



OPERATIONS RESEARCH FOR UNMANNED SYSTEMS

Edited by **JEFFREY R. CARES** and **JOHN Q. DICKMANN, JR**

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WILEY

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1

Introduction

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1.1 Introduction

Given all the attention and investment recently bestowed on unmanned systems, it might seem surprising that this book does not already exist. Even the most cursory internet search on this topic will show professional journal articles, industry symposia proceedings, and technical engineering texts conveying broad interest, substantial investment, and aggressive development in unmanned systems. Yet an internet bookstore or library search for “operations research” combined with “unmanned systems” will come up blank. This book will indeed be the first of its kind.

Historians of military innovation would not be surprised. In fact, they point to a recurring tendency of the study of *usage* to lag *invention*. Such a hyper-focus on engineering and production might be perfectly understandable (for program secrecy, to work the “bugs” out of early production models, or simply because of the sheer novelty of radically new devices) but the effect is often the same: a delayed understanding of how operators could use new hardware in new ways. As the preeminent World War II scientist P. M. S. Blackett observed of the innovations of his time, “relatively too much scientific effort has been expended hitherto on the *production* of new devices and too little in the *proper use* of what we have got.” [1] It is ironic that a study of usage is one of the best ways to understand how to develop and improve a new technology; but engineering, not usage, gets the most attention early in an innovation cycle.

One does not need to be a student of military innovation to know that the study of usage is not the engineer’s purview. Blackett’s counterparts across the Atlantic, Morse and Kimball, noted that the “the branches ... of engineering ... are involved in the construction and production of equipment, whereas operations research is involved in its use. The engineer is the consultant to the builder, the producer of equipment, whereas the operations research worker

is the consultant to the *user* of the equipment.” [2] Engineering tells you how to build things and operations research tells you how things should be used. In development of new military hardware, however, engineering nearly always has a head start over operations research.

Three of the many ways that engineering overshadows usage early in unmanned systems development have delayed a book such as this from reaching professional bookshelves. The first is that most engineers have not yet recognized that unmanned systems can be so much more than merely systems without a human onboard. This *anthropomorphism*—creating in our own image—was the first fertile ground for engineers, and early success with this approach made it seem unnecessary to conceive of unmanned operations as any different than those studied by operations researchers for decades.

The second reason is that since engineers build *things*, not *operations*, the engineer’s approach to improving operations is to refine the vehicles. Such engineering-centered solutions have already been observed in existing unmanned programs, driving up vehicle complexity and cost—without regard to how modifying operational schemes might be a better way to increase operational performance.

The third reason is that since humans are the most expensive “total cost of ownership” (TOC) components of modern military systems, the military and defense industries have been content to lean on “manpower cost avoidance” as the overriding value proposition for unmanned systems. For now, unmanned systems are convincingly sold on cost alone—there is no reason for program managers to answer questions about operational value that no one is yet asking. The engineer’s present task is to keep development and production costs lower than equivalent manned systems for a given level of performance—not to explore the performance–cost trade space.

The historian of military innovation would be quick to clarify that usage lags invention mostly in the *initial* phases of maturation. Engineers and program managers pre-occupied with production can indeed be quite successful. In the case of unmanned vehicle development, second- and third-generation variants have already replaced prototypes and initial production models in the fleet, field, and flightline. Major acquisition programs (such as the Global Hawk and Predator systems) are already out of adolescence. Now that well-engineered platforms are employed on a much larger scale, a growing cadre of operations research analysts are at last being asked to answer operational questions—questions of usage.

While the three reasons cited above are among those that have heretofore preempted this book, they also constitute an initial set of topics for the operations researcher. What we might now call “operations research for unmanned systems” is emerging with three main themes:

- *The Benefits of “Unmanning”*: While the challenges of removing humans from platforms are still manifold and rightfully deserve our attention, operations researchers are now looking past the low hanging fruit of “unmanning” these systems—such as less risk to humans, longer sortie duration, higher *g*-force tolerance—to develop entirely new operations for unmanned systems and to discern new ways of measuring effectiveness.
- *Improving Operations*: The introduction of large numbers of unmanned vehicles into a legacy order of battle may transform warfare in profound ways. Some authors in the defense community have coined the term “Age of Robotics” to refer to this transformation, but from an analytical perspective, this term (like “Network Centric Warfare” and others of their ilk) is still more rubric than operational concept. While a full appreciation of such a new age may remain elusive, operations researchers are approaching the study

of unmanned collectives in a more modest way. Through careful study and operational experimentation with smaller groups of vehicles, these analysts are starting to build evidence for claims of increasing returns and show why and how they may be possible (or, just as importantly, not).

