

# Advances in Industrial Mixing

*A Companion to the Handbook of Industrial Mixing*

*Edited by*

*Suzanne M. Kresta*

*Arthur W. Etchells III*

*David S. Dickey*

*Victor A. Atiemo-Obeng*

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## A COMPANION TO THE HANDBOOK OF INDUSTRIAL MIXING

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Edited by

**Suzanne M. Kresta**

Professor  
University of Alberta  
Edmonton, Canada

**Arthur W. Etchells III**

DuPont Fellow Emeritus, Adjunct Professor  
Chemical Engineering, Rowan University, Glassboro, NJ

**David S. Dickey**

MixTech, Inc., Coppel, Texas, USA

**Victor A. Atiemo-Obeng**

Fellow, Core Research & Development  
The Dow Chemical Company (Retired), Midland, MI



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Published by John Wiley & Sons, Inc., Hoboken, New Jersey  
Published simultaneously in Canada.

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***Library of Congress Cataloging-in-Publication Data is available.***

ISBN: 978-0-470-52382-7

Printed in the United States of America  
10 9 8 7 6 5 4 3 2 1

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## CONTRIBUTORS LIST

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**Otute Akiti** Drug Substance Manufacturing & Development, Relypsa, Inc.,  
Redwood City, CA

**Albert Alexander** Department of Chemical and Biochemical Engineering, Rutgers  
University, Piscataway, NJ

**Mario M. Alvarez** Rutgers University, New Brunswick, NJ

**Ashraf Amanullah** Merck & Co., Inc., West Point, PA

**Engin B. Arik** VioSense Corporation, Pasadena, CA

**Piero M. Armenante** Pharmaceutical Engineering Program, Department of  
Chemical, Biological and Pharmaceutical Engineering, New Jersey Institute of  
Technology, Newark, NJ

**Gabriel Ascanio** Universidad Nacional Autonoma del Mexico

**Victor A. Atiemo-Obeng, Co-editor** The Dow Chemical Company, retired,  
Midland, MI

**Joëlle Aubin** Laboratoire de Génie Chimique, Université de Toulouse, Toulouse,  
France, and CNRS, Toulouse, France

**André Bakker** Fluent, Inc., Lebanon, NH

**Chad P. J. Bennington (deceased)** NSERC/Paprican Chair in Chemical Pulping  
Technology, The University of British Columbia, Vancouver, BC, Canada

**David A. R. Brown** BHR Group, The Fluid Engineering Centre, Cranfield,  
Bedfordshire, UK

**Barry C. Buckland** Merck & Co., Inc., West Point, PA

**Richard V. Calabrese** Department of Chemical and Biological Engineering,  
University of Maryland, College Park, MD

**Robin Kay Connelly** Solae, LLC, St. Louis, MO

**Daniel R. Crookston** Nalco Champion, an Ecolab Company, Singapore

**Reid B. Crookston** Retired Engineer, League City, TX

**P. J. Cullen** Dublin Institute of Technology, Dublin, Ireland

**Michael K. Dawson** The Fluid Engineering Centre, Cranfield, Bedfordshire, UK

**Shrikant Dhodapkar** Solids Processing Lab, Dow Chemical Company, Freeport, TX

**David S. Dickey, *Co-editor*** MixTech, Inc., Coppell, TX

**Jay Dinnison** Sharpe Mixers, Seattle, WA

**Thomas Dziekonski** SPX Flow Technology, LIGHTNIN Operation, Rochester, NY

**Arthur W. Etchells III, *co-editor*** The DuPont Company, DuPont Engineering Technology (retired), Philadelphia, PA

**Julian B. Fasano** Chemineer, Inc., Dayton, OH

**Louis Fradette** Chemical Engineering Department, Polytechnique Montreal, Montreal, Canada

**James F. Gilchrist** Department of Chemical Engineering, Lehigh University, Bethlehem, PA

**Chris Goodridge** Department of Chemical and Biochemical Engineering, Rutgers University, Piscataway, NJ

**Richard K. Grenville** DuPont Engineering Wilmington DE (Handbook, Chapter 9a) and Mixing Technology, Philadelphia Mixing Solutions, Ltd., Palmyra, PA (Advances, Chapter 10b)

**Ramesh R. Hemrajani** ExxonMobil Research and Engineering Company, New Jersey

**Todd Hutchinson** Philadelphia Mixing Solutions, Palmyra, PA

**Karl Jacob** Solids Processing Lab, Dow Chemical Company, Freeport, TX

**Eric Janz** Chemineer, Inc., Dayton, OH

**Pip N. Jones** BHR Group, The Fluid Engineering Centre, Cranfield, Bedfordshire, UK

**Richard Kehn** SPX Flow Technology (LIGHTNIN Mixers), Rochester, NY

**Suzanne M. Kresta, *Chief Editor*** Department of Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta, Canada

**Douglas E. Leng** Leng Associates, Midland, MI

**Minye Liu** DuPont, Engineering Research & Technology, Wilmington, DE

**Márcio B. Machado** Department of Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta, Canada

**Konanur Manjunath** Solids Processing Lab, Dow Chemical Company, Freeport, TX

**Elizabeth Marden Marshall** Fluent, Inc., Lebanon, NH

**Thomas Martin** Technische Hochschule Merseburg, Merseburg, Germany

**Chris F. Meyer** Sulzer Chemtech USA, New Milford, NJ

**John C. Middleton** BHR Group, The Fluid Engineering, Centre, Cranfield,  
Bedfordshire, UK

**Michael Midler** Merck & Co., Inc. Rahway, NJ

**Fernando J. Muzzio** Department of Chemical and Biochemical Engineering,  
Rutgers University, Piscataway, NJ

**Kevin J. Myers** Chemical and Materials Engineering, University of Dayton,  
Dayton, OH

**E. Bruce Nauman, deceased** Rensselaer Polytechnic Institute, Troy, NY

**Alvin W. Nienow** School of Chemical Engineering, University of Birmingham,  
Birmingham, UK

**José Roberto Nunhez** Universidade Estadual de Campinas (UNICAMP), Cidade  
Universitária Zeferino Vaz, Campinas, SP, Brazil

**Gul Özcan-Taşkin** BHR Group, Cranfield, Bedfordshire, United Kingdom

**George Papadopolous** Dantec Dynamics, Inc., Mahwah, NJ

**Gary K. Patterson** Department of Chemical Engineering, University of  
Missouri—Rolla, Rolla, MO

**Edward L. Paul** *editor emeritus* Chemical Engineering Research and  
Development (retired), Merck & Co., Rahway, NJ

**W. Roy Penney** Department of Chemical Engineering, University of Arkansas,  
Fayetteville, AR

**Kayla Preston** Sharpe Mixers, Seattle, WA

**Aaron Sarafinas** The Dow Chemical Company, Collegeville, PA

**Elizabeth Shen** Department of Chemical and Biochemical Engineering, Rutgers  
University, Piscataway, NJ

**Troy Shinbrot** Department of Chemical and Biochemical Engineering, Rutgers  
University, Piscataway, NJ

**Thomas A. Simpson** The DuPont Company, Wilmington, DE

**John H. Smith (deceased)** University of Surrey

**Patrick T. Spicer** School of Chemical Engineering, University of New South  
Wales, Sydney, Australia

**Abraham D. Stroock** School of Chemical and Biomolecular Engineering, Cornell University, Ithaca, NY

**Yongkui Sun** Merck & Co., Inc., Rahway, NJ

**Edit S. Szalai** Rutgers University, New Burnswick, NJ

**Philippe Tanguy** Scientific Development Directorate, Total SA, Paris, France

**Gary B. Tatterson** North Carolina A&T State University, Greensboro, NC

**Cheryl I. Teich** The Dow Chemical Company, Collegeville, PA

**David B. Todd (deceased)** New Jersey Institute of Technology, Princeton, NJ

**Wesley Twombly** Solae, LLC, St. Louis, MO

**Ron J. Weetman** Lightnin, retired, Rochester, NY

**Gord Winkel** Faculty of Engineering, University of Alberta, Edmonton, Alberta, Canada

**Ryuichi Yatomi** Sumitomo Heavy Industries, Saijyo, Ehime, Japan

**Henry Zhang** Department of Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta, Canada

## EDITORS' INTRODUCTION

---

SUZANNE M. KRESTA

*University of Alberta*

ARTHUR W. ETCHELLS, III

*DuPont, retired, and Rowan University*

DAVID S. DICKEY

*MixTech*

VICTOR A. ATIEMO-OBENG

*The Dow Chemical Company, retired*

### 1 ORGANIZATION OF THE ADVANCES VOLUME

The *Handbook of Industrial Mixing* (hereafter referred to as “the *Handbook*”) was published in 2004 and has become a standard reference in the field. In designing this volume on advances in the field, we decided to provide the full original text of the *Handbook* in electronic form, and provide only the new and substantially revised material in print form. In this way, we hope to both reduce the shelf space taken up by multiple editions of favorite reference texts and make the key material in the *Handbook* fully portable for all readers—even when today’s excellent virtual libraries are out of reach. This volume of *Advances in Industrial Mixing* has brand new chapters for six areas of industrial application, additional sections or examples for five chapters, updates for pipelines, turbulence, and computational fluid dynamics (CFD), and five completely new fundamentals chapters. The videos have been completely revamped and updated so that the user interface is robust and portable across as many platforms as possible. The trouble-shooting charts that were included in the Introduction to the *Handbook* are now placed within individual chapters for easier reference. Copies are also provided on the accompanying DVD.

In all, there are 21 new chapters or sections in this advances volume. To provide continuity in cross references and the strongest possible links between the two volumes, the original chapter numbers have been retained for the present volume. Where the chapter is unchanged and not reprinted, the table of contents and

Introduction from the *Handbook* are reprinted in this volume. The reader is referred to the searchable pdf for the rest of the chapter contents.

Mixing as a discipline has evolved from early foundations that were laid in the 1950's, culminating in the publication of works by Uhl and Gray (1967) and Nagata (1975). Over the last 30 years, many engineering design principles have been developed and design of mixing equipment for a desired process objective has become possible. Later books by Harnby et al. (1992) and by Baldyga and Bourne (1999), stand as definitive landmarks in the field of industrial mixing. The present volume is a compilation of the experience and findings of those who have been most active in the development of the current state of mixing science and practice. Together, the authors' experiences extend over more than 1500 years of research, development, and consulting work.

This book is written for the practicing engineer who needs to both identify and solve mixing problems. In addition to a focus on industrial design and operation of mixing equipment, it contains summaries of the foundations on which these applications are based. In order to accomplish this, most chapters have paired an industrialist and an academic as co-authors. Discussions of theoretical background are necessarily concise, and applications contain many illustrative examples. To complement the discussions and enhance awareness of the complexity of mixing phenomena, a DVD is included that contains 21 instructional videos compiled from 130 raw clips of mixing processes.

**The core mixing design topics include the following:**

- **Homogeneous blending in tanks and in-line mixers**
- **Suspension and distribution of solids in liquids**
- Dispersion of gases in liquids with subsequent mass transfer
- Liquid–liquid dispersions
- **Reactions: both homogeneous and heterogeneous**
- **Heat transfer**
- Mixing of two or more solids

The following underlying principles are presented:

- **Technical definition of mixing**
- Residence time distribution and **mean age**
- **Turbulence**
- **Laminar blending** and dynamical systems
- **Rheology**

**Additional information is provided on ways of investigating mixing performance:**

- Experimental measurement techniques
- **Computational fluid dynamics**

**These topics are augmented by chapters on specific industrial mixing topics:**

- Mixing equipment: vessels, **vessel heads**, rotor-stators, **micromixers**, and **pipeline mixers**
- Solid–solid blending
- Polymer processing
- **Food mixing**
- Fine chemical and pharmaceutical processes
- **Pharmaceutical validation**
- **Crystallization**
- Fermentation and cell culture
- **Water treatment**
- Petroleum
- Pulp and paper
- Mechanical aspects of mixing equipment
- **Commissioning of mixing equipment**
- **Safety for mixing processes**
- The vendor's role

Topics in bold are entirely new or updated in this companion volume. The *Handbook* has been included for the readers' reference as a searchable pdf file on the accompanying DVD.

## **2 MIXING IN PERSPECTIVE**

***What is mixing?*** Since the publication of the *Handbook*, a technical definition of mixing has been proposed that considers three constraints: the degree of homogeneity in concentration, the scale of segregation (usually of a second phase), and the rate of mixing or the mixing time. The most difficult mixing problems are those where several constraints must be achieved at the same time. An example of this would be dissolution of a solid reagent into a liquid–liquid system where a reaction occurs at the interface of the two liquids. The reaction is mixing sensitive, so the product distribution depends on rapidly achieving homogeneity of concentration.

***What constitutes a mixing problem?*** Process objectives are critical to the successful manufacturing of a product, but if the mixing scale-up fails to produce the required product yield, quality, or physical attributes, the costs of manufacturing may be increased significantly, and, perhaps more importantly, marketing of the product may be delayed or even canceled in view of the cost and time required to correct the mixing problem.

Some mixing problems cannot be solved without changes to either the process steps or mixing equipment. Many batch operations, even formulation processes without a chemical reaction, can involve multiple steps through process conditions

with different fluid properties. A single type of mixing equipment will not work equally well in all process conditions. Some compromises must be made between mixing success and modifications to the process and equipment that create situations that cannot be solved by mixing alone. Understanding what is possible is an important part of mixing expertise, as represented by the diversity of information in this handbook.

Although there are many industrial operations in which mixing requirements are readily scaled-up from established correlations, many operations require a more thorough evaluation. In addition to presenting the state of the art on the traditional topics, this book presents methods for recognition of more complex problems and alternative mixing designs for critical applications. The updated turbulence chapter provides context for scale-up protocols, and an example of the Bourne protocol is provided as an update to Chapter 13, Mixing and Chemical Reaction of the *Handbook*. Many of the other chapters contain advice or additional protocols for specific process objectives.

Failure to provide the necessary mixing may result in severe manufacturing problems on scale-up, ranging from costly corrections in the plant to complete failure of a process. The costs associated with these problems are far greater than the cost of adequately evaluating and solving the mixing issues during process development. Conversely, the economic potential of improved mixing performance is substantial. Consider the following numbers:

- *Chemical Industry*: In 1989, the cost of poor mixing was estimated at \$1 to 10 billion in the U.S. chemical industry alone. In one large multinational chemical company, lost value due to poor mixing was estimated at \$100 million per year in 1993. Yield losses of 5% due to poor mixing are typical.
- *Pharmaceutical Industry*: Three categories should be considered: costs due to lower yield (costing on the order of \$100 million, industrywide), costs due to problems in scale-up and process development (costing on the order of \$500 million, industrywide), and costs due to lost opportunity, where mixing problems prevent new products from ever reaching the market (a very large number).
- *Pulp and Paper Industry*: Following the introduction of medium-consistency mixer technology in the 1980's, a CPPA survey documented chemical savings averaging 10 to 15% (Berry, 1990). Mills that took advantage of the improved mixing technology saw their capital investment returned in as little as three months.

From these numbers, the motivation for this two volume handbook and for the research efforts that it documents become clear. The reader will almost certainly profit from the time invested in improved understanding of design of mixing equipment. Mixing equipment design must go beyond mechanical and costing considerations, with the primary consideration being how best to achieve the desired process objectives.



*Mixing solutions seldom emphasize minimizing equipment costs or optimizing power consumption: They are more likely to focus on critical issues in process performance.*

***How much mixing is enough and when could overmixing be damaging to yield or quality?*** These critical issues depend on the process and the sensitivity of selectivity, physical attributes, separations, and/or product stability to mixing intensity and time. The nonideality of residence time distribution (RTD) effects combined with local mixing issues can have a profound effect on a continuous processes. A new tool for assessing the local nonideality of RTD's, the mean age, is described in Chapter 1b.

Useful methods for mixing process development effort have been evolving in academic and industrial laboratories over the past several decades. They include improvements to traditional correlations as well as increasingly effective methods both for experiments and for simulation and modeling of complex operations. The combination of these approaches is providing industry with greatly improved tools for development of scalable operations. This handbook provides the reader with all of the information required to evaluate and use these technologies effectively in process development and scale-up.

***How should new mixing problems be solved?*** Solutions for new mixing problems require answers to the question "Why?" as well as the very pressing question of "How?" This question is best addressed with a good understanding of both the process and the underlying fundamentals. This requires discussion with both operations and developmental chemists. It is often well served by reposing the question "How can we scale this up?" as "How can we scale down the process equipment to closely replicate plant conditions in the lab?" The importance of this question should never be underestimated, as it often opens the door for discussions of geometric similarity and matching of mixing conditions. Good experimental design based on an understanding of mixing mechanisms is critical to obtaining useful data and robust solutions. The engineer who ignores the fundamentals always does so at their own peril. Even when geometric similarity is followed and the mixer speed is adjusted according to some meaningfully equivalent mixing intensity, the laboratory or pilot plant model will not exactly match all of the production scale process conditions. Different mixing phenomena scale differently, which makes understanding the fundamentals and multiple process objectives important. It is our hope, in writing this book, that mixing fundamentals will become accessible to a much wider audience of engineers, chemists, and operators whose processes are affected by mixing issues.

