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Colin Beveridge

Maths Tutor

AS & A Level Maths For Dummies

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by Colin Beveridge

For Dummies

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***AS & A Level Maths* For Dummies®**

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Contents at a Glance

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Introduction

So, you've chosen to study maths for A level? Fantastic. Welcome to the course!

It's a bit of a cliché that A level is a big step up from GCSE – but only because it's at least partly true. That said, if you know everything from your GCSE, you could probably pick up about a third of the marks in Core 1 without doing any more study.

In reality, though, not many people know *everything* from GCSE, and if your teacher assumes you do when you don't, well, then you have a problem. Worse still, it's a problem that's up to you to put right!

Luckily, you've done something smart and got hold of a book to help you. You're not going to find exercises in it or nice pictures of Carl Friedrich Gauss, but you will find plenty of advice and instructions on how to tackle various types of questions.

As for me, I've been tutoring A level maths since 2008, so I know what students tend to struggle with – and the kinds of explanations that tend to make complicated ideas make sense. Throughout the book, I try to break everything down into simple, repeatable steps that will work even if the question is a bit different from the example I've chosen.

About This Book

This book is for you if you're about to start studying for your AS- or A level in maths – or if you're already doing the course and need to improve your grade.

I take you through the topics that you can expect to come up in the exams, starting with the GCSE knowledge you're expected to carry through to A level and ending with differential equations – which sound scarier than they really are.

The book is in five parts:

- **Getting started:** This part covers the things that will make your life easier if you know them before you start studying. If you're already some way through your course, they're still useful to learn or revise.
- **Algebra:** Here, you develop your algebra techniques further, building on what you learnt at GCSE to solve more-involved problems. The biggest new ideas are, I think, infinite series and logarithms.
- **Geometry:** Similarly, the geometry you need for A level builds on what you've done before. In this part, you take trigonometry to new levels and do more interesting things with vectors than you ever imagined.
- **Calculus:** For most students, calculus is a new topic (although if you've done a Further Maths GCSE, Additional Maths or even the IGCSE, you may be familiar with some of the concepts). Calculus is loosely the study of slopes and areas, and it's probably the most important mathematical concept you get from A level maths if you're planning to study a science at university. (For some subjects, statistical reasoning may push it close – but that's not in this book, so it's obviously not *that* important!)
- **The Part of Tens:** Lastly, I offer some quick hints and tips to help you get started on evil problems and avoid some of the mistakes everybody else makes.

Because the topics covered in the book are complicated enough, I've tried to keep the writing conventions simple. Here are the ones you should know:

- *Italics* are used for emphasis or to highlight new words or phrases in each chapter. They also indicate variables.
- **Boldfaced text** indicates key words in bulleted lists or the key steps in action lists. Vectors also appear in bold.
- Internet and email addresses appear in `monofont` to help them stand out.

Foolish Assumptions

Making assumptions is always risky, but it's unavoidable in this kind of book. Knowing where I'm coming from should help put you at ease, though. Here's what I've assumed:

- You've done reasonably well at GCSE, or (if you're coming back to maths after a break) you remember enough of your secondary school maths that you'd get at least a B. Don't worry, though – in Part I of the book, I review the skills you may have missed.

- You're competent with the basics of algebra (solving linear equations), arithmetic (you know how to add, subtract, multiply, divide and take powers of numbers), and geometry (you know the names of shapes, and you won't be surprised by the equations attached to them).
- You've chosen to do Maths A level, you have some enthusiasm for the subject, and you want to do well in it.

Icons Used in This Book

Here are the icons I use to draw your attention to particularly noteworthy paragraphs:

Theories are fine, but anything marked with a Tip icon in this book tells you something practical to help you get to the right answer.

Paragraphs marked with the Remember icon contain the key takeaways from the book and the essence of each subject.

The Warning icon highlights mistakes that can cost you marks or your sanity – or both!

You can skip anything marked with the Technical Stuff icon without missing out on the main message, but you may find the information useful for a deeper understanding of the subject.

Beyond the Book

In addition to all the great content provided in this book, you can find even more of it online. Check out `www.dummies.com/cheatsheet/asalevelmaths` for a free Cheat Sheet that provides you with a quick reference to trigonometric tricks and calculus rules.

You can also find several bonus articles on topics such as the link between arithmetic series and straight lines, and why the derivatives of the trigonometric functions are what they are, at `www.dummies.com/extras/asalevelmathsuk`.

