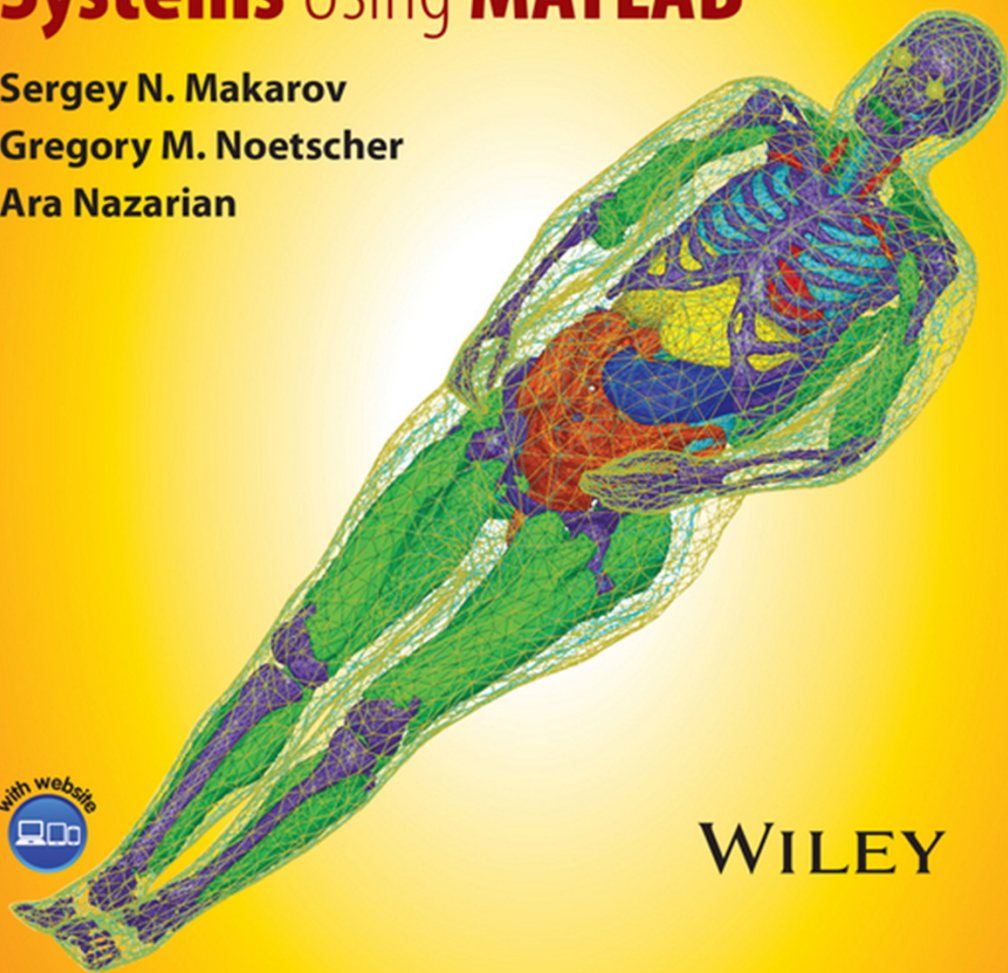


Low-Frequency Electromagnetic Modeling for **Electrical** and **Biological** Systems Using **MATLAB®**

Sergey N. Makarov
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Ara Nazarian



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MODELING FOR
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To Natasha, Yen, and Rosalynn

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PREFACE

SUBJECT OF THE TEXT

This text provides a systematic, detailed, and design-oriented course on electromagnetic modeling at low frequencies for electrical and biological systems. Low-frequency electromagnetic modeling, which is also known as a static or quasistatic approximation, is a well-established theoretical subject. Today, the role of low-frequency electromagnetic modeling in system design and testing is dominant in many disciplines. Examples include capacitive touchscreens in cellphones, the near-field wireless link between two cellphones or in implanted devices, power electronics, various bioelectromagnetic stimulation setups, modern biomolecular electrostatics, and many others. The text is divided into five parts:

Part I	Low-Frequency Electromagnetics. Computational Meshes. Computational Phantoms
Part II	Electrostatics of Conductors and Dielectrics. Direct Current Flow
Part III	Linear Magnetostatics
Part IV	Theory and Applications of Eddy Currents
Part V	Nonlinear Electrostatics

DISTINCT FEATURES

A unique feature of this text is the combination of fundamental electromagnetic theory and application-oriented computation algorithms realized in the form of distinct MATLAB[®] modules. The modules are stand alone open-source simulators,

which have a user-friendly and intuitive GUI and a highly visualized interactive output. They are accessible to all MATLAB users. No additional MATLAB toolboxes are necessary. The modules may be either employed along with this text or used and modified independently, for both research and demonstration purposes.