***What can be done to improve existing processes?*** The most numerous, if not the greatest, opportunities for process improvement through better mixing can be found in existing processes and equipment. Much existing mixing equipment is more than 25 years old. With near certainty, the process running in older equipment is not the same as the process for which that equipment was purchased. Even if the product is the same, the competition, knowledge, and equipment have changed, and better mixing technology may provide opportunities for improvement. Even when replacing an old mixer with a new mixer does not make economic sense, equipment modifications and process adjustments can reduce mixing problems. Improvements can come from

a better, more up-to-date, understanding of mixing processes and equipment. Old technology is often based on old ideas or limited information, which can be updated through the advances provided in this volume. The greatest advantage to working on existing processes is the certain knowledge that it is possible, so improvement is always an opportunity.

### 3 SCOPE OF THE *HANDBOOK* AND *ADVANCES VOLUME*

Mixing plays a key role in a wide range of industries:

- Fine chemicals, agrichemicals, and pharmaceuticals
- Petrochemicals
- Biotechnology
- Polymer processing
- Paints and automotive finishes
- Cosmetics and consumer products
- Food
- Drinking water and wastewater treatment
- Pulp and paper
- Mineral processing

In all of these industries, the components of mixing problems can be reduced to some fundamental concepts and tools. The key variables to identify in any mixing problem are:

1. The time available to accomplish mixing (the time scale)
2. The required homogeneity and the required scale of homogeneity
3. In the case of a multi-phase system, the scale of segregation

In the Introduction to the *Handbook*, the major areas of mixing were introduced and reviewed and the reader is referred to that section and the entertaining *conversations overheard in a chemical plant* for an overview of the skill sets that a mixing expert can leverage against process problems. In this volume, we provide instead an overview of the sections in the *Handbook* and the highlights of the *Advances Volume*.

#### 3.1 Fundamentals

The fundamentals section of the handbook begins with a discussion of the technical definition of mixing, which has been developed and enhanced over the last 10 years. This is followed by the origins of mixing theory, namely the residence time distribution, and the spatial distribution of mean age, which is a recent development on the classical residence time calculations. The mean age is appealing because it provides local information about mixing from the mean velocity field arising from a

CFD simulation. Following this are summaries of turbulence theory, as it applies to mixing, and chaos theory, again, as it applies to industrial mixing processes. These chapters are now augmented with a new chapter on the important question of fluid rheology and complex fluids, where the process fluid does not have a single viscosity but a range of properties that respond to both shear rate and, in some cases, shear history. The fundamentals section is rounded out with chapters on the key experimental methods used for mixing evaluation, and two chapters on CFD—the first providing a comprehensive review of the various tools available, and the second, much shorter, chapter providing perspective on where CFD can—and *also cases where it should not*—be applied for gaining understanding of industrial processes.

## **3.2 Mixing Equipment**

The second section of the *Handbook* reviewed the design and operation of the major types of mixing equipment: stirred tanks, pipelines with and without static mixers, micromixers, and rotor-stators. There are new sections on vessel heads (head depth, volume, and area calculations), flow patterns in stirred tanks, some very useful developments on mixing in pipelines, and an introduction to the relatively new area of micro-mixers.

## **3.3 Core Mixing Objectives**

The heart of the handbook is made up of the seven fundamentals: blending, solid–liquid mixing, gas–liquid mixing, liquid–liquid mixing, mixing and reactions, heat transfer, and solid–solid mixing. Of these seven chapters, three have been updated. A substantial chapter on laminar blending has been added. Additional examples and calculations have been provided for the heat transfer chapter and the reactions chapter. Most importantly, the solid–liquid mixing chapter has been completely rewritten due to very significant changes in our fundamental understanding of this area. It is strongly recommended that readers actively involved with solid–liquid mixing designs review this new chapter in detail.

## **3.4 Applications**

The *Handbook* pressed the binding limits of a single volume and left out several important industrial applications. Addressing this deficiency was one of the three major goals of the advances volume, so while none of the seven original applications chapters have been updated, there are six new chapters that fill out the rest of the major areas of interest. Chapter 17 on fine chemicals and pharmaceuticals is now complemented by chapters on crystallization and pharmaceutical process validation. The chapters on mechanical design and the role of the vendor are complemented by new chapters on safety and commissioning of equipment. Mixing operations in two major industries, food and water treatment, are now integrated with the rest of the handbook.

### 3.5 Using the *Handbook* and the *Advances Volume*

The *Handbook* and this companion volume are not meant to be read from beginning to end. They are designed as a reference, with extensive cross referencing and indexing—not just between chapters, but now between books. Many examples are included to aid the reader in understanding the fundamentals as well as some case histories of mixing issues in industrial practice. Authorship of most of the chapters includes both academic and industrial contributors for the purpose of providing a broad perspective on each topic. Also included is a DVD with instructional videos to illustrate, wherever possible, specific mixing issues and examples. The first chapter, *A Technical Definition of Mixing*, will help the reader new to the field of mixing in identifying what is meant by a mixing problem.

The text and examples include guidance in troubleshooting mixing problems based on understanding the fundamental issues, aided by drawing on the experiences cited. It is often assumed that mixing scale-up is accomplished by direct scaling to a larger pot. This approach may work in some cases but is doomed to failure in others. The key question is the determination of process requirements for which direct scale-up will be inadequate. Another overall concern is to beware of the fact that multiple process objectives often must be realized in a single piece of mixing equipment, thereby requiring selection of a design basis compatible with the most critical scale-up issue(s).

## 4 MIXING NOMENCLATURE AND UNIT CONVERSIONS

Table I-1 includes the common nomenclature used in mixing correlations and calculations. Many of the chapters in this book have more detailed lists of nomenclature for specific topics. Table I-1 is intended for general reference. Where a symbol is used for more than one purpose, the common multiple uses are given. The nomenclature follows that outlined by Oldshue (1977), Buck (1978), and the AIChE Equipment Testing Procedure for Mixing Equipment (2001) of symbols for use in the SI system. There are a few exceptions that are commonly used in mixing terminology. The European Federation of Chemical Engineering Working Party on Mixing Terms, Symbols, and Units has also published a comprehensive list of nomenclature (Fort et al., 2000). Conversions are given in Table I-2 and Table I-3.

## 5 ACKNOWLEDGMENTS

The editors would first like to acknowledge the contributions of the many authors whose efforts in writing their respective chapters have made publication of these two handbooks possible. The reader will appreciate the difficulty of finding time in very full professional lives to write authoritative chapters on fundamental ideas: chapters that required both reflection and compilation of the vast quantities of information in the technical literature. In addition, we would like to acknowledge the reviewers

**Table I-1** Mixing Nomenclature

Common Symbol	Quantity	Units
A, B, R, S	Reactants	(—)
A, B, R, S	Reactant concentrations, $C_A$ , $C_B$ , etc.	mol/m <sup>3</sup>
B	Baffle width	m
C	Impeller off-bottom clearance	m
C	Reaction conversion, $(A_o - A)/A_o$	%
$C_p$	Specific heat	J/kg K
D	Impeller diameter	m
$D_{AB}$	Diffusivity	m <sup>2</sup> /s
Da	Damkohler number (see Chapter 13)	(—)
$d_{32}$	Sauter mean diameter	m
Fr	Froude number, $N^2 D/g$	(—)
$g_c$	Gravitational correction for British units	32.2 lb <sub>m</sub> /lb <sub>f</sub> × ft/s <sup>2</sup>
H	Liquid height	m
$k_1, k_2, \dots$	Reaction rate constants	(mol/m <sup>3</sup> ) <sup>1-n</sup> /s
k	Thermal conductivity	W/m K
$k_g, k_l$	Mass transfer coefficient	m/s
L	Length scal	m
N	Impeller rotational speed	rps or rpm
$N_c$	Impeller critical rotational speed	rps or rpm
$N_E$	Just entrained speed for gas entrainment from surface	rps or rpm
$N_{js}$	Just suspended rotational speed	rps or rpm
$N_{jd}$	Just dispersed speed for liquid–liquid mixing	rps or rpm
$N_{min}$	Just suspended speed for liquid drops	rps or rpm
Nu	Nusselt number, $hT/k$	(—)
$N_p$ or Po	Power number, $Pg_c/\rho N^3 D^5$	(—)
P	Power, $N_p \rho N^3 D^5/g_c$	W
P	Pressure	Pa
Pr	Prandtl number, $C_p \mu/k$	(—)
Q	Heat transfer rate, $UA\Delta T_{lm}$	W
$Q_L$	Pumping rate of impeller, $N_Q ND^3$	m <sup>3</sup> /s
R	Gas constant	J/mol K
R	Impeller radius	m
Re	Reynolds number, $\rho DV/\mu$	(—)
Re	Impeller Reynolds number, $\rho ND^2/\mu$	(—)
S	Reaction selectivity (see Chapter 13)	(—)
Sc	Schmidt number, $\mu/D_{AB}\rho$	(—)
T	Tank diameter	m
T	Temperature	K, C
t	Time	s
To or $T_Q$	Torque	$N_p \rho N^2 D^5$ W/s
U	Overall heat transfer coeff, $Q/(A\Delta T_{lm})$	J/m <sup>2</sup> s K
$u'$	Fluctuating velocity	m/s
$V_{imp}$	Impeller swept volume	m <sup>3</sup>

(continued)

**Table I-1**    (Continued)

Common Symbol	Quantity	Units
V	Volume	m <sup>3</sup>
V	Velocity	m/s
W <sub>b</sub>	Baffle width	m
X <sub>s</sub>	Impurity selectivity, 2S/R + 2S	%
Y	Reaction yield, R/A <sub>o</sub>	%
Z	Vessel straight side	m
<b>Greek Symbols</b>		
α	Blade angle	°
γ	Shear rate	s <sup>−1</sup>
δ	Width of shear gap, rotor and stator	m
ε	Void fraction	(−)
ε	Local rate of dissipation of turbulent kinetic energy per unit mass	m <sup>2</sup> /s <sup>3</sup> or W/kg
ε <sub>avg</sub>	Power input per mass of fluid in the tank, power per volume, P/ρV <sub>tan</sub>	m <sup>2</sup> /s <sup>3</sup> or W/kg
ε <sub>imp</sub>	Power input per mass of fluid in the impeller swept volume, P/ρV <sub>Impeller</sub>	m <sup>2</sup> /s <sup>3</sup> or W/kg
η (also λ <sub>K</sub> )	Kolmogorov scale, (ν <sup>3</sup> /ε) <sup>1/4</sup>	m
θ <sub>B</sub> , t <sub>blend</sub>	Blend time	s
θ	Angle of impeller blade with axis of rotation	°
λ	Taylor microscale of turbulence	m
λ	Wavelength	m
λ <sub>B</sub>	Bachelor length scale, (νD <sup>2</sup> <sub>AB</sub> /ε) <sup>1/4</sup>	m
λ <sub>K</sub> (also η)	Kolmogorov scale, (ν <sup>3</sup> /ε) <sup>1/4</sup>	m
μ	Dynamic viscosity	Pa s
ν	Kinematic viscosity, μ/ρ	m <sup>2</sup> /s
ρ	Density	kg/m <sup>3</sup>
σ	Interfacial tension	N/m
τ <sub>M</sub>	Mixing time constant	s
τ <sub>D</sub>	Diffusion time constant	s
τ <sub>R</sub>	Reaction time constant	s
τ	Shear stress	Pa
τ (also T <sub>Q</sub> )	Torque	N m
φ	Volume fraction of dispersed phase	(−)
φ	Particle shape factor	(−)

whose careful evaluations have been instrumental in helping the authors and editors to evaluate the technical content and relevance of each chapter:

- Harry van den Akker,
 TU Delft
- Mark Allen,
 Zeon Chemicals LP
- David Asay,
 PPG Industries
- Inci Ayranci,
 Middle East Technical University

**Table I-2** Conversion from British to SI Units

Non-SI Unit	Quantity	To Convert to SI Unit	Multiply by
Btu	Heat	Joule (J)	1.0551 E +03
Btu/lbm °F	Heat capacity	J/kg K	4.1868 E +3
Btu/hr	Heat flux	Watt (W)	2.9307 E -01
Btu/hr ft <sup>2</sup> °F	Heat transfer coefficient	W/m <sup>2</sup> K	5.6782 E +00
Btu/ft hr °F	Thermal conductivity	W/m K	1.7307 E +00
cal	Calorie	Joule (J)	4.1868 E +00
centipoise	Viscosity	Pa s	1.0000 E -03
centistoke	Kinematic viscosity	m <sup>2</sup> /s	1.0000 E -06
°F	Temperature	°C	(°F - 32)/(5/9)
dyne	Force	Newton (N)	1.0000 E -05
erg	Energy	Joule (J)	1.0000 E -07
ft	Length	Meter (m)	3.0480 E -01
ft <sup>2</sup>	Area	m <sup>2</sup>	9.2990 E -02
ft <sup>3</sup>	Volume	m <sup>3</sup>	2.8316 E -02
ft lb <sub>f</sub>	Work	Joule (J)	1.3558 E +00
ft lb <sub>f</sub>	Torque	Newton-meter (N·m)	1.3558 E +00
gallon (U.S. liquid)	Volume	m <sup>3</sup>	3.7854 E -03
horsepower	550 ft-lb <sub>f</sub> /sec	Watt (W)	7.4570 E +02
inch	Length	Meter (m)	2.5400 E -02
inches Hg (60°F)	Pressure	Pascal (Pa)	3.3768 E +03
inches H <sub>2</sub> O (60°F)	Pressure	Pascal (Pa)	2.4884 E +02
in lb <sub>f</sub>	Torque	Newton-meter (N·m)	1.1298 E -01
kilocalorie	Energy	Joule (J)	4.1868 E +03
micrometer	Length	m	1.0000 E -06
mmHg (0°C)	Pressure	Pascal (Pa)	1.3332 E +02
poise	Absolute viscosity	Pa s	1.0000 E -01
lb <sub>f</sub>	Force	Newton (N)	4.4482 E -00
lb <sub>m</sub>	Mass	Kilogram (kg)	4.5359 E -01
lb <sub>m</sub> /ft <sup>3</sup>	Density	kg/m <sup>3</sup>	1.6018 E +01
lb <sub>m</sub> /ft-sec	Viscosity	Pa s	1.4882 E +00
psi	Pressure	Pascal (Pa)	6.8948 E +03
rpm	Rotational speed	rps (s <sup>-1</sup> )	1.6667 E -02
Stoke	Kinematic viscosity	m <sup>2</sup> /s	1.0000 E -04
tonne (long, 2240 lb <sub>m</sub> )	Mass	Kilogram (kg)	1.0160 E +03
ton (short, 2000 lbm)	Mass	Kilogram (kg)	9.0718 E +02
torr (mmHg, 0°C)	Pressure	Pascal (Pa)	1.3332 E +02
Watt	Power	Watt (W)	1.0002 E +00
kW-hr	Energy	Kilojoule (kJ)	3.6000 E +03

Hua Bai, *The Dow Chemical Company*

Francois Bertrand, *Polytechnique Montreal*

Marco Caggioni, *P&G Corporate Engineering*

Jos Derksen, *University of Aberdeen*

**Table I-3** Conversion of SI Units

SI Unit	To Convert to	Multiply by
Joule (J)	Btu	9 E -4
	ft-lb <sub>f</sub> /sec	0.7375
Watt (W)	Btu/hr	3.436
Volume (m <sup>3</sup> )	ft <sup>3</sup>	35.32
	Liter	1000
	Gallon	264.2
Meter (m)	Angstrom	1.000 E +10
	Micron (μm)	1.000 E +6
Viscosity (Pa s)	Centipoise	1.000 E +3
Power (W)	Horsepower	0.0013
Pressure (Pa)	Inch Hg	0.2953 E -3
	psi	0.1451 E -3
	torr (mmHg at 0 K)	7.5006 E -3

Sean Donkin, *Cleveland Mixer*

Steve Drury, *Sharpe Mixers*

Lydia Fenley, *Illes Seasonings and Flavors*

Bernie Gigas, *Lightnin SPX*

Andrew N. Hrymak, *Western University, London, Ontario*

Dan Hickman, *The Dow Chemical Company*

Celso Fernandes Joaquim Jr., *CEETEPS/FATEC, Brazil*

Matthew D. Jordan, *Conagra Foods*

John Kaiser, *Mars*

Richard Kehn, *Lightnin SPX*

Paul Larsen, *The Dow Chemical Company*

Marcio Machado, *University of Alberta*

Alan Myerson, *MIT*

Lennart Myhrberg, *Metanova AB*

Edward L. Paul, *Merck, retired*

Eric Powell, *Wild Flavors*

Timothy Ratkowski, *Pressure Products Industries-Milton Roy LLC*

Luis Sierra, *Merck*

James N. Tilton, *DuPont*

Peter Veenstra, *Shell*

Ronald Weetman, *Lightnin SPX, retired*

Michael Yianneskis, *University College London*



The editors have drawn extensively on the considerable resources of the North American Mixing Forum (NAMF) for the authors and reviewers. These two handbooks would not have been possible without the unwavering support of this rather remarkable organization. The remarkable cover image, provided by Clara Gomez and her team at Coanda is one small example of the resources willingly brought into play for the benefit of all. All of the royalties from the sale of these books are returned to NAMF for the promotion of mixing research and education. The encouragement of all of the presidents of NAMF, both past and current, is gratefully acknowledged. Mixing is a learning process for the student and teacher alike. Our colleagues and collaborators continue to provide wonderful questions and challenges for the development of mixing technology. This work, more than anything else, has informed the contents of the *Handbook* and this *Advances* volume. Finally, the work of our editorial assistants, Kathy van Denderen and Jeanne Haley, was indispensable, as were the efforts of the production team at Wiley, and the team of graduate students and researchers whose sharp eyes reviewed the page proofs: Nitin Arora, Tianxin Bao, Akshay Bhalerao, Hena Farooqi, Alexandra Komrakova, Marcio Machado, Francesco Maluta, Colin Saraka, and Fatemeh Safari-Alamuti. Your eagle eyes were indispensable to all of us.