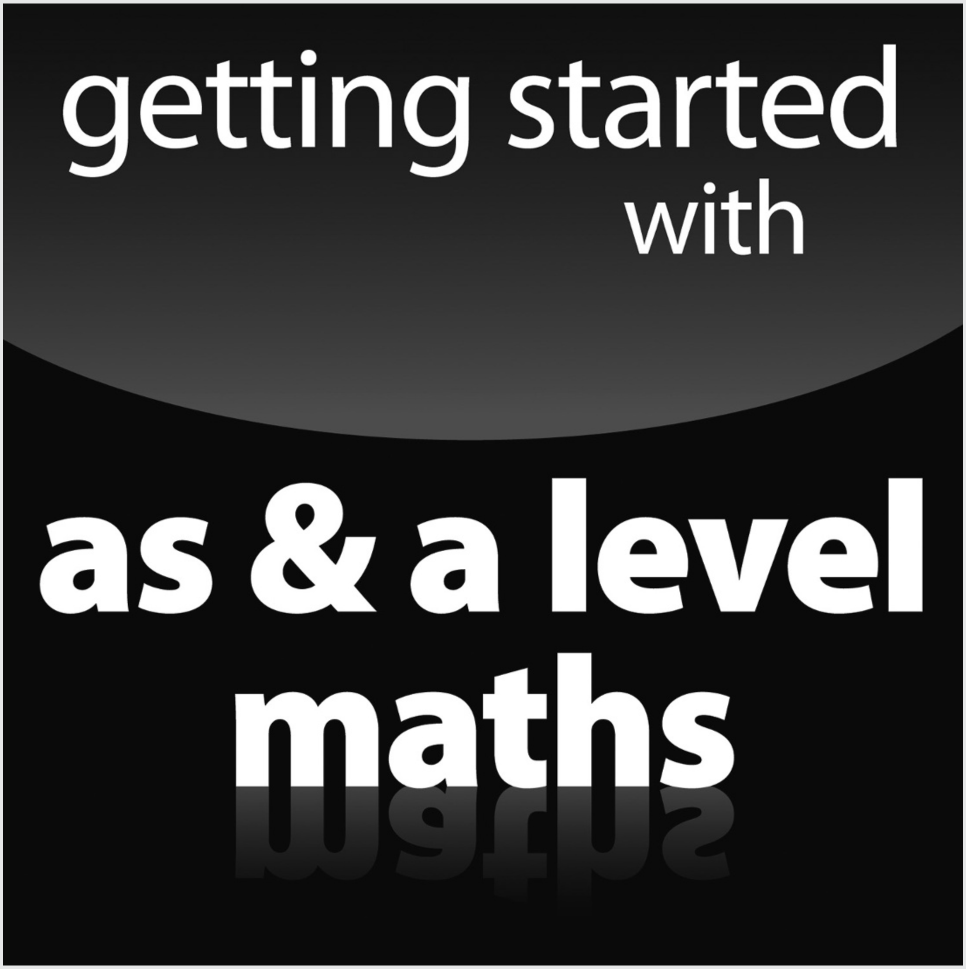
Where to Go from Here

Head to Chapter 1 for an overview of what you can expect in your A level course. Or if you want to get your study habits straight first, Chapter 2 ought to be your first port of call.

You can also use the table of contents and index to find the specific areas you want to work on. This book is meant as a reference guide, so keep it handy with your study gear and look things up whenever you're confused.

I wish you all the best with your studies! If there's something that isn't clear, get in touch – I'm always happy to try to make things make sense. The best way to reach me is on Twitter (I'm `@icecolbeveridge`) or through my website (`www.flyingcoloursmaths.co.uk`).

Part I
Getting Started



For Dummies has great info on lots of different topics. Check out `www.dummies.com` to find out how you can learn more and do more with *For Dummies*.

In this part . . .

- Set off towards mathematical mastery.
- Put yourself in a position to do well on the exam.
- Refresh your arithmetic and algebra.
- Get your shapes ship-shape.
- Sketch graphs with aplomb.

Chapter 1

Moving towards Mathematical Mastery

In This Chapter

- Understanding the overlap with GCSE
- Doing advanced algebra
- Building on geometry
- Diving into calculus

It's a big step up from GCSE to A level – especially if you're coming in with a B or a marginal A. The pace is pretty frenetic, and there's a fair amount of A and A* material from GCSE that's assumed knowledge at A level. If you're not especially happy about algebraic fractions or sketching curves, for example, you're likely to have a bit of catching up to do.

Luckily, this book has a whole part devoted to catching up with the top end of GCSE, as well as the stuff you'll need to learn completely fresh. In this chapter, I note where the content overlaps with GCSE and introduce you to A level algebra, trigonometry and calculus.

Reviewing GCSE

The good news is that if you've got a solid understanding of everything in your GCSE, quite a lot of Core 1 and a fair amount of Core 2 will be old news to you. Possibly less good news is that if you've got gaps in your knowledge, you need to fill them in pretty sharpish.

The four key areas where there's an overlap between the two qualifications are algebra, graphs and powers (all in Core 1) and trigonometry (which comes up a lot in Core 2). There are other bits and pieces, too – your arithmetic needs to be pretty decent in Core 1, where you don't have a calculator, and parts of Core 2 are likely to test your knowledge of shapes other than triangles – but generally speaking, these are the big four. The first part of this book is all about making sure you're up to speed with them.

