COMPUTATIONAL METHODS in ELECTROMAGNETIC COMPATIBILITY

Antenna Theory Approach Versus Transmission Line Models

DRAGAN POLJAK
KHALIL EL KHAMLICHI DRISSI

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Computational Methods in Electromagnetic Compatibility

Computational Methods in Electromagnetic Compatibility: Antenna Theory Approach versus Transmission line Models

Dragan Poljak and Khalil El Khamlichi Drissi

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To my lifetime inspiration, to my beloved daughters, my wife, my sister, my mother, and to the everlasting memory of my father who recently passed away and who will never be forgotten…

Dragan Poljak

To my dear parents, for all their sacrifices, their love, their tenderness, their support and their prayers. A special thought to my mother whom I miss terribly, I think of you everyday and I will probably never come to terms with the way your life ended on this earth.

Khalil El Khamlichi Drissi

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Preface

Electromagnetic Compatibility (EMC) as a topic has become very important in the last few decades. The vitality of EMC nowadays can be seen in many academic activities, as there are many universities worldwide offering undergraduate or graduate EMC courses, either obligatory or optional. Moreover, today, in the world of wireless communication and Internet of Things (IoT), many electronic products, devices, or systems are required to pass immunity and emission testing regarding EMC standards. Accordingly, there are dozens of books related to various EMC aspects currently available from major scientific publishers. Nevertheless, books rarely deal with EMC computational models and related numerical methods.

The previous book by Dragan Poljak, *Advanced Modeling in Computational Electromagnetic Compatibility*, was published by Wiley in February 2007. The present book authored by Dragan Poljak and Khalil El Khamlichi Drissi provides an overview of the further advances in the area of computational electromagnetics arising from a decade of very close and highly intensive collaboration between the Dragan research group from the University of Split, Croatia, and the Khalil group from Universitė Clermont Auvergne, France.

This rather fruitful collaboration resulted in successful joint projects and numerous journal and conference papers. The beauty of this collaboration reflects in merging two research teams tackling similar problems with different approaches related to antenna theory models (Dragan group) and transmission line methods (Khalil group). Furthermore, there is the benefit of discussing different solution methods related to boundary integral equation techniques and finite difference techniques. Moreover, throughout the book a trade-off between the different formulations and numerical solution methods is provided.

While the previous Wiley book by Dragan was primarily focused on academic examples, the present book by Dragan and Khalil deals with many practical engineering problems. The most significant topics covered in the book are related to realistic antenna systems, such as antennas for air traffic control or ground penetrating radar (GPR) antennas, grounding systems, such as grounding systems for wind turbines and biomedical applications of electromagnetic fields, such as transcranial magnetic stimulation. The book includes a large number of illustrative computational examples and reference list at the end of each chapter. Rigorous theoretical background and mathematical details of various formulations and solution methods being used throughout the book are presented in detail.

The authors hope that the present book gives not only a useful description of their expertise related to computational EMC but also updated information on the latest advances in this area.

The book is divided in two parts. The first part deals with electromagnetic field coupling to thin wire configurations of an arbitrary shape covering the following topics: introductory aspects of computational electromagnetics, antenna theory versus transmission line approximation, electromagnetic field coupling to overhead and buried wires, transient analysis of grounding systems and lightning channel modeling. An important goal of this part of the book is to provide a trade-off between a highly efficient transmission line approach, rather widely used by EMC community researchers and engineers, and antenna theory models providing the most rigorous analysis of high frequency (HF) and transient phenomena.

The second part of the book deals with advanced modeling of bioelectromagnetics phenomena featuring the method of moments (MoM), boundary element method (BEM) and hybrid finite element method (FEM)/ BEM, respectively. Of particular interest is not only human exposure to low frequency (LF) and HF electromagnetic fields but also some biomedical applications of electromagnetic fields.

We hope that this book will be useful material for undergraduate, graduate and postdoc students to learn about advanced EMC computational models and that it will also enable engineers in industry to solve some demanding practical problems. We also think that the book could be used for various university courses involving not only computational EMC models but also computational electromagnetics in general or numerical modeling in engineering itself.

The book requires a general background in electrical engineering, involving mainly basic electromagnetics. Fundamental EMC concepts such as numerical modeling principles are given in this book. Thus, the book is convenient for students, specialists, researchers and engineers.

To sum up, we are glad we have managed to compose this material stemming from more than a decade of very intensive collaboration in the areas of EMC and bioelectromagnetics. Of course, there are many rather challenging problems we plan to deal with together in days to come.

Split, Croatia–Clermont-Ferrand
France, June 2017

Dragan Poljak
Khalil El Khamlichi Drissi

Part I

Electromagnetic Field Coupling to Thin Wire Configurations of Arbitrary Shape

1

Computational Electromagnetics – Introductory Aspects

This introductory section deals with the character of a physical model and the corresponding mathematical method to solve the problem of interest. The models are characterized to be simplified imaginary simulations of the real-world systems one attempts to understand. However, models include only those properties and relationships required to understand aspects of real systems that are of interest at the given moment, i.e. those aspects of real systems one knows, or those one is aware of after all. The rest of the information about a real system is simply neglected. Furthermore, this introductory section discusses the fundamental framework to describe electromagnetic phenomena – Maxwell's equations, wave equations, and conservation laws.

1.1 The Character of Physical Models Representing Natural Phenomena

1.1.1 Scientific Method, a Definition, History, Development … ?

Scientists create tools, that's what they do...
C.P. Snow

Science could be considered as an entire set of facts, definitions, theorems, techniques, and relationships, and is tested on phenomena in the *real*, *objective*, and *external* world and, itself, has many elements of imagination, logic, creativity, judgment, metaphor, and instrumentations.

The essence of science is definitely more in *research methods* and specific *way of reasoning*, and less in particular facts and results.

Scientific insight starts with observing a certain phenomenon, and then organizing the collected observations in a sort of hypothesis that is tested on additional observations, and if necessary, modified. Then, predictions based on these modified hypotheses are carried out, and some experiments are

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performed to test the predictions. When the range of predictions provided by the hypothesis is considered to be satisfactory for the scientific community, the hypothesis is referred to as a *scientific theory* or *natural law*.

This rather successful methodology, more than four centuries old, is called *science* or *scientific method*.

Scientific method was born in the beginning of the seventeenth century with Galileo having abandoned Aristotle's theory of motion. It was Galileo who came up with the principle of the relativity of motion and with the statement that only change in motion required force.

At the same time, a separation of science from philosophy began in the form of shift from consideration of the *nature of phenomenon* (*essence*) to explanation of the *behavior of a phenomenon*. Namely, the *Aristotelian essentialistic approach* to the explanation of natural phenomena was replaced by the *mathematical predictive approach*. Instead of asking the question *why* scientists started to ask *how* [1]. As once Kelvin pointed out – *to know something about phenomena means to measure them and express them in terms of numbers*.

What is considered to be one of the crucial issues in the analysis of a natural phenomenon is related to the development and application of a physical model enabling one to predict the behavior of a system with a certain level of accuracy.

One of the crucial aspects of the scientific method and related technological progress is definitely *the physical model* of a natural phenomenon of interest.

1.1.2 Physical Model and the Mathematical Method to Solve the Problem – The Essence of Scientific Theories

> *Therefore, the goal of the scientific method is to establish the model of a physical phenomenon and to develop related mathematical methods for the analysis of the given problem.*

Various theoretical and experimental procedures are used while developing a model. *Models* are simplified imaginary simulations of the natural systems one attempts to understand and include only those properties and relationships required to understand aspects of real systems that are currently of interest, i.e. those aspects one knows, or, those one is aware of after all. The rest of the details about a real system are simply neglected from a model.