As we complete the final pages of this manuscript, the editors would like to extend personal thanks to Eileen Etchells for understanding and support, and excellent elevenses; to colleagues at the Dow Chemical Company for the challenge and opportunity to develop an expertise in mixing, and to Linda C. Atiemo-Obeng for empathy and encouragement; to MixTech's consulting clients, short course students, and trade journal readers who provide a diversity of practical challenges and opportunities for the use and development of mixing technology; to Jim Kresta and the Princesses of the Stirred Tank for support in many forms, practical, personal, and increasingly, technical; to Ed Paul, ex-officio editor par excellence—who sadly but stubbornly clings to total ignorance of the Beatles and Paul Simon in favor of the remarkable bass section in the Chautauqua Orchestra; to Jim Tilton for many thoughtful and useful technical discussions; to friends who shared meals and laughter as we pushed to the finish line; and to the University of Alberta Department of Chemical and Materials Engineering.

The objective of this book is to provide mixing practitioners with the current state of mixing knowledge: both in terms of fundamentals and from the perspective of industrial practice and experience. Many of the chapters are absolutely definitive in their area. We hope that readers find as much to stimulate and fascinate them in these pages as we have found during their editing.

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## CONTENTS OF THE DVD, INCLUDING INSTRUCTIONAL VIDEOS

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ARTHUR W. ETCHELLS, III

*Rowan University*

DAVID S. DICKEY

*MixTech*

SUZANNE M. KRESTA

*University of Alberta*

THOMAS MARTIN

*Technische Hochschule Merseberg*

HENRY ZHANG

*University of Alberta*

The contents of the DVD are broken down into four sections. The first two sections provide electronic copies of all of the figures which were originally in colour, and the troubleshooting charts. These may be used in presentations, as long as credit is given to the original authors. The third section includes 22 instructional videos. These videos are compiled from highlights from the library of 130 video clips, which are organized by topic in the fourth section of the DVD.

## ELECTRONIC (PDF) VERSION OF THE HANDBOOK OF INDUSTRIAL MIXING

A major file on the DVD is a complete electronic copy of the *Handbook of Industrial Mixing*. All errata are noted directly in this searchable file. The index for the *Advances* volume contains two columns of page numbers, one for the digital first edition (the *Handbook*), and one for the paper pages (the *Advances* volume). Any chapters with no substantial changes were not reprinted in this edition, but provided as part of this pdf for the reader's convenience. Many readers will already have, or may wish to buy, a paper copy of the *Handbook* for ease of reference.

## COLOR FIGURES AND TROUBLESHOOTING CHARTS

These color figures that show more in color than is possible in the printed black and white version. Some of them are not included in the color plates:

- Chapter 1b- Figures 2, 3, 4, and 5.
- Chapter 2b- Figures 2, 6b, 17, and 18.
- Chapter 3b- Figure 21.
- Chapter 7b- Figures 6, 8, 9, 11, and 16.
- Chapter 9b- Figures 12, 15, 18, 21, 34, and 45.
- Chapter 26- Figures 13, 18, and 19.

Color figures from the *Handbook of Industrial Mixing*, and the original video library, are on the DVD included with that edition.

There are also six trouble shooting or diagnostic charts which cover some of the most common mixing problems in industry:

- Dip Pipes (for mixing sensitive, single phase reactions)
- Gas-Liquid Reactions
- Liquid-Liquid Dispersion with Reaction
- Liquid-Liquid Extraction
- Solid Reagents (suspension plus mass transfer may limit the reaction rate)
- Solids Withdrawal (e.g. draining the tank while maintaining slurry suspension)

## 22 INSTRUCTIONAL VIDEOS

*Compiled and edited by Arthur Etchells, Rowan University, David Dickey, Mix-Tech, Suzanne Kresta, University of Alberta, Thomas Martin, Technische Hochschule Merseberg, and Henry Zhang, University of Alberta*

In the first volume of *Handbook of Industrial Mixing* a DVD was included with some of the best videos and animations available at the time. This very popular section has been greatly expanded in this volume. Twenty-two new videos are designed as brief instructional clips, roughly 2-3 minutes long, which are built around highlight clips from the video library. Explanations have been added in this edition.

The compilations are the result of contributions made by a number of people. They can be used for independent study, to demonstrate points to operations staff, or to explain phenomena in mixing classes. They can also be cut up at the readers' pleasure and be inserted in their own presentations. Again, we ask that the original source be noted in the presentation.

A descriptive list of the videos is provided in Table 1. The numbers in column 1 indicate the chapter content matching each video.

**Table 1** Instructional Videos on the DVD. The section in the book corresponding to each video is given in the title.

Title	Description
2.1 Jet Mixing	– positively and negatively buoyant jets
6.1 Circulation Patterns	– flow fields produced by different impellers
6.2 Baffles	– solids suspension and off-bottom clearance
6.3 Impeller Diameter and Circulation	– the importance of baffles in producing top to bottom circulation in a stirred tank
6.4 Angled Mixer Shafts	– limits on D/T ratio for effective flow
7.1 Laminar Blending in Static Mixers	– an introduction to terminology and set-up of portable mixers on angled shafts
7.2 Turbulent Blending in Static Mixers	– limitations for solids suspension
7.3 Laminar Drop Breakup in Static Mixers	– the classic video clip of laminar mixing in an SMX
7.4 Turbulent Drop Breakup in Static Mixers	– particle tracks from a CFD simulation
9.1 Turbulent Blending in Stirred Tanks	– radial vortices produced by HEV tabs
9.2 Laminar Blending with a Helical Ribbon	– blending in an SMV mixer
9.3 Compartmentalization	– drop break-up in an SMX mixer with varying pipe diameter and fluid velocity
9.4 Cavern Formation	– drop break up, gas dispersion, and heat transfer using static mixers in turbulent flow
10.1 Solids Suspension below $N_{js}$	– multiple impellers for large H/T
10.2 Cloud Height	– effect of impeller speed on blend time
10.3 Drawdown of Floating Solids	– effect of impeller geometry on blend time
11.1 Gas Liquid Mixing	– a full length video of laminar mixing
12.1 Laminar Drop Breakup	– the most effective geometry is shown
12.2 Turbulent Drop Breakup at Impellers	– limited mixing between multiple impellers in a tank where $H \gg T$
	– configurations which minimize this effect
	– formation of isolated caverns in yield stress fluids
	– state of solids suspension at 40%, 60%, 80% and 100% of $N_{js}$
	– effect of impeller speed on cloud height
	– comparison of cloud height and $N_{js}$ condition
	– effect of solids concentration on cloud height
	– effect of vessel base shape
	– effect of baffle design on the draw down of floating solids
	– difficulties due to a stable vortex
	– operating regimes and the effect of impeller pumping direction on gas-liquid dispersion
	– computer simulations of drop break-up at the critical capillary number
	– original H. Grace videos are included in the video library
	– details of drop break-up around impeller blades

*(continued)*

**Table 1** (Continued)

Title	Description
13.1 Mesomixing - Feed Time with Reaction	– illustration of the effects of different feed times on the formation of zones of high local concentration when surface feed is used
13.2 Changing Feed Location	– CFD simulation of the effect of feed point using particle tracks
15.1 Solid Solid Mixing	– comparison of tote blender, V-blender, and double cone blender

**VIDEO LIBRARY**

Fluid mixing is, by its very nature, a visually beautiful field of study. It involves complex fluid motion in complex geometries. Therefore the study of mixing often benefits from visualization. Computational Fluid Dynamics is one popular tool for visualization because it produces detailed pictures of flow from graphical and analytical analysis. Many experimental methods also exist, and can often give similar data with less time and effort—or much more time and effort—invested.

The DVD includes an extensive library of 140 raw video clips. Some of the raw clips are long high-definition runs of important process results. For those who wish to dig deeper into the processes, or to use only a small portion of a video, an afternoon spent browsing the library will be time well invested.

The videos are catalogued by chapter. The reader is encouraged to explore the directories and videos since some videos will be useful for several different topics. The behavior of non-Newtonian fluids in blending applications is shown under Chapter 9, Blending but also applies to Rheology. Laminar blending is included with Chapter 9, but also applies to Chapter 9b. The chapters and major directories in the library are given in Table 2.

The authors wish to acknowledge and thank those who contributed video clips to the library:

- British Hydromechanical Research Group (BHRG) and their Fluid Mixing Processes consortium
- Chemineer
- Steve Curran, Sujit Bhattacharya, Kevin Bittorf, Bob Hayes, Oscar Khazam, and Suzanne Kresta, University of Alberta
- Steve Boesch, Rowan University
- David Dickey, MixTech
- Lightnin Mixers
- Minye Liu, Richard Laroche and Clay Andreasson, Hypertrace
- Sulzer Chemtech

**Table 2** Directories in the Video Library

Chapter	Title	Directories	Video Clips
2	Turbulence	1	3
6	Stirred Tanks		
	6.1 Circulation: Clearance and Baffles	11	24
	6.2 Diameter, Power, and Torque	3	17
	6.3 Angled Shafts	5	11
7	Static Mixers – blending and drop breakup	4	10
9	Blending		
	9.1 Turbulent Blending	2	7
	9.2 Laminar Blending with a Helical Ribbon	1	1
	9.3 Compartmentalization	1	5
	9.4 Cavern Formation	1	7
10	Solid-Liquid Mixing		
	10.1 Solids Suspension Below Njs	1	11
	10.2 Cloud Height	1	7
	10.3 Drawdown of Floating Solids	1	6
11	Gas-Liquid Mixing	1	5
12	Liquid-Liquid Mixing – laminar and turbulent breakup	2	11
13	Mixing and Reaction – feed time and location	2	6
15	Solids Mixing	1	3
	Mixing Luminaries	1	5

- Thomas Martin, Kai Köhler-Terz, Sebastian Lebioda, Hanka Haschke and the rest of the video production team at Technische Hochschule Merseberg
- Maher Moakher, Troy Shinbrot and Fernando Muzzio, Rutgers University

## SOURCES OF ADDITIONAL VIDEOS

Other sources of mixing videos available to the reader include the various vendor web sites on the Internet. YouTube also has some interesting videos by university groups e.g. a great demonstration of non-Newtonian fluids and some interesting solids suspension videos by Rowan University.

The North American Mixing Forum is establishing a repository for mixing images and video on the NAMF website ([www.mixing.net](http://www.mixing.net)). Some of the longer clips are to be stored there. These videos will continue to be updated even after the publication date of the book.

Most manufacturers of industrial mixers have informative web sites often with pictures and video clips. A partial list is provided in Table 1, with dry solids mixer manufacturers in Table 4. For additional videos, the reader should search YouTube using the name of the manufacturer + mixer (e.g. Flygt mixer). If a site has inadvertently been left out please contact the editors so it can be posted on the North American Mixing Forum website ([www.mixing.net](http://www.mixing.net)).

**Table 3** Websites for Some Liquid Mixer Manufacturers and Other Mixing Videos

Ansys (CFD)	<a href="http://ansys.com/">http://ansys.com/</a>
CHEMINEER – Robbins and Meyers (also Chemineer Mixer on YouTube)	<a href="http://www.chemineer.com/">http://www.chemineer.com/</a>
Colorful Fluid Mixing Gallery – Andre Bakker	<a href="http://www.bakker.org/">http://www.bakker.org/</a>
EKATO (also Ekato Mixer on YouTube)	<a href="http://www.ekato.com">http://www.ekato.com</a>
Flygt Xylem	<a href="http://www.flygtus.com/">http://www.flygtus.com/</a>
Fusion Fluid Equipment	<a href="http://www.fusionfluid.com/">http://www.fusionfluid.com/</a>
IKA	<a href="http://www.ikausa.com">http://www.ikausa.com</a>
Kadyinternational	<a href="http://www.kadyinternational.com/">http://www.kadyinternational.com/</a>
Lightnin-SPX (also Lightnin Mixer on YouTube)	<a href="http://www.spx.com/en/lightnin/">http://www.spx.com/en/lightnin/</a>
Mythbusters (non-Newtonian fluid)	<a href="http://www.youtube.com">www.youtube.com</a> (search Mythbusters non-Newtonian fluid)
Solids mixing and viscous liquid mixing – F. Muzzio	<a href="http://www.muzzio.rutgers.edu/">http://www.muzzio.rutgers.edu/</a>
ProQuip	<a href="http://proquipinc.com/">http://proquipinc.com/</a>
Philadelphia Mixing Solutions	<a href="http://www.philamixers.com/">http://www.philamixers.com/</a>
Pulsair	<a href="http://pulsair.com/">http://pulsair.com/</a>
Statmixco	<a href="http://www.stamixco-usa.com/">http://www.stamixco-usa.com/</a>
Silverson	<a href="http://www.silverson.com">http://www.silverson.com</a>
Sulzer ChemTech	<a href="http://www.sulzer.com/">http://www.sulzer.com/</a>

**Table 4** Websites for Some Manufacturers of Dry Solids Mixing Equipment

Company	Website	Mixer
A & J Mixing International	<a href="http://www.ajmixing.com">http://www.ajmixing.com</a>	Phlauer
American Process Systems, Eirich Machines	<a href="http://www.americanprocesssystems.com">http://www.americanprocesssystems.com</a>	Ribbon blenders & plough blenders
B&P Process Equipment and Systems	<a href="http://www.bpprocess.com/">http://www.bpprocess.com/</a>	Baker Perkins Machinery
Charles Ross & Son Co.	<a href="http://www.mixers.com/">http://www.mixers.com/</a>	Ribbon, paddle, & vertical cone blenders
Christy & Norris, Ltd.	<a href="http://www.christy-norris.co.uk">www.christy-norris.co.uk</a>	Beken
Continental Products Corporation	<a href="http://www.continentalrollomixer.com/">http://www.continentalrollomixer.com/</a>	Rollo-Mixer
Design Integrated Technology	<a href="http://www.ditusa.com/">http://www.ditusa.com/</a>	SC Helicone Mixer
FEECO International	<a href="http://www.feeco.com">http://www.feeco.com</a>	Pin Mixers, Paddle Mixers, Pug Mills
GEA Niro	<a href="http://www.niro.com">http://www.niro.com</a>	Vertical Bowl Granulators
GEMCO	<a href="http://www.okgemco.com/">http://www.okgemco.com/</a>	
Glatt	<a href="http://www.glatt.com">http://www.glatt.com</a>	Vertical Bowl Granulators



