
Farhan A. Faruqi

# Differential Game Theory with Applications to Missiles and Autonomous Systems Guidance

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*Farhan A. Faruqi*

*Defence Science & Technology Organisation*
*Australia*

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*To my wife, Yasmeen Mahnaz, and*
*my daughters, Mariam Fazal and Nasheed Qamar*

# Contents

# Preface

This book entitled *Differential Game Theory with Applications to Missiles and Autonomous Systems Guidance* is an outgrowth of many years of the author's experience in missile guidance and control research and development in aerospace and defense organizations in the UK, the USA and Australia. Some of the material included in the book is the result of courses taught to undergraduate and post-graduate students in universities in the USA and Australia. The purpose of this book is to bring to the attention of researchers and engineers working in the field of aerospace guidance and control systems recent developments in the field. There are a number of excellent books on the topic of classical missile guidance theory. In this book the author has endeavored to approach the topic of missile guidance from the optimum game theory perspective. It is shown that the classical guidance approach is closely linked to this approach; in fact, it is demonstrated in Chapter 3 that the classical approach is simply a special case of the modern optimal game theory. This approach offers researchers and engineers a wider choice of system analysis and synthesis options to effectively deal with continuously evolving challenges of current and future missile and aircraft combat scenarios.

As noted in Chapter 1, the game theory has its origins in the field of economics, business, politics and social sciences. These developments have found their way into solving complex and challenging problems in engineering, operations research, and combat mission systems. Readers and practitioners in fields other than engineering will also find this book useful, particularly Chapter 2 which lays down formal mathematical foundations of the differential game theory. This should provide a useful background for readers whose interests encompass economics, business or other areas. Game theory approaches to problem solving, algorithms and their applications to various fields are progressing rapidly; evolutionary and quantum game theories, stochastic games, and diagnostic medicine applications are some examples of this trend. This book has been written to provide a formal and integrated text on the topic of differential game theory and should provide essential background to undergraduate and postgraduate research students in engineering, mathematics and science subjects. Missile guidance simulation examples are given in Chapter 6 and a simulation demonstration website (MATLAB, *.m files) is included with this book (program listing is given in the addendum). This resource should provide the reader with hands-on experience and with a tool to reinforce learning in topics covered in the book.

While this book is focussed on the application of the differential game theory to the missile guidance problem, there are other applications which are closely linked to this and are currently the subject of intense research. These applications include autonomous and intelligent vehicle control; unmanned vehicle formation strategies; UAV and aircraft collision avoidance; surveillance and reconnaissance; and electronic counter-measure and counter-countermeasure deployment. It is hoped that students, researchers and practicing engineers in industry and government as well as interested readers in other fields will find this text both interesting and challenging.

Farhan A. Faruqi

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Dr Farhan A. Faruqi

Senior Research Scientist
Information Processing and Human Systems Group
Combat and Mission Systems, WCSD
DST Group, Edinburgh, South Australia

Adjunct Professor
ITEE Division, University of South Australia
Mawson Lakes Campus, South Australia

# About the Companion Website

Don't forget to visit the companion website for this book:

**www.wiley.com/go/faruqi/game**

There you will find valuable material designed to enhance your learning, including:

- MATLAB codes
- DEMO content

# 1

# Differential Game Theory and Applications to Missile Guidance

## Nomenclature

| | |
|---|---|
| **k:** | is the epoch (in a discrete time game). |
| P: | is the set of players in a game. |
| **U:** | is the set of strategies available to all the players. |
| $\mathbf{U^i}$: | is the set of strategies available to player **i**. |
| $\mathbf{J_{ij}}(\cdots)$: | is the objective function for players **i** and **j**. |
| $\mathbf{X_k}$: | is the set of current state of a game at epoch **k**. |
| $\mathbf{U_k}$: | is the set of strategies available to a player at epoch **k**. |
| $\underline{\mathbf{u}}_{\mathbf{ij}}(\mathbf{k})$: | is the strategy vector (input vector) available to player **i** against player **j** at epoch **k**. |
| $\mathbf{C_k}$: | is the set of constraints at epoch **k**. |
| $\mathbf{G_k}$: | is the set of elements of a discrete-time game. |
| **t:** | is the time in a continuous time (differential) game. |
| $\mathbf{X_t}$: | is the set of states of a game at time **t**. |
| $\mathbf{U_t}$: | is the set of strategies at time **t**. |
| $\underline{\mathbf{u}}_{\mathbf{ij}}(\mathbf{t})$: | is the strategy vector (input vector) available to player **i** against player **j** at time **t**. |
| $\mathbf{C_t}$: | is the set of constraints at time **t**. |
| $\mathbf{G_t}$: | is the set of elements of a continuous time (differential) game. |
| $\underline{\mathbf{x}}_{\mathbf{ij}}(\mathbf{t})$: | is the relative state vector of player **i** w.r.t. player **j** at time **t**. |
| $\underline{\mathbf{u}}_{\mathbf{i}}(\mathbf{t})$: | is the strategy vector (input vector) of player **i**. |
| **F**: | is the state coefficient matrix. |
| **G:** | is the input coefficient matrix. |
| **Q:** | is the PI weightings matrix on the current relative states. |
| **S:** | is the PI weightings matrix on the final relative states. |
| $\{\mathbf{R_i}, \mathbf{R_j}\}$: | are PI weightings matrices on inputs. |