The concept of physical model represents the essence of reductionistic approach within the scientific method. How much the model of a given physical phenomenon is satisfactory depends on what is required from the model. In the language of mathematics, almost all problems arising in electromagnetics can be formulated in terms of differential, integral, or variational equations.

Generally, there are two basic approaches to solving problems in electromagnetics – the *differential* (*field) approach* and the *integral* (*source) approach*.

The field approach deals with a solution of a corresponding differential equation with associated initial and boundary conditions, specified at a boundary of a computational domain. Solving some differential equation type one obtains the spatial and temporal distribution of the corresponding field or potential.

Historically, this approach has been developed by Boskovic, Faraday, Maxwell, and others, and is generally very useful for handling the problems with closed domains and clearly specified boundary conditions, the so-called *interior field problems*.

The source or integral approach is based on the solution of a corresponding integral equation that yields the distribution of electromagnetic filed sources in terms of charge or current distribution, respectively.

In the past, this approach was promoted by Franklin, Cavendish, and Ampere, among others, and is convenient for the treatment of the *exterior* (unbounded) *field problems*.

Thus, a classical boundary-value problem can be formulated in terms of the operator equation:

$$L(u) = p \tag{1.1}$$

on the domain Ω with conditions

$$F(u) = q|_{\Gamma} \tag{1.2}$$

prescribed on the boundary Γ.

L is the linear differential operator, u is the solution of the problem, and p is the excitation function representing the known sources inside the domain. Note that u usually represents potentials (such as scalar potential φ) or fields (such as electric field E).

The character of the differential approach is depicted in Figure 1.1 [2].

Methods for the solution of the interior field problem are generally referred to as *differential methods* or *field methods*.

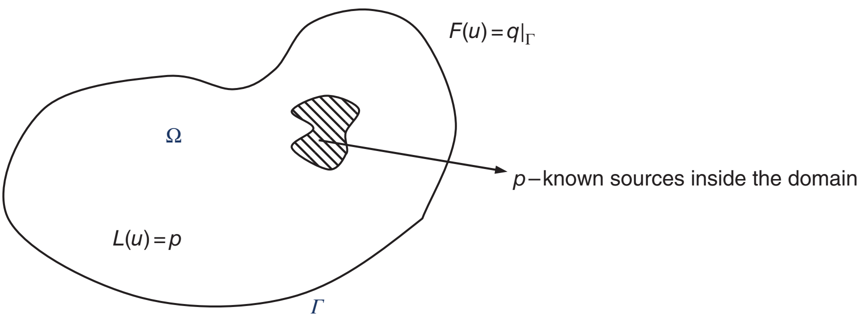

Figure 1.1 Differential approach concept.

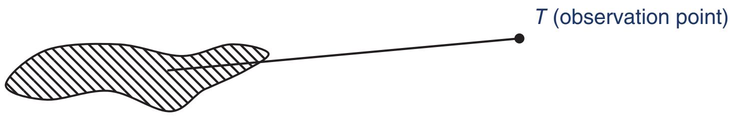

Figure 1.2 Integral approach concept.

Essentially, a differential approach isolates the calculation domain from the rest of the world. The interaction of the domain of interest with the rest is expressed (*de facto* replaced) by a set of prescribed boundary conditions.

If instead of differential operator L one considers an integral operator g, then unknowns are related to field sources (charge or current densities, respectively), distributed along the boundary Γ'. Namely, it can be written as

$$g(u) = h, \tag{1.3}$$

where h denotes the excitation function.

Figure 1.2 illustrates the character of the integral approach [2].

Solution methods for the exterior field problem are generally referred to as integral methods or source methods. In this case, the domain of interest is unbounded (infinite). However, the source distribution represents all that exists, i.e. all interactions coming from the outer world are neglected. For example, when a basic electromagnetic model is developed for a dipole antenna in engineering electromagnetics the antenna is assumed to be insulated in free space [2].

Finally, for dynamic phenomena the initial condition of a physical system has to be considered. Basically, any law of nature represents physical states in a mathematical form (written in terms of differential, integral, or variational equations). Thus, a prescribed initial condition (behavior of the considered physical quantity at $t = 0$) by definition implies that nothing exists earlier than $t = 0$. This is also considered to be the origin of time asymmetry in physical laws.

Generally, techniques for the solution of operator equations can be referred to as analytical, numerical, or hybrid methods. Analytical solution methods provide *exact solutions* but are, on the other hand, limited to a narrow range of applications, mostly related to canonical problems.

Unfortunately, there are not many realistic scenarios in physics or practical engineering problems that can be worked out using these techniques.

Numerical techniques are applicable to almost all scientific engineering problems, but the main drawbacks are related to the limits governed by the approximation contained in the model itself.

Moreover, the criteria for accuracy, stability, and convergence are not always straightforward and clear to the researcher in a particular area [2].

1.1.3 Philosophical Aspects Behind Scientific Theories

One of the crucial questions in the philosophy of science is how physical models really work, or, more generally, how scientific theories are developed or "upgraded."

Looking back into the history of physics, the development of Maxwell's kinetic theory of gases and electromagnetic field theory was not motivated by experimental findings that were not compatible with the existing paradigm (in the sense of Kuhn [3]), as was the case with relativity and quantum mechanics.

In the case of electromagnetism almost all facts, known in Maxwell's time, were interpreted satisfactorily within the Newton paradigm and incorporated into a powerful theoretical frame.

This theory was intensively in use till Hertz experimentally verified one of the main goals of the Maxwell theory – the existence of electromagnetic waves.

However, the principal motivation in the background of Maxwell's work was essentially philosophical, or even metaphysical in nature, i.e. the consequence of his own point of view. The origin of Maxwell's ideas came from Michael Faraday and his study of electromagnetic induction. Faraday, together with Boskovic, was one of the first scientists who came up with the idea of *field* versus action at a distance concept.

Some rather old, but still important questions from philosophy of science are as follows:

- Is scientific insight into the absolute truth possible, taking into account limitations of our conceptual, language, and mathematical tools?
- Is the rise of knowledge cumulative in nature and is it clearly directed to the objective truth?

According to *logical positivism*, *Popper's falsificationism,* and *Kuhn's social relativism* the objective truth is out of reach for the human mind. Furthermore, for Ernst Mach and Vienna circle followers, theories are systems of quantitative relationships between measurable phenomena, and are not directed toward the absolute and objective truth. Moreover, Mach and other empiricists claim that only theories directly testable with experiments should be accepted [4, 5]. For Niels Bohr, theory is a tool to explain various experimental data.

No universal theory exists for Popper that would be conclusively proved in an inductive sense. Theory is alive while its disadvantages are not found. For Kuhn, the *natural selection* of scientific theories is driven by the request for problem solving. For Wittgenstein, the origin of scientific triumph is aspiration for generality.

Gödel incompleteness theorem has destroyed the basis of *the axiomatic method*. Stephen Hawking lost faith in the existence of *Theory of Everything* as *Gödel theorem* had convinced him that any system could not be complete if it was consistent. There can always be a proposition that cannot be proved or refuted.

Einstein's reasoning was also affected by the strong philosophical background, classical education, and culture of dialogue. For Einstein, many professional scientists of his time have seen thousands of trees, but have never seen a wood [4, 5]. Einstein esteems that the knowledge of historical and philosophical background of science could set one free of prejudices of which most of the generation suffers. Thus, after initial respect for the Vienna circle, Einstein's attitudes began to differ from the circle ideas. The circle refused any element of a theory, as metaphysical, if there was no clear connection with an experience.

