



Farhan A. Faruqi

Differential Game Theory with Applications to Missiles and Autonomous Systems Guidance

Aerospace Series

Editors **Peter Belobaba, Jonathan Cooper
and Allan Seabridge**

WILEY

**Differential Game Theory with Applications to
Missiles and Autonomous Systems Guidance**

Aerospace Series

Differential Game Theory with Applications to Missiles and Autonomous Systems Guidance	Faruqi	May 2017
Introduction to Nonlinear Aeroelasticity	Dimitriadis	April 2017
Introduction to Aerospace Engineering with a Flight Test Perspective	Corda	March 2017
Aircraft Control Allocation	Durham, Bordignon and Beck	January 2017
Remotely Piloted Aircraft Systems: A Human Systems Integration Perspective	Cooke, Rowe, Bennett. Jr. and Joralmon	October 2016
Adaptive Aeroservoelastic Control	Tewari	March 2016
Theory and Practice of Aircraft Performance	Kundu, Price and Riordan	November 2015
The Global Airline Industry, Second Edition	Belobaba, Odoni and Barnhart	July 2015
Modeling the Effect of Damage in Composite Structures: Simplified Approaches	Kassapoglou	March 2015
Introduction to Aircraft Aeroelasticity and Loads, 2nd Edition	Wright and Cooper	December 2014
Aircraft Aerodynamic Design: Geometry and Optimization	Sóbestor and Forrester	October 2014
Theoretical and Computational Aerodynamics	Sengupta	September 2014
Aerospace Propulsion	Lee	October 2013
Aircraft Flight Dynamics and Control	Durham	August 2013
Civil Avionics Systems, 2nd Edition	Moir, Seabridge and Jukes	August 2013
Modelling and Managing Airport Performance	Zografos, Andreatta and Odoni	July 2013
Advanced Aircraft Design: Conceptual Design, Analysis and Optimization of Subsonic Civil Airplanes	Torenbeek	June 2013
Design and Analysis of Composite Structures: With Applications to Aerospace Structures, 2nd Edition	Kassapoglou	April 2013
Aircraft Systems Integration of Air-Launched Weapons	Rigby	April 2013
Design and Development of Aircraft Systems, 2nd Edition	Moir and Seabridge	November 2012
Understanding Aerodynamics: Arguing from the Real Physics	McLean	November 2012
Aircraft Design: A Systems Engineering Approach	Sadraey	October 2012
Introduction to UAV Systems 4e	Fahlstrom and Gleason	August 2012
Theory of Lift: Introductory Computational Aerodynamics with MATLAB and Octave	McBain	August 2012
Sense and Avoid in UAS: Research and Applications	Angelov	April 2012
Morphing Aerospace Vehicles and Structures	Valasek	April 2012
Gas Turbine Propulsion Systems	MacIsaac and Langton	July 2011
Basic Helicopter Aerodynamics, 3rd Edition	Seddon and Newman	July 2011
Advanced Control of Aircraft, Spacecraft and Rockets	Tewari	July 2011
Cooperative Path Planning of Unmanned Aerial Vehicles	Tsourdous et al	November 2010
Principles of Flight for Pilots	Swatton	October 2010
Air Travel and Health: A Systems Perspective	Seabridge et al	September 2010
Design and Analysis of Composite Structures: With applications to aerospace Structures	Kassapoglou	September 2010
Unmanned Aircraft Systems: UAVS Design, Development and Deployment	Austin	April 2010
Introduction to Antenna Placement & Installations	Macnamara	April 2010
Principles of Flight Simulation	Allerton	October 2009
Aircraft Fuel Systems	Langton et al	May 2009
The Global Airline Industry	Belobaba	April 2009
Computational Modelling and Simulation of Aircraft and the Environment: Volume 1 - Platform Kinematics and Synthetic Environment	Diston	April 2009
Handbook of Space Technology	Ley, Wittmann and Hallmann	April 2009
Aircraft Performance Theory and Practice for Pilots	Swatton	August 2008
Aircraft Systems, 3 rd Edition	Moir & Seabridge	March 2008
Introduction to Aircraft Aeroelasticity And Loads	Wright & Cooper	December 2007
Stability and Control of Aircraft Systems	Langton	September 2006
Military Avionics Systems	Moir & Seabridge	February 2006
Design and Development of Aircraft Systems	Moir & Seabridge	June 2004
Aircraft Loading and Structural Layout	Howe	May 2004
Aircraft Display Systems	Jukes	December 2003
Civil Avionics Systems	Moir & Seabridge	December 2002