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## Abbreviations

| | |
|---|---|
| APN: | augmented PN |
| CF: | cost function |
| LQPI: | linear system quadratic performance index |
| OF: | objective function |
| PI: | performance index |
| PN: | proportional navigation |
| UF: | utility function |
| 4-DOF: | four degrees of freedom |
| w.r.t.: | with respect to |

## 1.1 Introduction

Over the last few decades a great deal of material has been published covering some of the major aspects of game theory. The well-known publications in this field include "Games and Economic Behaviour" by John von Neumann and Oskar Morgenstern.[1] Since then there has been a significant growth in publication on both the theoretical results and applications. A total of eight Nobel Prizes were given in Economic Sciences for work primarily in game theory, including the one given in 1994 to John Harsanyi, John Nash, and Reinhard Selten for their pioneering work in the analysis of non-cooperative games. In 2005, the Nobel Prizes in game theory went to Robert Aumann and Thomas Schelling for their work on conflict and cooperation through game-theory analysis. In 2007, Leonid Hurwicz, Eric Maskin, and Roger Myerson were awarded the Nobel Prize for having laid the foundations of mechanism design theory. These and other notable works on game theory are given in the references.[2–7]

Cooperative game theory application to autonomous systems with applications to surveillance and reconnaissance of potential threats, and persistent area denial have been studied by a number of authors; useful references on this and allied topics are given at the end of this chapter.[8–15] Usually, the (potential) targets and threats in a battlefield are intelligent and mobile, and they employ counter-strategies to avoid being detected, tracked, or destroyed. These action and counteraction behaviors can be formulated in a game setting, or more specifically, by pursuit/evasion differential games (with multiple players). It is noteworthy that application of differential games to combat systems can be considered to have been started by Rufus P. Isaacs when he investigated pursuit/evasion games.[8] However, most of the theoretical results focus on two-player games with a single pursuer and a single evader, which has since been extended to a multi-player scenarios.

### 1.1.1 Need for Missile Guidance—Past, Present, and Future

Guided missiles with the requirement to intercept a target (usually an aircraft) at a long range from the missile launch point have been in use since WWII. Guidance systems for missiles are needed in order to correct for initial aiming errors and to maintain intercept flight trajectory in the presence of atmospheric disturbances that may cause the missile to go off course. Traditionally, the use of the so-called proportional navigation (PN) guidance (law) provided the means to enable an attacking missile to maintain its intercept trajectory to its target. As aircraft became more agile and capable of high-g maneuvers, which they could use for evading an incoming threat, the PN guidance law was upgraded to the augmented PN (APN) guidance law that compensated for target maneuvers. Zarchan[24] gives a comprehensive explanation of PN and APN guidance implementation and performance. With advances in missile hardware and computer processing (on-board target tracking sensor and processors), most modern missiles now use the APN guidance. Rapid advances in autonomous system technologies have opened up the possibility that next generation aircraft will be pilotless and capable of performing “intelligent” high-g evasive maneuvers. This potential development has prompted missile guidance designers to look at techniques, such as game theory-based guidance and “intelligent” guidance to outwit potential adversaries.

Earlier reported research[16–27] on the application of game theory to the missile guidance problem has concentrated on engagement scenarios that involve two parties, comprising an attacking missile (pursuer) aimed against another missile or aircraft referred to as a target or an evader. In this book, the above approach is extended to a three-party engagement scenario that includes the situation where an attacking missile may have dual objectives—that is, to evade a defending missile and then continue its mission to engage its primary designated high-value target. The role of the defending missile is only to intercept the attacking missile; the attacking missile, on the other hand, must perform the dual role, that of evading the defending missile, as well as subsequently intercepting its primary target—the aircraft. Since participants in this type of engagement are three players (the aircraft target, the attacking missile, and the defending missile), involved in competition, we shall refer to this type of engagement scenario as a three-party game.

Game theory-based linear state feedback guidance laws are derived for the parties through the use of the well-known linear system quadratic performance index (LQPI) approach. Guidance commands generated are lateral accelerations that parties can implement in order either to intercept a target, or to evade an attacker. A missile/target engagement model has been developed, and feedback gain values are obtained by solving the matrix Riccati differential equation. Preliminary simulation results to demonstrate the characteristics of intercept and evasion strategies are included in Chapter 6. Simple (rule-based) intelligent strategies are also considered for enhancing evasion by a target or for improving the chances of intercept for an attacker.

## 1.2 Game Theoretic Concepts and Definitions

Game theory is concerned with studying and characterizing the dynamics of interactions between players involved in a collective and competitive activity or contest, where each player is required to make decisions regarding his/her strategy, and implement this strategy in order to gain an advantage. These decision makers will be referred to as players or parties. Each player’s choice of the strategy, and the advantage gained by implementing this strategy, is defined through an objective function (OF), which that player tries to maximize. The OF in this case is also referred to as a utility function (UF), or pay-off. If a player sets out to minimize the objective function, it is referred to as a cost function (CF) or a loss function. The objective function of a player depends on the strategies (control or input variable) that a player implements in order to optimize the objective function. This involves action of at least one or more players involved in a game. The strategy that each party implements determines the strategies that the other