Einstein claims that veracity of a theory can never be proved, as it is never known if future experience will contradict its conclusions. Einstein moves aside Schlick and Reichenbach as new empirical philosophy, according to Einstein, turns science into something like engineering. Einstein's own experience leads him to a strong attitude that creative theoretical thinking cannot be replaced with algorithm for building and testing theories. Passion for knowledge, according to Einstein, creates the illusion that the objective world can be comprehended rationally, without any empirical foundation – in short, by means of metaphysics.

Therefore, the old question still of interest in both philosophy and science is this: Does scientific knowledge come from out-of-mind reality, or it is necessarily just a reflection of the mind and is it limited by its own insight abilities?

1.1.4 On the Character of Physical Models

Physical model represents the fundamental concept within the framework of the scientific method for the representation and understanding of natural phenomena. Physical models are simplified imaginary simulations of the real-world systems one attempts to understand, including only those properties and relationships required to understand the aspects of real systems one considers, i.e. those aspects of real systems one knows, or is generally aware of. The rest of the facts about a real system are simply neglected from the model. As a matter of fact, how much the model of a given physical phenomenon is satisfactory then strongly depends on what is required from the particular model. Thus, one draws conclusions from an incomplete information set.

Therefore, models are tools for capturing particular insights of the phenomena and they do not represent a full proof for a system behavior under all circumstances. Moreover, mathematically described physical models are abstractions of the natural world, while the related computational models, convenient for implementation on a digital computer, are eventually abstraction of the physical world.

Therefore, physical models and related solution methods are *problem dependent*.

1.2 Maxwell's Equations

In his hands electricity first became a mathematically exact science and the same might be said of other larger parts of physics.
Sir James H. Jeans

It was not possible to incorporate an increasing knowledge on electricity and magnetism through the nineteenth century into Newton's physics framework. Thus, contributions of Faraday, Maxwell, Heaviside, Hertz, and others led to the revolutionary concept of field in classical physics. The field was shown not to be just mathematical abstract entity, but pure physical reality. Consequently, by adopting the field notion the *action at a distance* concept was abandoned [5].

With James C. Maxwell, not only a rigorous electromagnetic field theory came along but also a grand unification of electricity, magnetism, and light. Namely, almost a quarter century before the Hertz experimental verification Maxwell theoretically anticipated the existence of electromagnetic waves. Light is just an electromagnetic wave, visible to the human eye propagating through ether. After Maxwell, H. A. Lorentz extended Maxwell's theory with electrodynamics of charged particle.

1.2.1 Original Form of Maxwell's Equations

Maxwell's equations were modified a few times [6] in the last 150 years since they were originally formulated by Maxwell and published for the first time [7]. The changes were regarding the physical interpretation, mathematical expression, and general approach to the solution methods for different problems. In the mid-1860s Maxwell originally derived 20 scalar equations. What is today considered to be modern Maxwell's equations are a set of vector equations independently derived by Heaviside and Hertz by the end of the nineteenth century.

Historically speaking, with Maxwell's equations, a rigorous mathematical basis has been established for a proper description of electromagnetic phenomena. Moreover, the appearance of Maxwell's equations has provided the paradigm shift from the old *action at a distance* concept to the *field approach*. Maxwell's equations have undergone significant changes twice [8]. First, it was when Heaviside reduced the scalar form into the vector notation. He also made important modifications, having abandoned the potentials in favor of fields. Next time, a significant improvement was initiated by Larmor due to the discovery of the electron.

Important advancements of Maxwell's theory in the mid-1880s were carried out by Poynting, FitzGerald, and Heaviside. Lorentz's contribution is related to the development of microscopic theory by means of Maxwell's equations and inclusion of the force acting on a charged particle arising from the existence of fields.

1.2.2 Modern Form of Maxwell's Equations

The laws of classical electromagnetism can be expressed very concisely by a set of four differential equations. There is also an equivalent integral form of these equations. The differential form of Maxwell's equations is commonly used in solving engineering problems while their integral forms are convenient in providing a deeper insight into the underlying physical laws.

The *first Maxwell equation* is the differential form of Faraday law (the time-varying magnetic flux density $\vec{B}$causes the curl of electric field $\vec{E}$) given by

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \tag{1.4}$$

Hence, the time-varying magnetic fields are vortex sources of electric fields.

The *second Maxwell equation* is the differential form of generalized Ampere's law stating that either a current density $\vec{J}$ or a time-varying electric flux density $\vec{D}$ gives rise to a magnetic field $\vec{H}$. This can be expressed as

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}. \tag{1.5}$$

It is worth noting that the term $\partial \vec{D}/\partial t$ was originally added by Maxwell to the original expression for Ampere's law, thus making the law consistent with the electric charge conservation. This term is usually referred to as a displacement current density.

The *third Maxwell equation* states that electric monopoles exist, so that

$$\nabla \cdot \vec{D} = \rho, \tag{1.6}$$

i.e. charge densities ρ are the monopole sources of the electric field.

Finally, the *fourth Maxwell equation* states that magnetic poles always occur in pairs and are due to electric currents; no free poles can exist. This is expressed by the divergence Maxwell equation:

$$\nabla \cdot \vec{B} = 0, \tag{1.7}$$

which implies that the magnetic field is always solenoidal.

The integral form of the Faraday law states that any change of magnetic flux density B through any closed loop induces an electromotive force around the loop. Taking the surface integration over (1.4) and applying the Stokes theorem yields

$$\oint_c \vec{E}\,\mathrm{d}\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{S}, \tag{1.8}$$

where the line integral is taken around the loop and with $\mathrm{d}\vec{S} = \vec{n}\mathrm{d}S$.

The voltage induced by a varying flux has a polarity such that the induced current in a closed path gives rise to a secondary magnetic flux, which opposes the change in time-varying source magnetic flux.

The integral form of the Ampere law is derived by integrating (1.5) and applying the Stokes theorem:

$$\oint_c \vec{H}\,\mathrm{d}\vec{s} = \int_S \vec{J}\,\mathrm{d}\vec{S} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \mathrm{d}\vec{S}. \tag{1.9}$$

Equation (1.6) is the Ampere circular rule with Maxwell addition of the second term on the right-hand side (the displacement current). The generalized Ampere law states that either an electric current or a time-varying electric flux gives rise to magnetic field.

Taking the volume integral over (1.6) and applying the Gauss divergence theorem results in

$$\oint_S \vec{D}\,\mathrm{d}\vec{S} = \int_V \rho\,\mathrm{d}V, \tag{1.10}$$

where the right-hand side represents the total charge within the volume V.

Equation (1.6) is the Gauss flux law for the electric field stating that the flux of D vector corresponds to the total electric charge within the domain.

The Gauss flux law for the magnetic field can be derived by taking the volume integral of (1.7) and applying the Gauss divergence theorem, i.e.

$$\oint_S \vec{B}\,\mathrm{d}\vec{S} = 0, \tag{1.11}$$

stating that the flux of B vector over any closed surface S is identically zero.

What is also necessary for a description of electromagnetic phenomena in a linear medium are the constitutive equations:

$$\vec{D} = \varepsilon\vec{E}, \tag{1.12}$$

$$\vec{J} = \sigma\vec{E}, \tag{1.13}$$

$$\vec{B} = \mu\vec{H}, \tag{1.14}$$

and the Lorentz force equation

$$\vec{F} = q(\vec{v} \times \vec{B}), \tag{1.15}$$

where q denotes the charged particle, v is the particle velocity, ε is permittivity, σ is conductivity, and μ is permeability of a medium, respectively.