- *The True Costs of Unmanned Systems:* The only “unmanned” part of today’s unmanned systems are the vehicles—the humans have been moved somewhere else in the system. The life-cycle cost savings accrue to the platforms, but is the overall system cheaper? In some systems, centralized human control and cognition may be a much more costly approach, requiring substantially more technological investment, greater manning, and networks with much higher capacity than legacy manned systems. Analyzing this trade space is an area of new growth for operations research.

1.2 Background and Scope

As recent as the late 1990s, unmanned vehicles were still seen as a threat to the legacy defense investments of the world’s leading defense establishments. Even the mildest endorsements of their value to the warfighter for anything but the most mundane military tasks were met with derision, suspicion, and resistance. At the same time, more modest militaries and their indigenous industries—unconstrained by the need to perpetuate big-ticket, long-term acquisition strategies—began to develop first-generation unmanned platforms and capabilities that could no longer be denied by their bigger counterparts.

Concurrently and independently, innovations in secure, distributed networking and high-speed computing—the two most basic building blocks of advanced unmanned systems—began to achieve the commercial successes that made unmanned military vehicles seem more viable as a complement to legacy platforms in the fleet, field, and flight-line. But while the war on terror has seen focused employment of surveillance drones and explosive ordinance disposal robots, defense budget reductions are spurring a more widespread use of unmanned military systems more for the cost savings they provide than for the capabilities they deliver.

The five-year future of unmanned systems is uncertain, except in one respect: every new operational concept or service vision produced by the world’s leading militaries *expect* that unmanned vehicles will be a major component of future force structures. The details of this expectation—which platforms will garner the most investment, what technological breakthrough will have the most impact or where unmanned systems will have their first, game-changing successes—are the subject of intense speculation. This book will be successful if it helps bring some operational focus to the current debate.

While it is common to assert that increasing returns must surely accrue as more unmanned hardware is connected to a larger “network-enabled” systems of systems, engineers still concentrate on the robotic vehicles, unable to conceive of how unmanned collectives might indeed perform better than merely the sum total of all the vehicles’ individual performance. Without better analyses of group operations, the engineer’s solution to improving the performance of a collective is simply to engineer better performance into each vehicle of the group. Network engineers have been the loudest advocates for “networked effects,” but like the hardware engineers they have largely ignored operations research, devoting their efforts to engineering architectural standards and interconnection protocols. To make matters worse, in many cases the *process* (engineering activity) has become the *product*.

This book will benefit readers by providing them with a new perspective on how to use and value unmanned systems. Since there is no other place where these types of analyses are yet assembled, this book will serve as a seminal reference, establishing the context in which operations research should be applied to unmanned systems, catalyzing additional research into the value of unmanned platforms, and providing critical initial feedback to the unmanned systems engineering community.

Good operations research analysis is at once digestible by operators and informative to specialists, so we have attempted to strike a balance between the two. Fortunately, nearly all defense community operators have a solid technical education and training (albeit somewhat dated), and can follow the main arguments from college-level physics, statistics, and engineering. Chapters in this book should briefly refresh their education and bring it into operational context. Defense engineers, by contrast, are expert at their applicable “hard science,” but must be informed of operational context. This book should confirm for a technical audience that the writers understand the most important technical issues, and then show how the technical issues play out in an operational context. Both will buy this book expecting to learn something more than they already know about unmanned systems; this book will have to approach this learning experience from both of these perspectives.

1.3 About the Chapters

Fourteen chapters follow this introduction. Considering that unmanned vehicle systems development is by nature multidisciplinary, there are certainly many ways that these chapters might be appropriately arranged. The editors opted to arrange the chapters on a continuum from individual problems to analyses of vehicle groups, then to organizational issues, and finally to broad theoretical questions of command and control. Some of the topics may be new intellectual ground for many readers. For this reason, the editors have tried to ensure each chapter has enough basic context for a general audience with some mathematical background to digest each chapter, no matter what the subject. They also hope that this will satisfy readers who come to this book for, say, unmanned vehicle routing techniques, to stay for a discussion of Test and Evaluation or TOC.