Differential Game Theory with Applications to Missiles and Autonomous Systems Guidance

Farhan A. Faruqi

*Defence Science & Technology Organisation
Australia*

WILEY

This edition first published 2017

© 2017 John Wiley & Sons Ltd

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, except as permitted by law. Advice on how to obtain permission to reuse material from this title is available at <http://www.wiley.com/go/permissions>.

The right of Farhan A. Faruqi to be identified as the author of this work has been asserted in accordance with law.

Registered Offices

John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

Editorial Office

The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

Limit of Liability/Disclaimer of Warranty

MATLAB® is a trademark of The MathWorks, Inc. and is used with permission. The MathWorks does not warrant the accuracy of the text or exercises in this book. This work's use or discussion of MATLAB® software or related products does not constitute endorsement or sponsorship by The MathWorks of a particular pedagogical approach or particular use of the MATLAB® software. While the publisher and authors have used their best efforts in preparing this work, they make no representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives, written sales materials or promotional statements for this work. The fact that an organization, website, or product is referred to in this work as a citation and/or potential source of further information does not mean that the publisher and authors endorse the information or services the organization, website, or product may provide or recommendations it may make. This work is sold with the understanding that the publisher is not engaged in rendering professional services. The advice and strategies contained herein may not be suitable for your situation. You should consult with a specialist where appropriate. Further, readers should be aware that websites listed in this work may have changed or disappeared between when this work was written and when it is read. Neither the publisher nor authors shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

Library of Congress Cataloging-in-Publication Data

Names: Faruqi, Farhan A., 1944– author.

Title: Differential game theory with applications to missiles and autonomous systems guidance / Farhan A. Faruqi.

Other titles: Aerospace series (Chichester, England)

Description: Hoboken, NJ : John Wiley & Sons, 2017. | Series: Aerospace series | Includes bibliographical references and index.

Identifiers: LCCN 2017000484 (print) | LCCN 2017008046 (ebook) | ISBN 9781119168478 (cloth) |

ISBN 9781119168508 (pdf) | ISBN 9781119168492 (epub)

Subjects: LCSH: Guided missiles—Guidance systems—Mathematical models. | Guided missiles—Control systems—Mathematical models. | Differential games. | Game theory.

Classification: LCC UG1310 .F37 2017 (print) | LCC UG1310 (ebook) | DDC 623.4/519—dc23