Setting up for study success

Forgive me if you think it's patronising to tell you how to study – after all, you must have done pretty well with exams to get this far. I go into studying because A level is a much tougher beast than GCSE – it's possible for a reasonably smart student to coast through GCSE and get a good grade without needing to work too hard; by contrast, it's unusual to see someone glide through A level. And presumably, if you were finding it straightforward, you wouldn't be buying books like this to help you through it.

In this kind of scenario, everything you can do to optimise your working environment, your note-taking and your revision translates to quicker understanding and more marks in the exam.

Also, if you're studying at sixth form, there's likely to be a bit more going on socially than at secondary school. The more quickly you can absorb your studies, the sooner you can get out to absorb the odd lemonade with your friends. That is what you're drinking, isn't it?

All about the algebra

You've probably been manipulating algebraic expressions for years by now, and some of it will be second nature. However, just as a checklist, here are some of the topics you need to have under your belt:

- Solving linear equations (such as $7x+9=3x-7$)
- Expanding quadratic brackets (for example, $(x+3)(2x-5)$)
- Factorising and solving quadratics (such as $x^2-5x-36=0$)
- Solving linear and nonlinear simultaneous equations
- Simplifying algebraic fractions

I recap all of these in Chapter 3.

All this algebra isn't just for the sake of jumbling letters around and feeling super-smug when your answer matches the one in the mark scheme (although that can be a nice motivator). Algebraic competence underpins just about everything in A level. Even in places where you'd normally expect to use only numbers (for example, Pythagoras's theorem), you may be asked to work with named constants (such as *k*) instead of given numbers (such as 3).

Grabbing graphs by the horns

Somewhat related to algebra are graphs. You rarely need to draw an *accurate* graph at A level; it's far more common to be asked to *sketch* a graph.

That's good news: sketching is much quicker and more generously marked than plotting. However, you can no longer rely on painstakingly working out the coordinates and joining them up with a nice curve. Instead, you need to know the shapes of several kinds of graphs you've come across at GCSE: the straight line, the quadratic and the cubic graphs as well as the reciprocal and squared-reciprocal graphs.

You'll frequently be asked to work out where a graph crosses either of the coordinate axes (which is really an algebra question), and you'll be expected to be on top of curve transformations. I cover all these in Chapters 4 and 10.

Taming triangles and other shapes

Triangles, obviously, are the best shape of all, which is why you spend so much time on SOH CAH TOA, the sine and cosine rules, and finding areas at GCSE.

Oh, and Pythagoras's theorem. If there were a usefulness scale, Pythagoras's theorem would be *way* off it. I can't think of a more important equation at A level, and you can read about it in Chapter 4.

Those skills are extremely useful in A level maths. Pretty much every Core 2 paper I've ever seen has used a triangle somewhere, and triangles frequently crop up in other modules. (If you're doing Mechanics, having strong trigonometry skills is a massive help.)

It's not just triangles you need to know about, though. Be sure you know the areas and perimeters of basic two-dimensional shapes like rectangles, trapeziums and circles as well as the surface areas and volumes of three-dimensional shapes such as cuboids and prisms. I recap these shapes in Chapter 15.

Attacking Advanced Algebra

As you'd expect, the algebra you're expected to do at A level gets a bit more involved than what you did at GCSE. It comes down to learning some new techniques and linking some new notation to ideas you may already have a decent grasp of.

You start with powers and surds, a GCSE topic that sometimes gets glossed over. You'll need to be fairly solid on these, as they come up over and over again in A level. In later modules, you have a calculator that will happily tell you that the square root of 98 can be written as $7\sqrt{2}$, but in Core 1, you need to be able to work that out on paper.

You also deal with sequences and series (extending the work you've done in the past), solidify the ideas of factorising polynomials, and do some work on functions – one of the most important ideas in maths.

Picking over powers and surds

Working out combinations of powers is one of the most critical skills for A level maths. I wouldn't say it's more important than topics like algebra, but a student's skill here is a strong indicator of how easy a student is going to find A level. If you're a bit rusty on the power laws, you're going to have to sort that out in fairly short order.

You also need to be pretty hot on your surds, especially in Core 1, when you're one calculator short of a pencil case. Throughout your course, you'll need to work square roots out in *simplified surd* form or, more generally, in *exact* form – examiners want to see things like $\pi\sqrt{2}$ rather than 4.443.

A step up from powers and surds are *logarithms*, which are handy functions for turning equations with unknown powers into equations with unknown multipliers. For example, without logarithms, $3^x = 100$ is hard to solve (you know the answer is 4-and-a-bit but not necessarily what the bit is), but with logarithms, getting the answer is a simple bit of algebra: $x\log(3) = \log(100)$, where log(3) and log(100) are just numbers you can get from your calculator.