Huang Teng Tan and Dr. Raymond R. Hill of the US Air Force Institute of Technology provided the first chapter, *The In-Transit Vigilant Covering Tour Problem for Routing Unmanned Ground Vehicles*. One might rightly wonder why Air Force researchers care about unmanned ground vehicles, but the answer is simple: the US Air Force has a significant Force Protection mission at its many bases worldwide, and the total ownership costs of human sentries are high. Unmanned sentries can augment and replace humans at lower costs. This chapter provides a formal discussion of how to efficiently address the covering tour problem, or in other words, what is the best way for a robotic sentry to “make its rounds.” There are obvious border patrol and civilian security applications of this research.

The next chapter, *Near-Optimal Assignment of UAVs to Targets Using a Market-Based Approach* by Dr. Elad Kivelevitch, Dr. Kelly Cohen, and Dr. Manish Kumar, is an application of “market-based” optimization to sensor–target pairings. This family of optimization techniques is inspired by economic markets, such as in this example where unmanned vehicles act as rational economic agents and bid for targets using a valuation and trading scheme. The authors show the benefits and limits of this approach to obtaining a fast, reliable optimization under conditions of high uncertainty.

A chapter discussing naval applications of unmanned underwater vehicles comes next. In *Considering Mine Countermeasures Exploratory Operations Conducted by Autonomous Underwater Vehicles*, Dr. Bao Nguyen, David Hopkin, and Dr. Handson Yip look at ways to evaluate the performance of Commercial Off-The-Shelf (COTS) unmanned underwater vehicles in searches for underwater mines. They present and discuss measures of effectiveness and compare and contrast different search patterns.

Optical Search by Unmanned Aerial Vehicles: Fauna Detection Case Study, by Raquel Prieto Molina *et al.*, is a very interesting chapter that harkens back to some of the very early operations research work from World War II. Readers familiar with Koopman's *Search and Screening*[3], for example, will note the strong parallel between this chapter and World War II research on lateral range curves and the inverse cube law. Both were trying to describe the basic physics of visual detection (Prieto *et al.*, are, of course dealing with artificial visual detection), and how it impacts search patterns and detection probabilities.

There are many cases in modern military operations where a clever scheme or algorithm devised *in silico* unravels when it is placed in operation in a real environment. Recognizing this, Dr. Matthew J. Henchey, Dr. Rajan Batta, Dr. Mark Karwan, and Dr. Agamemnon Crassidis show how algorithms might compensate for environmental effects in *Flight Time Approximation Model for Unmanned Aerial Vehicles: Estimating the Effects of Path Variations and Wind*. While crafted for air vehicles, this research could be adapted for any unmanned vehicles operations where delay or resistance are encountered (such as set and drift at sea, reduced trafficability on land, or interruption by an adversary in any medium).

For many militaries and corporations, unmanned vehicles are now major acquisition programs, requiring high-level analyses of alternatives (AOAs) not just between vehicles, but between human-vehicle hybrid systems. Fred D. J. Bowden, Andrew W. Coutts, Richard M. Dexter, Luke Finlay, Ben Pietsch, and Denis R. Shine present a template for these types of studies in *Impacts of Unmanned Ground Vehicles on Combined Arms Team Performance*. While specific to Australian Army trade-off analyses, this chapter is certainly useful for other AOA analyses in both cabinet departments and corporate executive suites.

With respect to human-vehicle hybrids, how much human work should robots actually do? All senior military officials will insist that a human must always be in the loop, but in many cases this is just to confirm an automated solution before weapons are employed. But how much was this human involved in the automated solution, and how reliable is the robot's "thinking"? Patrick Chisan Hew's chapter, *Processing, Exploitation, and Dissemination: When is Aided/Automated Target Recognition "Good Enough" for Operational Use?*, offers a formal mathematical treatment to this and similar questions of the operational and ethical impact of automated cognition.

Also exploring the man-machine trade-space is *Analyzing a Design Continuum for Automated Military Convoy Operations*, by David M. Mahalak. This chapter used logistics convoy operations to show how automated control can supplant human control in a continuum of increasingly automated convoys. This is yet another chapter that applies to a broader range of vehicles and operations.

Continuing along the continuum to higher levels of organizational problems, another chapter from Dr. Raymond R. Hill (this time with Brian B. Stone, also of the US Air Force Institute of Technology), *Experimental Design for Unmanned Aerial Systems Analysis: Bringing Statistical Rigor to UAS Testing*, addresses new operational test and evaluation issues wrought by the introduction of unmanned systems.