To solve Maxwell's equations for a given problem the continuity conditions at the interface of two media with different electrical properties must be specified [2]:

$$\vec{n} \times (\vec{E}_1 - \vec{E}_2) = 0, \tag{1.16}$$

$$\vec{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s, \tag{1.17}$$

$$\vec{n}(\vec{D}_1 - \vec{D}_2) = \rho_s, \tag{1.18}$$

$$\vec{n}(\vec{B}_1 - \vec{B}_2) = 0, \tag{1.19}$$

where $\vec{n}$ is a unit normal vector directed from medium 1 to medium 2, and subscripts 1 and 2 denote fields in regions 1 and 2. Equations (1.16) and (1.19) state that the tangential components of E and the normal components of B are continuous across the boundary. Equation (1.17) represents that the tangential component of H is discontinuous due to the surface current density J_s induced on the boundary. Equation (1.18) means that the discontinuity in the normal component of D is the same as the surface charge density ρ_s on the boundary.

In the case of a perfect conductor, the electric field E and magnetic field H vanish within the perfectly conducting medium. These fields are replaced by the surface charge density ρ_s and surface current density J_s. At higher frequencies, there is a well-known effect that confines current largely to surface regions. The so-called skin depth in common situations is often sufficiently small for the surface phenomenon to be an accurate representation. Therefore, the familiar rules for the behavior of time-varying fields at a boundary defined by good conductors follow directly from consideration of the limit condition, i.e. when the conductor is perfect.

As no time-varying field exists in a perfect conductor, the electric flux density is entirely normal to the conductor and supported by a surface charge density at the interface.

$$D_n = \rho_{s.} \tag{1.20}$$

The magnetic field is entirely tangential to the perfect conductor and is equilibrated by a surface current density:

$$H_s = J_s. \tag{1.21}$$

Conditions at the extremes of the boundary value problem are obtained by extending the interface conditions.

1.2.3 From the Corner of Philosophy of Science

Essentially, development of Maxwell kinetic theory of gases and electromagnetic field theory was not motivated by experimental findings which were not compatible with existing paradigm (in a sense of T.S. Kuhn [3]), as was the case with relativity and quantum mechanics.

As already explained in *1.1.3*, almost all facts from electrodynamics, known in Maxwell time, were interpreted relatively satisfactory within the Newton paradigm. That approach was standardly used till Hertz experimental verification of the very existence of electromagnetic waves.

The origin of Maxwell's ideas to replace the *action at a distance* concept with the concept of *physical field*, was essentially philosophical, or a pure abstract, mathematical thought, and came from previous works of Faraday and Boskovic.

Maxwell introduced a revolution not only in electromagnetics but also in thermodynamics [9]. His approach to represent a physical phenomenon in terms of statistical function was a remarkable improvement in science generally. Such an approach led not only to the statistical nature of the second law of thermodynamics, but also provided the development of mathematical description of quantum mechanics.

Furthermore, his introduction of today's famous Maxwell's demon having questioned the second law of thermodynamics contributed to the development of information theory in the twentieth century.

No doubt, Maxwell, himself, was and is one of the greatest scientists of all times.

1.2.4 FDTD Solution of Maxwell's Equations

One of the widely used approaches for a direct solution of Maxwell's equations in the last few decades is the use of the finite difference time domain (FDTD) method. FDTD solution of Maxwell's equations is based on discretizing the differential equations by means of pulse basis approximation and converting them into a finite difference equation. FDTD is a highly versatile method enabling the analysis of objects with a wide range of size and complexity, from a microstrip circuit to a helicopter or the human body. The method discretizes a domain of interest into unit cells, leading to the use of the so-called staircase approximation for smoothly curved surfaces or volumes. The unit cell, usually called "the Yee cell" after Yee who introduced the first FDTD algorithm [10], is shown in Figure 1.3.

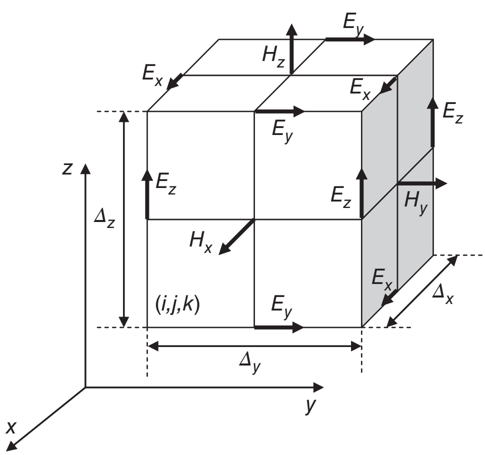

Figure 1.3 Configuration of electric and magnetic fields in a cell.

Assuming isotropic and nondispersive media, Maxwell's equations (1.4) and (1.5) can be written as

$$\nabla \times \vec{E}(r,t) = -\mu \frac{\partial \vec{H}(r,t)}{\partial t}, \tag{1.22}$$

$$\nabla \times \vec{H}(r,t) = \sigma \vec{E}(r,t) + \varepsilon \frac{\partial \vec{E}(t,r)}{\partial t}. \tag{1.23}$$

Using the Yee cell [10] and performing the space–time finite difference discretization one obtains [11]

$$\begin{aligned} E_x^{n+1}\left(i+\frac{1}{2},j,k\right) &= A \cdot E_x^{n}\left(i+\frac{1}{2},j,k\right) \\ &+B \cdot \left[\begin{array}{c} \dfrac{H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2},k\right) - H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y} \\ -\dfrac{H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right) - H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z} \end{array} \right], \end{aligned} \tag{1.24}$$

$$\begin{aligned} H_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right) &= H_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right) \\ &-\frac{\Delta t}{\mu} \cdot \left[\begin{array}{c} \dfrac{E_z^{n}\left(i,j+1,k+\frac{1}{2}\right) - E_z^{n}\left(i,j,k+\frac{1}{2}\right)}{\Delta y} \\ -\dfrac{E_y^{n}\left(i,j+\frac{1}{2},k+1\right) - E_y^{n}\left(i,j+\frac{1}{2},k\right)}{\Delta z} \end{array} \right], \end{aligned} \tag{1.25}$$

where A and B are given by

$$A = \frac{1 - \dfrac{\sigma \cdot \Delta t}{2 \cdot \varepsilon}}{1 + \dfrac{\sigma \cdot \Delta t}{2 \cdot \varepsilon}}, \quad B = \frac{\dfrac{\Delta t}{\varepsilon}}{1 + \dfrac{\sigma \cdot \Delta t}{2 \cdot \varepsilon}}. \tag{1.26}$$

Other electromagnetic field components are obtained by a simple circular permutation of the space variables. It is possible to improve the original FDTD formulation to provide more accurate modeling of smooth curves, but with the price of increasing the complexity of the algorithms. Also, the FDTD method encounters some difficulties in modeling thin wires and the related feed-gap concept [9]. Moreover, within the FDTD method E and H fields are not computed exactly at a distance of half a cell from each other.

The thin wire itself is defined as a conductive object with radius smaller than the size of an FDTD cell [9, 11].

In many engineering applications, such as antenna analysis and design and grounding systems studies, it is necessary to model electrically thin conducting cylinders requiring the radii r_0 of such conducting structures to be smaller than the smallest Yee cell dimensions. Thus, a special formulation must be implemented to accurately represent these radii. The details can be found elsewhere, e.g. in [11].

Contour integral approach [9], derived from Maxwell's equations in integral form, is used to account for the air–ground interface. It is convenient to treat the problems including inhomogeneous media and a nonuniform spatial discretization by using Maxwell's equations in their integral form [11].

