

Harnessing Bistable Structural Dynamics

For Vibration Control, Energy
Harvesting and Sensing

Ryan L. Harne and K. W. Wang



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Preface

This book describes a recent perspective that seeks to strategically harness the unique dynamics of bistable structural systems for engineering advances, with focus on the three technical areas of vibration control, energy harvesting, and sensing.

When a structural system exhibits two statically-stable configurations, it is said to be *bistable*. This class of structures has been employed in various mechanical, civil, marine, and aerospace engineering applications for many years. The bistable components include arches and post-buckled beams/columns, panels or shells having a shallow curvature, and curved microbeam or dome diaphragm transducers, to name just a few examples. In many historical assessments, it was undesirable for the bistable structure to “snap” to the second state of static equilibrium – a phenomenon referred to as *snap-through* – because the consequence might be unfavorable to the health and performance of the overall engineered system. Thus, the structures or materials were used in ways to avoid static or dynamic loading (e.g., pressure on a shell) that could cause the bistable system to switch from the original, stable configuration to the other stable equilibrium (e.g., an inverted shell).

It is from such a perspective that the focus of this book departs. Recently, researchers have been challenged to reconsider bistable structural systems within a variety of emerging engineering contexts. Many scientists and engineers have discovered and explored the dynamics of bistable structures that may be deliberately exploited to the advantage of certain applications. Innovative ideas have been proposed to intelligently induce snap-through behaviors such that the performance of the overall system is enhanced and/or new functionality is realized. This new spirit of engineering system development is the foundational viewpoint of this book: *harnessing bistable structural dynamics*.

The new ideas have been found to be well-suited for many applications across a variety of engineering disciplines. Among them, researchers have particularly investigated the exploitation of bistable structural dynamics in the areas of (i) *vibration control*, (ii) *vibration energy harvesting*, and (iii) *sensing and detection*. In the first area, the dynamics of bistable devices integrated with the structural system are used so as to isolate, dissipate, or reactively attenuate the input energies that excite the system. Depending on the performance objective, the multi-faceted dynamics of the bistable members are strategically employed to best control the vibrations and improve the operational integrity of the system.

The aim of energy harvesting is to electromechanically convert ambient vibrations into usable electrical power resources. To this end, maintaining persistent snap-through behaviors is a common aim because of the large dynamic mechanical and electrical response amplitudes that they induce in the energy harvesting systems. Hence, the energetics of snap-through promotes a significant potential for energy conversion. In the context of sensing and detection, the ability to recognize small changes in monitored parameters, which represent time-varying system characteristics, is critical in providing the earliest indicator of structural change. Thus, by harnessing the sudden transition between low amplitude oscillations around a stable equilibrium to energetic snap-through vibrations spanning the static equilibria, bistable dynamics-based detection strategies have been found as promising, novel approaches well-suited for a variety of sensing contexts.

The recent interest in harnessing bistable structural dynamics for engineering advances in vibration control, energy harvesting, and sensing has led to a flourishing research development that ranges from rigorous theoretical formulations to exploratory experimental studies. The comprehensive investigations have drawn a variety of conclusions regarding the exploitation of bistable dynamics that may best promote the aims of the respective technical area and applications, and the continued active research engagements suggest that much is left to be discovered.

The insights on the effective exploitation of bistable dynamics to enhance the aims of the aforementioned three technical areas are currently scattered amongst numerous scientific publications and proceedings, including a selection of works by the authors. To derive the greatest benefits from the assorted findings, there is a need to consolidate the results and, with the organized evidence, to derive coherent conclusions that are informed by the broad range of research. This book seeks to meet this need through the presentation of extensive topical reviews and detailed case studies which complement the overall conclusions. For the benefit of the ongoing efforts, this book also seeks to identify the emerging areas and outstanding needs that require future attention before the exploitation of bistable dynamics meets its fullest potential for vibration control, energy harvesting, and sensing and detection.

A large body of scientists and engineers is represented among these technical areas. The approaches and conclusions described in detail in this book will inspire researchers investigating the exploitation of bistability in engineering systems, as well as enlightening a broad range of readers interested in vibration control, energy harvesting, and sensing to the attractive opportunities that may be engendered via harnessing bistable structural dynamics. By reviewing the developments and presenting specific studies as examples, a goal of this book is to provide an accessible avenue for a large readership to an appreciation of the recent findings from the engineering objective lens. We assume that readers have a college level undergraduate education in an engineering curriculum including engineering mathematics and structural dynamics or vibrations. Exposure to more advanced topics such as electromechanical systems, nonlinear dynamics, and stochastic vibrations is beneficial but not essential. Additionally, this book focuses on all-mechanical, electromechanical, or all-electrical bistable systems,

which may be realized or adequately modeled as one-dimensional systems; such platforms represent a large number and wide variety of bistable devices that have been investigated. For readers interested in more details on a specific topic or concept, references to many books and papers are provided throughout the book.

We are indebted to financial support from sponsors in order to conduct and document the research included in this book. Parts of the research were supported by grants from the Air Force Office of Scientific Research, the Defense Advanced Research Projects Agency, the National Science Foundation, by the University of Michigan Collegiate Professorship Fund, and by funds from the Department of Mechanical and Aerospace Engineering at The Ohio State University. We also recognize the contributions from our graduate students and co-workers so that the breadth of research topics in this book may be so comprehensively investigated. These individuals include David Johnson, Jinki Kim, Fabio Semperlotti, Manoj Thota, Zhen Wu, and Kai Yang. In addition, we wish to acknowledge our many colleagues within the professional communities of adaptive materials and structures, dynamics and vibration, energy harvesting, structural sensing and health monitoring, and systems and controls who have cultivated a vigorous environment of search and discovery within which we have enthusiastically undertaken and compiled the efforts of this book. Finally, our deepest appreciations go to our families and loved ones for their tremendous support throughout the years.

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1

Background and Introduction

This chapter provides a background to the common realizations and dynamics of bistable structures which have recently been harnessed to advance the aims of the three technical areas of interest in this book: vibration control, vibration energy harvesting, and sensing and detection. The structural forms and dynamical behaviors of bistable systems are first introduced to demonstrate the broad versatility of design and response which are commonly exploited, and to highlight the dynamics which are enabled via leveraging bistability. Then, two aspects of bistable structural dynamics are identified as trademark features which serve as common rationales for exploiting bistability in engineering applications. These aspects are elaborated upon through concise descriptions of example implementations within the technical areas considered here. Finally, an outline of the remaining chapters is provided.

1.1 Examples of Bistable Structures and Systems

To introduce the essential static characteristics of a bistable structure, a schematic of an example one-dimensional, mechanical bistable system is shown in Figure 1.1. To set a clear convention, hereafter the terms *bistable structures* and *systems* will be used interchangeably, irrespective of whether the object considered is all-mechanical, electromechanical, or all-electrical. For the bistable system shown in Figure 1.1, two identical springs of undeformed lengths l_0 connect a lumped mass to a surrounding frame of span $2d$. It is assumed that all displacements of the system are in a horizontal direction, such that the frame motions z and mass displacements X move along one axis. When the undeformed spring length is less than or equal to half of the span of the frame, $l_0 \leq d$, the system is monostable and the mass will come to rest at the zero displacement position, $X = 0$. In contrast, when the undeformed spring length is greater than half of the span of the frame, $l_0 > d$, the springs exert a force on the mass such that the mass cannot be easily maintained in the central configuration. The zero displacement configuration is unstable, while two stable positions of the mass are adjacent (and, here, symmetric) to the central, unstable state. As a result of the geometric design condition $l_0 > d$, the mass-spring and frame system is said

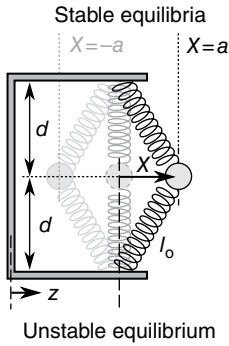


Figure 1.1 Schematic of a bistable system composed of springs, mass, and frame. Here $l_0 > d$ which makes the central configuration $X = 0$ an unstable position of static equilibrium.

to be *bistable*. The stable equilibria of the structure are shown to be configurations of the mass such that the displacements are $X = \pm a$.

Figure 1.2a,b illustrates the force, $F(X)$, and potential energy, $U(X)$, of the bistable system, respectively, as functions of the mass displacement. Potential energy is determined by $U = \int F \cdot dX$. In this example, the bistable structure is symmetric and the only forces resisting the horizontal mass displacements are due to the identical springs. Figure 1.2a shows that the total restoring force in the X -axis is zero when the inertial mass is positioned at any of the equilibria. On the other hand, Figure 1.2b shows that the potential energy is locally maximized at the unstable central configuration of the inertial mass $X = 0$, while the adjacent, stable equilibria at $X = \pm a$ locally minimize the potential energy of the system. Therefore, based on the principle of minimum total potential energy, disturbances to the inertial mass when it is originally positioned at the unstable equilibrium will propel the mass towards one of the stable equilibria.

An instructive analogy is that of a ball on a terrain with elevation profile shaped like the potential energy plot in Figure 1.2b. While situated precisely at the peak of the central hill (at $X = 0$), the ball will remain stationary even though the gravitational potential energy is high. But if given a slight perturbation, the ball will roll into the nearest valley where it settles into the displacement position which minimizes potential energy (specifically, gravitational potential energy in this analogy).

By Hooke's law, the stiffness of a spring element is determined by the spatial derivative of the restoring force, dF/dX . Considering the total spring force profile at the unstable equilibrium in Figure 1.2a, it is apparent that the bistable spring is characterized as having a *negative stiffness* for this mass position. In contrast to a spring which *resists* the motion of the mass in a given direction, a spring exhibiting negative stiffness over a range of displacements will *assist* the motion of the mass. As a result, the small perturbation to the inertial mass when precisely positioned at the unstable equilibrium will lead the bistable spring to propel the mass away from the central location to one of the stable system configurations.