LC record available at <https://lcn.loc.gov/2017000484>

Cover Design: Wiley

Cover Image: ©Makhnach_M/Gettyimages

Set in 10/12pt WarnockPro by Aptara Inc., New Delhi, India

Printed in Great Britain by TJ International Ltd, Padstow, Cornwall

10 9 8 7 6 5 4 3 2 1

*To my wife, Yasmeen Mahnaz, and
my daughters, Mariam Fazal and Nasheed Qamar*

Contents

Preface *xi*

Acknowledgments *xiii*

About the Companion Website *xv*

1	Differential Game Theory and Applications to Missile Guidance	1
	Nomenclature	1
	Abbreviations	2
1.1	Introduction	2
1.1.1	Need for Missile Guidance—Past, Present, and Future	2
1.2	Game Theoretic Concepts and Definitions	3
1.3	Game Theory Problem Examples	4
1.3.1	Prisoner's Dilemma	4
1.3.2	The Game of Tic-Tac-Toe	6
1.4	Game Theory Concepts Generalized	8
1.4.1	Discrete-Time Game	8
1.4.2	Continuous-Time Differential Game	9
1.5	Differential Game Theory Application to Missile Guidance	10
1.6	Two-Party and Three-Party Pursuit-Evasion Game	11
1.7	Book Chapter Summaries	11
1.7.1	A Note on the Terminology Used In the Book	13
	References	14
2	Optimum Control and Differential Game Theory	16
	Nomenclature	16
	Abbreviations	17
2.1	Introduction	17
2.2	Calculus of Optima (Minimum or Maximum) for a Function	18
2.2.1	On the Existence of the Necessary and Sufficient Conditions for an Optima	18
2.2.2	Steady State Optimum Control Problem with Equality Constraints Utilizing Lagrange Multipliers	19
2.2.3	Steady State Optimum Control Problem for a Linear System with Quadratic Cost Function	22
2.3	Dynamic Optimum Control Problem	23
2.3.1	Optimal Control with Initial and Terminal Conditions Specified	23

2.3.2	Boundary (Transversality) Conditions	25
2.3.3	Sufficient Conditions for Optimality	29
2.3.4	Continuous Optimal Control with Fixed Initial Condition and Unspecified Final Time	30
2.3.5	A Further Property of the Hamiltonian	35
2.3.6	Continuous Optimal Control with Inequality Control Constraints—the Pontryagin's Minimum (Maximum) Principle	36
2.4	Optimal Control for a Linear Dynamical System	38
2.4.1	The LQPI Problem—Fixed Final Time	38
2.5	Optimal Control Applications in Differential Game Theory	40
2.5.1	Two-Party Game Theoretic Guidance for Linear Dynamical Systems	41
2.5.2	Three-Party Game Theoretic Guidance for Linear Dynamical Systems	44
2.6	Extension of the Differential Game Theory to Multi-Party Engagement	50
2.7	Summary and Conclusions	50
	References	51
	Appendix	53

3 Differential Game Theory Applied to Two-Party Missile Guidance Problem

	Nomenclature	63
	Abbreviations	64
3.1	Introduction	64
3.2	Development of the Engagement Kinematics Model	67
3.2.1	Relative Engage Kinematics of n Versus m Vehicles	68
3.2.2	Vector/Matrix Representation	69
3.3	Optimum Interceptor/Target Guidance for a Two-Party Game	70
3.3.1	Construction of the Differential Game Performance Index	70
3.3.2	Weighting Matrices S , R^p , R^e	72
3.3.3	Solution of the Differential Game Guidance Problem	73
3.4	Solution of the Riccati Differential Equations	75
3.4.1	Solution of the Matrix Riccati Differential Equations (MRDE)	75
3.4.2	State Feedback Guidance Gains	76
3.4.3	Solution of the Vector Riccati Differential Equations (VRDE)	77
3.4.4	Analytical Solution of the VRDE for the Special Case	78
3.4.5	Mechanization of the Game Theoretic Guidance	79
3.5	Extension of the Game Theory to Optimum Guidance	79
3.6	Relationship with the Proportional Navigation (PN) and the Augmented PN Guidance	81
3.7	Conclusions	82
	References	82
	Appendix	84