Figure 1.4 shows a grid of FDTD discretization of the air–ground interface. Thus, from the generalized Ampere's law (1.9) one obtains

$$\oint_c \vec{H} \cdot \mathrm{d}\vec{s} = \left[H_z^{n+\frac{1}{2}}(i+1/2, j+1/2, k) - H_z^{n+\frac{1}{2}}(i+1/2, j-1/2, k)\right] \cdot \Delta z + \left[H_y^{n+\frac{1}{2}}(i+1/2, j, k-1/2) - H_y^{n+\frac{1}{2}}(i+1/2, j, k+1/2)\right] \cdot \Delta y, \quad (1.27)$$

$$\iint_s \left(\sigma \vec{E}_x + \varepsilon \frac{\partial \vec{E}_x}{\partial t}\right) \mathrm{d}\vec{s} = \frac{\Delta y \Delta z}{2} \cdot \varepsilon_0 \cdot \frac{\partial E_x}{\partial t} + \frac{\Delta y \Delta z}{2} \cdot \left(\varepsilon_s \frac{\partial E_x}{\partial t} + \sigma_s E_x\right). \quad (1.28)$$

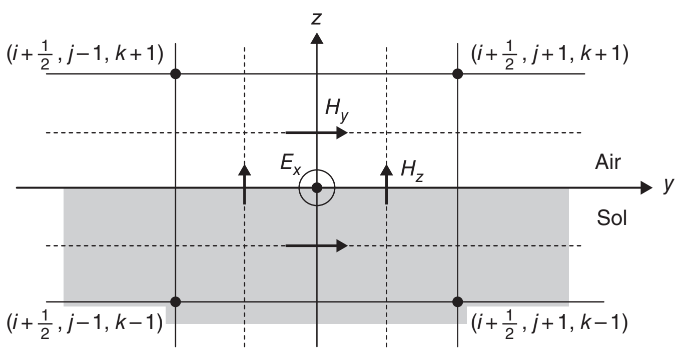

Figure 1.4 Treatment of the air–ground interface by the "the contour integral approach" method.

Combining (1.27) and (1.28) yields

$$E_x^{n+1}(i+1/2,j,k) = \frac{K}{N}\cdot E_x^n(i+1/2,j,k) + \frac{1}{N\cdot\Delta y} \cdot \left[H_z^{n+\frac{1}{2}}(i+1/2,j+1/2,k) - H_z^{n+\frac{1}{2}}(i+1/2,j-1/2,k)\right] - \frac{1}{N\cdot\Delta z}\cdot\left[H_y^{n+\frac{1}{2}}(i+1/2,j,k+1/2) - H_y^{n+\frac{1}{2}}(i+1/2,j,k-1/2)\right], \tag{1.29}$$

where

$$K = \frac{1}{2\Delta t}(\varepsilon_0+\varepsilon_s) - \frac{\sigma_s}{4},\quad N = \frac{1}{2\Delta t}(\varepsilon_0+\varepsilon_s) + \frac{\sigma_s}{4}. \tag{1.30}$$

The other field components can be derived in a similar manner. It is also necessary to truncate the calculation domain. Therefore, absorbing regions must be implemented at the domain's limits, simulating the wave propagation thus avoiding non-natural reflections in infinite domain. In this context, the absorbing conditions are used [12].

1.2.5 Computational Examples

Computational examples are related to the transient behavior of grounding systems in two-layer soil. FDTD solution of Maxwell's equations is performed by taking into account the variation in conductivity between the conductive layers of the soil. The obtained FDTD results are compared to the numerical results calculated via the TL approach [11].

The first example deals with a horizontal grounding electrode in two-layer stratified soil. Figure 1.5 shows the electrode buried horizontally at depth

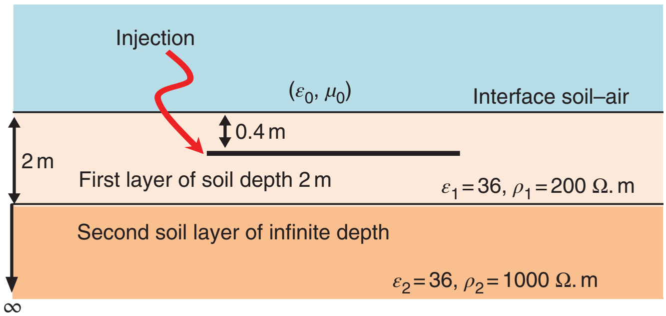

Figure 1.5 Horizontal grounding electrode in two-layer soil.

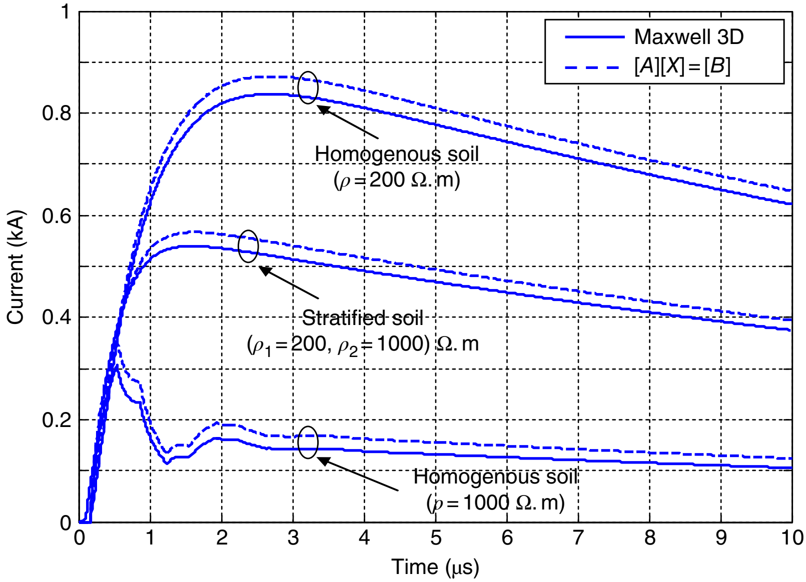

Figure 1.6 Transinet current induced at the middle of the buried electrode.

$d = 0.4$ m from the soil–air interface. The radius of the conductor is $a = 7$ mm and its length is $L = 20$ m, while the height of the upper layer is $D = 2$ m.

The electrode is excited by a voltage source given by double exponential function [11]

$$V(t) = V_0(e^{-at} - e^{-bt}), \tag{1.31}$$

where $V_0 = 30$ kV, $a = 45\,099$ s^{-1}, $b = 9\,022\,879$ s^{-1}.

Physical constants ε, ρ of the medium are depicted in Figure 1.5.

Figure 1.6 shows the transient current induced along the horizontal grounding electrode calculated by FDTD and the TL approach, respectively.

The results obtained via different approaches seem to agree satisfactorily. The next example is a grounding grid in two-layer stratified soil. Figure 1.7 shows the grid buried at $d = 0.8$ m depth in the stratified soil. The size of the grounding grid is 20 m × 20 m. The physical constants (ε, ρ) of the medium are shown in Figure 1.7. The radius of the conductors is $a = 7$ mm. The grounding grid system is excited by double exponential voltage impulse (1.31). Two cases are considered, with the injection point being on the corner and in the middle of the grounding grid system, respectively, as shown in Figure 1.7.

