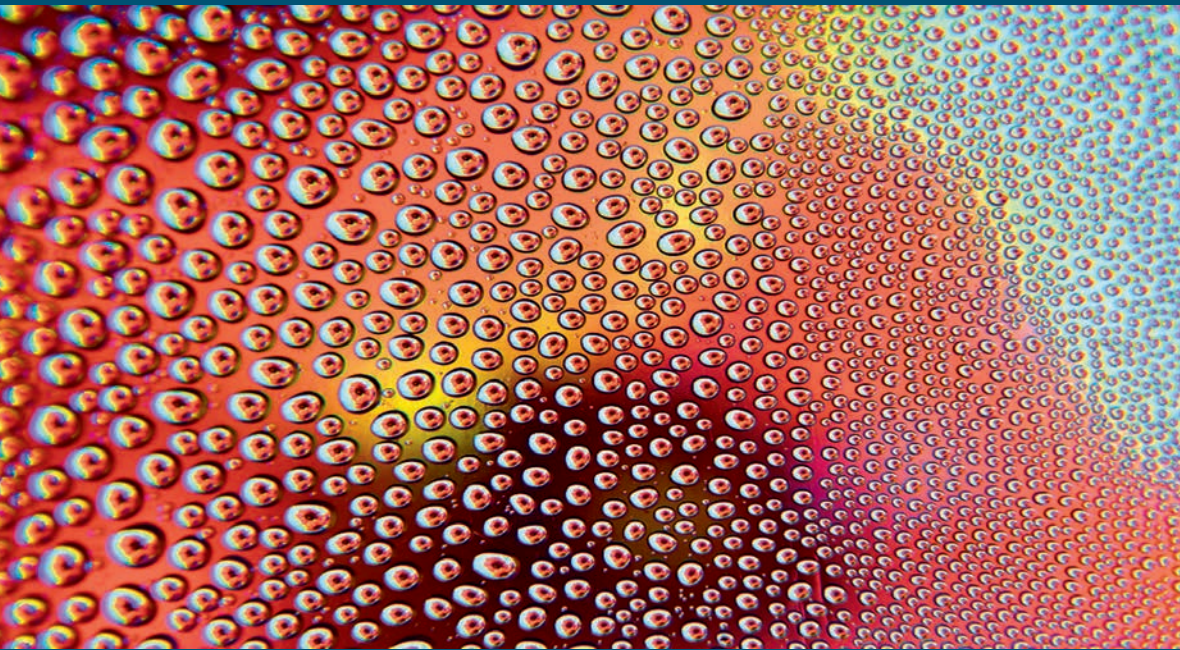


FLUID MECHANICS SERIES

Flows and Chemical Reactions in an Electromagnetic Field

Roger Prud'homme



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Preface

This book – a continuation of the previous publications *Flows and chemical reactions* [PRU 12], *Flows and chemical reactions in homogeneous mixtures* [PRU 13] and *Flows and chemical reactions in heterogeneous mixtures* [PRU 14a] – is devoted to *Flows and chemical reactions in an electromagnetic field*.¹

Part One, entitled Introduction, is made up of four chapters. *Chapter 1* gives an introduction to the equations of electromagnetism in the Minkowski timespace. This mode of presentation is extended to the balance equations, first in non-polarized homogeneous mixtures in *Chapter 2*, and then in a polarized homogeneous fluid medium in *Chapter 3*.

¹ Remember that the volume *Flows and chemical reactions* comprised three parts: 1. Fluid media with a single component, 2. Reactive mixtures, and 3. Interfaces and lines, that the volume *Flows and Chemical Reactions in Homogeneous Mixtures* comprised: 1. Pipe flows, 2. Chemical reactors, and 3. Laminar and turbulent flames, and that the volume *Flows and Chemical Reactions in Heterogeneous Mixtures* comprised: 1. Generation of multi-phase flows, 2. Problems at the scale of a particle, 3. Simplified model of a non-reactive flow with particles, 4. Simplified model of a reactive flow with particles, and 5. Radiative phenomena.

Chapter 4 is given over to heterogeneous media in the presence of an electromagnetic field. In that chapter, the balance equations at the interfaces are established.

Part Two of this volume is entitled Introduction. It too has four chapters. *Chapter 5* presents a study of the influence of diverse fields on flames; *Chapter 6* discusses a classic application of the Peltier effect; *Chapter 7* is devoted to metal/plasma interaction, and more specifically to the Langmuir probe, and finally *Chapter 8* discusses space propulsion by the Hall effect.

The *Appendix* gives supplementary information about the balance laws with an electromagnetic field, before going on to describe the methodology used to establish one-dimensional equations for a flow with active walls, as is the case with certain Hall effect thrusters.

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Roger PRUD'HOMME
September 2014

List of Main Symbols

Latin characters

A	chemical affinity; or monatomic species
A_2	diatomic species
\underline{A}	vectorial potential
A, B	Arrhenius coefficients
\mathbf{B}	magnetic field
c	speed of sound; or speed of light in a vacuum in the absence of a field
C	total number of moles per unit volume
C_j	molar concentration per unit volume
C_p, C_v	specific heat at constant pressure or constant volume respectively (c_p, c_v per unit mass)
d	distance; or differential
D	diffusion coefficient; or diameter
$D_{\alpha,i}$	diffusion coefficient of species α at quantum level i
$\vec{\mathbf{D}}$	electric displacement
$\vec{\vec{\mathbf{D}}}$	strain rate tensor