Lastly, under the 'powers' heading, you need to work with one of the most interesting numbers in all of maths, *e*. It's a constant (exactly $1+\frac{1}{1}+\frac{1}{2\times1}+\frac{1}{3\times2\times1}+\ldots$, or roughly 2.718281828459045…) with the lovely property that if you work out the tangent line of $y = e^x$ at any point, you find its gradient is equal to e^x.

Sorting out sequences and series

A *sequence* is simply a list of mathematical objects (often numbers, sometimes expressions). A *series* is what you get if you add them up.

You probably did some work on sequences in the past (finding the *n*th term, for instance, or deciding whether a term belonged to a sequence), and that will stand you in good stead. At A level, though, there's a lot more to it (who would have thought?).

As well as the arithmetic sequences you know and love, there are geometric sequences (where each term is a constant multiple of the one before). There are also explicitly and recursively defined functions, which *sound* like a horrible wild-card but in fact are quite nice because you're told precisely how they behave in the question.

And there are binomial expansions, which are a really neat way of expanding expressions like $(1+2x)^6$ without needing to multiply out huge numbers of brackets. It's a particularly useful technique when you get to Core 4 and have to expand monsters like $\frac{1}{\sqrt{4+3x}}$ and use the result to approximate $\frac{1}{\sqrt{4.03}}$.

Of course, the binomial expansion is one of many things you do much more often in exams than you ever will in the outside world. (We have machines for that.) However, the idea of approximating things using polynomial series is a powerful tool for doing serious maths if you take the subject beyond A level. Oh, and it's handy for doing mental arithmetic tricks that make you look like a god, too.

Finding factors

I keep coming back to a theme in this book: things in brackets are (usually) happy things. In most cases, if you can put something in brackets – a quadratic expression, or a cubic, or a fraction – you almost certainly should. If you have to solve for where an expression is 0, the factorising makes it very easy; if you need to sketch a curve, the bracketed form is much easier to work with than the expanded one.

All the work you did in learning to factorise quadratics over the last few years will serve you quite well with this – although, as you may expect, you take it a few steps further at A level.

In Core 2, you learn to identify factors of cubics (and higher-degree polynomials) using the factor theorem. You use polynomial division to take these factors out so you can factorise the remaining expression. You also use its cousin, the remainder theorem, to find out what's left over without having to work through the whole division.

Sometimes, though, you need to do the whole division or, depending on your preferred method, find a way to work around it. I like to turn the problem on its head by coming up with a template answer and seeing which numbers have to go in the template, but your mileage may vary.

All this factor fun shows up in Chapter 7, along with the Core 4 topic of partial fractions. Since you started working with fractions, you've been adding and subtracting them using common denominators. Partial fractions is the reverse process of taking a fraction that's been combined and splitting it up into the parts that once made it up. Why would you do such a thing? Two reasons: it makes things much easier to integrate, and it means you can apply the binomial expansion much more easily.

Functions

A mathematical *function* is, roughly speaking, a recipe for taking one or more values and spitting out another. They're a big deal, mathematically speaking: being able to talk about functions in the abstract, without explaining what the recipe is, means you can do interesting things with graphs and calculus without getting bogged down in the details. For example, if you compare the graph of $y = f\left(x-2\right)$ with the graph of $y = f\left(x\right)$, you can say, 'The graph has moved two units to the right' without caring whether the function is quadratic, reciprocal, trigonometric or other – quite a handy trick!

In Chapter 9, you learn about the slightly esoteric notation you use for defining functions. You find out how to combine functions with each other, how to *invert* (undo) functions and how to solve equations involving functions.

You also get to play with *iteration*, which falls into the dull-but-usually-straightforward category. The idea is that you set up a recursive process, doing the same thing over and over again, until it converges on a specific value.

Getting to Grips with Geometry

Geometry – literally 'measuring the Earth' – has developed over time to mean the study of shapes. At A level, the most important shapes are triangles (clearly the best shape) and circles (which are really triangles in disguise), although you will need to deal with rectangles and trapeziums and all manner of three-dimensional shapes in good time.

The four main areas of A level geometry are

- **Coordinate geometry,** which is about dealing – as you might expect – with coordinates; that includes midpoints, distances and equations of curves
- **Circles,** including their equations and some theorems
- **Triangles**, including advanced trigonometry
- **Vectors**, including vector lines, angles between vectors, and triangles in three dimensions

Conquering coordinate geometry

Coordinate geometry is a big topic at A level. You need to be super-confident with your *x*s and *y*s. You've covered the basics at GCSE – the equation of a line, finding midpoints and distances between points using Pythagoras's theorem, and so on – but it all gets taken a bit further at A level.

There are curves to sketch and shapes whose areas need to be known. After you do some differentiation, there are tangents and normals to find the equations of.

Setting up circles and triangles

I'm always surprised when I ask for the equation of a circle and someone says, 'πr^2'. First of all, that's not an equation (he or she means $A = \pi r^2$, where A is the area and r is the radius), and second, that's the *area* of the circle, not the circle itself.