4 Three-Party Differential Game Theory Applied to Missile Guidance Problem

	Nomenclature	102
	Abbreviations	103
4.1	Introduction	103

4.2	Engagement Kinematics Model	104
4.2.1	Three-Party Engagement Scenario	105
4.3	Three-Party Differential Game Problem and Solution	107
4.4	Solution of the Riccati Differential Equations	111
4.4.1	Solution of the Matrix Riccati Differential Equation (MRDE)	111
4.4.2	Solution of the Vector Riccati Differential Equation (VRDE)	112
4.4.3	Further Consideration of Performance Index (PI) Weightings	115
4.4.4	Game Termination Criteria and Outcomes	116
4.5	Discussion and Conclusions	116
	References	117
	Appendix	118
5	Four Degrees-of-Freedom (DOF) Simulation Model for Missile Guidance and Control Systems	125
	Nomenclature	125
	Abbreviations	126
5.1	Introduction	126
5.2	Development of the Engagement Kinematics Model	126
5.2.1	Translational Kinematics for Multi-Vehicle Engagement	126
5.2.2	Vector/Matrix Representation	128
5.2.3	Rotational Kinematics: Relative Range, Range Rates, Sightline Angles, and Rates	128
5.3	Vehicle Navigation Model	130
5.3.1	Application of Quaternion to Navigation	131
5.4	Vehicle Body Angles and Flight Path Angles	133
5.4.1	Computing Body Rates (p_i , q_i , r_i)	134
5.5	Vehicle Autopilot Dynamics	135
5.6	Aerodynamic Considerations	135
5.7	Conventional Guidance Laws	136
5.7.1	Proportional Navigation (PN) Guidance	136
5.7.2	Augmented Proportional Navigation (APN) Guidance	137
5.7.3	Optimum Guidance and Game Theory–Based Guidance	137
5.8	Overall State Space Model	138
5.9	Conclusions	138
	References	139
	Appendix	140
6	Three-Party Differential Game Missile Guidance Simulation Study	150
	Nomenclature	150
	Abbreviations	150
6.1	Introduction	151
6.2	Engagement Kinematics Model	151
6.3	Game Theory Problem and the Solution	154
6.4	Discussion of the Simulation Results	157
6.4.1	Game Theory Guidance Demonstrator Simulation	157
6.4.2	Game Theory Guidance Simulation Including Disturbance Inputs	160
6.5	Conclusions	162

6.5.1	Useful Future Studies	162
	References	163
	Appendix	164
	Addendum	165

Index	189
--------------	-----

Preface

This book entitled *Differential Game Theory with Applications to Missiles and Autonomous Systems Guidance* is an outgrowth of many years of the author's experience in missile guidance and control research and development in aerospace and defense organizations in the UK, the USA and Australia. Some of the material included in the book is the result of courses taught to undergraduate and post-graduate students in universities in the USA and Australia. The purpose of this book is to bring to the attention of researchers and engineers working in the field of aerospace guidance and control systems recent developments in the field. There are a number of excellent books on the topic of classical missile guidance theory. In this book the author has endeavored to approach the topic of missile guidance from the optimum game theory perspective. It is shown that the classical guidance approach is closely linked to this approach; in fact, it is demonstrated in Chapter 3 that the classical approach is simply a special case of the modern optimal game theory. This approach offers researchers and engineers a wider choice of system analysis and synthesis options to effectively deal with continuously evolving challenges of current and future missile and aircraft combat scenarios.

As noted in Chapter 1, the game theory has its origins in the field of economics, business, politics and social sciences. These developments have found their way into solving complex and challenging problems in engineering, operations research, and combat mission systems. Readers and practitioners in fields other than engineering will also find this book useful, particularly Chapter 2 which lays down formal mathematical foundations of the differential game theory. This should provide a useful background for readers whose interests encompass economics, business or other areas. Game theory approaches to problem solving, algorithms and their applications to various fields are progressing rapidly; evolutionary and quantum game theories, stochastic games, and diagnostic medicine applications are some examples of this trend. This book has been written to provide a formal and integrated text on the topic of differential game theory and should provide essential background to undergraduate and postgraduate research students in engineering, mathematics and science subjects. Missile guidance simulation examples are given in Chapter 6 and a simulation demonstration website (MATLAB, *.m files) is included with this book (program listing is given in the addendum). This resource should provide the reader with hands-on experience and with a tool to reinforce learning in topics covered in the book.

While this book is focussed on the application of the differential game theory to the missile guidance problem, there are other applications which are closely linked to this and are currently the subject of intense research. These applications include

autonomous and intelligent vehicle control; unmanned vehicle formation strategies; UAV and aircraft collision avoidance; surveillance and reconnaissance; and electronic counter-measure and counter-countermeasure deployment. It is hoped that students, researchers and practicing engineers in industry and government as well as interested readers in other fields will find this text both interesting and challenging.