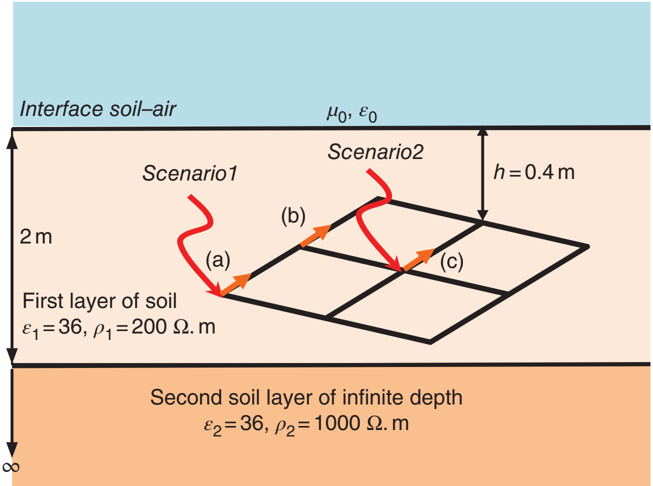

Figure 1.7 Horizontal grounding grid in two-layer soil.

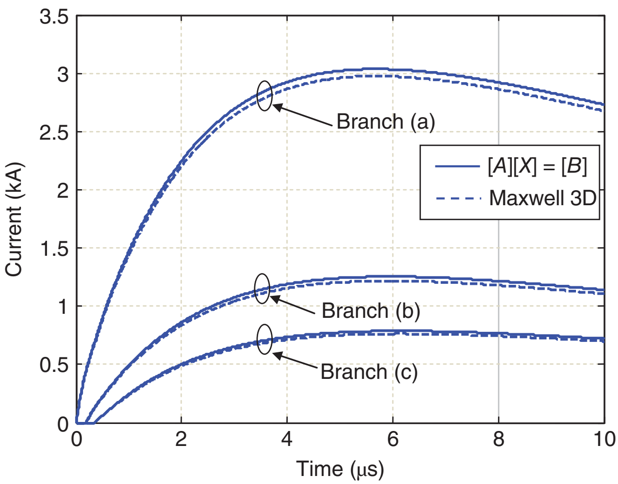

Figure 1.8 Transient currents induced at different points for the horizontal grounding grid (Scenario 1).

Figures 1.8 and 1.9 show transient currents induced on the horizontal grounding grid obtained as FDTD solution of Maxwell's equations and using the TL approach, respectively.

Two different scenarios are studied (corner injection – scenario 1, and central point injection – scenario 2 of the current source).

The numerical results obtained by different approaches are in satisfactory agreement, again.

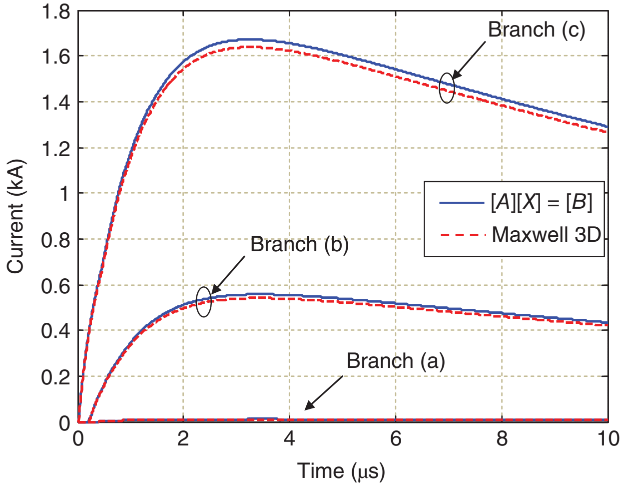

Figure 1.9 Transient currents induced at different points for the horizontal grounding grid (Scenario 2).

1.3 The Electromagnetic Wave Equations

Maxwell's equations are coupled first order space–time partial differential equations that are very difficult to apply when solving boundary-value problems. One way to overcome the difficulty of solving coupling equations is to decouple these first order equations, thereby obtaining the second order electromagnetic wave equations.

The wave equations are readily derived from the Maxwell curl equations, by differentiation and substitution. Taking curl on both sides of equation (1.5) leads to

$$\nabla \times \nabla \times \vec{H} = \nabla \times \vec{J} + \frac{\partial}{\partial t}(\nabla \times \vec{D}). \tag{1.32}$$

Using constitutive equations (1.12) and (1.13) and assuming uniform scalar material properties yields

$$\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \varepsilon \frac{\partial}{\partial t}(\nabla \times \vec{E}). \tag{1.33}$$

According to the Maxwell equation (1.4) curl of E is replaced by the rate of change of magnetic flux density, and using (1.14) it follows that

$$\nabla \times \nabla \times \vec{H} = -\mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}. \tag{1.34}$$

Performing some mathematical manipulations, the same equations can be derived for the electric field.

Using the standard vector identity valid for any vector *E*,

$$\nabla \times \nabla \times \vec{H} = \nabla \cdot (\nabla \vec{E}) - \nabla^2 \vec{H}. \tag{1.35}$$

Taking into account the solenoidal nature of the magnetic field (1.7) yields the final form of the wave equation:

$$\nabla^2 \vec{H} - \mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0. \tag{1.36}$$

If a linear, isotropic, homogeneous, source-free medium is considered then the set of equations (1.36) simplifies into

$$\nabla^2 \vec{H} - \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0, \tag{1.37}$$

where v denotes the wave propagation velocity in lossless homogeneous medium:

$$v = \frac{1}{\sqrt{\mu\varepsilon}}. \tag{1.38}$$

The velocity of wave propagation in free space is the velocity of light:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}, \tag{1.39}$$

where $c = 3 \times 10^8$ m s^{-1}, approximately.

1.4 Conservation Laws in the Electromagnetic Field

A general relationship for power and energy expressed in terms of electric and magnetic fields is given in the form of Poynting theorem.

The conservation law of electromagnetic energy can be obtained from curl Maxwell equations.

An equivalence of vector operators yields

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot \nabla \times \vec{E} + \vec{E} \cdot \nabla \times \vec{H}. \tag{1.40}$$

Combining Equations (1.4), (1.5), and (1.40), one has

$$\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = -\vec{E} \cdot \vec{J} + \vec{E} \cdot (\nabla \times \vec{H}) - \vec{H} \cdot (\nabla \times \vec{E}), \tag{1.41}$$

or in the alternative form,

$$\frac{\partial w}{\partial t} = -\vec{E} \cdot \vec{J} - \nabla \cdot (\vec{E} \times \vec{H}), \tag{1.42}$$

where the w term represents the energy storage per unit volume for an electromagnetic field:

$$w = \frac{1}{2}(\vec{E}\vec{D} + \vec{H}\vec{B}). \tag{1.43}$$

Integrating (1.41) over some finite region in space one obtains the integral form of the electromagnetic energy conservation law:

$$\int_V w\,\mathrm{d}V = -\int_V \vec{E}\cdot\vec{J}\,\mathrm{d}V - \int_V \nabla\cdot(\vec{E}\times\vec{H})\mathrm{d}V. \tag{1.44}$$

The left-hand side term is the time rate of the stored energy in the electric and magnetic fields of the region.

The first term on the right-hand side represents the Joule heat (the ohmic power loss if J is a conduction current density or the power required to accelerate charges if J is a convection current arising from moving charges). If there is an energy source then the product EJ is negative for that source and represents energy flow out of the region.

The other term on the right-hand side gives the flow into the domain boundary.

Applying the Gauss integral theorem to the last term of (1.44)

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\mathrm{d}V = \oint_S (\vec{E}\times\vec{H})\cdot\mathrm{d}\vec{S}, \tag{1.45}$$

the volume integral transforms to the surface integral over the boundary, where $\mathrm{d}\vec{S}$ is the outward drawn normal vector surface element.