**Table 4** (Continued)

Company	Website	Mixer
Glen Mills Inc.	<a href="http://www.glenmills.com/">http://www.glenmills.com/</a>	Turbula
Hosokawa Bepex	<a href="http://www.bepex.com/">http://www.bepex.com/</a>	Bepex, Shugi
Jaygo Inc.	<a href="http://www.jaygoinc.com/">http://www.jaygoinc.com/</a>	
JR Johanson, Inc, Diamondback Technology	<a href="http://www.jrjohanson.com/">http://www.jrjohanson.com/</a>	Diamondback
Kemutec, Inc.	<a href="http://www.kemutec.com/">http://www.kemutec.com/</a>	Gardner
Littleford Day, Inc.	<a href="http://www.littleford.com/">http://www.littleford.com/</a>	Littleford
Lödige Process Technology	<a href="http://www.loedige.de/">http://www.loedige.de/</a>	
M.P.E. Group U.S.A.	<a href="http://mpegroupusa.com">http://mpegroupusa.com</a>	Bolz Summix
Marion Mixers, Inc.	<a href="http://www.marionmixers.com/">http://www.marionmixers.com/</a>	
MIXACO	<a href="http://www.mixaco.com/">http://www.mixaco.com/</a>	
Mixer Systems, Inc.	<a href="http://www.dustmaster.com/">http://www.dustmaster.com/</a>	TURBIN Mixer XL
Munson Machinery Company, Inc.	<a href="http://www.munsonmachinery.com/">http://www.munsonmachinery.com/</a>	
Patterson Industries (Canada) Ltd.	<a href="http://www.pattersonindustries.com/">http://www.pattersonindustries.com/</a>	
Patterson Process Equipment Corporation	<a href="http://www.pattersonprocess.com/">http://www.pattersonprocess.com/</a>	
Patterson-Kelley Co.	<a href="http://buflovak.com/PKProcess">http://buflovak.com/PKProcess</a>	twin-shell & cross-flow blenders
Paul O. Abbé Inc.	<a href="http://www.pauloabbe.com/">http://www.pauloabbe.com/</a>	
Peerless Dough Mixing & Make-up	<a href="http://www.peerlessfood.com/">http://www.peerlessfood.com/</a>	
Readco Kurimoto, LLC.	<a href="http://www.readco.com/">http://www.readco.com/</a>	
Semi-Bulk Systems, Inc.	<a href="http://www.semi-bulk.com/">http://www.semi-bulk.com/</a>	
TOTE Systems International	<a href="http://www.totesystems.com/">http://www.totesystems.com/</a>	
Vector Corporation	<a href="http://www.vectorcorporation.com">http://www.vectorcorporation.com</a>	Vertical Bowl Granulators



# A Technical Definition of Mixing

JOËLLE AUBIN

*Université de Toulouse, Laboratoire de Génie Chimique CNRS/INPT, Toulouse, France*

SUZANNE M. KRESTA

*University of Alberta, Edmonton, Canada*

If you have picked up this book, you probably already suspect that you have a mixing problem. While blending to homogeneity is the first mixing problem most people recognize, the blending of low-viscosity miscible fluids is also the easiest mixing application. The classical model of the perfectly mixed CSTR (continuous stirred tank reactor) is most likely to be a good assumption if the tank contains low-viscosity miscible fluids. Mixing frequently gets worse on scale-up, so if you are trying to scale up a new process that ran well at the bench scale but is now performing poorly in the plant, this volume is a good place to look for a solution. There is a wealth of technical information in this book about more difficult mixing problems, such as the dispersion of one phase into another. Multiphase mixing problems involve many scales of mixing, frequently mass transfer, and always incompletely understood physics. Competing rate processes, such as two competing reactions, or reaction with mass transfer, are among the most difficult processes to design. Competing rates often occur when a reaction happens in parallel with some other mixing objective, or when the surface or physical properties of the system are changing as mixing progresses.

In this companion volume to the *Handbook of Industrial Mixing* (the *Handbook*), the first chapters are dedicated to discussion of areas where the field has changed significantly since 2004, when that book was published. One of these areas is the development of a technical definition of mixing, which can be applied to any industrial mixing process to help identify and understand the key physical and chemical phenomena, as well as the process objectives. Early mixing researchers clearly identified many of the key elements of the definition, but the early experimental tools did not allow direct measurement of the key variables, so much of this literature

fell into disuse by the 1990s. With new digital imaging and computational methods, and continuing frustration with the limitations of simply measuring the coefficient of variation (CoV), this early work has been revisited, integrating both some of the literature on spatial statistics and standard content from mixing courses. The key results are presented here to begin the discussion of mixing.

To become better acquainted with these objectives, first hold the problem that brought you to the handbook in mind but also consider four everyday examples of mixing: stirring cream into coffee, dissolving sugar in water, making salad dressing by mixing oil and vinegar, and making gravy or its simpler cousin, a cream sauce. All of these processes are either batch or semibatch, and they cover the range of mixing process objectives: blending, multiphase dissolution (mass transfer) or dispersion (scale reduction), and reaction with a change in viscosity. They are discussed in more detail in the example at the end of this chapter.

## RANGE OF INDUSTRIAL MIXING APPLICATIONS

The classical range of industrial mixing includes six core mixing operations, which were covered by chapters in the *Handbook*:

- Blending of miscible liquids
- Solid–liquid mixing
- Gas–liquid mixing
- Immiscible liquid–liquid mixing
- Mixing and chemical reaction
- Solid–solid mixing

Heat transfer is affected by agitation but does not involve mixing of species or phases, so it is not generally seen as a core mixing operation. These chapters can be grouped into three major areas: blending, multiphase, and reaction. In evaluating a new mixing application, the following process-based questions should be asked:

1. What is the key process objective?
2. What phases are present? Is there a need to suspend solids, disperse one immiscible liquid into another, disperse gas to achieve mass transfer, or draw down a second phase from the liquid surface? Is there mixing between two or more solid phases or two or more liquid phases?
3. Does the viscosity change during the process? Will the flow regime be laminar, turbulent, or transitional?
4. Is continuous or batch processing the best choice?

These are all good questions, but none of them address a quantitative definition of the mixing objective.

### THREE DIMENSIONS OF SEGREGATION: A TECHNICAL DEFINITION OF MIXING

The goal of this section is to replace an intuitive sense of “good mixing” with a technical understanding of how to define “well mixed” for the purposes of process development, scale-up, or scale-down, equipment selection and design, and troubleshooting. Good mixing has three possible objectives:

- Uniform concentration
- A specified scale of segregation, for example particle size, drop size, or striation thickness
- A required mixing time or rate of mixing

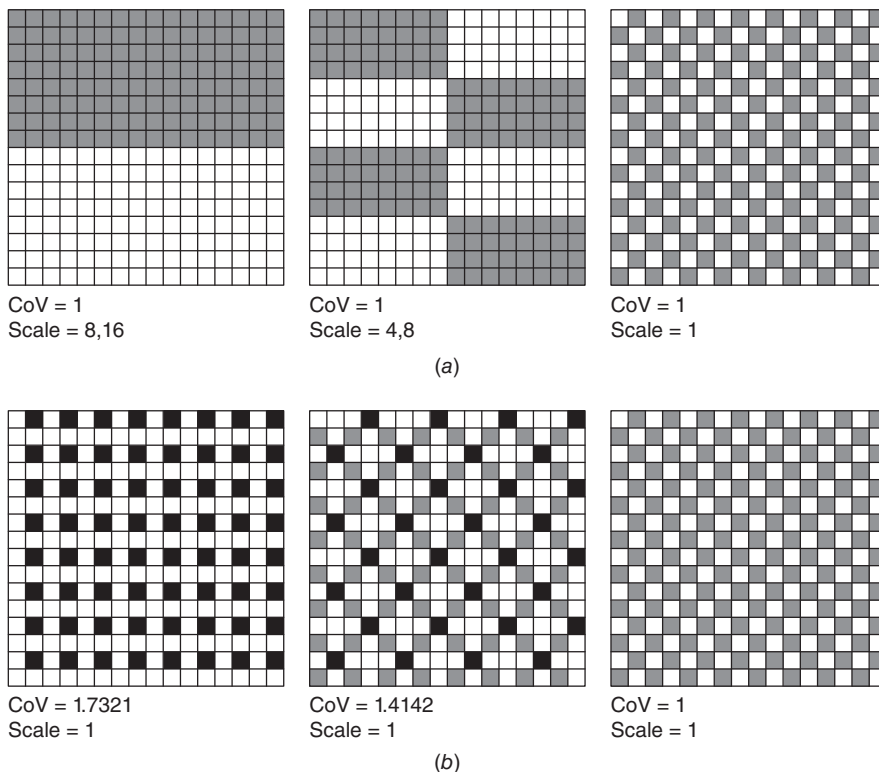
The concentration uniformity, also known as the intensity of segregation, is measured using the concentration variance or the coefficient of variation (CoV):

$$\text{CoV} = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( \frac{C_i - C_{\text{mean}}}{C_{\text{mean}}} \right)^2}$$

(see Chapter 4 of the *Handbook* for more information). As the concentration becomes more uniform, the CoV drops toward zero. The scale of segregation typically decreases as mixing proceeds, but in some cases it may also increase over the course of the process, for example, floc formation or crystal growth. A quantitative illustration of the difference between the intensity and scale of segregation is given in the checkerboard illustration in Figure 0-1.

Figure 0-1a shows three different checkerboard patterns. To both the novice and expert, it is clear that mixing of the gray and white species improves from left to right. However, if the CoV is calculated, it is equal to 1 for all three cases! In fact, it is the scale of segregation that decreases from left to right. Clearly, the CoV contains no information on the spatial arrangement and the proximity of the different species. Taking the inverse example in Figure 0-1b, where the scale of segregation is constant, it is a trivial matter to generate different intensities of segregation for the identical length scale. It is evident that the intensity of segregation and the scale of segregation are independent variables. These points have been discussed in detail in Kukukova et al. (2009) and were touched on in Chapter 2 of the *Handbook*.

The measured CoV also depends on the scale of measurement. Take solid–solid mixing as an example: If the measurement is done at the scale of the particle size, the CoV will remain at the initial completely segregated value for all times. If, however, the scale of measurement gives a sample volume that includes a large number of particles, say 500, the CoV will drop from its initial value to zero as the mixture reaches homogeneity at this scale (Kukukova et al., 2008). If the scale of measurement is not well matched to the desired scale of segregation, it is difficult to know what CoV means. This issue is also discussed in detail in the solids mixing chapter (15a).



**Figure 0-1** Illustration of the difference between the scale of segregation and the intensity of segregation (*a*) constant CoV, or intensity of segregation, and (*b*) constant scale of segregation.

Measurement of the scale of segregation has rarely been addressed explicitly in the recent mixing literature; however, it was discussed in 1952 by Danckwerts (1952), as well as in early solids mixing studies (Lacey, 1954; Lacey and Mirza, 1976; Harnby, 1967; Hersey, 1970) and polymer processing applications (Mohr et al., 1957). At that time, it was extraordinarily labor intensive to collect the data densities needed to calculate scales of segregation. As a result, the authors concluded that this measure was not accessible for the solution of realistic problems and was therefore most often abandoned. Today, however, powerful tools such as computational fluid dynamics (CFD), tomography, particle image velocimetry (PIV), and image analysis exist, enabling large data sets to be obtained and now making the measure of scale of segregation possible. Due to these modern resources, some more recent work has focused on the scale of segregation in chaotic laminar flows indirectly via stretching functions and Lyapunov exponents (see Chapter 3 in the *Handbook*). In the polymer processing literature (Chapter 16 in the *Handbook*), the scale of segregation is addressed through the concept of dispersive mixing, which may be thought of as the dispersion of small clumps of additive particles into single particles or the dispersion

of large drops into smaller drops. Distributive mixing is analogous to the intensity of segregation and refers to the distribution of any additive throughout the volume. Experimental examples of the scale of mixing are given in Kukukova et al. (2011).

The third mixing objective is a specified rate of mixing, or mixing time, and there are a number of mixing times to consider. The first mixing time is the blend time (see Chapter 9 of the *Handbook*). This is a macromixing time: How long does it take for the whole tank to become well mixed? The goal is to eliminate large islands of unmixedness (distributive mixing). The second set of mixing times is an array of mesoscale rates, usually related to reducing the scale of segregation (dispersive mixing). One example of this is the time required to form a liquid–liquid dispersion with a certain mean drop size. In a typical process, the blend time might be 5 s, with 80% of the drop breakup complete within 5 min, but a full 30 min required to achieve the equilibrium drop size. These processes happen at scales smaller than the tank diameter but much larger than the diffusional scales. At the smallest scales of mixing, referred to as the microscale, several rates compete: mass transfer across films, molecular diffusion through striations, and chemical reaction rates. The micromixing time is the time needed to reduce the scale of segregation to any of the microscales given above. *If the micromixing rate is slower than the reaction or mass transfer rate, micromixing will affect the process outcome.* The mixing time is the integral of the instantaneous rate of mixing, or the rate of change of segregation, which we can also call exposure. Exposure depends on three factors: the strength of interaction (typically a rate constant), the distance from the minimum segregation state (this depends both on the physical properties of the fluid and on the equipment design) and the opportunity the two species have to interact (the simplest example of this is the interfacial area for mass transfer). Exposure, or the instantaneous rate of change of segregation, is a nonlinear combination of the intensity and the scale of segregation (Kukukova et al., 2009).

## IDENTIFYING MIXING PROBLEMS: DEFINING THE CRITICAL SCALES AND PROCESS OBJECTIVES

Before digging more deeply into some applications and examples, recall the three major categories of mixing problems: blending, multiphase, and reaction. Most blending problems are macroscale problems, although a small number involve product defects down to the micrometer scale. Most multiphase mixing problems occur at the mesoscale. The first multiphase mixing objective is to eliminate large-scale segregation by suspending sinking solids, drawing down liquids and floating solids, or by dispersing gas while avoiding the flooding regime of the impeller. The second multiphase mixing objective involves dispersion of the second phase. The final multiphase mixing objective is to create a homogeneous suspension or dispersion, where the concentration and size distribution of the dispersion is the same throughout. This final, most demanding mixing objective is usually not achievable in large-scale industrial tanks but is often a requirement for the production of consumer products such as pastes and creams. This limits the batch size for some products. The most

**Table 0-1** Multiphase Mixing Objectives Classified by the Dominant Scale of Mixing

	Macroscale	Mesoscale	Microscale
<b>Solid-Liquid</b>	Suspend all solids off the tank bottom Draw down floating solids from the free liquid surface Eliminate stratified layers, which are visible as a solids cloud height Uniformly distribute solids throughout the tank Blend pastes to uniformity	Disperse clumping solids Flocculate fines Aggregate primary particles	Dissolve solids Avoid the production of fine particles by solids attrition Reduce particle size to produce fine particles in media mills Control crystal size through the conditions needed for nucleation and diffusion-limited growth
<b>Liquid-Liquid</b>	Eliminate stratified layers by drawing down liquids from the free surface Predict and control phase inversion	Form liquid-liquid dispersions with a specified drop size distribution Form stable emulsions Coalesce impurities	Dissolve partially miscible drops
<b>Gas-Liquid</b>	Select and control the gas-liquid flow regime Avoid impeller flooding	Disperse gas to reduce bubble size Entrain gas from the head space	Maximize the rate of gas-liquid mass transfer
<b>Solid-Solid</b>	Eliminate large-scale segregation and solids demixing	Disperse clumping solids	Blend powders to uniformity

common multiphase mixing objectives are given in Table 0-1, where they are classified by the dominant mixing length scales. The final category of mixing problem is reactions. Mixing-sensitive reactions are limited by the micromixing time scale and may also be limited by mass transfer or by mesomixing and by-product formation. In most reactor design cases, the primary mixing design objective is to minimize the mass transfer and mesomixing limitations so that the final design is only limited by the microscale. This is driven by two considerations: First, it eliminates mixing limitations in the process, and, second, it provides the conditions needed for reliable scale-up from pilot process to fully industrial-scale production. When the chemistry is severely limiting at the small scale, selection of an alternate reaction route may be considered (see Chapter 13 of the *Handbook*) or equipment that gives a very high mixing intensity (e.g., micromixers) might be implemented.



Mixing involves three distinct specifications: degree of homogeneity; scale of segregation, and mixing time. The three major classes of process objectives that depend on mixing are blending, multiphase mixing and dispersion, and mixing-sensitive reactions. Each of these major classes of process objectives is associated with a scale of mixing. Blending is associated with the macroscale, with segregation on scales as large as the scale of the equipment, that is, the pipe diameter or the tank diameter. Mesoscale mixing occurs when the scale of segregation is reduced from macro- to micro-. In this case it is important to consider how the scales of mixing will interact with the desired process scales (e.g., drop size distribution). At the micromixing scale the scale of segregation is small enough that viscous dissipation, molecular diffusion, and surface effects become significant. All mixing processes have associated mixing times, or rates. In some cases, achieving uniform concentration is important, multiple scales of mixing interact, and the rate of mixing can be critical.