Farhan A. Faruqi

Acknowledgments

I gratefully acknowledge the opportunity, support and encouragements by the Defence Science and Technology (DST) Group, Australia, in making the writing of this book possible. I would also like to thank the Chief of Weapon and Combat Systems Division (WCSD) and the staff for their support in completing this work. Many other people have helped me and offered advice that has made this book possible. In this regard I wish to thank Jim Repo (formerly DST Group staff) and Arvind Rajagopalan (DST Group) for their help, advice and suggestions that contributed much to the book, particularly the guidance simulation test bed. I would like to thank the many staff members working with guidance and control technology at DST Group and with whom I have worked in the past and learnt much from them. I would like to thank Dr Sanjeev Arulampalam, Dr Mark Krieg and Mr Paul Heuer of DST Group for reviewing the technical contents of the book and for their comments and suggestions. I would also like to thank the many academics from the University of South Australia, and others from the University of Adelaide, for useful discussions and inspirational ideas on the topic of game theory. Finally I would like to thank my daughter Nasheed Qamar for taking on the unenviable task of editing this book and for her suggestions and advice.

Dr Farhan A. Faruqi

Senior Research Scientist
Information Processing and Human Systems Group
Combat and Mission Systems, WCSD
DST Group, Edinburgh, South Australia

Adjunct Professor
ITEE Division, University of South Australia
Mawson Lakes Campus, South Australia

About the Companion Website

Don't forget to visit the companion website for this book:

www.wiley.com/go/faruqi/game



There you will find valuable material designed to enhance your learning, including:

- MATLAB codes
- DEMO content

1

Differential Game Theory and Applications to Missile Guidance

Nomenclature

k :	is the epoch (in a discrete time game).
P :	is the set of players in a game.
U :	is the set of strategies available to all the players.
U^i :	is the set of strategies available to player i .
$J_{ij}(\dots)$:	is the objective function for players i and j .
X_k :	is the set of current state of a game at epoch k .
U_k :	is the set of strategies available to a player at epoch k .
$\underline{u}_{ij}(k)$:	is the strategy vector (input vector) available to player i against player j at epoch k .
C_k :	is the set of constraints at epoch k .
G_k :	is the set of elements of a discrete-time game.
t :	is the time in a continuous time (differential) game.
X_t :	is the set of states of a game at time t .
U_t :	is the set of strategies at time t .
$\underline{u}_{ij}(t)$:	is the strategy vector (input vector) available to player i against player j at time t .
C_t :	is the set of constraints at time t .
G_t :	is the set of elements of a continuous time (differential) game.
$\underline{x}_{ij}(t)$:	is the relative state vector of player i w.r.t. player j at time t .
$\underline{u}_i(t)$:	is the strategy vector (input vector) of player i .
F :	is the state coefficient matrix.
G :	is the input coefficient matrix.
Q :	is the PI weightings matrix on the current relative states.
S :	is the PI weightings matrix on the final relative states.
$\{R_i, R_j\}$:	are PI weightings matrices on inputs.

Abbreviations

APN:	augmented PN
CF:	cost function
LQPI:	linear system quadratic performance index
OF:	objective function
PI:	performance index
PN:	proportional navigation
UF:	utility function
4-DOF:	four degrees of freedom
w.r.t.:	with respect to

1.1 Introduction

Over the last few decades a great deal of material has been published covering some of the major aspects of game theory. The well-known publications in this field include “Games and Economic Behaviour” by John von Neumann and Oskar Morgenstern.^[1] Since then there has been a significant growth in publication on both the theoretical results and applications. A total of eight Nobel Prizes were given in Economic Sciences for work primarily in game theory, including the one given in 1994 to John Harsanyi, John Nash, and Reinhard Selten for their pioneering work in the analysis of non-cooperative games. In 2005, the Nobel Prizes in game theory went to Robert Aumann and Thomas Schelling for their work on conflict and cooperation through game-theory analysis. In 2007, Leonid Hurwicz, Eric Maskin, and Roger Myerson were awarded the Nobel Prize for having laid the foundations of mechanism design theory. These and other notable works on game theory are given in the references.^[2–7]