It has long been assumed that automated systems are cheaper than manned systems. As this introduction has stated, however, this has not been as well investigated as investments in unmanned vehicle systems should warrant. Dr. Ricardo Valerdi and Captain Thomas R. Ryan, Jr., US Army, address this issue and provide costing techniques in *Total Cost of Ownership (TOC): An Approach for Estimating UMAS Costs*.

Part of the TOC of any system are the costs associated with logistics and maintenance. Major Keirin Joyce, Australian Army, discusses modeling techniques for logistics operations with a focus on how well current logistics models can support unmanned vehicle operations. In a very important section of this chapter, *Logistics Support for Unmanned Systems*, Major Joyce extrapolates from current models and operations to address logistics support challenges for future unmanned systems.

As more systems are automated and dispersed throughout the battlespace or commercial work environment, there is an increasing need to understand how networks of collectives are effectively operated and controlled. *Organizing for Improved Effectiveness in Networked Operations* by Dr. Sean Deller, Dr. Ghaith Rabadi, Dr. Andreas Tolk, and Dr. Shannon R. Bowling combines concepts of interaction patterns in biochemistry with modern agent-based modeling techniques to explore a general model of command and control in a distributed, networked system.

Two chapters addressing theoretical topics complete the volume. *An Exploration of Performance Distributions in Collectives* by Jeffrey R. Cares compares individual and collective performance in competition, using baseball as a proxy. In *Distributed Combat Power: The Application of Salvo Theory to Unmanned Systems*, the same author shows how Hughes' Salvo Equations might be modified to evaluate outcomes from missile combat between large platforms when advanced unmanned vehicles are employed.

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2

The In-Transit Vigilant Covering Tour Problem for Routing Unmanned Ground Vehicles*

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2.1 Introduction

The Maximization of Observability in Navigation for Autonomous Robotic Control (MONARC) project within the Air Force Institute of Technology (AFIT) has an overarching goal to develop an autonomous robotic, network-enabled, Search, Track, ID, Geo-locate, and Destroy (Kill Chain) capability, effective in any environment, at any time. One area of interest in the MONARC project is mission planning for base security protection of Key Installations (KINs) from adversarial intrusions using autonomous Unmanned Ground Vehicles (UGVs). This UGV mission planning task is multifaceted and requires the consolidation of intelligence, management of system readiness, centralized operational planning and dissemination of Command and Control (C2) information. Sensory data from various locations around the KINs are fused into a Recognized Ground Situation Picture (RGSP) and augmented with intelligence from various agencies. A centralized C2 center consolidates and manages the real-time system serviceability and readiness state of the UGVs. The mission planners input the security requirements, such as key surveillance points, potential intrusion spots, and Rules of Engagement (ROE) into a Ground Mission Planning System which provides a surveillance approach for use by the team of UGVs.

The protection of a large KIN, such as a military airbase, requires a team of UGVs patrolling along certain routes to effectively cover numerous intrusion spots. The surveillance approach of the security defense task can be formulated as a combinatorial optimization model, in which

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multiple security entities must visit multiple locations, while covering certain adversarial locations, in the shortest total distance traveled for all entities. When adversarial locations are sensed by UGVs at their route location, the problem is a Covering Tour Problem (CTP) [1–3]. The CTP is a Traveling Salesman Problem (TSP) with a Set Covering Problem (SCP) structure. The multiple vehicle variant is a natural extension. Not addressed in prior CTP research, but quite applicable in the current context, is the vehicle covering capability while transiting along edges between route locations. This in-transit vigilance component is important to the current mission planning environment. A new variant of the multiple vehicle Covering Tour Problem (mCTP) model called the in-transit Vigilant Covering Tour Problem (VCTP) is used as a mission planning tool for base security applications. The single vehicle version is evaluated to assess the viability of the VCTP.

2.2 Background

The CTP model has been applied extensively in the health care industry, especially in planning deployments of mobile health care units traveling in developing countries [4]. Mobile health care units have access to a limited number of villages due to factors such as infrastructure restrictions, unit capacity, and cost. Therefore, it is infeasible to travel to all villages. Instead, a tour route is planned so that the unvisited villages are within reasonable walking distance of the visited villages thereby providing greater access for those in need of health care. The vehicle routes of the health care units are efficiently planned to reduce the amount of travel required, but to visit enough villages to provide sufficient overall medical coverage. A real-life problem associated with the planning of mobile health care units was in the Suhum district, Ghana [5] and solved by Hachicha *et al.* [6] as a CTP.