When evaluating an existing process for mixing problems, begin with the initial questions in this section, which are now modified to include information needed for the mixing specifications:

1. What is the equipment? Is the process continuous or batch?
2. What is the continuous phase? Is the flow regime laminar, turbulent, or transitional? Does the viscosity change during the process?
3. What other phases are present? Is there a need to suspend solids, disperse one immiscible liquid into another, disperse gas to achieve mass transfer, draw down a second phase from the liquid surface, promote agglomeration or growth, or mix two solids together? Are there rheological effects?
4. Is a reaction present? Is it mass transfer limited? Is it mixing sensitive? Is it limited by heat transfer?

Continue to define the mixing specifications:

5. Is a degree of homogeneity or intensity of segregation needed? Where? How fast? How uniform? At what scale of observation?
6. Does the scale of segregation need to be reduced? What size? Does the shape of the distribution matter? Does the rate matter? Is accurate data available for the specification?
7. What are the competing rates in the process? What rate is critical and is it limited by mixing?

**Example: Definition of Mixing Objectives in the Kitchen** Returning to the four examples presented at the beginning of the section, consider them now in terms of mixing problem specification:

1. Stirring cream into coffee: This is a simple macroscale blending problem. When observing this mixing in action, try taking a knife and using it as a baffle. You will be able to feel the additional resistance to flow (increased power draw on the motor) and see the dramatic change in circulation pattern.

2. Dissolving sugar into water: This is a multiphase problem, where the goal is to lift the sugar off the bottom by mixing, and then stir long enough for the sugar to dissolve. The mixing time depends on the particle size (try using icing sugar or large raw sugar crystals), the water temperature, and the solubility of the solid (try changing the solid from sugar to something less soluble—like gelatin or laundry soap).
3. Mixing oil and vinegar: Another multiphase problem, this time with no mass transfer but with both a macroscale segregation component (initially all of the oil floats on the top) and a scale of segregation requirement (a small drop size gives an even coating of oil and vinegar over the salad). Depending on the dressing, the oil drops may be very stable (e.g., mayonnaise, which is stabilized with egg yolk) or very unstable (a simple oil and balsamic vinaigrette). Addition of herbs (small biwetable solids), mustard, or salt tends to stabilize the dispersion. Note that most store-bought dressings contain surfactants and/or stabilizers to extend their shelf life. An experiment to try: The oil tends to disperse into the vinegar, even if there is much less vinegar than oil. Why? What is the smallest volume fraction of vinegar that will form a continuous phase?
4. Making gravy: This is both an advanced cooking lesson and quite a difficult mixing problem that is analogous to a number of industrial processes. There are a number of steps in the semibatch process, which involves changing viscosity, paste mixing, reaction, heat transfer, and mass transfer. The process objective is to achieve a stable, uniform dispersion of meat juices, fats, and seasonings, which has an appealing consistency and flavor. To understand the mixing steps, it is necessary to take the process apart step by step.
  - Heat transfer and fouling: Don't burn the gravy or let it stick to the bottom of the pan. This requires continuous agitation with good circulation close to the heated surfaces and possibly a close clearance mixer made with heat resistant material. A silicon spatula will do nicely.
  - Mass transfer: Dissolve the juices from the bottom of the pan into the stock. This requires patience, gentle heating, and agitation. The dissolution time varies depending on the thickness of the solid layer and the degree of local agitation.
  - Mix flour with cold water: Nonwetting and clumping solids. You may start this by making a paste and then diluting it or by mixing rapidly and intensely with small-scale turbulence. If warm water is used, the solids will tend to clump together and more mixing intensity will be required to make a smooth dispersion. *Complete dispersion of the individual particles of flour at this stage is critical and requires high shear at the particle scale, either due to turbulence or due to local shearing of a thick paste followed by dilution.* The mixing time is not important, as long as the solids are fully dispersed (and your guests are not too hungry).
  - Reaction: The starches in the flour react when heated and thicken the stock. The flour is a complex solid, and breaking up the flour granules requires vigorous mixing, again at the small scale, if the starches are to be released

and completely react (McGee, 2004). Thus the degree of mixing changes the flavor of the gravy (and the extent of reaction) from one cook to another. In the case of incomplete mixing, the remaining (<1%) unreacted starches can have a large impact on the quality of the final product. As the heating and reaction progresses, the homogeneous mixture of flour, water, stock, and fat changes from a waterlike consistency to a thick non-Newtonian paste with a yield stress. The mixing intensity must be carefully controlled to quickly disperse the non-Newtonian paste without forming lumps. Excessive mixing will cause splashing and a safety hazard, as well as a dirty kitchen.

- Emulsifying the remaining fats: The flour also acts as an emulsifier, dispersing the melted fat into the stock. When the roasted meat or bird has given off a lot of fat, the first addition of flour may not capture all of the fat in the dispersion. In this case, the excess fat will separate on the surface, and the mixing expert has several choices: drain the fat (perhaps a healthier solution), stir intermittently to redisperse the second phase (a lower quality, but cheaper product), or add more stabilizer. If more stabilizer is added, it must be premixed and diluted to avoid lump formation, and vigorous mixing will be required to completely disperse the organic phase into the dispersion. The gravy is now a smooth slightly viscous liquid–liquid dispersion, and the final additives can be incorporated to finish the product (add salt and pepper to taste!). When cooled, the fat solidifies and the dispersion becomes a gel with a yield stress.

Notice how the mixing requirements change for the different steps! This is a very tricky problem. Some industrial mixers have multiple functions, such as high-shear or rotor-stator mixers in combination with close clearance impellers to provide for changing mixing requirements.

## NOTATION

$C_i$	Concentration in sample $i$
$C_{\text{mean}}$	Mean concentration in the volume
CoV	Coefficient of variation
$N_t$	Total number of samples

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# Residence Time Distributions

THE LATE E. BRUCE NAUMAN

*Rensselaer Polytechnic Institute*

The material in this chapter is largely unchanged since the printing of the *Handbook of Industrial Mixing* (hereafter referred to as the *Handbook*). The original chapter is provided on the DVD attached to the back cover as a searchable pdf. Included below are the table of contents and the chapter introduction.

A new chapter on mean age and local residence time distributions follows as Chapter 1b.

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## 1a-1 INTRODUCTION

The concept of residence time distribution (RTD) and its importance in flow processes first developed by Danckwerts (1953) was a seminal contribution to the emergence of chemical engineering science. An introduction to RTD theory is now included in standard texts on chemical reaction engineering. There is also an extensive literature on the measurement, theory, and application of residence time distributions. A literature search returns nearly 5000 references containing the concept of residence time distribution and some 30,000 references dealing with residence time in general. This chapter necessarily provides only a brief introduction; the references provide more comprehensive treatments.

The residence time distribution measures features of ideal or nonideal flows associated with the bulk flow patterns or *macromixing* in a reactor or other process vessel. The term *micromixing*, as used in this chapter, applies to spatial mixing at the molecular scale that is bounded but not uniquely determined by the residence time distribution. The bounds are extreme conditions known as *complete segregation* and *maximum mixedness*. They represent, respectively, the least and most molecular-level mixing that is possible for a given residence time distribution.

Most of this handbook treats *spatial mixing*. Suppose a sample of fluid is collected and analyzed. One may ask: Is it homogeneous? Standard measures of homogeneity such as the striation thickness in laminar flow or the coefficient of variation in turbulent flow can be used to answer this question quantitatively. In this chapter we look at a different question that is important for continuous flow systems: When did the particles, typically molecules but sometimes larger particles, enter the system, and how long did they stay? This question involves *temporal mixing*, and its quantitative answer is provided by the RTD (Danckwerts, 1953).

To distinguish between spatial and temporal mixing, suppose that a flow system is fed from separate black and white streams. If the effluent emerges uniformly gray, there is good spatial mixing. For the case of a pipe, the uniform grayness corresponds to good mixing in the radial direction. Now suppose that the pipe is fed from a single stream that varies in shade or grayness. The effluent will also vary in shade unless there is good temporal mixing. In the context of a pipe, spatial mixing is equivalent to *radial mixing* and temporal mixing is equivalent to *axial mixing*.

In a batch reactor, all molecules enter and leave together. If the system is isothermal, reaction yields depend only on the elapsed time and on the initial composition. The situation in flow systems is more complicated but not impossibly so. The counterpart of the batch reaction time is the age of a molecule. Aging begins when a molecule enters the reactor and ceases when it leaves. The total time spent within the boundaries of the reactor is known as the exit age, or *residence time*,  $t$ . In real flow systems, molecules leaving the system will have a variety of residence times. The distribution of residence times provides considerable information about homogeneous, isothermal reactions. For single, first-order reactions, knowledge of the *RTD* allows the yield to be calculated exactly, even in flow systems of arbitrary complexity. For other reaction orders, it is usually possible to calculate fairly tight limits, within which the yield

must lie (Zwietering, 1959). If the system is nonisothermal or heterogeneous, the RTD cannot predict reaction yield directly, but it still provides a general description of the flow that is not easily obtained by velocity measurements.

Residence time experiments have been used to explore the hydrodynamics of many chemical processes. Examples include fixed and fluidized bed reactors, chromatography columns, two-phase stirred tanks, distillation and absorption columns, and trickle bed reactors.





# Mean Age Theory for Quantitative Mixing Analysis

MINYE LIU

*E. I. DuPont de Nemours*

## 1b-1 INTRODUCTION

The concept of residence time and age has been widely used in chemical reactor engineering for mixing and flow distribution studies. The theory of residence time distribution is based on this concept and has been an integral part of modern chemical engineering (Levenspiel, 1999). The theory has been successfully used in many chemical reactor designs. The strength and weaknesses of the theory have been briefly reviewed in the Introduction and Chapter 1 of *Handbook of Industrial Mixing* (Paul et al., 2004). Although the theory is very useful, it is also limited in several aspects. The main limitation is perhaps that residence time distribution (RTD) function is based on the probability distribution of age. In the process of obtaining this function, the spatial distribution of tracer concentration, which is critical for characterizing mixing and reaction process in the reactor, has been lost. Therefore, an RTD function cannot determine the local mixing states inside the reactor, such as the locations of dead corners, bypassing channels, and so forth.

Recently, a new theory based on the spatial distribution of tracer mean age has emerged. In this theory, mean age is governed by a transport equation in a similar form to the Navier–Stokes (NS) equation. This equation can thus be solved using the same solver as for the NS equation in a computational fluid dynamics (CFD) code. With the spatial distribution of mean age known, the mixing state in a reactor can therefore be defined. This theory can be considered a major extension of the RTD.

This chapter provides a brief introduction to the mean age theory for steady incompressible flows. The relationship between mean age theory and residence time theory will be described. The application of mean age theory to quantitative characterization of mixing processes in both continuous flow reactors and batch reactors will also be discussed. Methods to compute the degree of mixing and tracer concentration history and distribution in a continuous flow reactor and blend time in a batch reactor will be presented.

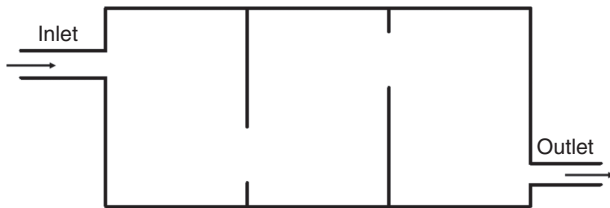
## 1b-2 AGE AND TIME IN A FLOW SYSTEM

An RTD frequency function is obtained by mixing-cup averaging of tracer concentration at the exit of a reactor. This is also a process of *spatial* integration of the concentration distribution, which is a function of both space and time. After integration, the resulting function varies with time only. A different function can be obtained if *time* integration is applied to the local concentration distribution function. The result is then a function of space only. When this time integration is done for the time-weighted concentration at every point in space in the reactor, a spatial distribution of local mean age can be obtained. This is the essence of the mean age theory to be discussed in this chapter.

Consider a simple flow system with one inlet and one outlet as shown in Figure 1b-1. The flow through the system is assumed to be steady. At time 0, the tracer concentration at the inlet is suddenly changed with a pulse or to a new constant. A time-dependent tracer concentration will be obtained at any selected spatial location. Based on the measured concentration, several different definitions of age have been used in the literature. Before we proceed, we would like to clarify these different definitions and unify the terminology of age used in the literature.

Usually, the term *residence time* is reserved for tracer molecules exiting the vessel, as the elapsed time since they entered (Nauman and Buffham, 1983). Suppose that we can measure the tracer concentration at every point at the exit,  $c(\mathbf{x}, t)$ . Then a frequency function can be defined with this concentration by “mixing-cup” averaging:

$$f(t) = \frac{C_e(t)}{\int_0^\infty C_e(\theta) d\theta} = \frac{1}{Q} \int_e u_e(\mathbf{x}, t) dA \quad (1b-1)$$



**Figure 1b-1** Steady flow reactor with one inlet and one outlet.

where  $f(t)$  is called the residence time distribution function. It is sometimes denoted by  $E(t)$ , thus often called E-curve. *Mean residence time* is defined as the first moment of this function:

$$\tau = \int_0^{\infty} tf(t) dt = \frac{V}{Q} \quad (1b-2)$$

Accompanying the RTD function is the accumulative residence time distribution function  $F(t)$ , which is also frequently used in the literature:

$$F(t) = \int_0^t f(\theta) d\theta \quad (1b-3)$$

and

$$E(t) = \frac{dF(t)}{dt} \quad (1b-4)$$

The *age* of a fluid identity, a molecule or fluid particle, as initially defined by Danckwerts (1953), is usually reserved to describe the elapsed time since the entrance of the identity still in the vessel. When the identity reaches the exit, its age equals the residence time. For this reason, Danckwerts also referred to this age as *internal age*. Since age is tagged to an individual identity, it will increase at the same rate as the time. Therefore, if we use  $\alpha$  to denote age, we can write

$$\frac{d\alpha}{dt} = 1 \quad (1b-5)$$

This derivative is a substantial derivative. The age distribution, whether inside the reactor or at the exit, is referred to as a probability distribution, not spatial distribution.

*Mean age*, denoted by  $a$ , as we will discuss in this chapter, is the averaged age of all molecules in a sample taken at a spatial location  $\mathbf{x}$ . For a steady flow, mean age is independent of time coordinate and is a function of space only. This is perhaps the most important difference between mean age and the traditional residence time.

*Internal residence time* is the time when a molecule inside the flow eventually leaves the system (Nauman and Buffham, 1983).

In some studies, *local residence time* is used to refer to internal age at a spatial location inside a flow system and *mean local residence time* as mean age. Throughout this chapter, we will restrict the use of the term *residence time* to the exit.

### 1b-3 GOVERNING EQUATIONS OF MEAN AGE AND HIGHER MOMENTS

Let's again consider the steady continuous flow system in Figure 1b-1. At time  $t = 0$ , a pulse of passive tracer is suddenly injected uniformly into the flow at the inlet.

Due to the spatially nonuniform velocity field, the tracer material will be convected and diffused throughout the flow and generate a nonuniform spatial distribution of concentration. If we measure the concentration of the tracer at a spatial location  $\mathbf{x}$  over time, we will obtain a time history of the concentration at the location,  $c(\mathbf{x}, t)$ . The distribution of  $c(\mathbf{x}, t)$  is governed by the convection-diffusion equation:

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (\mathcal{D} \nabla c) \quad (1b-6)$$

If we integrate this equation over time from 0 to  $\infty$ , we will obtain (Spalding, 1958)

$$\nabla \cdot (\mathbf{u} \mathbf{I}) - \nabla \cdot (\mathcal{D} \nabla \mathbf{I}) = 0 \quad (1b-7)$$

where

$$\mathbf{I} \equiv \int_0^{\infty} c(\mathbf{x}, t) dt \quad (1b-8)$$

Spalding argued that  $\mathbf{I}$  is an invariant of the steady flow, and  $\mathbf{I} = q/Q$  with  $q$  the total volume of tracer in the pulse and  $Q$  the volumetric flow rate of the continuous flow. Using this invariant, a probability frequency function, called age distribution function, can be defined as

$$\varphi = \frac{c(\mathbf{x}, t)}{\mathbf{I}} \quad (1b-9)$$

The mean age at  $\mathbf{x}$  can then be defined as

$$a = \int_0^{\infty} t \varphi(\mathbf{x}, t) dt = \frac{1}{\mathbf{I}} \int_0^{\infty} t c(\mathbf{x}, t) dt \quad (1b-10)$$

The governing equation for mean age  $a$  can be found as

$$\nabla \cdot (\mathbf{u} a) = \nabla \cdot (\mathcal{D} \nabla a) + 1 \quad (1b-11)$$

Liu and Tilton (2010) gave the complete steps of derivation for this equation, following the approach of Sandberg (1981) and Spalding (1958).