All the bistable systems considered in this book share these fundamental, static characteristics illustrated using the mechanical example in Figure 1.1. In fact, the existence of two statically-stable equilibria configurations and one unstable configuration make it straightforward to identify bistable structures or systems. However, the type of geometrical constraints exemplified in the mechanical

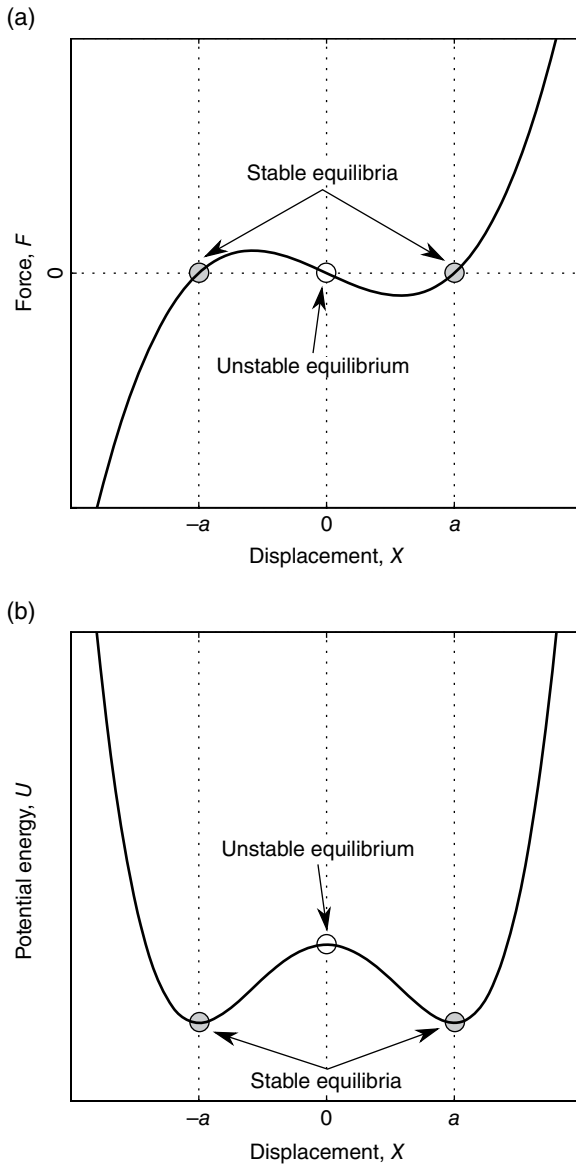


Figure 1.2 Dependence of (a) spring force and (b) stored potential energy on the displacement position of the inertial mass.

device shown in Figure 1.1 are just one possible approach to effect bistability. For the technical areas of vibration control, energy harvesting, or sensing, numerous and diverse methods are employed to realize bistability.

To sample the many approaches, Figure 1.3 shows recently investigated engineering systems that utilize one of the various methods to effect bistability. In Figure 1.3a, modules of series- and parallel-assembled double-beam units are compressed within a housing support near the threshold of buckling. When the

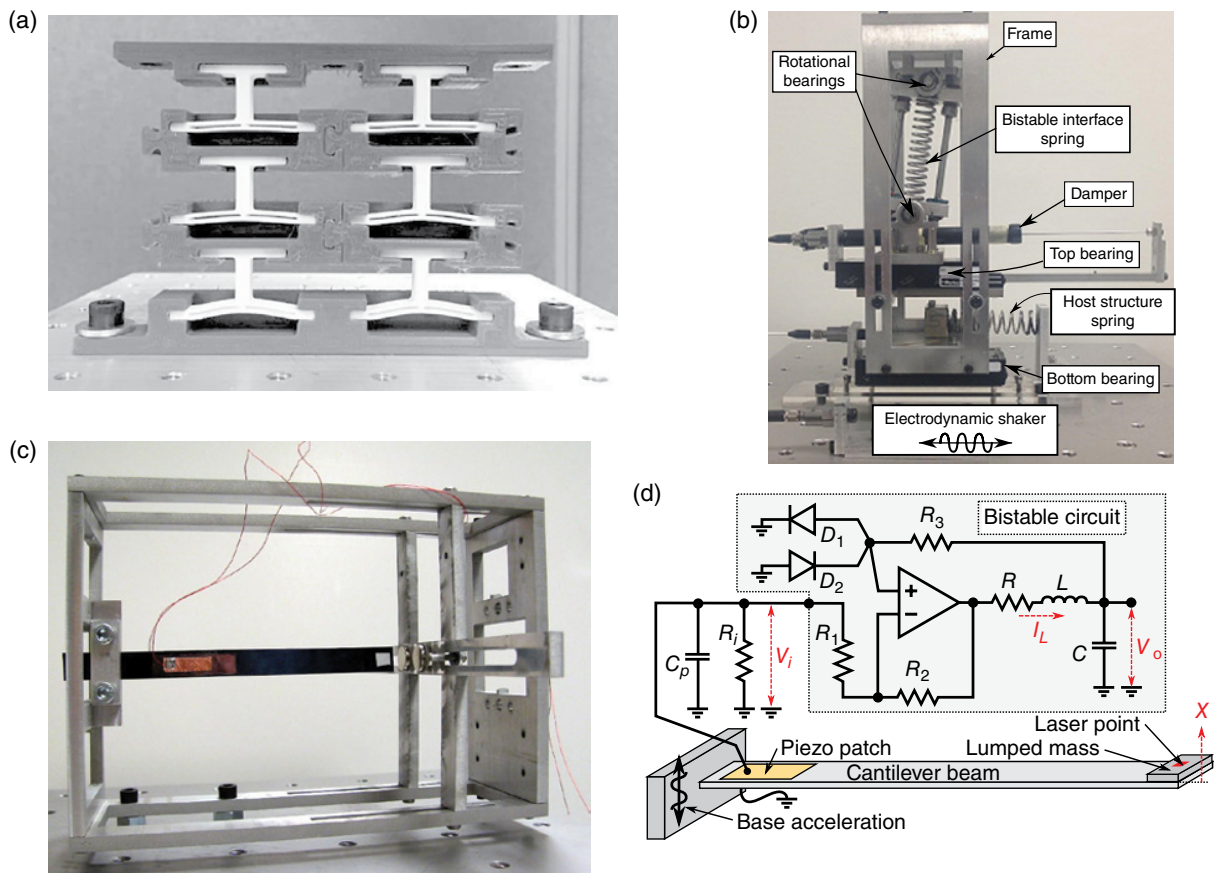


Figure 1.3 (a) Damping module of buckling beams arranged in series and parallel within a housing frame for energy attenuation between the ends of the module. (b) Bistable device for vibration isolation of suspended top bearing mass. (c) Cantilevered beam with piezoelectric PVDF patches and magnetic attraction induced bistability for energy harvesting. (d) Bistable circuitry attached to host beam structure via piezoelectric transducer for sensing structural change.

statically compressed modular structure is excited with periodic dynamic loads, the modules provide high damping due to the transitions among the various stable topologies. Such transition phenomena result in a large dissipation of the input energy according to the number of state changes that occur. In Figure 1.3b, a two-degrees-of-freedom (DOF) vibration suspension concept is shown [1]. Here, bistability is effected between the DOF (the moving frame and top bearing mass) via geometric relations, comparing the length of the interfacing spring to the distance between the frame and top bearing mass. Due to the activation of unique bistable dynamics, the investigations of the two DOF suspension system uncover an exceptional reduction of motion transmissibility as compared to the counterpart two DOF linear suspension [1]. Note that the geometrical constraints in the two DOF architecture shown in Figure 1.3b which lead to bistability between the two moving bodies are similar to those constraints employed in the translational single DOF bistable system example shown in Figure 1.1.

In some vibration energy harvesting applications [2], a composite plate with attached piezoelectric patches is made bistable through the generation of a static stress for the flattened plate configuration, such that the plate maintains one of two stable equilibria shapes having finite curvatures. There are numerous design and fabrication parameters which may be adjusted to tailor the two stable plate curvatures, including the composite material layer selection, lay-up order, layer relative rotations, and thermal conditions under which the layers are stacked and cured [2,3]. When the plate is excited at its center by external vibrations, the energetic snap-through actions of the plate from one stable curvature to the next greatly strain piezoelectric materials attached to the plate surface. In consequence, the input vibrational energy is converted by the piezoelectric material to an oscillating flow of current which can be exploited for energy storage purposes (e.g., battery charging) or conditioned for direct utilization as a supply for low-power microelectronics. In another vibration energy harvesting investigation, bistability is realized using a combination of elastic and magnetic restoring forces on a cantilever beam, as shown in Figure 1.3c [4]. Motions of the ferromagnetic cantilever beam tip are resisted via the beam's inherent elasticity but the attractive influences of the magnet pair are tuned so as to draw the beam away from the original cantilevered configuration. The piezoelectric PVDF (polyvinylidene fluoride) patches attached to the beam surface are significantly strained as ground motions excite the cantilever base and cause the beam to oscillate between the stable beam positions. Similar to the case of the bistable plate, the charge generation on the piezoelectric material electrodes may then be harnessed for energy harvesting purposes. The broad frequency bandwidth and high sensitivity at low frequencies of snap-through dynamics set bistable energy harvesters apart from other harvester platform designs and strongly justify the recent attention given to bistable structures for energy harvesting applications [5,6].