The equation of a circle, like the equation of a line, gives you a relationship between its coordinates. If the equation is $(x-3)^2 + (y-4)^2 = 25$, one of A level's favourite circles, you can tell whether a particular point is on the circle by putting its coordinates into the equation as x and y.

In Chapter 11, I show you how to work out the equation of a circle as well as work out the area of a sector, the length of an arc and other things related to a circle.

Circles are closely linked to triangles; apart from all the trigonometry you know from GCSE, you'll also need to be able to do it all in *radians*, a much better measurement of angles than the degree. Fortunately, much of it is just a case of switching your calculator mode and relabelling your graphs.

Taking trigonometry further

If you split the word *trigonometry* up into its parts, you get 'tri', meaning three; 'gon', meaning 'knee' or corner; 'o', which means nothing; and 'metry', meaning 'measuring.' Trigonometry is about measuring things with three corners.

However, that's not all you use it for. It also has applications in any situation where things are periodic – measuring tides, modelling daylight lengths and analysing sounds, just off the top of my head. For that reason, you need to be able to take trigonometry further. Some of the things you'll be doing include

- Finding all the possible solutions to simple trigonometric equations in a given interval
- Using trigonometric identities to turn trigonometric equations into something you can solve
- Exploring what happens to sine, cosine and their friends when you add angles together
- Adding sine and cosine waves together to get another sine or cosine wave
- Working with the minor trigonometric functions – the reciprocals of sine, cosine and tangent, which are the cosecant, secant and cotangent (respectively); usually, they're denoted $\text{cosec}(x)$, $\sec(x)$ and $\cot(x)$

You also need to be up to speed on proving that two trigonometric expressions are equivalent. That generally involves combining fractions, applying identities, factorising things cleverly and understanding the symmetries of the various functions. Because these problems bring so many areas together, they're one of the most demanding (but also most rewarding) bits of Core 3.

Vanquishing vectors

You've done some work with vectors at GCSE – although you may not have done as much as you'd like, because vectors are usually in the A* questions at the end of the paper. It's OK, though: your GCSE vectors work isn't essential to your A level studies.

Vectors at A level can seem intimidating, with new vocabulary, new ways of multiplying things together, new equations of lines. After you get past that and think about how vectors fit together, they're really powerful – and straightforward.

In Chapter 13, I show you the following:

- How to come up with the vector equation of a line
- How to find the distance between two points (the length of a vector)
- How to find the angle between two vectors
- How to tell whether two vector lines cross
- How to find a point on a line so that a vector involving it makes a right angle with the line

Additionally, in case you're studying for the OCR MEI board, I take you through the gory details of vector equations of a plane – usually a big part of a Section B question in Core 4.

Conquering Calculus

In terms of A level maths, calculus is pretty much what you've been building towards for your entire career. The discipline has plenty of rules to learn but is based very carefully on much of what you've done up until now in algebra, arithmetic and geometry.

You can think of calculus as the study of curves, very loosely speaking: How steep are they? How much area do they have underneath them?

Why is that important? In many processes in physics, chemistry, biology, economics, psychology and anywhere else you care to apply maths, either there's a relationship between how much of a thing there is and how quickly it's changing, or you're actually interested in how something is changing. For example, your speed is how quickly your position is changing, and your acceleration is how quickly your speed is changing.

To work that out, you *differentiate* an expression or an equation. *Integration* is the reverse process. And because Core maths isn't just about doing maths for the sake of it, you need to know how to apply calculus to situations, both in mathematical and real-life contexts.

Dashing off differentiation

Differentiation is the process of finding how a quantity is changing instantaneously – at a specific point in space or time. It's kind of a big deal. Two of the greatest scientists of the early eighteenth century, Isaac Newton and Gottfried Leibniz, squabbled for years over priority (Newton thought of it before Leibniz, but Leibniz published first, leading to Newton having a hissy fit and accusing Leibniz of pinching his ideas).

The history of calculus is interesting but not especially useful for understanding what to do. In the chapters on differentiation (14 and 15, to be precise), you learn the following:

- How to differentiate powers of x (or any other letter you put your mind to)
- How to differentiate the trigonometric functions $\sin(x)$, $\cos(x)$ and $\tan(x)$
- How to differentiate exponential and logarithmic expressions
- What to do when expressions are multiplied together, divided by each other or applied to each other

Here, you also learn about the geometrical interpretation of differentiation – how to find tangents and normals, how to determine and classify turning points of a function and how to find whether a turning point is a maximum or a minimum.

Inspiring yourself to integrate

Integration is the reverse process of differentiation: taking a gradient and turning it into a curve. It's also used for finding the area underneath a curve. In fact, all the formulas for the area and volume of shapes can be worked out by integration, but I leave that as an exercise for the very interested reader.

