time that Law was arranging to delink the livre from gold or silver, Newton was putting the pound on the gold standard, where it would remain for the next couple of hundred years.[1] While Law was issuing what some considered to be fake money, Newton was sending counterfeiters to their death. One wonders what he would have said about the situation in France, from his position at the Tower of London. Perhaps he felt some sympathy with Law's fall from grace; he did manage to lose £20,000 himself (over £2 million in today's money) on his investment in the South Sea Company, the British version of the Mississippi Company.

[1] By accident. He set the exchange rate for silver too high, so silver coins left the country.

The two certainly had completely different personalities. Here is a portrait of the young John Law by journalist John Flynn: "He got access to the smartest circles. He was a young man of education and culture, handsome, quick-witted, a good athlete excelling at tennis, a graceful dancer, and a redoubtable talker. He spent his mornings in the city, where he got a reputation for skill in speculating in government paper. He passed his afternoons in the parks, his evenings at the opera or theater, and the later hours at the routs, balls, masquerades, and gaming houses. He played for high stakes and won large sums. He was a man with a system. Had he lived in our time he would have been in Wall Street with an infallible formula for beating the market."[2] Perhaps he would have launched a hedge fund, or penned a bestseller about his "system."

Isaac Newton, in contrast, was a decidedly more solitary type. As a child, he showed great talent at making models, such as a working windmill. This skill later came in useful while constructing his own experimental apparatus, including a new design of telescope. He attended Cambridge University, but his most creative period came when the university was closed for two years because of the advancing plague, and Newton returned to his home in Lincolnshire to work alone. It was there that he claimed to have been prompted to discover the law of gravity after seeing an apple fall from a tree. Throughout his life he had a passion for alchemy and mysticism; in fact, most of his output consisted of religious writings, including a 300,000-word tract on the Book of Revelation.[3] He was famously anti-social and incommunicative; if no one showed up for his lectures, he just gave them to the empty room. There is no record of him being an expert dancer, or really fun at parties. As economist John Maynard Keynes wrote, he became instead the "Sage and Monarch of the Age of Reason."[4]

Researchers at Oxford and Cambridge have suggested that Isaac Newton may have had Asperger's Syndrome.[5] There is quite a business in such historical psych evaluations nowadays (see Box 1.1), but this one has a ring of truth about it. Often those with Asperger's Syndrome have a very narrow field of interest, with little curiosity in or appreciation of the bigger picture. They can exhibit intense concentration and understanding, and in many cases there is increased

[2] Flynn (1941).
[3] Manuel (1974).
[4] Keynes (1946).
[5] Muir (2003).

intelligence in areas such as mathematics. Which perhaps would explain why Newton was better with celestial mechanics than the financial sort.

These two contemporaries, Law and Newton, represent two aspects of the relationship between mathematics and finance. Mathematical finance is about using objective, rational, Newtonian models to simulate markets and make predictions about their future evolution. Quants are often described as modern-day wizards, hidden away in secret laboratories, who use mind-bending techniques inspired by areas such as quantum physics and string theory, coupled with the power of massive computers, to find hidden patterns in the markets. As Scott Patterson puts it in his book *The Quants*: "Think of white-coated scientists building ever more powerful devices to replicate conditions at the moment of the Big Bang to understand the forces at the root of creation."[6]

However, these scientists are trying to make money, not discover the next Higgs boson. (Juan Maldacena, Professor of Theoretical Physics at the Institute for Advanced Studies at Princeton and winner of many prizes for his work on such things as black holes, has said that finance is harder than physics. However, he has also given a public lecture in which he uses exchange rates as an analogy to explain the very same boson.) Mathematicians, like Law, are attracted to practical finance because they think they can use a system to beat the market, or even create an entirely new one. As seen later, their financial innovations often amount to creating new forms of credit, which like Law's scheme boost the money supply, at least for a while. In place of paper money, they invent credit default swaps or collateralized debt obligations. ("Make your very own 'credit default swap' and find out how to create money out of thin air!" as guides in a bus tour around the City of London now shout.[7]) They see the markets, with their rhythms and patterns, as a kind of music, which they can shape and control – and would agree with former CitiGroup CEO Chuck Prince who famously said, in the midst of the credit crunch, that "As long as the music is playing, you've got to get up and dance."