Bistability may also be effected via electromechanical or electrical means. For instance, due to electrostatic actuation including bias and oscillating voltages, a microbeam having an initial curvature may be excited to oscillate between two stable beam shapes [7]. In this example, the beam may or may not be inherently bistable due to elastic influences alone. Nevertheless, the electrostatic forces provide a means to ensure bistability and then to actuate the beam between the

two stable shapes. The applications for micro- and nanoscale bistable systems are numerous, and include their implementation as electromechanical signal filters, switches, actuators, or as novel mechanical memory elements, to name only a few of the functions for which they have been explored [7–10]. Additionally, the circuit schematic shown in Figure 1.3d realizes bistability simply by the circuit design [11]. In the absence of an input voltage V_i , the output voltage V_o across the capacitor C will settle on a finite positive or negative value. Bistable circuitry is a well-established tool in the study of many physical sciences [12], but the exploitation of such circuit designs to advance the aims in engineering contexts is an emerging area of technical interest. For example, when the circuit input is connected to a structural transducer, such as a piezoelectric patch as shown in Figure 1.3d, the activation of the high amplitude snap-through response of the bistable circuit output voltage V_o can be harnessed as an indicator of change in the structural system. By this strategy, one may monitor shifts in important parameters, such as a reduction in stiffness which could indicate damage. Thus, small variations in structural responses of the beam are tracked using large changes in bistable circuit voltage dynamics. Through a relation between the changing structural model parameters and the critical conditions that activate the circuit snap-through dynamics, one realizes a novel pathway for robust and sensitive detection of change [11].

As exemplified by the cross-disciplinary sample described above, the bistable systems considered in this book represent diverse mechanical, electromechanical, and electrical platforms that span a vast range of length scales. Yet, regardless of the specific manifestation and inducement of bistability, the existence of two statically-stable configurations and one unstable configuration is the common denominator for all bistable systems. However, the mechanics of bistable structures are historically-established science [13]. Thus, following the previous introduction of the essential static features of bistable systems and how they have been realized in structural or system forms, the next section introduces the numerous, characteristic *dynamics* of bistable structures. These are the behaviors which have been recently harnessed to the advantage of the technical areas of interest in this book.

1.2 Characteristics of Bistable Structural Dynamics

Regardless of the method by which bistability is effected, each platform shown in Figure 1.3 exemplifies dynamics common to all bistable systems. Moreover, these dynamics have particularly unique manifestations as compared to the responses exhibited by linear or monostable nonlinear systems, which justifies the recent research attention given to bistable structures in the technical areas of interest in this book. The following sections review the distinct dynamic characteristics of bistable structures and provide illustrative and exemplary plots of the behaviors. Before elaborating on the unique dynamical features, a brief description is provided below regarding the means to model and predict such dynamics and the interpretations of the parameters that have been employed to generate the illustrative results.

The illustrative plots presented in the following sections are computed as the dynamic solutions to the governing equation of motion of a bistable system. The analytical and numerical techniques to solve the equation, so as to generate the following plots, are described in detail in Chapter 2. A conventional governing equation construction is used which is known to accurately model the dynamics of numerous bistable structure realizations that have been examined across a wide range of sciences and engineering fields. The equation may be expressed in a normalized form as

$$\frac{d^2x}{d\tau^2} + \gamma \frac{dx}{d\tau} - x + \beta x^3 = p \cos \omega \tau \quad (1.1)$$

where $x(\tau)$ is considered to be the non-dimensional, generalized displacement of the bistable structural system which is a function of non-dimensional time τ ; γ is a damping factor; β is a degree of nonlinearity; p is the excitation level. The term ω is the excitation frequency which is normalized according to the system's natural frequency computed using the magnitude of the linear stiffness and the mass of the non-normalized system [14]; thus, $\omega = 1$ indicates a resonance-like excitation, although the analogy is imperfect. The transient solution to Eq. (1.1) also depends on the initial conditions of displacement and velocity, $x(0)=x_0$ and $\dot{x}(0)=\dot{x}_0$, respectively, where an overdot indicates differentiation with respect to the normalized time, $d/d\tau$.

Although the following examples describe the characteristic dynamics computed from Eq. (1.1) in regards to a bistable *mechanical* system response, in which case terms such as displacement and velocity will be used, it is noted for completeness that Eq. (1.1) also models bistable systems realized in electrical domains. Thus, the coordinate x may represent a normalized voltage or current and the corresponding system and excitation parameters $(p, \omega, \gamma, \beta)$ may be described in terms of the appropriate counterpart electrical components.

The unforced, static solution to Eq. (1.1) leads to the determination of the equilibria configurations of the bistable system. In this case, one solves an equation expressed by

$$-x + \beta x^3 = 0 \quad (1.2)$$

The solutions to the polynomial in Eq. (1.2) are $x = \pm 1$ and 0. Further mathematical treatment of these results, detailed in Chapter 2, reveals that the equilibria at $x = \pm 1$ are *stable* whereas the equilibrium at $x = 0$ is *unstable*.

Using the analytically or numerically computed solutions to the governing equation and by taking into consideration the bistable system equilibria configurations, the following sections elaborate on the characteristics of bistable structural dynamics. While some of the general dynamical features are exhibited by other types of nonlinear systems, the unique ways in which they are realized by bistable structures are detailed to highlight the important distinctions. For brevity, the following sections describe and elucidate the plotted results while the figure captions detail the specific parameter set combinations and relevant computational aspects to generate the data.

1.2.1 Coexistence of Single-periodic, Steady-state Responses

In contrast to linear systems, when driven by single-frequency excitations many nonlinear systems may potentially exhibit one or more forms of steady-state response that *coexist* as a result of the same excitation conditions. These vibrations occur primarily at the excitation frequency. The coexistence of dynamic responses means that for a set of design parameters and prescribed harmonic excitation level and frequency, the nonlinear system may undergo different dynamics in time, often one of two distinct harmonic response amplitudes. The specific dynamic that occurs is dependent upon the system initial conditions at the starting time of the single-frequency excitation.

In the case of bistable systems, there are numerous types of single-periodic, steady-state dynamics which may coexist. First, one may classify the dynamics into two regimes: *intrawell* oscillations which occur around one of the two stable equilibria, or *interwell* oscillations which vibrate across the unstable equilibrium twice per excitation cycle. Because the potential energy profile of bistable systems, shown in Figure 1.2b, is conventionally referred to as the *double-well* potential, the terms *intrawell* and *interwell* denote that the oscillations of the inertial mass remain confined to one of the local wells of potential energy or cross back-and-forth between them, respectively. One may then separate these two dynamic regimes into low and high amplitude versions of the intra- and interwell dynamics. As a result, there are *four* forms of single-periodic, steady-state dynamics exhibited by a bistable structure, some of that may coexist for the same excitation and system design parameters.

As examples of the two forms of *intrawell* dynamics, Figure 1.4a presents numerically computed displacement responses using Eq. (1.1). Here, the different initial conditions lead to unique steady-state dynamics while all other parameters remain the same. It is clear that the intrawell dynamics, plotted as the black or gray solid curves, oscillate around one of the two stable equilibria shown as thick dotted lines. Although not able to be observed using one example, the low and high amplitude oscillations, as predicted from Eq. (1.1), may occur around either stable equilibrium. In the example presented in Figure 1.4a, the initial conditions are such that the high amplitude oscillations vibrate around the stable equilibrium at $x = -1$ while the low amplitude responses occur around $x = +1$. Figure 1.4b plots the displacement amplitude across all frequencies as a consequence of the excitation at frequency ω . The data are computed using the fast Fourier transform (FFT) of the displacement time series shown in Figure 1.4a; the final 50 of 100 excitation periods are used in the computation. The spectra shown in Figure 1.4b plainly indicate that the different intrawell dynamics are distinct in their amplitude at the excitation frequency, shown as the circle points. The input energy at the frequency of ω is apparently diffused to other harmonics; further discussion of this feature is deferred until Section 1.2.6.

Figure 1.5a plots an example in which the low and high amplitude *interwell* dynamics are found to coexist for the same excitation frequency and level. The interwell behaviors are distinct from the intrawell responses in that the prior oscillate across the unstable equilibrium position, which was identified above to be the normalized position of $x = 0$. Apart from the difference in the amplitudes