Cooperative game theory application to autonomous systems with applications to surveillance and reconnaissance of potential threats, and persistent area denial have been studied by a number of authors; useful references on this and allied topics are given at the end of this chapter.^[8–15] Usually, the (potential) targets and threats in a battlefield are intelligent and mobile, and they employ counter-strategies to avoid being detected, tracked, or destroyed. These action and counteraction behaviors can be formulated in a game setting, or more specifically, by pursuit/evasion differential games (with multiple players). It is noteworthy that application of differential games to combat systems can be considered to have been started by Rufus P. Isaacs when he investigated pursuit/evasion games.^[8] However, most of the theoretical results focus on two-player games with a single pursuer and a single evader, which has since been extended to a multi-player scenarios.

1.1.1 Need for Missile Guidance—Past, Present, and Future

Guided missiles with the requirement to intercept a target (usually an aircraft) at a long range from the missile launch point have been in use since WWII. Guidance systems for missiles are needed in order to correct for initial aiming errors and to maintain intercept flight trajectory in the presence of atmospheric disturbances that may cause the missile to go off course. Traditionally, the use of the so-called proportional navigation (PN) guidance (law) provided the means to enable an attacking missile to maintain its

intercept trajectory to its target. As aircraft became more agile and capable of high-g maneuvers, which they could use for evading an incoming threat, the PN guidance law was upgraded to the augmented PN (APN) guidance law that compensated for target maneuvers. Zarchan^[24] gives a comprehensive explanation of PN and APN guidance implementation and performance. With advances in missile hardware and computer processing (on-board target tracking sensor and processors), most modern missiles now use the APN guidance. Rapid advances in autonomous system technologies have opened up the possibility that next generation aircraft will be pilotless and capable of performing “intelligent” high-g evasive maneuvers. This potential development has prompted missile guidance designers to look at techniques, such as game theory-based guidance and “intelligent” guidance to outwit potential adversaries.

Earlier reported research^[16–27] on the application of game theory to the missile guidance problem has concentrated on engagement scenarios that involve two parties, comprising an attacking missile (pursuer) aimed against another missile or aircraft referred to as a target or an evader. In this book, the above approach is extended to a three-party engagement scenario that includes the situation where an attacking missile may have dual objectives—that is, to evade a defending missile and then continue its mission to engage its primary designated high-value target. The role of the defending missile is only to intercept the attacking missile; the attacking missile, on the other hand, must perform the dual role, that of evading the defending missile, as well as subsequently intercepting its primary target—the aircraft. Since participants in this type of engagement are three players (the aircraft target, the attacking missile, and the defending missile), involved in competition, we shall refer to this type of engagement scenario as a three-party game.

Game theory-based linear state feedback guidance laws are derived for the parties through the use of the well-known linear system quadratic performance index (LQPI) approach. Guidance commands generated are lateral accelerations that parties can implement in order either to intercept a target, or to evade an attacker. A missile/target engagement model has been developed, and feedback gain values are obtained by solving the matrix Riccati differential equation. Preliminary simulation results to demonstrate the characteristics of intercept and evasion strategies are included in Chapter 6. Simple (rule-based) intelligent strategies are also considered for enhancing evasion by a target or for improving the chances of intercept for an attacker.

1.2 Game Theoretic Concepts and Definitions

Game theory is concerned with studying and characterizing the dynamics of interactions between players involved in a collective and competitive activity or contest, where each player is required to make decisions regarding his/her strategy, and implement this strategy in order to gain an advantage. These decision makers will be referred to as players or parties. Each player’s choice of the strategy, and the advantage gained by implementing this strategy, is defined through an objective function (OF), which that player tries to maximize. The OF in this case is also referred to as a utility function (UF), or pay-off. If a player sets out to minimize the objective function, it is referred to as a cost function (CF) or a loss function. The objective function of a player depends on the strategies (control or input variable) that a player implements in order to optimize the objective function. This involves action of at least one or more players involved in a game. The strategy that each party implements determines the strategies that the other