In Chapters 16 and 17, I show you how to

- Integrate powers of x (apart from that pesky x^{-1})
- Remember to add a constant of integration
- Work with limits to evaluate definite integrals and areas
- Find the area between curves
- Deal with trigonometric functions such as sine and cosine
- Work with exponentials and (at last) that problematic $\frac{1}{x}$
- Manage functions of functions and expressions multiplied together

Applying the calculus

There are two main schools of thought when it comes to maths. The crazy pure mathematicians think maths should be done for its own sake, for the sheer beauty of it, because it's fun. The more sensible applied mathematicians do maths because it's useful.

And calculus, while it's beautiful and fun, is especially useful. It's useful geometrically: it allows you to draw tangents and normals, find turning points and classify them, and that sort of thing. But it's also useful in what exam boards think of as real life, too. Here are some examples:

- **Finding maxima and minima of functions:** This allows you to find the most efficient way to use material to package products, or at what price to sell something, or how to minimise the cost of a journey.
- **Finding rates of change:** Knowing how quickly your experiment is running or at what speed your car is going is often useful.
- **Finding areas and volumes:** You may need to know how big an irregularly shaped object is. For a slightly more abstract application, the area under a speed–time graph gives you the distance something has travelled.
- **Solving differential equations:** In many applications, the value of a function is linked to its rate of change. For instance, the amount of a radioactive substance left depends on how quickly it decays, and the position of a satellite depends on its acceleration. Solving the links between derivatives and values is a key skill, both in Core 4 and as you take maths further.

Chapter 2

Setting Yourself Up for Study Success

In This Chapter

- Getting the right equipment and people
- Believing in your abilities
- Creating a study plan
- Using revision techniques

Preparation is nine tenths of the battle for A level maths (and, for that matter, most other things). Your goal between now and the exam should be to put yourself into the best possible position so that whatever they throw at you, you have the best possible chance of doing yourself justice.

In this chapter, I go through the details of *how* to study – what equipment you need, what you need from your workspace and what you need from anyone you choose to work with. I also talk about the mental-resilience side of studying (it's a hard slog, and for most people, studying maths isn't a smooth ride). You need a good attitude, good perseverance and a good way of talking to yourself – as well as good recovery when you mess up.

Messing up is *totally normal* and part of learning maths. If you're not making mistakes, you're probably not challenging yourself enough. I made mistakes in writing this book, but I (or, more often, my minions at Dummies Towers) caught them and put them right.

The final things you need are the self-reliance and motivation to come up with your own study plan. You *can* blindly accept that your teachers know how you learn best, but the chances are that they're working in an order and at a pace that they judge will be okay for most people – not necessarily optimal for you! If you take control of what you learn when, you can get significantly ahead of the class and lose some of the 'I'm not sure we've covered this' stress.

Equipping Yourself

I interpret *equipment* very broadly in this chapter. It's not just the mathematical paraphernalia like calculators and sharp pencils but also your whole studying environment: where do you work? Who do you work with?

You're probably tempted to say, 'I've got my calculator, I've got my desk, and I work *alone*, dammit' and skip straight to the next section – but I reckon it's worth making sure those things are well-suited to what you need. Sometimes, they can be the difference between an easy homework and a hard homework (or between a hard homework and an impossible one).

Stuff you need

In terms of actual equipment, there's not a whole lot you need for A level maths. A supply of pens and paper. Some pencils for drawing. A good scientific calculator (the calculator tips in this book are geared towards the Casio FX-83 or -85, but other brands are available).

You want somewhere to store your notes – ring-binders and poly-pockets are a good way to do that, although I advise against carrying ring-binders around with you, because the rings tend to break. They're a storage solution, not a transport solution! Alternatively, notebooks are always an option.

Set squares? Nope, definitely don't need them. Compasses? Nah, not really, unless you want your circle sketches to look really neat. Rulers? Again, the only time you *need* a ruler is if you're drawing something accurately, and there's very little of that at A level. If you want your axes to look neat, I suppose there's not much harm in rulers, but I personally make a stand for proud scruffiness in sketches.

Possible study aids to consider are

- **Index cards:** Index cards (bits of cardboard, typically about 8 centimetres by 13 centimetres) are really popular in the USA for revising but are bizarrely underused here. Write a question on one side, the answer on the other and presto! A quick quiz so you can revise whenever you have a spare moment.

- **Coloured pens:** Some students find it easier to remember their notes if they're colourful. If you're one of those, treating yourself to nice felt-tips is a good idea.
- **Textbooks:** I know, I know, you've just bought a brilliant *For Dummies* book! However, the official textbooks for your course have questions of the specific style you can expect in your exam as well as precise details of what you're expected to know.