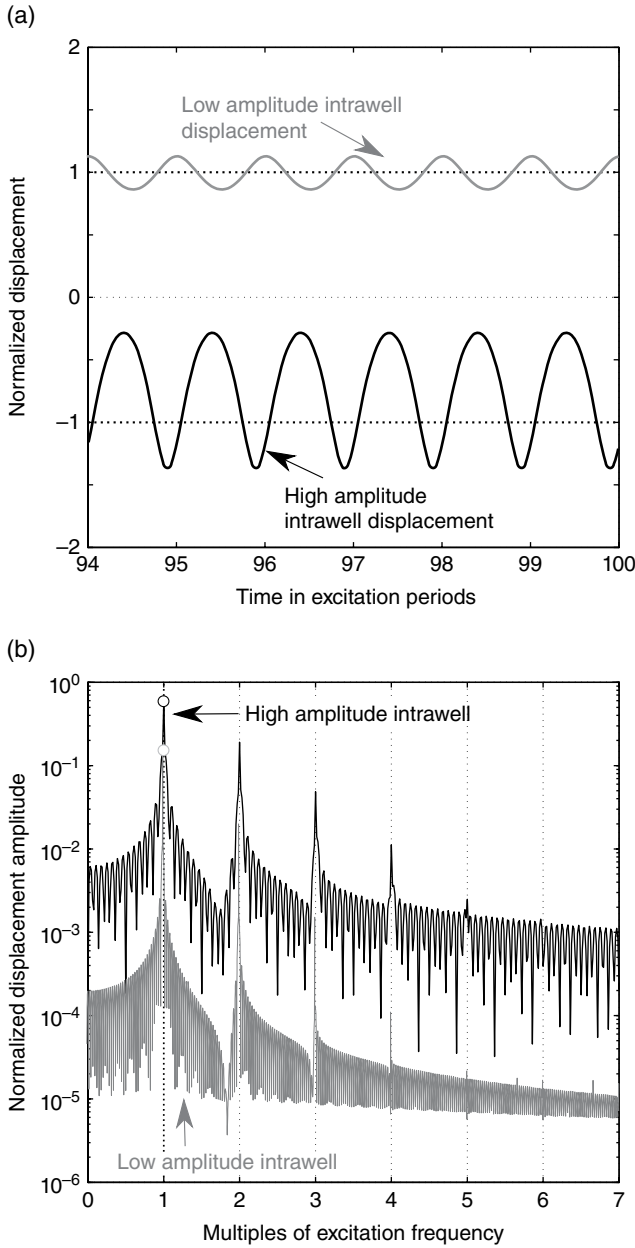


Figure 1.4 (a) For an excitation frequency at which both the low and high intrawell dynamics coexist, the time series of the displacement responses are presented. The thick dotted lines indicate the statically-stable equilibria. (b) The displacement magnitude spectral responses of the low and high amplitude intrawell dynamics are shown as computed from the time series data plotted in (a). The energy diffusion to other harmonics of the excitation frequency is apparent and distinct between the response forms. In (a,b), the parameters are $(p, \omega, \gamma, \beta) = (0.1, 1.1, 0.09, 1)$, with initial conditions $(x_0, \dot{x}_0) = (1.0041, 0.0961)$ for the low amplitude oscillations and $(2.2858, -0.9302)$ for the high amplitude dynamics.

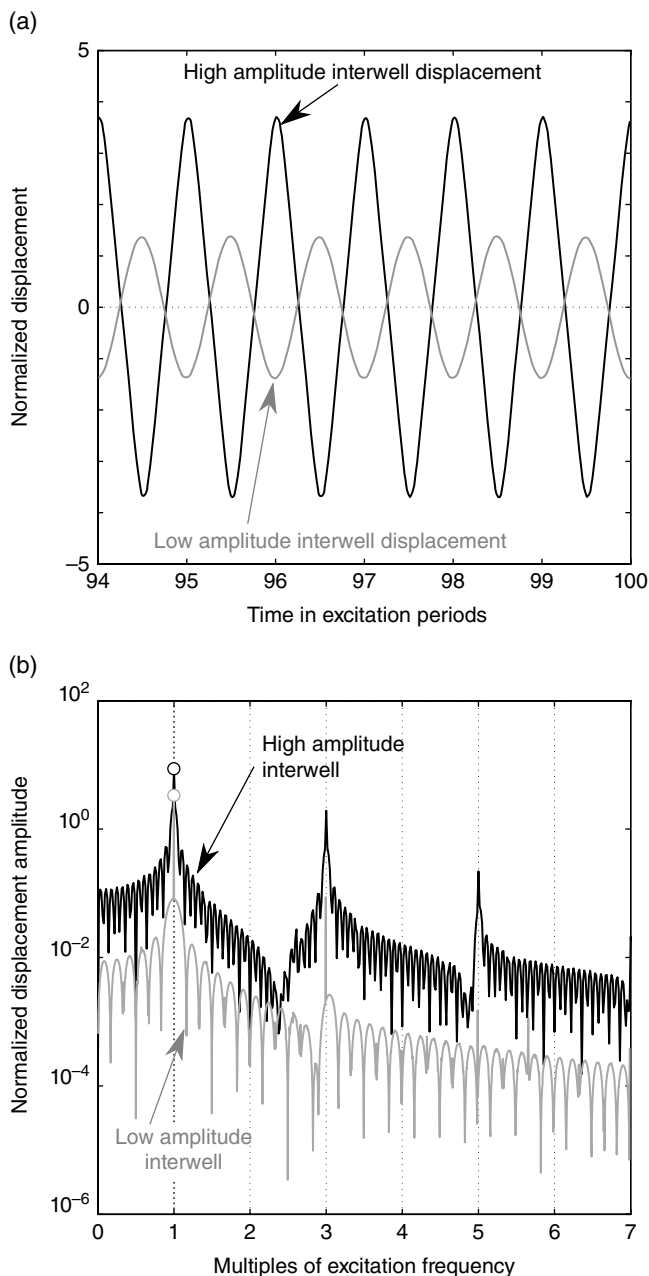


Figure 1.5 (a) For an excitation frequency at which both the low and high interwell dynamics coexist, the time series of the displacement responses is presented. (b) The displacement magnitude spectral responses of the low and high amplitude interwell dynamics are shown as computed from the time series data plotted in (a). The energy diffusion to other harmonics of the excitation frequency is apparent and distinct between the response forms. In (a, b), the parameters are $(p, \omega, \gamma, \beta) = (8, 2.5, 0.09, 1)$, with initial conditions $(-1.9297, -0.0944)$ for the low amplitude interwell responses and $(x_0, \dot{x}_0) = (-2.9443, -0.1791)$ for the high amplitude interwell vibrations.

of the two dynamic forms, it is clear that the responses are approximately 180 degrees out-of-phase. Although not shown in Figure 1.5a, the high amplitude snap-through dynamics occur with phase lags no greater than 90 degrees with respect to the excitation, while the low amplitude interwell oscillations lag the excitation typically by around 180 degrees. The unique phase relationships enable a practical means to distinguish the two forms of interwell dynamics during experimentation in which case they both oscillate back-and-forth across the unstable equilibrium but clearly have different phase relationships with respect to the excitation. Similar to the intrawell dynamic spectral features, Figure 1.5b shows that the two interwell dynamics of bistable systems are different in terms of response amplitude at the excitation frequency of ω (shown as the circle points) as well as in terms of the energy diffusion to other integer multiple harmonics of the excitation.

By evaluating Eq. (1.1) over a broad range of excitation frequencies using an analytical strategy, Figure 1.6a plots the normalized displacement amplitudes of a bistable system (shown as solid curves) as compared to the linear system governed by the corresponding normalized governing equation of motion $\ddot{x} + \gamma \dot{x} + x = p \cos \omega \tau$ (dash-dot curve). Additionally, Figure 1.6a shows thin gray curves which represent *unstable* dynamics of the bistable system. In other words, these are responses which are mathematical solutions to the governing equations but which are not physically realizable dynamics.

The amplitudes of the bistable system intrawell dynamics show resonance-like frequency dependence similar to the linear system, but the prior are “bent” towards lower frequencies. This is an indicator of a “softening” type of nonlinearity, further details of which are provided in later chapters of this book. It is also observed in Figure 1.6a that the high amplitude interwell dynamics exhibit large displacement magnitudes over a broad range of frequencies. In fact, the large amplitude snap-through dynamics are predicted to occur even for a progressively vanishing excitation frequency, in other words as $\omega \rightarrow 0$. This prediction may be intuitively appreciated through a basic consideration of the mechanics involved. Note that as the frequency approaches zero, the motions of the linear system mass are confined to how greatly the linear spring may be statically deformed for the applied load. In contrast, as the excitation frequency approaches quasi-static conditions, the displacements of a bistable system mass will still undergo a large stroke from one stable equilibrium to the other so long as the system is “pushed” with a large enough load level to induce a snap-through dynamic. As such, it may be stated that snap-through is a *non-resonant* dynamic because there is no reliance on resonance-like features to excite the energetic motions. The exceptional sensitivity to broadband and low frequency inputs is a repeatedly exploited feature of bistable systems in the technical areas described in this book.

A major difference between the responses of the bistable structure and the counterpart linear oscillator are the *bifurcations* in the bistable system responses, observed in consequence to variation in the excitation frequency, ω . A bifurcation is a large qualitative change in a system state (static and/or dynamic) due to an infinitesimally small change in an individual parameter (whether internal or external to the system) [15,16]. Three bifurcations in the dynamics of the bistable

system are apparent in Figure 1.6a. The first is seen to occur where the low amplitude intrawell oscillations destabilize if the excitation frequency exceeds $\omega \approx 1.1$. While the linear system exhibits smooth and continuous variation of displacement amplitudes as the excitation frequency is varied, the bistable system dynamics will undergo a sudden jump from low (point B) to high amplitude

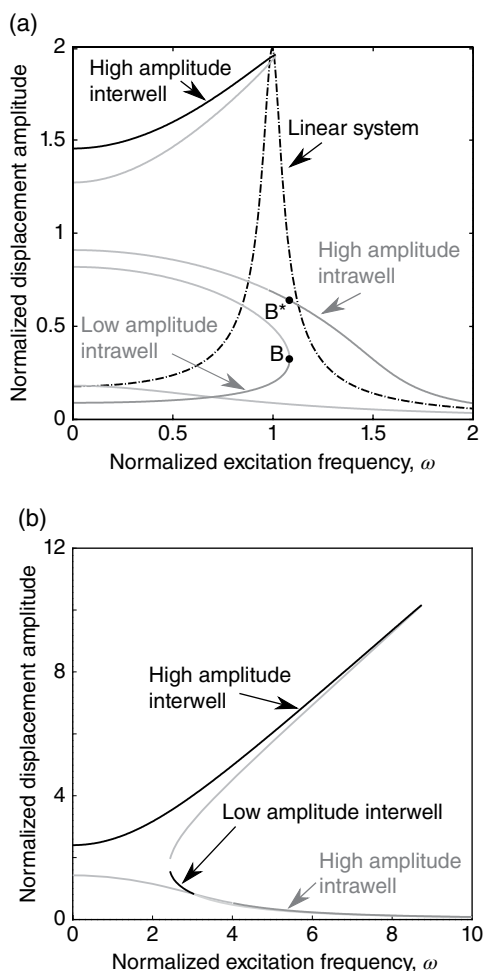


Figure 1.6 Depending on the excitation level, the single-periodic, steady-state response of a bistable system may exhibit non-unique dynamic forms. (a) A high amplitude interwell and both low and high amplitude intrawell responses may occur depending on the excitation frequency. Here, the parameters are $(p, \gamma, \beta) = (0.18, 0.09, 0.7)$. The corresponding linear system stationary response computed using the counterpart equation $\ddot{x} + \gamma \dot{x} + x = p \cos \omega \tau$. (b) A greater excitation level than that used in (a) may induce low or high interwell responses or an intrawell response. Here, the parameters are $(p, \gamma, \beta) = (8, 0.09, 1)$. (c) Responses of softening and hardening Duffing oscillators contrasted against the corresponding linear system. Here, the parameters are $(p, \gamma, \beta) = (0.18, 0.09, 0.7)$. The thin gray curves indicate unstable response forms.