Since all the energy changes must be supplied externally, this term represents the energy flow into the volume per unit time due to the minus sign of the surface integral. Changing sign, the rate of energy flow, or power flow, out through the enclosing surface is given by

$$P = \oint_S (\vec{E}\times\vec{H})\mathrm{d}\vec{S}, \tag{1.46}$$

where $\vec{E}\times\vec{H}$ is the Poynting vector representing power density flow – flow of energy per unit area per unit time at the surface (power density flow), known as Poynting vector:

$$\vec{P}_\mathrm{d} = \vec{E}\times\vec{H}. \tag{1.47}$$

The Poynting vector (1.47) gives the direction and magnitude of energy flow density at any point in space.

Power flow does not exist in the vicinity of a system of static charges having electric but no magnetic field. Also, in the vicinity of a perfect conductor there is a zero tangential component normal to the conductor and power flow into the perfect conductor is not possible.

Therefore, the final integral form of the conservation law in the electromagnetic field is then given by

$$\frac{\partial}{\partial t}\int_V \frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})\mathrm{d}V = -\int_V \vec{E}\cdot\vec{J}\ \mathrm{d}V + \oint_S (\vec{E}\times\vec{H})\cdot \mathrm{d}\vec{S}. \tag{1.48}$$

Therefore, the rate of increase of electromagnetic energy in the domain equals the rate of flow of energy in through the domain surface less the Joule heat production in the domain.

For a battery with a nonelectrostatic field E' pumping energy both into heat losses and into a magnetic field is considered; the corresponding current density can be written as

$$\vec{J} = \sigma(\vec{E}+\vec{E}') \tag{1.49}$$

and (1.48) becomes

$$\int_V \vec{E}'\vec{J}\ \mathrm{d}V = \frac{\partial}{\partial t}\int_V \frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})\mathrm{d}V + \int_V \frac{|\vec{J}|}{\sigma}\mathrm{d}V + \oint_S (\vec{E}\times\vec{H})\cdot \mathrm{d}\vec{S}, \tag{1.50}$$

where the term on the left-hand side represents the sources within the volume of interest.

The first and second terms on the right-hand side of (1.50) are the total energy stored in the electric and magnetic fields, respectively.

1.5 Density of Quantity of Movement in the Electromagnetic Field

According to the laws of classical mechanics, force $\vec{F}$ is equal to the change in quantity of movement of matter:

$$\vec{F} = \frac{\mathrm{d}\vec{G}_{\mathrm{meh}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(m\vec{v}) = \frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_m \vec{v}\,\mathrm{d}V, \tag{1.51}$$

where $\vec{G}_{\mathrm{meh}}$ is the momentum or quantity of movement, ρ_{m} g is the mass density, and $\vec{v}$ is the velocity.

If an electromagnetic system is subjected to an external force, where force density to charges and currents is given by

$$\vec{f} = \rho\vec{E} + \vec{J}\times\vec{B}. \tag{1.52}$$

The total force on the matter contained within volume V, i.e. to charges and currents, is defined by the expression

$$\vec{F} = \int_V \vec{f}\,\mathrm{d}V = \int_V (\rho\vec{E}+\vec{J}\times\vec{B})\mathrm{d}V. \tag{1.53}$$

Combining (1.51) and (1.53), Newton's second law yields

$$\frac{\mathrm{d}\vec{G}_{\mathrm{meh}}}{\mathrm{d}t} = \int_V (\rho\vec{E} + \vec{J} \times \vec{B})\mathrm{d}V. \tag{1.54}$$

Furthermore, for free space one has

$$\rho = \varepsilon_0 \nabla \vec{E} \tag{1.55}$$

and

$$\vec{J} = \frac{1}{\mu_0}\nabla \times \vec{B} - \varepsilon_0 \frac{\partial \vec{E}}{\partial t}. \tag{1.56}$$

So, it follows that

$$\vec{f} = \rho\vec{E} + \vec{J} \times \vec{B} = \varepsilon_0 \vec{E}\nabla\vec{E} + \frac{1}{\mu_0}\vec{B} \times \nabla \times \vec{B} - \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \times \vec{B}. \tag{1.57}$$

Now, the last term in (1.57) can be written in the following manner:

$$\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \times \vec{B} = \frac{\partial}{\partial t}(\varepsilon_0 \vec{E} \times \vec{B}) - \varepsilon_0 \vec{E} \times \frac{\partial \vec{B}}{\partial t} = \frac{\partial}{\partial t}(\varepsilon_0 \vec{E} \times \vec{B}) + \varepsilon_0 \vec{E} \times \nabla \times \vec{E}. \tag{1.58}$$

and the force density (1.57) becomes

$$\vec{f} = \varepsilon_0 \vec{E}\nabla\vec{E} - \frac{1}{\mu_0}\vec{B} \times \nabla \times \vec{B} - \frac{\partial}{\partial t}(\varepsilon_0 \vec{E} \times \vec{B}) - \varepsilon_0 \vec{E} \times \nabla \times \vec{E}. \tag{1.59}$$

As one has $\nabla\vec{H} = 0$, expression (1.59) can be written as

$$\vec{f} = \varepsilon_0 \vec{E}\nabla\vec{E} - \varepsilon_0 \vec{E} \times \nabla \times \vec{E} + \mu_0 \vec{H}\nabla\vec{H} - \mu_0 \vec{H} \times \nabla \times \vec{H} - \frac{\partial}{\partial t}(\varepsilon_0 \vec{E} \times \vec{B}), \tag{1.60}$$

and the relation for the total force within the volume becomes

$$\vec{F} = \int_V \vec{f}\,\mathrm{d}V = \varepsilon_0 \int_V (\vec{E}\nabla\vec{E} - \vec{E} \times \nabla \times \vec{E})\mathrm{d}V + \frac{1}{\mu_0}\int_V (\vec{B}\nabla\vec{B} - \vec{B} \times \nabla \times \vec{B})\mathrm{d}V - \frac{\mathrm{d}}{\mathrm{d}t}\int_V (\varepsilon_0 \mu_0 \vec{E} \times \vec{H})\mathrm{d}V. \tag{1.61}$$

As for an arbitrary vector function $\vec{A}$ one has

$$\int_V (\vec{A}\nabla\vec{A} - \vec{A} \times \nabla \times \vec{A})\mathrm{d}V = \oint_S \left[\vec{A}(\vec{n} \cdot \vec{A}) - \frac{1}{2}\vec{n}A^2\right] \mathrm{d}S, \tag{1.62}$$

it follows that

$$\vec{F} = \oint_S \vec{T}\,\mathrm{d}S - \frac{\mathrm{d}}{\mathrm{d}t}\int_V \frac{1}{c^2}\vec{E} \times \vec{H}\,\mathrm{d}V, \tag{1.63}$$

where

$$\vec{T} = \varepsilon_0 \left[\vec{E}(\vec{n} \cdot \vec{E}) - \frac{1}{2}\vec{n}E^2 \right] + \frac{1}{\mu_0} \left[\vec{B}(\vec{n} \cdot \vec{B}) - \frac{1}{2}\vec{n}B^2 \right] \tag{1.64}$$

is the force over the surface unit and is referred as stress tensor.