Where to work

The best place to work is wherever you feel comfortable working. As a teenager, my best place to work was spread out on a bed (which at least forced me to tidy up before going to sleep). You might prefer working at the kitchen table, or at a coffee shop, or on the bus – or even changing it up and never working in the same place twice. Here are some things to consider about your work environment:

- **Lighting:** It's written into the parenting contract that, at some point in a child's life, a parent will have to turn a light on and nag, 'You'll hurt your eyes.' I don't believe you will hurt your eyes if you work in poor light, but you will make more mistakes. It's harder to read the questions and your work if the lights are too low.
- **Noise and distraction:** If you can work in a noisy environment, I envy you. Go ahead and work in the café or at the station. If you find it difficult to concentrate when there's hubbub, bear that in mind – close your door, ask people to leave you alone, and/or put on headphones to close out distractions.
- **Music:** Talking of headphones, try working with and without music. I find music with lyrics distracts me, but classical or instrumental music keeps me focused. You can also find noise generators that give you a background hiss to drown out noise.

Who to work with

Every so often, I think about education, exams and what the system teaches, and I cry a little bit: the whole system seems to be set up to show you that maths (and other subjects) are individual pursuits – working with other people is somehow cheating.

No it [expletive deleted] well isn't!

Maths is a collaborative effort. Pythagoras had a school of mathematicians he worked with. The boffins at Bletchley Park figured out the maths behind computers and codes as a team. Euler, probably the greatest genius in all of maths, collaborated widely. For every Andrew Wiles, who proved Fermat's Last Theorem more or less on his own, there are ten Paul Erdőses, who worked with everyone he could find, and a thousand unheard-of mathematicians who tried to do stuff on their own and failed.

The point is that doing maths on your own is much harder and much less fun than doing it with friends. There are all sorts of advantages:

- **Several heads are better than one.** Between you, you can figure out the answer to problems much more efficiently than on your own.
- **You're good at different things.** You can learn from each other and really shore up your own knowledge by helping each other.
- **You're building teamwork skills.** Employers like that. Especially in maths, you'll stand out.

It's important to pick the right people to work with, though. If you work with people who don't want to put the effort in, or who want to monopolise the conversation, or have a really bad attitude (like the kind of attitude I tell you to avoid in the next section), it's not going to be so much fun.

Instead, make sure you work with people who are ready to work together, who can talk without fighting and who have a positive, can-do (or if-we-can't-do-then-maybe-we-can-work-it-out-together) attitude. I promise you, it'll be better than slogging through it on your own.

Getting Your Head On Straight

If you were playing a sport, you'd make sure you were in good physical shape before you started. You wouldn't play hockey with a broken arm or hobble onto the basketball court on crutches, I hope.

Studying maths is much the same, only it's more of a mental workout than a physical one. You need to make sure you're in good shape mentally, or else you'll underperform (and perhaps do yourself an injury – believe me, you don't want to end up with a twisted mind!)

In this section, I give you some ideas about how to make sure you're at peak mental fitness before you start studying.

Please do not confuse this with Brain Gym. We do not speak of Brain Gym.

Sorting out your attitude

It's OK, it's OK, it's OK: I'm not accusing you of having an attitude. Sheesh. What I mean is that you ought to have an attitude if you want to do well at A level (and I presume you do). As a tutor, I've noticed several things about the attitudes of students who tend to succeed and the ones who struggle. (***Please note:*** This is a personal observation rather than anything I've done a rigorous study on.) The students who do poorly say things like the following:

- 'We haven't been taught that.'
- 'I don't like [topic].'
- 'I'll just hope for an easy paper.'

That's an attitude, all right: it's passive, and it's based on hope and preference rather than effort. 'We haven't been taught that' is particularly telling – it says 'Education is something that happens to me' rather than 'I take responsibility for my learning.' Instead of these horrible phrases, the students who do well say things like this:

- 'I've not seen that before. Can you point me at some resources?'
- 'How can I avoid mistakes on [topic]?'
- 'What's the nastiest thing they can ask about [topic]?'

See the difference? If you want to succeed, you need to *engage*. Figure out what you need to know, and make sure you learn it. Your teachers are there to help you do that, but they can't learn it for you.

Talking yourself up

Explain to me something, would you, while I tug at my cloth cap and wave my walking stick in the air? Why do young people today come out of exams and say, 'That went *terribly!* I don't think I got a single question right!' when they know perfectly well they're going to get an A? I'm sure they did it in my day, too, but I never bothered to challenge them on it.

Do you do that? Not necessarily the 'going to get an A' bit, although I hope that's the case. If you do talk like that, then stop it. It's silly and unhelpful: if you keep telling yourself you're bad at maths or that you're not good in exams, you'll end up making yourself believe it – which just leads to stress and misery. Stress and misery? Those are bad things best avoided.

Instead, you should be saying good things after exams: 'I'm pretty sure I got that question right' or 'I got lost on question 8, but I think I've picked up a few marks.' You're a decent mathematician, or they wouldn't be letting you do A level – own that!