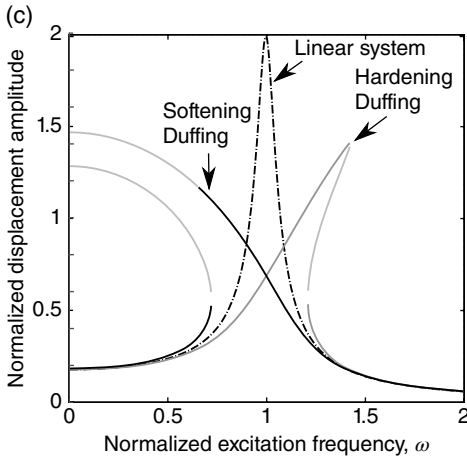


Figure 1.6 (Continued)

(point B*) intrawell oscillations when the frequency exceeds $\omega \approx 1.1$. Another bifurcation is evident in the high amplitude intrawell response if the excitation frequency decreases below $\omega \approx 0.95$; if the frequency is swept below this critical point, the system is predicted to undergo either low amplitude intrawell oscillations or snap-through. The final bifurcation featured in the plot occurs in this bandwidth when the bistable structure is initialized in the high amplitude interwell dynamic state and the excitation frequency exceeds $\omega \approx 1$. This latter bifurcation from an exceptionally large amplitude response (snap-through) to an intrawell dynamic is a distinct characteristic of bistable systems, and the significant response amplitude difference involved can be favorably exploited in several scientific and engineering contexts. For the previous two bifurcations where three, coexistent steady-state responses are reduced to two, the initial conditions at the time of bifurcation activation determine into which dynamic response the bistable system settles.

Figure 1.6b shows results predicted using different system and excitation parameters, and indicates that the low and high *interwell* dynamics may coexist near an excitation frequency of $\omega \approx 2.5$. It is found that the low amplitude interwell oscillations occur across a narrow bandwidth of excitation frequencies. The trend for the high amplitude interwell dynamics is a nearly linear increase in displacement magnitude as the excitation frequency progressively increases. This trend continues up to a critical frequency at which the single-periodic snap-through behaviors become destabilized: a similar bifurcation feature as that observed in Figure 1.6a. Finally, the results plotted in Figure 1.6b show that a high amplitude intrawell dynamic is possible for excitation frequencies greater than $\omega \approx 4$. The phase relationship of these intrawell oscillations with respect to the excitation is the characteristic factor to identify the dynamic regime.

The analytically predicted results in Figure 1.6 provide a clear example of the coexistence of numerous, steady-state dynamic forms of bistable structures as well as the possibility for sudden transitions in the dynamic regime due to small

changes in system or excitation parameters. The multiplicity of the response forms, characterized by their unique displacement amplitudes and excitation phase lags, makes bistable structural dynamics particularly distinct from those of other linear and nonlinear systems. Indeed, snap-through is truly a distinguishing feature of bistable systems.

It is worthwhile to compare the steady-state, single-periodic dynamics of bistable structures to those of the traditional hardening and softening Duffing oscillators, which are regularly implemented as archetypal models for engineering systems in the real-world [17,18]. These oscillators are governed by the equations $\ddot{x} + \gamma\dot{x} + x \pm \beta x^3 = p \cos \omega \tau$ where the positive (negative) nonlinear restoring force term denotes hardening (softening) behavior. It is evident that this governing equation is similar to Eq. (1.1) for the bistable system, repeated here for convenience: $\ddot{x} + \gamma\dot{x} - x + \beta x^3 = p \cos \omega \tau$. On the other hand, the resulting static and dynamic behaviors are considerably distinct. Note that the only static solution to the equations $\ddot{x} + \gamma\dot{x} + x \pm \beta x^3 = p \cos \omega \tau$ is $x = 0$, indicating that the Duffing systems have just one static equilibrium and are thus termed *monostable*. Using the methods presented in Chapter 2, the steady-state dynamics of the forced Duffing equations may be approximately solved and representative results are shown in Figure 1.6c using parameters of (p, γ, β) identical to those used in Figure 1.6a. Here again, the corresponding linear system dynamics are also shown, and light gray curves denote the unstable behaviors of the Duffing systems. Figure 1.6c illustrates that the hardening Duffing oscillator exhibits a “bending” or “leaning” of the resonance curve to higher frequencies with respect to linear system trends. In contrast, the softening Duffing oscillator shows response amplitude curves that lean to lower frequencies. Figure 1.6c indicates that the Duffing systems may exhibit *dynamic bistability*, in that more than one steady-state oscillation regime may occur for a given excitation frequency (similar results are obtained by varying the other parameters β, γ , and p). In comparing the different responses of bistable and these *monostable* Duffing oscillators between Figures 1.6a and 1.6c, respectively, it is evident that the intrawell oscillation regime of the bistable system is comparable to the softening Duffing response, while the Duffing oscillators exhibit no such dynamic behavior as snap-through since the Duffing systems do not possess *static bistability*. Truly, it is static bistability that enables snap-through and draws a dramatic distinction in the resulting, potential dynamic behaviors of bistable systems which the following sections continue to detail.

1.2.2 Sensitivity to Initial Conditions

It was shown in Figures 1.4 and 1.5 that system and excitation parameters as well as the initial conditions influence the resulting dynamic state of a bistable system. In fact, this is a characteristic of all nonlinear systems which may exhibit multiple coexistent steady-state dynamics. On the other hand, due to the two stable equilibria, there are important implications of the initial condition sensitivity demonstrated by bistable structures. A clear example of this feature is shown in Figure 1.7 where a difference in initial normalized displacement of 10^{-6} leads to completely different results as time elapses. It is seen that, based on the

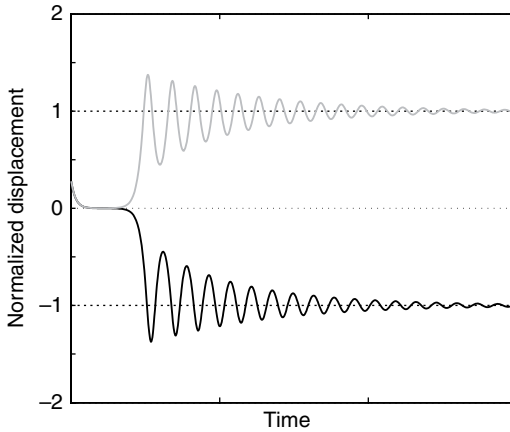


Figure 1.7 The transient responses of bistable systems may exhibit high sensitivity to initial conditions. In this example, the initial conditions are $(x_0, \dot{x}_0) = (0.292721, -0.3)$ for the response settling to the normalized displacement stable equilibria position of -1 (black curve), while the small change of initial conditions to $(x_0, \dot{x}_0) = (0.292722, -0.3)$ causes the transient dynamics to tend to the stable equilibrium of normalized displacement $+1$ (gray curve). Here, the parameters are $(\gamma, \beta) = (0.09, 1)$. The thick dotted lines indicate the statically-stable equilibria.

initial displacement, the unforced, transient dynamics of the oscillating mass decay and ultimately come to rest at one of the two stable equilibria. Thus, unlike the initial condition sensitivity of other nonlinear systems which becomes evident due to *steady-state* excitations, the *unforced* dynamics of bistable structures may be strongly sensitive to changes in initial conditions. The unique final value of normalized mass displacement is also relevant in the event that the bistable system is harmonically-excited. As was described in Section 1.2.1, the low or high amplitude intrawell dynamics may be realized as oscillations around either of the stable equilibria, which means that the average value of the steady-state dynamic is unique. All together, the initial condition sensitivities of bistable systems are manifest in distinct ways as compared to other nonlinear systems, because the offset or bias of the inertial mass due to the stable equilibria is a trademark characteristic of bistable structures.

1.2.3 Aperiodic or Chaotic Oscillations

When subjected to single-frequency excitations, certain classes of nonlinear systems may oscillate aperiodically. These motions are not to be confused with noise-induced behaviors and may indeed appear to be completely random. Yet, they can be deterministically predicted using the system governing equation and knowledge of the initial conditions. This phenomenon can be classified and quantified as *chaos*, and it occurs for a variety of physical, fluid, electrical, chemical, optical, and acoustic systems [19]. When it is possible, chaos is not always the dynamic form which is exhibited due to single-frequency excitation. Comparable to the potential for coexistent steady-state dynamic regimes exemplified in Figure 1.6, a specific set of system and excitation parameters can lead to aperiodic

responses in a bistable system. However, the analytical tools to predict the steady-state dynamics in Figure 1.6 are incapable of identifying chaotic dynamics because such analytical techniques assume the dynamics of the bistable system are periodic. Consequently, a common strategy is to numerically integrate the governing equations when chaos is anticipated to approximate the parameter dependence of realizing chaotic responses for bistable structures. The manifestation and importance of chaos for the engineering implementation of bistable systems is described below.