As we will see, it is the tension between these two aspects that drives mathematical finance, in both its inventiveness and creativity, and its tendency toward self-destruction.

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[6] Patterson (2009, p. 8).
[7] Gitlin (2014).

The Systems of Nature

After his losses in the South Sea debacle, Newton famously said: "I can calculate the movement of the stars, but not the madness of men." While Newton may not have tried to calculate the markets, and preferred chemical alchemy to the financial kind, he probably did more to shape the world of mathematical finance than any other scientist. His law of gravity, coupled with his three laws of motion, provided an archetype for a successful mathematical model that would influence not just areas such as physics and chemistry, but also social sciences including economics, and serve as an inspiration for quants to the present day.

One person who appreciated the power of Newton's approach was Adam Smith. He is of course best known for his book *The Wealth of Nations*,[8] which was the first to present economics as an objective, rational science, separate from areas such as ethics and political science. Some insight into his motivations is provided, however, by an earlier work on astronomy, written around 1758 but not published until after his death, in which his examination of "all the different systems of nature" culminates in a celebration of "The superior genius and sagacity of Sir Isaac Newton." He was less impressed by John Law. As he wrote in *The Wealth of Nations*, "The idea of the possibility of multiplying paper to almost any extent was the real foundation of what is called the Mississippi scheme, the most extravagant project both of banking and stock-jobbing that, perhaps, the world ever saw." (Smith would no doubt have been surprised to learn that we now organize our economies around Law's idea of a fiat currency, which was ahead of its time, rather than Newton's gold standard.)

Smith saw philosophy as a kind of calming device for making sense of the world, with its random events and its John Laws, its "chaos of jarring and discordant appearances." The beauty of Newton's method was the way in which it took a simple idea, such as gravity, and showed how "all the appearances, which he joins together by it, necessarily follow."

In the same book, Smith makes his first mention of the invisible hand. However, the passage was about the tendency for polytheistic religions to interpret events as being caused by gods: "the invisible hand of Jupiter." It was only later that he attributed this miraculous

[8] Smith (1776).

power to the markets. In *The Theory of Moral Sentiments*, he used the term in the context of wealth distribution: the rich "divide with the poor the produce of all their improvements. They are led by an invisible hand to make nearly the same distribution of the necessaries of life, which would have been made, had the earth been divided into equal portions among all its inhabitants, and thus without intending it, without knowing it, advance the interest of the society." (We wonder if he asked the poor.) Finally, and most famously, the phrase pops up again in *The Wealth of Nations*, in which – in a section on trade policy – an individual is again "led by an invisible hand to promote an end which was no part of his intention."

No one paid any attention to the metaphor until 1948, when Chicago School economist Paul Samuelson published his textbook *Economics*, which would go on to become the best-selling economics textbook of all time, translated into over 40 languages.[9] As he paraphrased: "Every individual, in pursuing only his own selfish good, was led, as if by an invisible hand, to achieve the best good for all, so that any interference with free competition by government was almost certain to be injurious." Which is when widespread use of the term, both in academic papers and general use, suddenly took off.[10]

Box 1.1 On the Couch

As mentioned above, it's unreliable to psychoanalyze people who aren't around to lie down on the couch, and sometimes it's annoying – as in the 2014 film *The Imitation Game*, in which Benedict Cumberbatch, the actor playing mathematician Alan Turing, might as well have worn a button saying "Hi, I have Asperger's!" Also, we're not psychologists and have no idea what we're talking about. But Adam Smith does seem worth a look.

From our case notes, it seems that tales abound of Smith's bizarre character. Friendly and good-tempered, he was also, according to one friend, "the most absent man in Company that I ever saw, Moving his Lips and talking to himself, and Smiling."[11] He did things like absentmindedly walk into a tanning pit, from which he needed to be rescued, or go for a stroll in his nightgown and end up 15 miles outside town. He was frequently ill and his doctors diagnosed him as a

[9] Samuelson (1973).
[10] Kennedy (2005).
[11] Alexander Carlyle, quoted in Özler (2012).