This goes for everything in maths – if you say, 'I can't do this', you're reinforcing that idea. Simply changing the statement to 'I haven't figured this out yet' or even to 'I'm missing a piece here. I wonder what it is' changes your perception: instead of a statement of static helplessness, you've turned it into a dynamic statement in which you're moving forwards and looking for a solution. It sounds a bit daft, but it makes a *big* difference to your outlook.

REMEMBER

Don't say things to yourself that you wouldn't say to a friend who was struggling. You wouldn't tell your best friend she was useless, so don't tell yourself that, either. You'd encourage her to keep at it and reassure her it'll come good in the end, and she'd feel a bit better. That works when you do it to yourself, too.

Coping when things go wrong

Things *do* go wrong when you're studying. You will make mistakes. You will find things difficult. *I* make mistakes. *I* find things difficult. *Stephen Hawking* makes mistakes and finds things difficult. It's normal. It's part of the process.

The main difference between a good mathematician and a mediocre one is how they handle mistakes, bad test scores, tellings-off and other general disappointments. There's a lot of talk among teachers and politicians about 'resilience', and this is what they mean: it's the ability to bounce back when things go wrong.

I think everyone has a day when they wake up and maths has suddenly become *really* hard. For some people, it's algebra; for others, calculus; for others, logarithms. For me, it was functions at university – I remember turning the paper over in the exam and realising that I had no idea on the first two questions and just about recognised some of the words in the third. Yes, I even checked to make sure I was sitting the right exam. Proper, full-blown panic attack in the exam, and only just about got on top of it in time to answer enough questions to get me through with a mediocre grade.

Here's how I handled it:

- **Take deep breaths.** If you're not breathing properly, your brain isn't working properly. Put everything down for a few seconds, try to clear your brain and breathe deeply. (I was later taught to count to seven on the in-breaths and to eleven on the way out, but just making sure you fill your lungs up is a good start.)
- **Sit up straight.** This helps with the breathing – and it also helps you feel more confident. If you behave like a confident person, sometimes you can borrow some confidence!
- **Talk yourself up.** I mention this earlier in the chapter, but saying, 'Oh my God, I can't do this, I'm going to *fail*, aargh!' isn't especially helpful. Instead, say things like 'I *was* paying attention' and 'There *is* something on here I can do' and 'Let me try this and see how far I get.' After you get going, you can often fool your brain into keeping going.
- **Change things around.** In that exam, I knew I was toast for the first couple of questions, so I started with the last question and worked backwards through the paper. By the time I'd calmed down and got up a head of steam, I'd figured out what the third question was asking.

If you get regular panic attacks, *please* go and see a doctor. I suffered from undiagnosed mental health problems for many years and would really rather you didn't. They're horrible.

Setting Up a Study Plan

By and large, you can expect your school classes to lead you through the content you need in a fairly logical order and at a pace that means you'll cover everything in time for the exam. And that's fine, although I've worked with many students whose teachers fell ill at a critical stage and were not able to cover everything, some students who missed school themselves and ended up far behind, and a few students whose schools messed up and were not able to cover the whole course. It can happen.

The key to avoiding these calamities is to take charge. It's your A level. Figure out what you need to know and when you need to know it by, and make sure you learn it. Not only will it give you a fallback in case things go pear-shaped at school, but when *you* decide what you're going to work on, it's much less of a chore to do it.

Being your own pacemaker

There is nothing – *nothing* – in the rules that says you have to go at the same pace as your class. Ideally, nothing your teacher teaches you in class should be a surprise: if you've read ahead in the textbook to familiarise yourself with what's in the module, you're much better-placed to understand what you're being told.

This is part of what I mean by 'taking responsibility for your learning'. The more active you are about deciding what to learn and when to learn it, the more effective you'll be at learning it! Rather than simply doing your homework and saying, 'I'm done,' it's worth taking ten minutes to see what's on the menu for the next class.

You may not be used to learning from a textbook rather than from a teacher, but it's a skill that'll serve you well, both in other courses and at university (should you choose to go there). Here are some tips on how to do it:

- **Read the text.** This might sound like an obvious thing, but course textbooks aren't just a source of homework questions. They also give plenty of description of what a topic is about, why it's important and what it's used for. So if another student tries to derail the class by saying, 'But when will we ever use *this*?' you can say, 'Well, it's a really big thing in physics' or similar and shut them up.
- **Make notes.** As you go through, write down anything you think is important. Put things into your own words, if you can. Again, it's all about active learning: if you're translating something from Mathematician into English, you're thinking about it rather than letting it wash over you.
- **Come up with questions.** Take special care over anything that's not 100 per cent obvious to you, and think about what questions you could ask to put it right. The more specific questions you can ask, the better help you'll get. 'I don't get logarithms' is a much harder thing to fix than 'I tried this question, and here's what I did. I suspect the mistake is around here, but I can't see it.'

Finding the topics

Deciding which topics to revise isn't quite as big a job as doing the actual revision, although for some people it seems to take just about as long! There are four main ways to find topics to work on: