Computation of Nonlinear Structures Extremely Large Elements for Frames, Plates and Shells

Debabrata Ray









COMPUTATION OF NONLINEAR STRUCTURES

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WILEY

This edition first published 2016 © 2016 John Wiley & Sons, Ltd

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John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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Library of Congress Cataloging-in-Publication Data applied for.

A catalogue record for this book is available from the British Library.

ISBN: 9781118996959

Set in 10/12pt Times by Aptara Inc., New Delhi, India

1 2016

To the memory of my dad & mom, Provanshu and Sushila Roy, with whose altruistic love it was nurtured

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the best thing ever happened to me, my wife, Anjana Ray, M.D., with whose infinite patience and eternal support it blossomed

&

the budding analysts and researchers like my sons, Dipanjan Ray, PhD and Shonket Ray, PhD, to whom it is offered

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Acknowledgements

I would like to sincerely thank my friend, **Anil K. Chopra**, Johnson Professor of Structural Engineering, Department of Civil and Environmental Engineering, University of California, Berkeley, who introduced me to structural dynamics, for his encouragement and generous support in presenting my work to **Eric Willner**, Executive Commissioning Editor of John Wiley & Sons Ltd, who graciously agreed to publish the book, for which I am deeply appreciative. I must also thank **Anne Hunt**, Associate Commissioning Editor, Mechanical Engineering, for her unstinting support, and, **Clive Lawson**, Project Editor, Content Capture, Natural Sciences, Engineering & Stats, Professional Practice and Learning, Wiley. My special thanks for meticulous scrutiny of the manuscript to copy editor, Paul Beverley, LCGI, and Baljinder Kaur, Project Manager, Professional Publishing, Aptara.

During the writing of the manuscript for the book, I was paralysed as a result of a botched surgical procedure; however, from the initial period of rehabilitation to the date of this writing, I have been extremely fortunate to be surrounded by innumerable friends and well-wishers. I would like to thank them all; especially, my deep appreciation goes to my friend, Prof. Amitabha Basu of Dept. of South Asian Studies, University of California, Berkeley, and, to my care-givers, Levi Soler and John Viray.

Finally, however, the faults and mistakes, if any, are entirely mine.

DR August 31, 2014

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Introduction: Background and Motivation

1.1 What This Book Is All About

The book introduces linear and nonlinear structural analysis through a combination of of mesh generation, solid mechanics and a new finite element methodology called *c-type finite element* method (Ray, 1999, 2003, 2004, 2005, 2007, 2008). The ultimate objective is to present the largest possible (curved) beam, plate and shell elements undergoing extremely large displacement and rotation, and to apply these to solve standard industrial problems. Any finite element method is only as strong as its weakest link. In other words, the book is not just about unification of mesh generation and finite element methodology but it strives to serve as a reference for budding researchers, engineers, analysts, upper division and graduate students and teachers by demonstrating what various interdisciplinary machinery has to be accurately harnessed to devise a solid and conducive theoretical framework upon which to build a robust, reliable and efficient numerical methodology for linear and nonlinear static and dynamic analysis of beams, plates and shells. As indicated, the principal goal of the book is to produce the largest possible arbitrary shaped elements (a) defined and restricted solely by the requirements of geometry, material, loading and support conditions, (b) avoiding computational problems such as locking in the conventional finite element methods and (c) presenting new, accurate and explicit expressions for resolution of the symmetry issue of the tangent operator for beams, plates and shells in areas of extreme nonlinearity. The 'mega-sized' elements may result in substantial cost saving and reduced bookkeeping for the subsequent finite element analysis, and a reduced engineering manpower requirement for the final quality assurance. For example, the explicit algebraic and symmetric expressions of the tangent operator, as presented in the book, are an absolute necessity for computational cost efficiency, especially in repetitive calculations that are commonly associated with nonlinear problems. It must be recognized that the requirements for numerical convergence should be purely adaptive and subservient to the main delineating factors already mentioned. However, this strategy of computer generation of mega-elements of arbitrary shape, as it turns out, takes its toll on the analyst. Firstly, only accurate theoretical formulation can be used for the underlying continuum or solid mechanics principles without unnecessary 'short-circuiting' by proliferation of ad hoc numerical manipulations. Secondly, it demands that the applicable finite element method be devised to successfully accept computer generated elements with arbitrarily distorted shapes, with edges (or faces) consisting of up to truly 3D curved boundaries (or surfaces)

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