At the inlet, the concentration is zero for  $t > 0$ , and so the boundary condition for mean age is

$$a = 0 \quad (\text{at} \quad \text{inlet}) \quad (1b-12)$$

On solid walls, the normal gradient of tracer concentration is zero (the tracer cannot diffuse into the wall), therefore,

$$\mathbf{n} \cdot \nabla a = \frac{\partial a}{\partial x_n} = 0 \quad (\text{on solid walls}) \quad (1b-13)$$

At the exit, zero normal gradient  $\partial c / \partial x_n = 0$  boundary condition is generally applied to the species conservation equation. This leads to the same boundary condition at the exit for the mean age:

$$\mathbf{n} \cdot \nabla a = \frac{\partial a}{\partial x_n} = 0 \quad (\text{at exit}) \quad (1b-14)$$

This boundary condition is consistent with the requirement for closed boundaries upon which the tracer method is dependent. These boundary conditions have been used in the literature before, but Liu and Tilton (2010) gave the first rigorous derivations.

By its definition, mean age is the first moment of the tracer age distribution function  $\varphi$ . Higher moments of age also provide important information about the properties of the function. For example, the second moment characterizes the spread of  $c(\mathbf{x}, t)$ , and the third moment describes the skewness. The  $i$ -th moment of age is defined as

$$M_i = \int_0^\infty t^i \varphi(\mathbf{x}, t) dt \quad (1b-15)$$

Transport equations for moments can be derived in the same way as the mean age transport equation,

$$\nabla \cdot (\mathbf{u} M_i) = \nabla \cdot (\mathcal{D} \nabla M_i) + i M_{i-1} \quad (1b-16)$$

The same boundary conditions as for mean age apply to all the moments.

For a Reynolds-averaged turbulent flow, eqs. (1b-11) and (1b-16) can still be used, but the molecular diffusivity will be replaced by the effective turbulent diffusivity  $D_t$ ,

$$\mathcal{D}_t = \frac{\nu_t}{Sc_a} + \mathcal{D} \quad (1b-17)$$

where  $\nu_t$  is the turbulent viscosity and  $Sc_a$  is the turbulent Schmidt number for mean age.  $Sc_a$  is the same as that for turbulent species diffusion since mean age is determined by tracer diffusion.

Although the above equations of moments are derived for a pulse input flow system, the same equations can be derived for other types of input (Liu, 2011a). The key step in deriving these equations is to define the age frequency function. For a pulse input system, the age frequency function is defined by eq. (1b-9) with a spatial invariant  $I$ . Liu (2011a) showed that for a step input, the age frequency

function can be defined with the time derivative of concentration. For a step-up input, the corresponding invariant is the concentration imposed at the inlet,  $I = C_{in}$ . For a step-down input, the invariant is the initial concentration in the system,  $I = C_0$ . For a steady tracer input, the age frequency function is still defined by eq. (1b-9), but the invariant is the steady tracer concentration at the inlet,  $I = C_{in}$ . Thus, the transport equations for the moments of age are independent of the type of tracer inputs. The equations describe the flow properties, and different types of tracer addition are just ways to measure these flow properties.

It is noted that the governing equations for mean age and higher moments are in the same conservative form as the transport equations for momentum, energy, and species and so can be solved with the same CFD solver. This is a great advantage for the mean age method over the particle tracking method.

## 1b-4 COMPUTATION OF MEAN AGE

### 1b-4.1 Validations of Numerical Solutions

The mean age and higher moments computed from the transport equations can be validated in two ways: experimental and computational. Both methods involve finding the full distribution of local concentration distribution as a function of time. Equations (1b-10) and (1b-15) can then be used to calculate the moments.

Baleo and Le Cloirec (2000) compared the mean age distribution from both experimental measurements and the numerical solution of eq. (1b-11) for an incompressible flow in a circular pipe with expansions and contractions. The mean age from the solution of the steady transport equation agreed well with the mean age calculated from the measured time history of tracer concentration.

Liu and Tilton (2010) computed time-dependent tracer concentrations by solving the transient convection-diffusion equation for tracer concentration, eq. (1b-6) for a two-dimensional (2D) test reactor. The mean residence time of the flow system is 1.0297 s. From the solution of the steady transport equation for mean age, eq. (1b-11), the flow-averaged mean age at the exit is identical to this mean residence time up to five effective digits, showing the excellent accuracy of the mean age method. The results confirm the spatial invariant defined in eq. (1b-8). The moments of age computed from the concentration history were compared with the solution of the steady transport equations of the moments, eq. (1b-16), at several selected spatial locations inside the flow domain. With a concentration history of 12 s, the results showed that the maximum difference in mean age from the two methods is less than 2%. The difference increases for higher moments. For the second moments, the difference is less than 3%. For third moments, the difference at some locations was higher than 10%. Among the selected locations, larger differences are for points on or near the inlet jet path. Moments up to the fifth were also compared at the exit of the flow, and excellent agreement was obtained between the two methods. Even for the fifth moment, the difference is less than 1%.

The computed discrepancies between the two methods are mainly due to the early truncation of the time history of concentration and the numerical errors in the numerical integration. A numerical study revealed that for mean age, a 10% error resulted when the time history was truncated at  $t = 2\tau$ . The error is 4% for  $t = 4\tau$  and 1% for  $t = 6\tau$ . For the third moments, the error was 71% at  $t = 2\tau$ , 25% at  $t = 4\tau$ , and 5.6% at  $t = 6\tau$ .

For the CPU time, 8 h was reported for a concentration history of 12 s. In contrast, the CPU time for each of the steady transport equations for the moments of age was only about 1 min. These results clearly show that the mean age method is a major step forward in both numerical accuracy and efficiency. The gain in both accuracy and efficiency is due to the fact that the time integration has been done analytically. It should be clear from the above discussion that the mean age method does not provide any time history of tracer variation. Therefore, the method cannot be used to obtain the residence time function. However, in principle, such a function can be reconstructed from the computed moments.

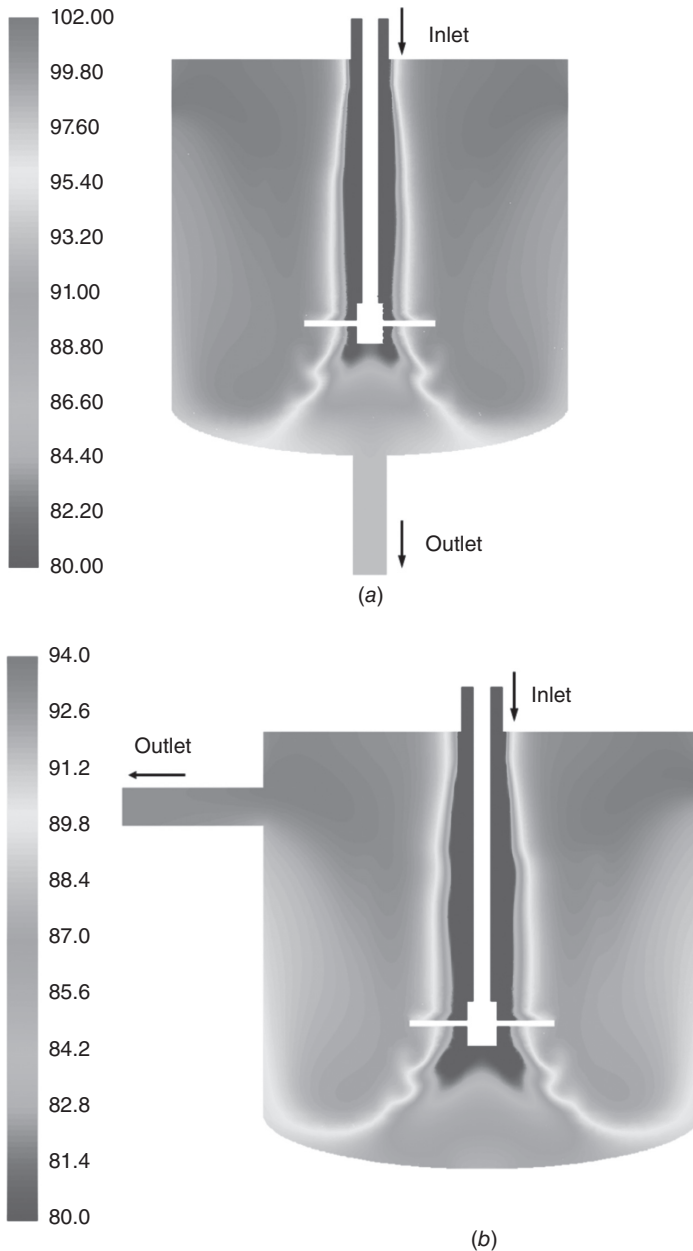
## 1b-4.2 Spatial Distribution of Mean Age in Mixing Devices

**1b-4.2.1 Stirred Tanks** The spatial distribution of mean age in a continuous flow stirred tank reactor can be used to analyze spatial nonuniformity of mixing. In the current literature, such nonuniformity is rarely discussed since there has not been an effective method for such analysis. Mean age is a powerful tool for both visual and quantitative characterization of such nonuniformity.

In designing a continuous flow stirred tank reactor (CFSTR), a key parameter is the locations of the inlet and outlet. Improperly placed inlets or outlets can cause strong bypassing. On a bypassing path, the mean age is much lower than the rest of the reactor. Figure 1b-2a (Liu, 2012a) shows clearly that when both the inlet and the outlet are placed on the axis of the reactor, even though the inlet flow passes through the impeller before reaching the outlet, strong bypassing can still exist. When the outlet is moved to the side of the reactor, the bypassing is avoided, as can be seen from Figure 1b-2b (case J; Liu, 2012a). For this case, although the direct distance of the inlet and the outlet is short, the flow path is the longest among all the cases studied in the Liu's (2012a) article.

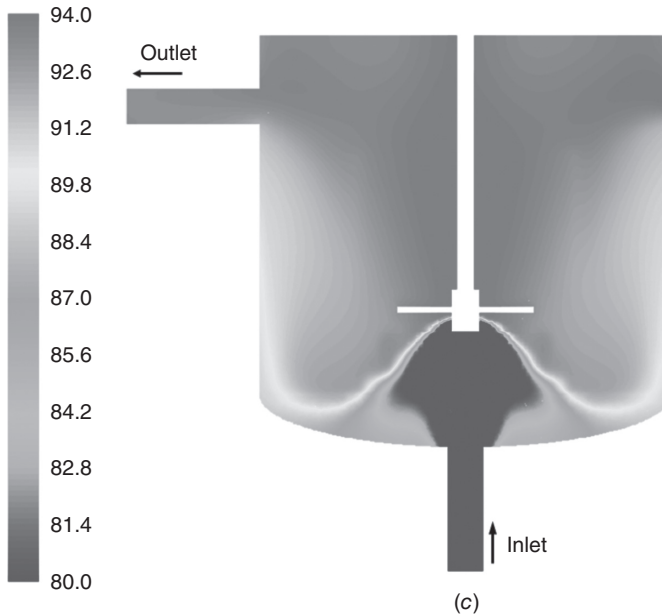
Fresh feed into a slower mixing zone can also cause problems, and an undesired side product or low grade product can often be the result. Figure 1b-2c shows the mean age distribution when the inlet is placed at the center of the bottom of the tank. The inlet jet sends the fresh material into the triangular (cone in 3D) zone below the PBT (pitched blade turbine) impeller. It is well known that this zone is outside of the impeller pumping stream and is a very slow mixing zone. The main mixing mechanism of the fresh feed to the rest of the reactor is the turbulent diffusion through the shear layer of the impeller jet. This process is much slower than the strong turbulent jet convection.

Slower mixing zones are those with higher mean age. In many stirred tank reactors, the last point where mixing is complete is behind the baffle if the test tracer is injected to the main circulation loop. The main mechanism for the tracer to be transported into



**Figure 1b-2** Mean age (s) contour plot in a stirred tank reactor equipped with a standard 45° PBT impeller. The color shade shows mean age distribution in seconds. The mean residence time of the reactor is 93.4 s. (a) Mean age distribution with short circuiting. (b) Mean age distribution without short circuiting. (c) Mean age distribution with fresh feed into slower mixing zone.



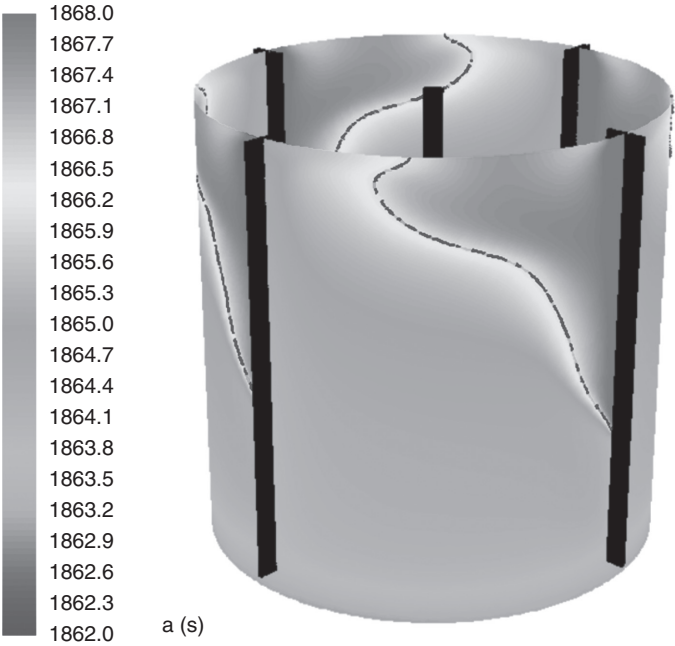


**Figure 1b-2** (Continued)

the separated region is by turbulent diffusion, a much slower process than convection by main circulation. Figure 1b-3 shows the mean age distribution on a constant radius surface in a reactor with a standard PBT. The contour plot clearly shows the high mean age inside the separated region behind the baffles. The mean residence time of this example is 1864 s, but the batch blend time of the same reactor is about 9 s. The relation of mean age distribution and batch blend time will be discussed in Section 1b-8.2.

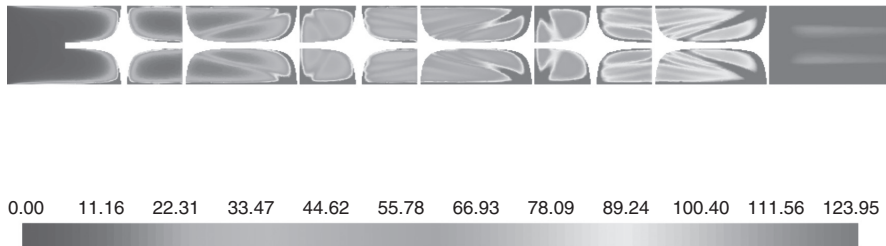
**1b-4.2.2 Kenics Static Mixers** Another example of using mean age to show spatial nonuniformity of mixing is the Kenics static mixer (Liu, 2012b). Liu studied age mixing in a laminar flow in a microscaled Kenics mixer. It is well known that numerical diffusion overwhelms molecular diffusion in a CFD solution of tracer concentration for a laminar flow in industrial scales. The small size of the mixer in this example helps to minimize the effect of numerical diffusion in the solution. Six elements are considered in the mixer plus an entrance region and exit region. The flow in the mixer is dominated by convection.

Figure 1b-4 shows the mean age distribution on the center plane across the full length of the mixer. This picture clearly shows the striations of different mean age as the material flows down the mixer. These spatial structures in mean age reveal the nonuniform mixing in the mixer. Such information cannot be seen from velocity solutions. In general, material age increases as it flows along the mixer. However, older aged material appears even in the first element. This is due to the slow motion near

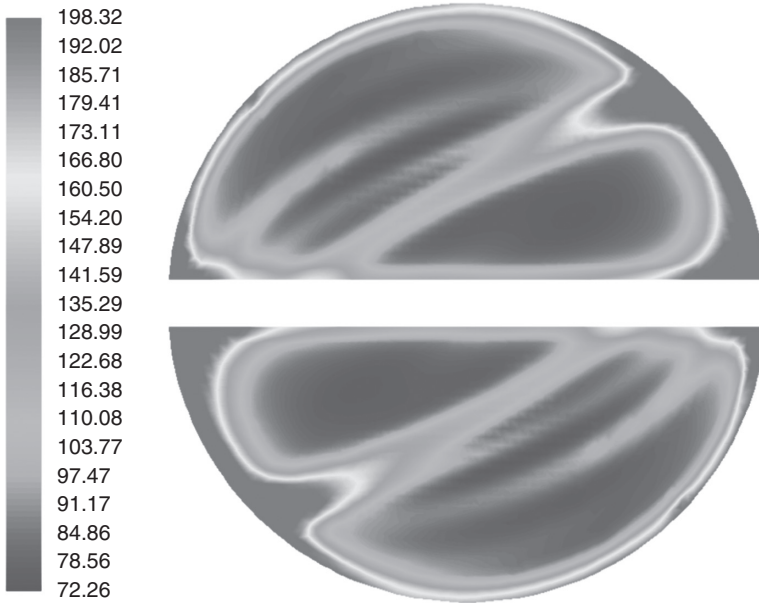


**Figure 1b-3** Mean age contour plot showing the slow mixing zone behind baffles with higher mean age in red. The mean residence time of the reactor is 1864 s.

the mixer pipe and element walls. In polymer processing or biomaterial processing, such older aged material could be the cause of low-quality product or mixer fouling. The spatial structure of mean age distribution can also be seen on a cross section of the mixer as shown in Figure 1b-5. This is on a cross section in the middle of the fourth element. The striations of higher mean age are due to the mixing mechanism of the element. This picture clearly shows the location of older material. At the end of each element, some of the older material sheds off from the element and enters the main flow to form new striations.



**Figure 1b-4** Mean age (s) contour plot in a micro-Kenics static mixer with six elements.



**Figure 1b-5** Mean age (s) contour plot on a cross section in the middle of the fourth element of the micro-Kenics static mixer.

### 1b-5 RELATIONS OF MEAN AGE AND RESIDENCE TIME DISTRIBUTION

Although both mean age and residence time describe age distributions, they differ in several ways. First, the RTD function is defined at the exit of a flow system while the mean age function is defined at a point in space inside the system. Second, RTD is a function of time only, while both mean age and mean age function are dependent on spatial locations. Therefore, the local mean age describes the *spatial* distribution of age, and the RTD describes the *probability* distribution of age. Third, the mean age is a scalar, a dependent variable governed by a convection-diffusion equation, while residence time is tagged to time, an independent variable. While mean age is a function of space, mean residence time is an integrated quantity at the exit. Referring to eq. (1b-1), the RTD function is also defined by the tracer concentration function. Therefore, the two variables are also related.

With the RTD function, the moments of the residence time distribution can also be defined:

$$\bar{t}^i = \int_0^{\infty} t^i f(t) dt \quad (1b-18)$$

Liu and Tilton (2010) have shown that the moments of residence time distribution are equal to the mass-averaged moments of age at the exit.

$$\bar{t}^i = \bar{M}_{i,e} \quad (1b-19)$$

where

$$\bar{M}_{i,e} = \frac{\int_{S_e} u M_i dA}{\int_{S_e} u dA} = \frac{1}{Q} \int_{S_e} u M_i dA \quad (1b-20)$$

This relationship allows us to compute the moments of the residence time distribution using the mean age method.

The relationship between the moments of age in the interior of the flow and the moments of residence time at the exit can also be established. If we integrate eq. (1b-16) over the flow domain and use Stokes theorem, we will have

$$\int_S \mathbf{u} M_i dS = \int_S \mathcal{D} \nabla M_i dS + i \int_V M_{i-1} dV \quad (1b-21)$$

Since the first term on the right-hand side vanishes on all boundaries, we have

$$Q \bar{M}_{i,e} = V i \bar{M}_{i-1,V} \quad (1b-22)$$

With eq. (1b-19), and recall that the mean residence time  $\tau = V/Q$ , we then have

$$\bar{M}_{i,V} = \frac{\bar{t}^{i+1}}{(i+1)\tau} \quad (1b-23)$$

This equation can also be obtained by integrating the relation of internal age frequency function and the accumulative residence time function over the flow domain, as shown in Liu (2012a).

Liu (2011a) has also shown that the volume-averaged moment of age is equal to the moment of internal age as defined by Zwietering (1959):

$$\bar{\alpha}^i = \int_0^\infty \alpha^i \phi(\alpha) d\alpha = \frac{1}{V} \int_V M_i(\mathbf{x}) d\mathbf{x} = \bar{M}_{i,V} \quad (1b-24)$$

where the internal age distribution function  $\phi(\alpha)$  is (Danckwerts, 1953)

$$\phi(\alpha) = 1 - F(\alpha) \quad (1b-25)$$

This relation will allow us to extend the discussions of Danckwerts and Zwietering on internal age distribution to the mean age theory.

## 1b-6 VARIANCES AND THE DEGREE OF MIXING

The current mean age theory offers several measures to quantify the state of mixing in a flow system.

### 1b-6.1 Variance of Residence Time Distribution

Traditionally, nonideal mixing in a continuous flow reactor is analyzed using RTD theory (Levenspiel, 1999; Nauman and Buffham, 1983). One measure for flow distribution is the variance of the RTD:

$$\bar{\sigma}_e^2 = \frac{1}{\tau^2} \int_0^{\infty} (t - \tau)^2 f(t) dt = \frac{\overline{t^2} - \tau^2}{\tau^2} \quad (1b-26)$$

The RTD function  $f(t)$  is obtained from the time history of tracer concentration at the exit of a reactor by either experimental measurements or numerical computation. In many industrial reactors, RTD function  $f(t)$  has a long tail. As discussed earlier in Section 1b-4.1, early truncation of  $f(t)$  often causes large errors in the resulting variance. On the other hand, computing  $f(t)$  for a long time often causes large computing cost. Using the mean age method, the variance can be computed without tracking  $f(t)$ .

From eq. (1b-19) for  $i = 1$ ,  $\tau = \bar{M}_{1,e} \equiv \bar{a}_e$ , and for  $i = 2$ ,  $\overline{t^2} = \bar{M}_{2,e}$ . The variance then becomes

$$\bar{\sigma}_e^2 = \frac{\bar{M}_{2,e} - \tau^2}{\tau^2} \quad (1b-27)$$

Using eq. (1b-23), the computing process of this variance can be further simplified, requiring the solution of mean age only:

$$\bar{\sigma}_e^2 = \frac{2\bar{a}_V - \tau}{\tau} \quad (1b-28)$$

Thus, the volume-averaged mean age can be used as a measure for a mixing system. Since only the first moment of age is needed, the computing cost of this method is often orders of magnitude lower than tracking the long time history of tracer concentration computationally. It should be mentioned that the volume-averaged mean age  $\bar{a}_V$  is an important quantity in the mean age theory. Later we will see that it is also the time constant for tracer concentration decay rate in a continuous flow reactor.

### 1b-6.2 Variances of Age

The distribution of age inside a reactor can also be measured with a variance. A variance for the internal molecular age distribution inside the flow can be defined:

$$\overline{(\alpha - \bar{\alpha})^2} = \int_0^{\infty} (\alpha - \bar{\alpha})^2 \phi(\alpha) d\alpha \quad (1b-29)$$

This variance is called the variance of internal age, or the variance of age. Danckwerts (1958a) and Zwietering (1959) discussed this variance in defining a relative measure for the degree of segregation, or the degree of mixing. Liu (2012a) has shown that this variance can be computed as

$$\overline{(\alpha - \bar{\alpha})^2} = \overline{M}_{2,V} - \bar{a}_V^2 \quad (1b-30)$$

This variance measures the probability distribution of internal age distribution, or molecular age distribution. However, it offers little information about the spatial distribution of age and mixing state.

In order to quantify spatial distribution of age, the variance of mean age can be used (Liu, 2012a):

$$\overline{(a - \bar{\alpha})^2} = \overline{a_V^2} - \bar{a}_V^2 \quad (1b-31)$$

A coefficient of variance (CoV) can be defined for each of the variances in eqs. (1b-30) and (1b-31). The CoV for molecular age is

$$\bar{\sigma}_\alpha \equiv \frac{\sqrt{\overline{(\alpha - \bar{\alpha})^2}}}{\bar{\alpha}} = \frac{\sqrt{\overline{M}_{2,V} - \bar{a}_V^2}}{\bar{a}_V} \quad (1b-32)$$

And the CoV for mean age is

$$\bar{\sigma}_a \equiv \frac{\sqrt{\overline{(a - \bar{\alpha})^2}}}{\bar{\alpha}} = \frac{\sqrt{\overline{a_V^2} - \bar{a}_V^2}}{\bar{a}_V} \quad (1b-33)$$

### 1b-6.3 Degree of Mixing

Danckwerts (1958a) proposed the concept of degree of segregation as the ratio of two different variances of age in the flow system. Zwietering (1959) further defined the two variances as the variance of age  $(\alpha - \bar{\alpha})^2$  and the variance of mean age  $(a - \bar{\alpha})^2$  and called the ratio the degree of mixing:

$$J_Z = \frac{\overline{(a - \bar{\alpha})^2}}{\overline{(\alpha - \bar{\alpha})^2}} \quad (1b-34)$$

The degree of mixing is a relative measure of the state of mixing between a plug flow and a perfect mixer. For a plug flow, there is no diffusion and the system is completely segregated. Thus the mean age  $a$  is the same as the molecular age  $\alpha$  and, therefore,  $J_Z = 1$ . For an ideal mixer the mean age is the same everywhere in the system as  $\bar{\alpha}$  and  $(a - \bar{\alpha})^2 = 0$ . Thus  $J_Z = 0$ . According to Zwietering's maximum mixedness theory (Zwietering, 1959), for any practical mixing device with a given RTD, a minimum value of  $J_Z > 0$  exists for the best mixing state.

From eqs. (1b-30) and (1b-31), the Danckwerts–Zwietering degree of mixing can be computed as

$$J_Z = \frac{\overline{a_V^2} - \bar{a}_V^2}{\overline{M_{2,V}} - \bar{a}_V^2} \quad (1b-35)$$

The Danckwerts–Zwietering degree of mixing is defined with variances about the average age inside the system. This average age varies with the mixing condition of the system. When this measure is used to compare different systems, the comparison may not be on the same basis. In studying mixing efficiency in continuous flow stirred tank reactors, Liu (2012a) found that this measure failed to characterize the differences in mixing states of different designs of inlet and outlet. To compare mixing efficiency with an invariant reference, another set of variances were defined about an invariant reference, the mean residence time of the corresponding ideal mixer. The new variances of age are then

$$\overline{(\alpha - \tau)^2} = \overline{M_{2,V}} - 2\tau\bar{a}_V + \tau^2 \quad (1b-36)$$

$$\overline{(a - \tau)^2} = \overline{a_V^2} - 2\tau\bar{a}_V + \tau^2 \quad (1b-37)$$

With these two variances, two new CoVs can be defined as

$$\bar{\sigma}_{\alpha-\tau} = \frac{1}{\tau} \sqrt{\overline{(\alpha - \tau)^2}} = \frac{1}{\tau} \sqrt{\overline{M_{2,V}} - 2\tau\bar{a}_V + \tau^2} \quad (1b-38)$$

$$\bar{\sigma}_{a-\tau} = \frac{1}{\tau} \sqrt{\overline{(a - \tau)^2}} = \frac{1}{\tau} \sqrt{\overline{a_V^2} - 2\tau\bar{a}_V + \tau^2} \quad (1b-39)$$

With these two variances, a new degree of mixing can be defined as

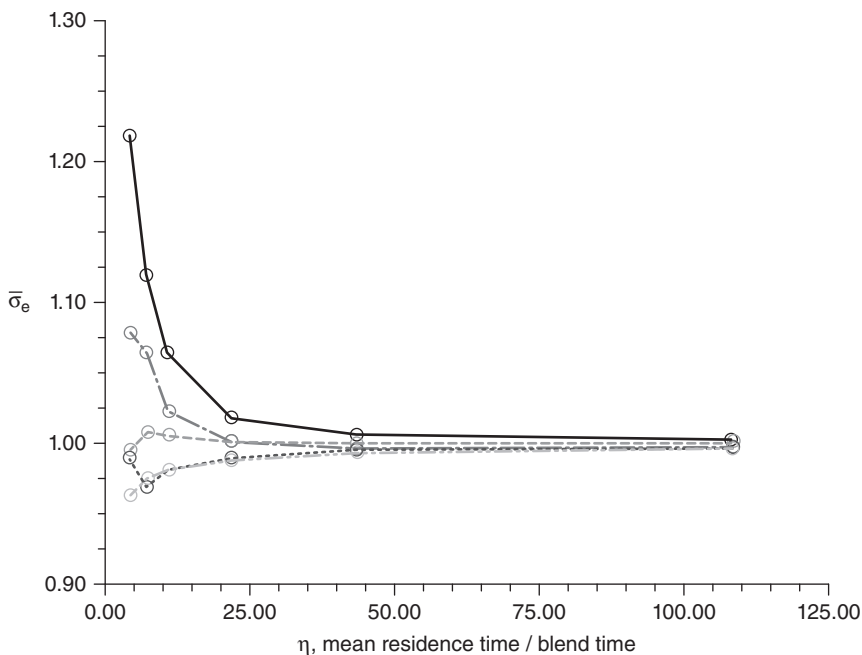
$$J_\tau = \frac{\overline{(a - \tau)^2}}{\overline{(\alpha - \tau)^2}} = \frac{\overline{a_V^2} - 2\tau\bar{a}_V + \tau^2}{\overline{M_{2,V}} - 2\tau\bar{a}_V + \tau^2} \quad (1b-40)$$

where  $J_\tau$  has the same value as  $J_Z$  for the plug flow and the ideal mixer but different values for a nonideal flow. We will see later in this chapter that this new degree of

mixing more accurately measures the relative mixing efficiency with the effects of short circuiting and plug flow.

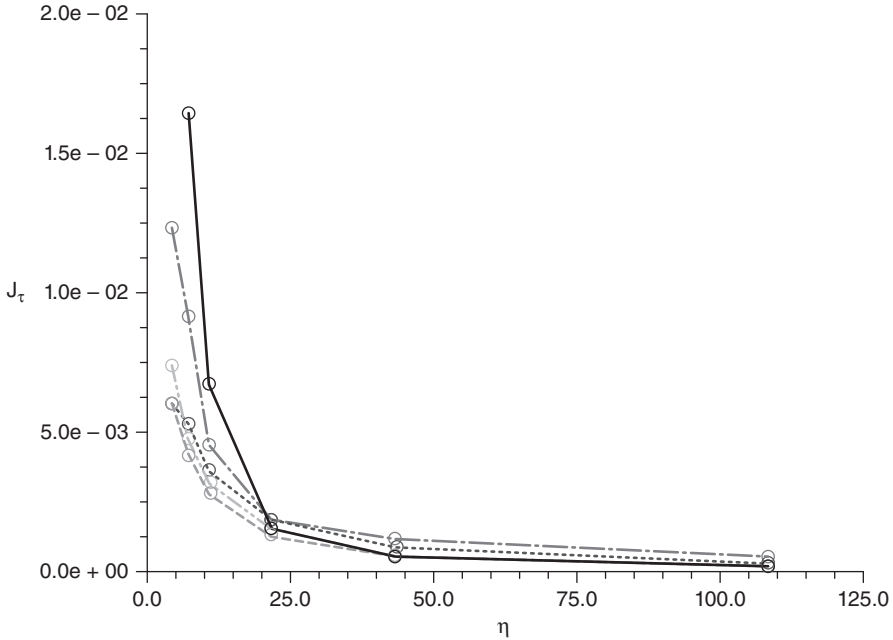
1b-6.4 Spatial Nonuniformity in CFSTRs

The spatial nonuniformity in mixing in a CFSTR can be characterized with the above-discussed variances and degree of mixing. Liu (2012a) studied five different configurations of inlet and outlet for a CFSTR. The variances and the degrees of mixing were computed to measure the performance of mixing. Figure 1b-6 shows the standard deviation of residence time  $\bar{\sigma}_e$  as a function of  $\eta$ , the ratio of mean residence time to the batch blend time of the same tank. Also shown in the figure by the thin solid line is the standard deviation for the ideal mixer,  $\bar{\sigma}_e = 1$ . It can be seen that all the standard deviations approach to one as the time ratio  $\eta$  increases. At shorter residence time, which corresponds to higher continuous flow rate, the curve with highest value of  $\bar{\sigma}_e$  is the one with short circuiting shown in Figure 1b-2a. Some curves approach 1 from above and others from below the solid line. This is due to the



**Figure 1b-6** CoV of residence time at the exit of stirred tank reactors with different layouts of inlet and outlet. Solid line (black): with short circuiting, as shown in Figure 1b-2a; dash-double dots (tan): without short circuiting, as shown in Figure 1b-2b; dotted line (green): fresh feed to slower mixing zone, as shown in Figure 1b-2c. Thin dashed line (dark gray): ideal mixer. See Liu (2012a) for dashed line (red) and dash-dot (blue) cases.





**Figure 1b-7** Degree of mixing of stirred tank reactors with different layouts of inlet and outlet. Same cases as in Figure 1b-6.

different feature of the flow, short circuiting, or plug flow. Detailed discussion can be found in Liu (2012a).