According (9.4) and (9.13) it can be written that

$$\frac{\mathrm{d}\vec{G}_{\text{meh}}}{\mathrm{d}t} = \oint_S \vec{T}\,\mathrm{d}S - \frac{\mathrm{d}}{\mathrm{d}t} \int_V \frac{1}{c^2} \vec{E} \times \vec{H}\,\mathrm{d}V, \tag{1.65}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\vec{G}_{\text{meh}} + \int_V \frac{1}{c^2} \vec{E} \times \vec{H}\,\mathrm{d}V \right) = \oint_S \vec{T}\,\mathrm{d}S. \tag{1.66}$$

If one deals with an isolated system the term from the right-hand side in (9.16.), which accounts for the surface stress, vanishes and from (9.1) it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V (\rho_m \vec{v} + \varepsilon_0 \vec{E} \times \vec{B})\mathrm{d}V = 0. \tag{1.67}$$

So the expression for the laws of the conservation of quantity of movement in an isolated mechanic-electromagnetic system must be modified, and the expression

$$\vec{g}_{\text{EM}} = \varepsilon_0 \vec{E} \times \vec{B} = \frac{1}{c^2} \vec{E} \times \vec{H} \tag{1.68}$$

is considered to be the density of the quantity of movement of the electromagnetic field.

Electromagnetic momentum, in accordance to mechanics, is then given by

$$\vec{G}_{\text{EM}} = \int_V (\vec{D} \times \vec{B})\mathrm{d}V = \int_V \left(\frac{1}{c^2} \vec{E} \times \vec{H} \right) \mathrm{d}V. \tag{1.69}$$

Therefore, if an arbitrary isolated mechanical system of mass density ρ_{m} and velocity v within a small volume V is considered the quantity of movement is conserved. The law of conservation of quantity of movement, which accounts for *electromagnetic quantity of movement*, could be written in the form

$$\frac{\mathrm{d}}{\mathrm{d}t} (\vec{G}_{\text{meh}} + \vec{G}_{\text{EM}}) = 0, \tag{1.70}$$

i.e. it follows that

$$\vec{G}_{\text{meh.}} + \vec{G}_{\text{EM}} = \text{Const.} \tag{1.71}$$

The existence of stress in the electromagnetic field and the expression of electromagnetic quantity of movement prove the existence of the electromagnetic field as a real physical entity, thus confirming the vision of Michael Faraday.

1.6 Electromagnetic Potentials

Instead of using fields the analysis of electric and magnetic fields can be simplified by using auxiliary potential functions, such as the electric scalar potential φ, or the magnetic vector potential A. The potential functions are readily derived from the Maxwell equations.

Thus, the Maxwell equation (1.7) is satisfied if the flux density B can be expressed in terms of an auxiliary vector A, i.e.

$$\vec{B} = \nabla \times \vec{A}. \tag{1.72}$$

Maxwell curl equation (1.4) then becomes

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}), \tag{1.73}$$

and by rearranging (1.73), it follows that

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0. \tag{1.74}$$

Furthermore, the quantity within brackets in (1.74) can be written as the gradient of the scalar potential function φ:

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla\varphi \tag{1.75}$$

or

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla\phi. \tag{1.76}$$

Thus, knowing the potential functions A and φ, the magnetic and electric fields can be determined from Equations (1.72) and (1.76).

1.7 Solution of the Wave Equation and Radiation Arrow of Time

From the physical point of view natural laws expressed in the mathematical form are symmetric, i.e. they remain the same if one changes the direction of time [11, 13, 14].

Thus, Maxwell's equations appear to be time invariant (there is no preference regarding the time direction). However, the electromagnetic wave equations that are derived from Maxwell's equations have *two solutions* in the form of retarded and advanced potential, respectively. The *retarded* potential solution is considered to have physical meaning and is related to the electromagnetic waves detected at an observation point after they left the source, i.e. the time required for these waves to reach a receiver is delayed with respect to the time measured at the source. On the other hand, the advanced potential solutions pertaining to the waves that would propagate in such a way as to arrive at the detector before they leave the source are mathematically also possible. Such waves are never observed in nature and are eliminated by specifying certain set of boundary and initial conditions, respectively. Note that the physical laws formulated in terms of differential equations are not sufficient to fully describe the natural phenomena as the corresponding initial and boundary conditions, respectively, are to be specified [14].

Basically, this "wave asymmetry," i.e. nonexistence of convergent waves is due to the radiation. Thus, divergent radiated fields exist (related with accelerated charges) and their temporal inverses (convergent waves) are never observed in nature.

The radiation arrow of time could be analyzed by studying the solutions of wave equation for magnetic vector potential A:

$$\nabla^2\vec{A} - \mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2} = -\mu\vec{J}. \tag{1.77}$$

The wave equation (1.77) has *two solutions,* one in the form of *retarded potential*

$$\vec{A}_{\text{ret}}(r,t) = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}\,',t-R/c)}{R}\mathrm{d}V', \tag{1.78}$$

and the other one in the form of *advanced potential*

$$\vec{A}_{\text{adv}}(r,t) = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}\,',t+R/c)}{R}\mathrm{d}V', \tag{1.79}$$

where $R = |\vec{r} - \vec{r}\,'|$ is the distance from the source to the observation point, respectively.

Note that R/c is the time necessary for the signal to arrive from the source point to the observation point (Figure 1.10).

In the case of divergent waves a signal at the source point at time $t' = t - R/c$ is advanced for time R/c compared to the signal at the observation point, i.e. the signal at the observation point at time $t = t' + R/c$ is delayed for time R/c compared to the signal at the source point. Similarly, in the case of convergent waves a signal at the source point at time $t' = t + R/c$ is delayed for R/c compared to the signal at the observation point, or the signal at an observation point at time $t = t' - R/c$ is advanced for time R/c at an observation point with respect to the source point. The source and observation point, respectively, are depicted in Figure 1.10.

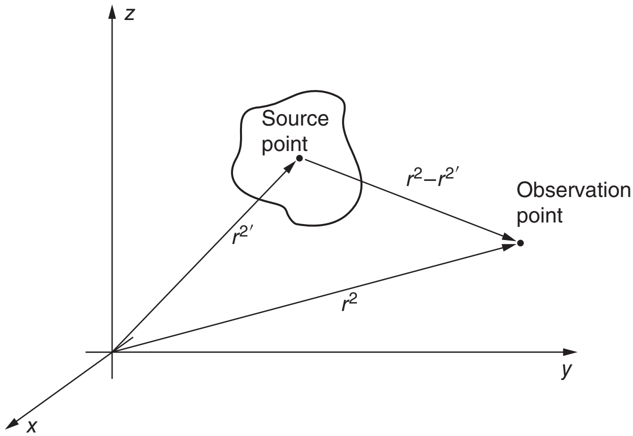

Figure 1.10 The source point and the observation point.

1.8 Complex Phasor Form of Equations in Electromagnetics

In many engineering scenarios, devices or systems are excited sinusoidally and a time-harmonic variation of electromagnetic fields is assumed. In such cases, it is convenient to represent the variables of interest in a complex phasor form. Also, if a transient response is of interest, inverse Fourier transform (IFT) is used to transform the frequency response in the time domain.

1.8.1 The Generalized Symmetric Form of Maxwell's Equations

For a simple medium the time-harmonic, *symmetric* form of Maxwell's equations, i.e. the form in which both electric and fictitious magnetic charges and currents are taken into account, is given by Poljak and Tham [15]

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} - \vec{M}, \tag{1.80}$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \vec{J}, \tag{1.81}$$

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon}\rho_{\mathrm{e}}, \tag{1.82}$$

$$\nabla \cdot \vec{H} = \frac{1}{\mu}\rho_{\mathrm{m}}, \tag{1.83}$$

in which the fictitious magnetic surface current M and magnetic charge density ρ_{m} are introduced. The time-harmonic factor $\mathrm{e}^{j\omega t}$, which is implied, has been omitted in the equations.