Yet another unique feature of the text is a large collection of computational human phantoms, including segmentation of the Visible Human Project[®] dataset performed over the last four years. In 2014, this model was evaluated and accepted by the IEEE International Committee on Electromagnetic Safety for the calculation of specific absorption rates. The computational human phantoms are an integral part of the present text. Simultaneously, they can be imported into major commercial electromagnetic software packages such as ANSYS, COMSOL, and CST.

AUDIENCE

The text is intended for use in courses on computational electromagnetics and in courses covering general electromagnetics and bioelectromagnetics. The targeted audience includes electrical and biomedical engineering students at the graduate or senior undergraduate levels as well as practicing researchers, engineers, and medical doctors working on sensor and bioelectromagnetic applications. The MATLAB modules can be used for demonstration purposes in any undergraduate classes.

NUMERICAL ALGORITHM

The three-dimensional method of moments (MoM), which is the surface charge boundary element method, is studied and utilized throughout the text. It is applicable to all *linear* static and quasistatic problems considered herein including heterogeneous objects such as human tissues. The major development steps of the MoM approach are not collected in a single chapter. They are specifically quantified and explained for each distinct physical problem pertaining to quasistatic electromagnetics. These steps include

- Poisson and Laplace equations in one, two, and three dimensions (Chapters 1, 4, 6, and 8–14);
- boundary conditions and the corresponding integral equations in terms of the relevant surface charge density (Chapters 1 and 4–12);
- construction of the MoM matrix, direct and iterative solutions (Chapters 4–8);
- methods for computations of potential integrals and their implementation, Gaussian quadratures (Chapters 4 and 6);
- Adaptive mesh refinement (Chapters 2, 4, and 7);
- MoM basis functions (Chapter 4);
- Cancellation error (Chapter 9).

For nonlinear problems, the situation complicates. An iterative solution may be applied. Chapters 13 and 14 study an important example of a nonlinear solution in one dimension for semiconductor pn-junction modeling.

APPLICATION EXAMPLES

Application examples included in this text are related to both electrical and biological problems. They cover all major subjects of low-frequency electromagnetic theory. These computational examples are applicable to many practical engineering problems and are designed to gain reader's interest and motivation in the subject matter. The examples include

- self-capacitance of a human body, modeling of ESD—electrostatic discharge (Chapter 5);
- capacitances of two or three arbitrary conductors—capacitive sensors (Chapter 5);
- human body computational phantom under a power line (Chapter 5);
- dielectric objects subjected to an applied electric field (Chapter 6);
- metal-dielectric capacitors (Chapter 6);
- modeling charges in cellphone capacitive touchscreens (Chapter 6);
- modeling two-dimensional single-ended and differential transmission lines (Chapter 7);
- electric impedance tomography—simple shapes and human tissues (Chapter 8);
- transcranial direct current stimulation of human tissues (Chapter 8);
- magnetic objects subjected to an applied magnetic field (Chapter 9);
- static magnetic shielding with a magnetic material (Chapter 9);
- computation of self- and mutual-coil inductances with/without magnetic core (Chapter 10);
- wireless inductive power transfer between two arbitrary inductors (Chapter 10);
- modeling gap field and leakage flux of a magnetic yoke (Chapter 10);
- eddy currents created by loop(s) of current in a conducting specimen (Chapter 11);
- upper estimate of eddy currents (Chapter 11);
- eddy currents excited in a human body (Chapter 12);
- nonlinear electrostatic modeling of a pn-junction—junction capacitance (Chapter 14).

Every application example is demonstrated with a distinct stand-alone MATLAB module, which can be extended and modified for relevant research purposes.

ANALYTICAL SOLUTIONS

Complete or summarized analytical solutions to a large number of quasistatic electromagnetic problems are presented throughout the text. These solutions provide a