The chaotic dynamics exhibited by bistable systems are realized as aperiodic interwell behaviors. As a first look into the features of chaos for a bistable system, Eq. (1.1) is numerically integrated using two slightly different sets of initial conditions and the computed displacement trajectories are plotted in Figure 1.8. There are several aspects to observe. First, the dynamics are clearly aperiodic although the input excitation is a single sinusoidal motion. Even after 500 cycles of excitation, Figure 1.8b shows that the displacement responses maintain an apparently random nature. The behaviors are not to be confused with a prolonged decay of transient dynamics related to initial conditions because the bistable system has appreciable damping. Continued numerical integration of the governing equation shows that the aperiodicity is maintained. In addition, the initial condition sensitivity of the chaotic responses as time progresses is exemplified in Figure 1.8a where two aperiodic displacement trajectories are seen to diverge after about 10 cycles of excitation. Following 500 periods of excitation, Figure 1.8b shows that there is little resemblance between the results. Although not shown in the example in Figure 1.8, a feature may be exhibited where chaos is realized in brief bursts. This *transient chaos* phenomenon occurs when intrawell dynamics of the bistable system appear to be the established steady-state response, but then a sudden and temporary bout of aperiodic interwell responses may be activated. The engineering implications of transient chaos are important because large amplitude chaotic dynamics may suddenly overtake an otherwise predictable intrawell dynamic of the bistable structure. In addition, once the transient chaos passes and the intrawell behaviors re-emerge, the system might in fact oscillate around the other stable equilibrium. Thus, an instance of transient chaos may lead to a change in the static state of the bistable system once the phenomenon has passed (should the excitations be removed). Such potential for state change is unique to the bistable structural systems which exhibit chaos. Although it is important to understand the potential for chaos, to date there have been few practical implementations of chaos for bistable systems in the technical areas of interest in this book. Engineers in vibration control, energy harvesting, and sensing typically avoid design and operational regimes that may induce chaotic dynamics. Interested readers are encouraged to refer to other works for further details regarding the chaotic dynamics of bistable structures [19–23].

1.2.4 Excitation Level Dependence

The forced, steady-state dynamics of nonlinear oscillators often exhibit an intricate dependence on the level of excitation. Whereas the response magnitudes of linear systems change with direct proportionality to the excitation level, the

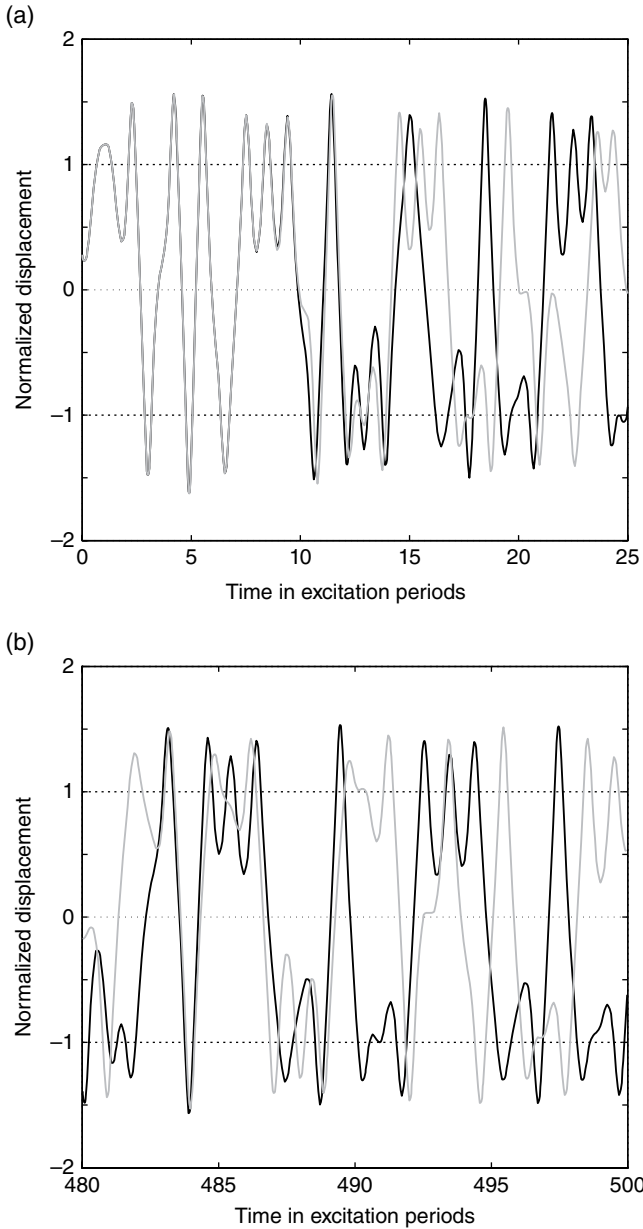


Figure 1.8 (a) An harmonically-excited bistable system may lead to aperiodic dynamics. This characteristic is also strongly sensitive to initial conditions. Thus, after only a small number of excitation periods, two nearly identical initial conditions show great divergence in the transient, aperiodic snap-through dynamics of the bistable systems. (b) After 500 excitation periods, the aperiodic responses bear little resemblance. Here, the parameters are $(p, \omega, \gamma, \beta) = (0.37, 1.29, 0.1, 1)$, with initial conditions $(x_0, \dot{x}_0) = (0.292707, -0.3)$ leading to the responses plotted in the black curves and with initial conditions $(x_0, \dot{x}_0) = (0.292706, -0.3)$ leading to the responses plotted in the gray curves. The thick dotted lines indicate the statically-stable equilibria. The normalized sampling frequency in the 4th-order Runge–Kutta numerical integration is 32.

dynamic amplitudes of nonlinear systems as well as the existence of a given dynamic form will, in general, vary non-proportionally due to excitation level change. Because of the multitude of single-periodic behaviors that bistable systems may exhibit, the excitation level dependence is critical to examine for a bistable structure applied in an engineering practice. In other words, it is essential to determine the suitable design and excitation conditions that help to ensure the desired dynamic form is activated and the target performance objectives are achieved.

Figure 1.9 provides an example of the excitation level dependence of a bistable system. The analytically predicted results are computed for three excitation frequencies. Figure 1.9a shows the results for frequency $\omega = 0.8$ while Figure 1.9b,c show results for $\omega = 1$ and $\omega = 1.2$, respectively. The data for the bistable system are shown as solid curves whereas the counterpart linear system responses are identified by the dash-dot curves. Comparable to the excitation *frequency* dependence illustrated in Figure 1.6, the excitation *level* is an important parameter to determine the occurrence of the unique dynamic forms of the bistable structure; in Figure 1.9 these dynamic types are the low and high amplitude intrawell oscillations and the high amplitude interwell, snap-through vibrations. For the case of excitation frequency at the resonance-like condition, $\omega = 1$, shown in Figure 1.9b, the peak amplitudes which may be achieved for the high amplitude intra- and interwell dynamics of the bistable system are nearly identical to the response of the linear system excited at resonance. On the other hand, when excited off-resonance such as $\omega = 0.8$ or 1.2 , Figure 1.9a,c, respectively, the dynamics of the bistable structure, and in particular the interwell snap-through vibrations, may achieve substantially greater amplitudes for the same excitation level. For the scientist or engineer who wishes to exploit the large, steady-state amplitudes of displacement enabled by a bistable system's snap-through characteristics, it is clear that an intelligent assessment of the excitation conditions under which the platform will operate – both the level and frequency of the forcing – is required to ensure the activation of the energetic behaviors.

Another informative assessment metric of the excitation level dependence for nonlinear systems is the frequency response function (frf), which is often computed as the ratio of a response magnitude (e.g., displacement, velocity, or acceleration) to the excitation level. As an example, Figure 1.10 plots the displacement frf magnitude of a bistable system (solid and dashed curves) as compared to that determined for a linear system (dash-dot curve). It is clear that the linear system response amplitudes are independent of the excitation level, as anticipated from a fundamental understanding of system dynamics. In contrast, due to the variation in excitation level, the bistable system displacements change in terms of frf amplitude as well as the frequency bandwidths across which each dynamic regime occurs. The arrows on the plot indicate the trend of change as the excitation level is increased. It is seen that increasing excitation level reduces the maximum frequency of existence for the low amplitude intrawell oscillations and increases the minimum frequency of existence for the high amplitude intrawell oscillations: a gap in the frequency bandwidth thus occurs across which only the snap-through dynamics are anticipated. At the same time, as the excitation level increases the snap-through dynamics are predicted to be sustained to higher frequencies. Additionally, the frf magnitude of the snap-through displacement

response reduces. Thus, from Figure 1.10 it is clear that the maximum amplitude of the frf of bistable system snap-through dynamics is most nearly equal to that of the linear system only for low excitation levels when considering the resonance-like excitation frequency $\omega = 1$. On the other hand, the realization of the snap-through dynamic is more greatly ensured as the excitation increases because the frequency bandwidth of coexistence with intrawell dynamics is found to reduce. The excitation level dependence characteristics illustrated in Figure 1.9 and Figure 1.10 are important to understand in order to appropriately harness the distinct bistable structural dynamics for engineering applications.

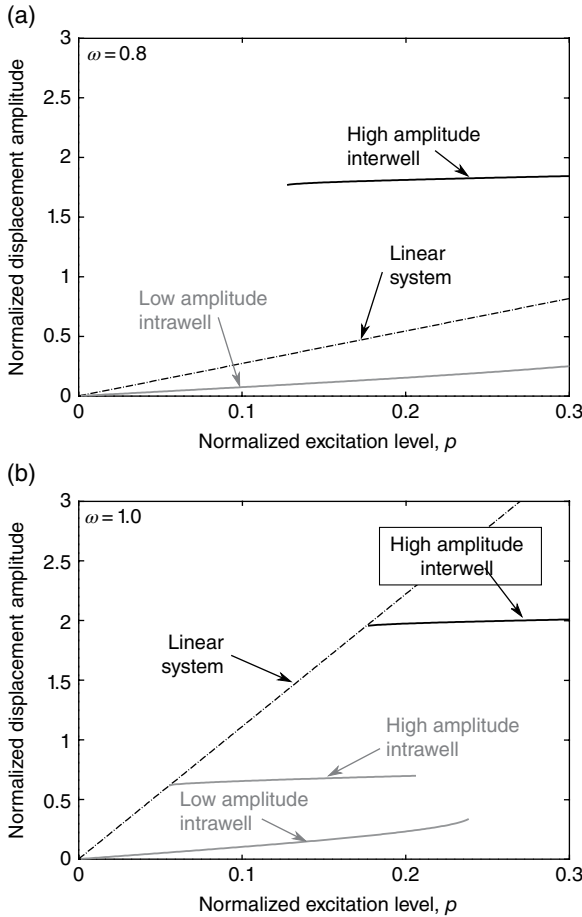


Figure 1.9 Depending on the excitation frequency, the single-periodic, steady-state response of a bistable system may exhibit non-unique dynamic forms. In the following examples, changes in normalized excitation frequency ω lead to variation in the amplitude and existence of the low and high amplitude intrawell dynamics and the high amplitude interwell vibrations of a bistable system, as functions of changing normalized excitation level p . In contrast, only the amplitude of the counterpart linear system responses is seen to vary. Results computed for (a) a normalized excitation frequency $\omega = 0.8$, (b) $\omega = 1.0$, and (c) $\omega = 1.2$. Here, the parameters are $(\gamma, \beta) = (0.09, 0.7)$. The corresponding linear system stationary response computed using the counterpart equation $\ddot{x} + \gamma\dot{x} + x = p \cos \omega\tau$.

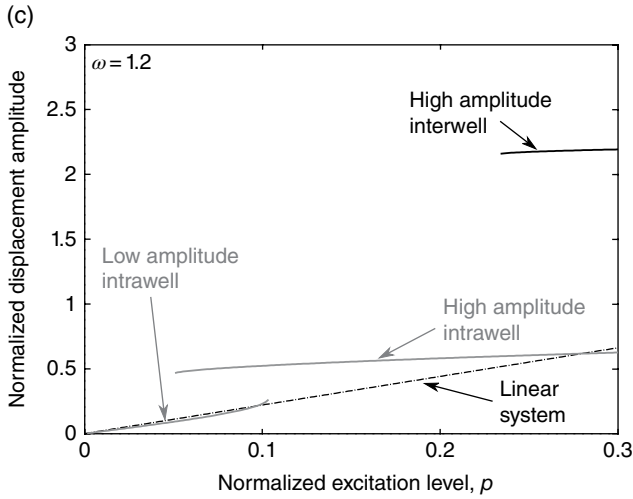


Figure 1.9 (Continued)

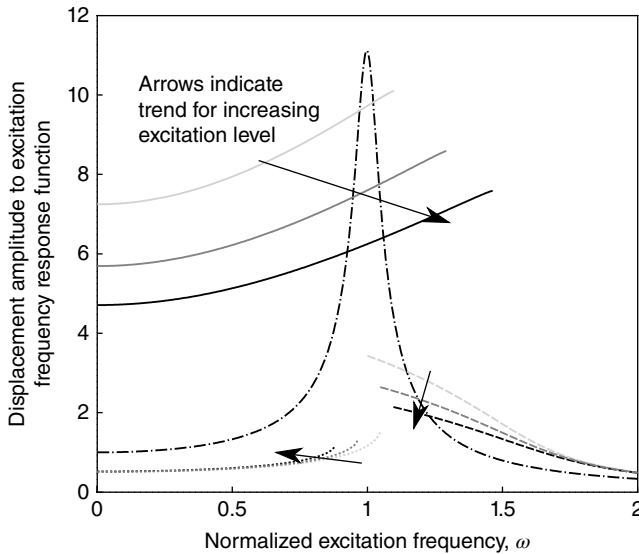


Figure 1.10 Examples of the frequency response function (frf) dependence of a bistable system as compared to a linear system. The frf is the displacement amplitude with respect to the excitation level. The linear system frf (dash-dot curve) is independent of excitation level. For bistable systems, the interwell response frfs (solid curves) reduce in amplitude but extend to higher frequencies as the excitation level increases. The high amplitude intrawell frfs (dashed curves) reduce in amplitude and frequency range as the excitation level increases, while the low amplitude intrawell dynamics (dotted curves) primarily reduce in frequency bandwidth without notable change in frf level. The light to dark curves for the bistable system are determined using p from 0.17, 0.22, to 0.27. Here, the parameters are $(\gamma, \beta) = (0.09, 1)$. The linear system frf is computed using damping factor $\gamma = 0.09$.

1.2.5 Stochastic Resonance

Although Section 1.2.4 indicated that greater excitation levels may help to consistently activate the snap-through dynamics of bistable systems by reducing the likelihood that intrawell dynamics could coexist, there is an additional avenue to realize the energetic interwell behaviors. The phenomenon is called *stochastic resonance* and it is a thoroughly investigated feature of bistable systems, particularly in physics-based research [24,25]. Stochastic resonance refers to the effect of exciting a bistable system with a combination of low-level, single-frequency harmonic input and low-level noise so as to activate snap-through dynamics. Individually, the two excitation inputs are insufficient to induce the interwell behaviors but their combination has dramatic influence.

Figure 1.11 shows an example of this dynamical feature. It is seen that the individual periodic or white Gaussian noise excitations upon the bistable system lead only to low amplitude oscillations around one of the two stable equilibria, whether they are periodic or stochastic in nature. However, in combination, the excitations activate snap-through dynamics. By considering the normalized time axis and the times at which the snap-through transitions occur, it is also apparent that the resulting large amplitude motions occur primarily at the same frequency as the periodic input. As a result, this phenomenon provides a novel avenue to magnify a low-level input by the “injection” of either noise or periodic excitations to activate the high amplitude interwell dynamics in the bistable system with a minimum of extra cost in terms of the input energy. Stochastic resonance is not unique to nonlinear systems exhibiting bistability [24], but the degree of response amplification that bistable structures may realize due to

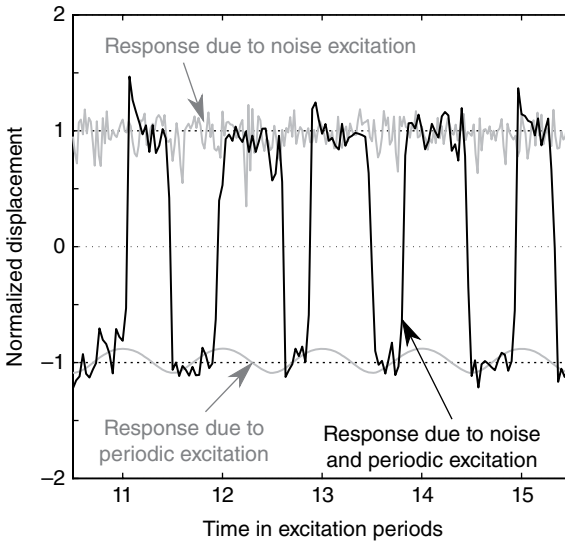


Figure 1.11 The stochastic resonance phenomenon occurs when combined periodic and stochastic excitations, which are individually insufficient to activate snap-through dynamics of a bistable system, lead to large amplitude interwell dynamics which closely follow the excitation period of the periodic excitations. Here, the parameters are $(p, \omega, \gamma, \beta) = (0.2, 0.05, 0.5, 1)$ for the cases involving the periodic excitations. The normalized, additive white noise variance is 0.04. All cases use initial conditions $(x_0, \dot{x}_0) = (0, 0)$.

stochastic resonance is notably greater because of the switching back and forth between the two stable equilibria. This distinction is one reason for the increased attention on stochastic resonance in the context of bistable structures and systems throughout the scientific and engineering fields.

1.2.6 Harmonic Energy Diffusion

The results presented in Figure 1.4b and Figure 1.5b indicate that the four, steady-state types of bistable structural dynamics are distinct in terms of their respective amplitudes. Looking closer at the spectral plots, it is also apparent that the input energy at frequency ω becomes manifest via dynamics of the bistable structure at other frequencies. In other words, the energy is *diffused* from the single-frequency input to other harmonics. Specifically, for the intrawell dynamics Figure 1.4b shows that the input energy becomes concentrated in the system dynamics at integer multiples of the excitation frequency. In this case, the input excitations at a frequency ω are exhibited by bistable system dynamics at frequencies $n\omega$ where $n = 1, 2, 3, \dots$. It is also evident in Figure 1.4b that the high amplitude intrawell dynamics exhibit a greater degree of such energy diffusion than the low amplitude responses.

Similar results are seen in Figure 1.5b where the coexistent low and high amplitude interwell dynamics diffuse the input energies at frequency ω to responses at frequencies $m\omega$ where $m = 1, 3, 5, \dots$. In the cases presented in Figures 1.4b and 1.5b, the harmonic energy diffusion is not dramatic because the corresponding time series plots, Figures 1.4a and 1.5a, show that the bulk of the dynamic behaviors is indeed at the same frequency of excitation, albeit with phase shifts. (One may verify this conclusion by comparing the primary periods of the displacement responses to the horizontal axes which are shown as time in normalized excitation periods). Nevertheless, the spectral results reveal that the bistable system redistributes a non-trivial amount of the input energy into dynamics at other frequencies.

Yet, in some cases, the harmonic energy diffusion may become extreme. Figure 1.12 provides clear examples of this possibility such that the single-frequency input at ω is mostly concentrated at other frequencies. Figure 1.12a shows that for some system and excitation parameters, the inputs at frequency ω become primarily concentrated at frequency $\omega/3$ (black curve) or 2ω (gray curve). The time series results plainly reveal that the dynamics which achieve such significant harmonic energy diffusion are interwell behaviors, and specifically large amplitude snap-through effects. In Figure 1.12b, the spectral results computed from the corresponding displacement time series verify the dramatic redistribution of input energy via the particular manifestation of bistable system dynamics. It is seen that in the one case, the energy is predominantly redistributed to frequencies $m\omega/3$ where $m = 1, 3, 5, \dots$ (black curve), while in the other case the spectral peaks occur at $2m\omega$ (gray curve). No less important than when the energy remains primarily concentrated at the excitation frequency, the extreme harmonic energy diffusion due to bistable system snap-through dynamics has critical engineering implications for their appropriate utilization.

In general, the redistribution of the dynamics from a single-periodic response to a variety of frequencies is not unique to bistable systems and can be exhibited

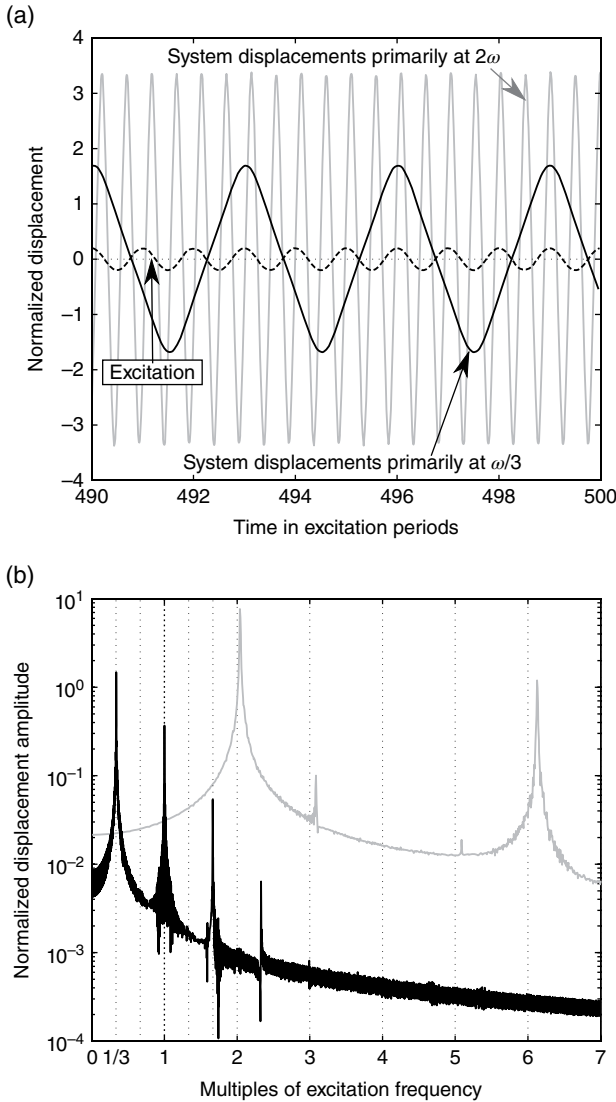


Figure 1.12 (a) Although bistable systems may be excited at a frequency ω , the steady-state dynamics may occur primarily at integer multiples or fractions of the excitation frequency. Two cases are shown for which the system primarily vibrates at a frequency of 2ω (gray curve) or $\omega/3$ (black curve). (b) The displacement magnitude spectral responses of the bistable system dynamics, as computed from the time series data plotted in (a), show clearly the unique spectral concentrations of the input excitation energies. In (a,b), for the dynamics tending to the frequency of 2ω , the parameters are $(p, \omega, \gamma, \beta) = (0.2, 1.3, 0.0001, 1)$, with initial conditions $(x_0, \dot{x}_0) = (-3.2987, 1.5132)$; while for the dynamics tending to the frequency $\omega/3$, the parameters are $(p, \omega, \gamma, \beta) = (0.2, 3, 0.01, 1)$ with initial conditions $(x_0, \dot{x}_0) = (-0.7507, 1.4841)$.

by other nonlinear systems. On the other hand, the multitude of the possible, coexistent steady-state dynamic forms of bistable structures lends itself to an intricate manifestation of harmonic energy diffusion that is not evident in other

nonlinear systems. This provides bistable systems with a much greater variety of multiharmonic dynamic characteristics that may be intelligently harnessed.

1.3 The Exploitation of Bistable Structural Dynamics

The review above indicates that the dynamics of bistable systems include a rich assortment of behaviors. All together, the numerous distinct features suggest that an attractive versatility of response may be exploited by intelligently utilizing or switching among the different types of dynamics that are realized by a single degree of freedom (DOF), bistable architecture. As compared to the considerably more uniform response characteristics of linear systems and the fewer opportunities to exploit with other nonlinear platforms, the dynamics of bistable structures and the various means by which to realize bistability motivate the scientific and engineering utilization of bistable system architectures to advance a variety of technical fields.

The rationale for exploiting bistable structural dynamics depends upon the application. In some cases, the reasons that encourage use of certain dynamic forms are the same that could discourage their use for other purposes. Therefore, the perspective of the researcher and the performance metrics of interest are the guiding factors for appropriately harnessing bistable system dynamics. Across the fields of vibration control, vibration energy harvesting, and sensing, there are certain shared or underlying aims that the dynamics of bistable systems have been found to meet effectively in ways and to degrees beyond what is possible when utilizing alternative system platforms.

The shared aims may broadly be described as achieving exceptionally *energetic responses* and enabling *adaptability*. For bistable structures, the basis for realizing notably energetic responses is the opportunity to activate snap-through dynamics, which do not have a counterpart in single DOF linear or other monostable nonlinear systems. The adaptability of bistable system dynamics is due to the potential for multiple, coexistent steady-state responses – whether they are single-periodic, multiharmonic, or aperiodic – such that an intelligent exploitation of system design and excitations can enable one to selectively trigger the dynamics that are preferred for a certain operating condition or application.

In the technical areas of interest in this book, the energetic responses and adaptability of bistable structural dynamics draw the several engineering applications around the general objectives of enhanced performance versatility and functionality. In the following sections, the individual aims of vibration control, energy harvesting, and sensing applications are briefly detailed. Within each context, a first look is taken at how the objectives of the areas have recently been achieved and advanced through the exploitation of bistable system dynamics, particularly as they relate to harnessing the energetic and adaptable responses.

1.3.1 Vibration Control

Dissipating, isolating, or absorbing the vibrations of structures is an historic goal: elastic carriage suspensions were employed during, if not also before, the Iron Age [26], while the first example of a vibration absorber for reactive attenuation

purposes is often dated to Hermann Frahm's patent in 1910 for an anti-roll tank for ships [27]. Apart from the assurance of comfort via limiting the jarring of carriage vibrations or alleviating the possibility of sea-sickness, the control of vibrations is more recently a matter of structural necessity.

The use of engineering systems in ways or in environments that cause significant vibration levels, for example space vehicle launches or even the ordinary operation of rotorcraft, requires that vibration control systems operate effectively and with high performance to ensure the success or integrity of the structure in its mission. The numerous costs associated with energy to fuel a wide variety of vehicular systems, including aerospace, marine, and automotive vehicles, have stimulated the use of lightweight structures and materials to minimize mass to allowable safety standards. This means that the inherent energy dissipation characteristics of the system are of great importance. Without sufficient, and oftentimes high, damping capacity in the lightweight systems, the day-to-day operation of many modern engineering structures and materials carries the risk of incurring a catastrophic failure due to working too closely to the limits of system integrity. On the other hand, although high damping is critical for some applications, low damping may be just as vital. For example, the need to obtain accurate wave- or vibration-based signal readouts from a structure to monitor its operating state, also termed structural health monitoring, benefits from a lightly damped structure at the time of data acquisition for the sensors to acquire clear measurements. As a result, modern implementation of engineering systems has led to a set of operational requirements for vibration control performance, some conflicting with each other: for instance, a lightweight structure or material solution that provides high damping during rotorcraft operation but low damping during a rotor blade condition monitoring examination.

From age to age, the achievement of these overall vibration control aims has been limited by the traditional employment of passive materials or devices having tightly prescribed capabilities, often as a result of operating the system within a linear dynamic regime. The introduction of nonlinear components for energy dissipation, such as fluid [28] or friction dampers [29], led to improved performance potential, particularly for high vibration levels. Nevertheless, the more extensive exploration and exploitation of nonlinear vibration control concepts, including the harnessing of bistable elements, is a comparatively recent initiative [30,31].

Indeed, there are a variety of advantages to be gained by employing a bistable device in its own right within the architecture of a vibration damping, isolation, or control system. The energetic snap-through dynamics lead to high velocity motions for the inertial mass, due to the large displacement strokes from one stable equilibrium to the other and associated low dynamic stiffness. In addition, snap-through is activated across a broad bandwidth of frequencies when compared to a linear system and its resonant dynamics, as exemplified by Figure 1.6. Collectively, these characteristics of large kinetic energy, low dynamic stiffness, and potential for adaptively triggering large amplitude, periodic motions provide opportunities to advance aims in dissipating, isolating, and absorbing vibrations in myriad contexts.

One idea recently explored to achieve damping characteristics uncommon in traditional bulk materials is the implementation of bistable elements into structural or material systems to take advantage of the snap-through dynamics.