Figure 1b-7 shows the new degree of mixing for different inlet and outlet configurations. Again, the value for the case with strong short circuiting is the highest, showing the most deviation from an ideal mixer. As  $\eta$  increases, all curves approach to 0, but the level of deviation from the ideal mixer can be measured by the different values of  $J_t$ . Liu (2012a) showed that the degree of mixing calculated from eq. (1b-35) for this case did not separate this curve from others, indicating that the method was unable to detect the difference in mixing performance among the cases. Thus, the new definition of degree of mixing, eq. (1b-40), is recommended for mixing evaluation among different design cases.

## 1b-7 MEAN AGE AND CONCENTRATION IN A CFSTR

### 1b-7.1 Time History of Tracer Concentration

In a stirred tank reactor, the ultimate mixing measure comes from tracer concentration. Therefore, it is important to relate mean age distribution to tracer concentration. By tracking the time history of tracer concentration in nonideal CFSTRs, Liu (2011b) found that concentration history at each location has two stages. The first stage is

the initial response of concentration at the measured location to the input pulse. This response is very much location dependent. The time span of this stage is very short, in the same order of magnitude as the batch blend time. In the second stage, concentration at all locations decays exponentially from its peak value. Liu called this stage as the stationary stage and found that the decay rate at every point is the same as the reciprocal of the volume averaged mean age,  $\bar{a}_V$ :

$$c(\mathbf{x}, t) = \tilde{c}(\mathbf{x})e^{-t/\bar{a}_V} \quad (1b-41)$$

where  $\tilde{c}(\mathbf{x})$  is a function of spatial location only. The exponential decay of concentration in a mixing device with back mixing has been observed for a long time by many researchers. Nauman and Buffham (1983) believed that it is due to diffusion between slower and faster moving fluids. Now we can quantify this exponential decay rate with the average mean age for a stirred tank.

One of the results of exponential decay with the same rate in the flow is that if the concentration is scaled with the volume-averaged concentration, it is a function of spatial position only:

$$\hat{c} = \frac{c(\mathbf{x}, t)}{\bar{c}_V(t)} = \hat{c}(\mathbf{x}) \quad (1b-42)$$

where  $\bar{c}_V(t)$  is the volume-averaged tracer concentration defined as

$$\bar{c}_V(t) = \frac{1}{V} \int_V c(\mathbf{x}, t) dV \quad (1b-43)$$

It can easily be found that

$$\bar{c}_V(t) = \bar{C}_V e^{-t/\bar{a}_V} \quad (1b-44)$$

with

$$\bar{C}_V = \frac{1}{V} \int_V \tilde{c}(\mathbf{x}) dV \quad (1b-45)$$

Thus,  $\hat{c}$  can be used to study the spatial nonuniformity of tracer concentration.

By comparing the scaled concentration with the scaled mean age, Liu (2011a) found that the two scaled fields are nearly identical,

$$\hat{c}(\mathbf{x}) = \frac{c(\mathbf{x}, t)}{\bar{c}_V(t)} \approx \frac{a(\mathbf{x})}{\bar{a}_V} = \hat{a}(\mathbf{x}) \quad (1b-46)$$

For the continuous flow reactors studied in Liu (2011a), the difference between  $\hat{c}$  and  $\hat{a}$  is less than 1%, and the difference is caused by the nonuniform response of the input

pulse in the initial stage. With eq. (1b-46), the scaled tracer concentration field can be approximated by the mean age solution. Since mean age can be computed from eq. (1b-11), a steady transport equation, this relation provides a significant advantage in computing efficiency.

Mean age solution can also be used to determine the complete history of concentration at any point in space in the stationary stage without tracking the time-dependent solution of concentration. From eqs. (1b-41) and (1b-46), Liu found that

$$c(\mathbf{x}, t) = \frac{Ia(\mathbf{x})}{\tilde{A}\tilde{a}_V} e^{-t/\tilde{a}_V} \quad t > t_i \quad (1b-47)$$

where  $I$  is the invariant defined in Section 1b-3 and  $t_i$  is the beginning of the stationary stage, that is, the time when the concentration decay becomes exponential. The only constant in this equation,  $\tilde{A}$ , can be determined from measured or computed solution of concentration at any time and location after the concentration history becomes exponential. Thus, tracking the complete concentration history is not necessary.

When  $\tilde{A}$  is determined using the residence time distribution at the exit, Liu found that  $\tilde{A} = \tau\tilde{a}_e/\tilde{a}_V$  and

$$c(\mathbf{x}, t) = \frac{Ia(\mathbf{x})}{\tau\tilde{a}_e} e^{-t/\tilde{a}_V} \quad t > t_i \quad (1b-48)$$

where  $\tilde{a}_e$  can be determined numerically at any instant  $t > t_i$  from

$$\tilde{a}_e = \frac{1}{c_e(t)} e^{-t/\tilde{a}_V} \quad (1b-49)$$

with  $c_e(t)$  the time history of flow-averaged concentration at the exit of the flow.

### 1b-7.2 Mixing Time in CFSTRs

To measure mixing performance of a CFSTR, Roussinova and Kresta (2008) defined a blend time for a system with a stepup input as the time when the tracer concentration at a measuring point reaches 95% of the inlet concentration. For an ideal mixer, the concentration in the system is

$$c(t) = C_{in}(1 - e^{-t/\tau}) \quad (1b-50)$$

The time for the concentration in the system to reach  $0.95C_{in}$  can easily be found as

$$\theta_c = -\tau \ln 0.05 = 3\tau \quad (1b-51)$$

For most practical stirred tank reactors, its mean residence time is much larger than the batch blend time. Therefore,  $\theta_c$  should not be a time measuring mixing performance

in the reactor but rather a time for purging performance. Nevertheless, it is a time to measure the system response to the inlet condition change.

For a pulse system, the same response time can be defined as the time when the tracer concentration at a measuring point reaches within 5% of the initial average concentration in the system immediately after the pulse is released. For the ideal mixer, the response of the concentration in the system is

$$c(t) = C_0 e^{-t/\tau} \quad (1b-52)$$

where  $C_0$  is the initial mean concentration in the reactor after the pulse is released:

$$C_0 = \frac{IQ}{V} = \frac{I}{\tau} \quad (1b-53)$$

Then, the response time is the same as eq. (1b-51). For a nonideal reactor, since the concentration in the stationary stage can be found from eq. (1b-48), the 95% blend time  $\theta_c$  can easily be found as

$$\theta_c(\mathbf{x}) = -\bar{a}_v \ln \left[ 0.05 \frac{\tilde{a}_e}{a(\mathbf{x})} \right] \quad (1b-54)$$

Liu (2011b) studied effects of different inlet–outlet layouts and continuous flow rates on this blend time. Notice that the blend time  $\theta_c$  is location dependent. The largest value of  $\theta_c$  at all locations should be chosen as the blend time of the reactor.

## 1b-8 PROBABILITY DISTRIBUTION FUNCTION OF MEAN AGE

### 1b-8.1 Definition

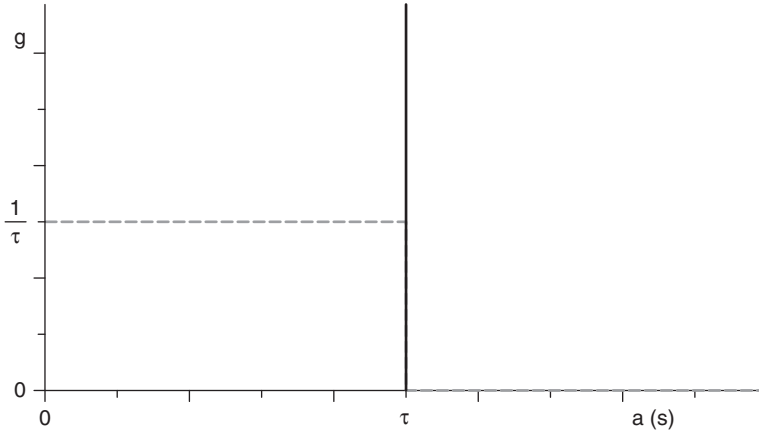
The probability distribution function is often a powerful tool to characterize a complex system. Like residence time distribution, the mean age distribution can also be quantitatively characterized with a PDF. Consider a differential volume  $dv(a)$  in the flow domain with mean age in the range of  $a$  and  $a+da$ . The fraction of the volume with mean age in this range out of the total volume of the flow domain is a function of  $a$ . We can then define a probability density function (PDF)  $g(a)$  for mean age as

$$g(a) da = \frac{dv(a)}{V} \quad (1b-55)$$

or

$$g(a) = \frac{1}{V} \frac{dv(a)}{da} \quad (1b-56)$$

It is trivial to show that  $g(a)$  satisfies the two conditions of a PDF:  $g(a) > 0$  and  $\int_0^\infty g(a) da = 1$ . Liu (2011c) has found the mean age PDF for a plug flow and an



**Figure 1b-8** Probability density functions of mean age in the plug flow (dashed line) and ideal mixer (solid line).

ideal mixer. For a plug flow with a length of  $L$  and velocity of  $U$ , the PDF is

$$g(a) = \frac{1}{V} \frac{dv}{da} = \frac{U}{L} = \frac{1}{\tau} \quad a \in [0, \tau]$$

$$g(a) = 0 \quad a > \tau \quad (1b-57)$$

Thus the mean age PDF for a plug flow is a constant,  $1/\tau$ .

For an ideal mixer, molecular age  $\alpha$  everywhere inside the reactor is the same as that at the exit and is an exponential function:

$$\phi(\alpha) = \frac{1}{\tau} e^{-\alpha/\tau} \quad (1b-58)$$

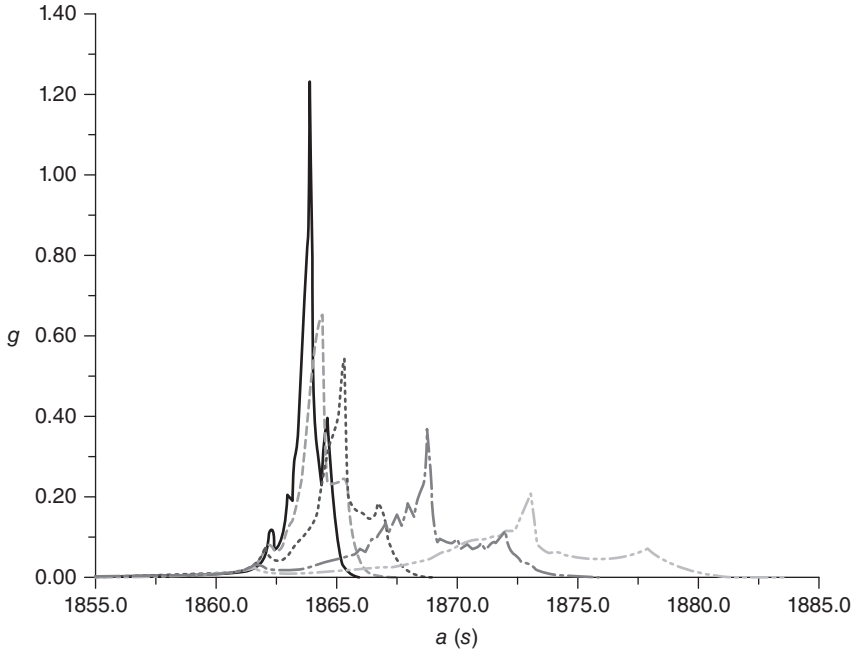
The mean age is, therefore, a constant throughout the device and equal to the mean residence time at the exit,  $a = \tau$ . Thus, the mean age PDF is a delta function at  $a = \tau$ :

$$g(a) = \delta(a - \tau) \quad (1b-59)$$

The PDFs of mean age for the plug flow and the ideal mixer are shown in Figure 1b-8.

### 1b-8.2 Scaling and Blend Time Estimation

Liu (2011c) computed mean age PDFs for a stirred tank reactor with small flow rate. He showed that as the continuous flow rate reduces, its effect on the flow field and mean age distribution becomes negligible. Thus, the computed mean age reflects the impeller performance on mixing in a batch reactor. The PDF of mean age can then be used to study the effects of impeller speed and diameter on mixing efficiency.



**Figure 1b-9** Probability density functions of mean age in the stirred tank reactor at different impeller speeds. Solid line (black): rpm = 600; dashed line (red): rpm = 450; dotted line (green): rpm = 300; dash-dot (blue): rpm = 150; dash-double dots (tan): rpm = 100.

Figure 1b-9 shows the PDF for different impeller speeds. The width of the curves clearly shows the strong effect of impeller speed on mixing rate. The curves are more spread out at low rpm than at higher rpm. As revolutions per minute (rpm) increases, the curve approaches the delta function for the ideal mixer. A similar effect of impeller diameter was also found.

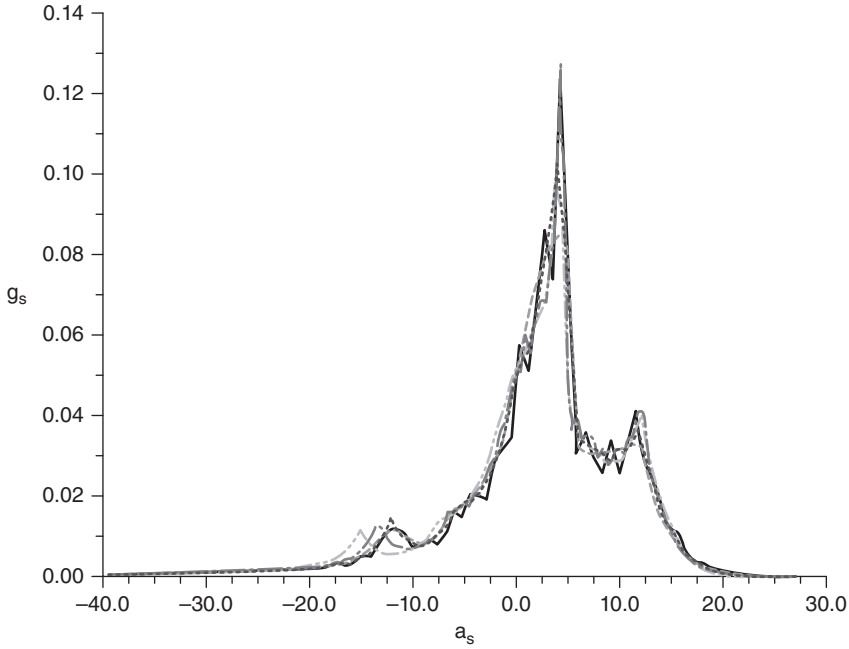
Although the PDF curves are affected by both the impeller speed and diameter, Liu (2011c) has found that the curves are scalable. He defined a new and scaled mean age as

$$\tilde{a} = N^\beta \left( \frac{D}{T} \right)^\gamma (a - \bar{a}_v) \quad (1b-60)$$

With the new scaled mean age, a new PDF can be found as

$$\tilde{g}(\tilde{a}) = \frac{1}{V} \frac{dv}{d\tilde{a}} = N^{-\beta} \left( \frac{D}{T} \right)^{-\gamma} g(a) \quad (1b-61)$$

For the stirred tank with a 45° PBT impeller with the bottom clearance at  $T/3$ , he found that  $\beta = 1$  and  $\gamma = 2$  give the best scaling of all the curves. Figure 1b-10 shows the curves of Figure 1b-9 and three other curves for 300 rpm and  $D/T = \frac{1}{4}, \frac{1}{3},$  and  $\frac{1}{2}$ .



**Figure 1b-10** Scaled probability frequency functions of mean age in the stirred tank reactors at different impeller speeds.

By comparing eq. (1b-60) with the blend time correlation in Chapter 9 of the *Handbook* (Grenville and Nienow, 2004),

$$N\theta_{95} \left( \frac{D}{T} \right)^2 = 5.2P\sigma^{-1/3} \quad (1b-62)$$

Liu (2011c) postulated that the mean age PDF curves can be used to estimate blend time of a batch mode mixing device. He defined this blend time as  $\theta_a = a_{\max} - a_0$  with  $a_{\max} - a_0$  as the width of the curve at the base. As shown in Figure 1b-9, the width of the mean age PDF is a function of the impeller speed. Similarly, it is found to be a function of impeller diameter. This suggests that the width may be used as a measure for blend time. Liu (2011c) defined the width of a PDF curve as  $a_{\max} - a_0 = 5\sigma$  or  $a_{\max} - a_0 = 6\sigma$  and compared with the blend time calculated from eq. (1b-62) and another correlation by Fasano and Penney (1991). For a normal distribution,  $5\sigma$  covers the 98.76% of the area and  $6\sigma$  covers 99.74% of the area under the PDF curve. Figure 1b-11 shows the comparison of  $\theta_a$  with  $\theta_{95}$  from correlations. It can be seen that the blend times calculated from the width of the mean age curves agree well with both correlations.

Another method to calculate the width of a mean age PDF curve for blend time is to integrate the area under the curve. From the definition of mean age PDF in eq. (1b-56), it can be found that this area is in fact the volume of the stirred tank. A blend