Another important application of the CTP is the placement of mailbox locations to reduce the traveling distance of the postal delivery service while ensuring maximum coverage [7]. Good locations of mailboxes to cover a region of users and an optimal route for mail distribution are constructed. Alternatively, this approach applies to the management of centralized post offices, that is, post offices are centralized in towns with larger populations while the smaller nearby towns are covered by the centralized post offices.

The CTP model has also been applied to the transportation industry such as in the design of bi-level, hierarchical transportation networks [8]. For an overnight mail delivery service provider, such as DHL, FedEx, and so on, the optimal tour route represents the route taken by the primary vehicle (aircraft) to the distribution centers and the coverage radius is represented by the maximum distance traveled by the delivery trucks from the distribution centers to the customers. The route ensures that the overall distribution cost is minimized and provides the required delivery service to its customers.

The mCTP [6] is defined as a complete undirected graph $G = (V, E)$ where V of size $n + 1$, and indexed as $v_0, \dots, v_n$, is the vertex set, and $E = \{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$ is the edge set. Vertex v_0 is a depot (base station), $U \subseteq V$ is the subset of vertices that must be visited ($v_0 \in U$), and W is the set of targets to cover of size p and indexed as $w_1, \dots, w_p$. Each element of V and W have a location given by their x and y coordinates. A distance matrix $C = (c_{ij})$ provides the edge length for each element in E . A final parameter is c , the pre-defined maximum size of the cover. A solution to the mCTP consists in defining a set of m vehicle routes of minimum total length, all starting and ending at the depot such that every target in W is covered. Target coverage is satisfied if it lies within distance c from a vertex in V in a tour route. Each vehicle uniquely visits a selected vertex within its route but vertices may overlap among the individual vehicle routes. Targets need only be covered one time although multiple coverage is permitted.

2.3 CTP for UGV Coverage

One defined MONARC scenario relates to the protection of KINs from potential adversarial intrusions. A team of UGVs are tasked to protect a critical installation. Their surveillance capabilities are augmented by static sensors located throughout the installation. An RGSP is available to a mission planner to assist in finding UGV tour routes. The UGVs should only patrol routes that cover all the required checkpoints and the overall route length should be minimized. All UGVs originate from a base station, the depot. There are certain checkpoints that the UGVs must visit (these checkpoints are usually critical ones requiring compulsory surveillance) and there are also checkpoints that may be visited. There are also potential spots where the adversary may appear and these spots must be covered by visiting a checkpoint that is within a fixed proximity distance. Each checkpoint is visited by a UGV and all UGVs return to the base station.

Coverage of targets by a UGV at a visited checkpoint (vertex) is defined as the circular area of a fixed radius, where any target within the area is covered by that checkpoint. The circular area of coverage is analogous to the effective range of a weapon or sensor system onboard the UGVs. Each UGV is modeled as an individual vehicle traveling on different routes of minimum-length tours. During the route, the UGV covers targets when at the vertices and all targets must be covered for there to be a feasible solution to the overall problem.

There are some key assumptions and limitations made in modeling the base defense security scenario as an mCTP:

1. All UGVs are homogeneous and have equal capabilities in movement and coverage.
2. UGVs can uniquely visit as many vertices as needed and transit as long as required.
3. UGVs travel in a straight line between vertices.
4. Potential adversarial spots are known at a specific point of time or are pre-defined and are thus part of the problem structure.

In reality, UGVs, or for that matter any sensor craft, can sense while traveling. Thus, coverage only at vertices is artificially limiting and coverage while in transit between vertices is reasonable. The CTP model is thus extended to include target coverage via traveled edges. This new variant of the CTP for a generic base security defense scenario, which considers *coverage by both visited vertices and traveled edges*, is described in the next section. The extension of the VCTP to an mVCTP (multiple vehicle Vigilant Covering Tour Problem) is discussed in the subsequent section.

2.4 The In-Transit Vigilant Covering Tour Problem

The CTP can be used to model a UGV assigned to protect a critical installation. However, a scenario may exist in which a potential adversarial spot is not covered by any vertex. In this case, the CTP is infeasible. Figure 2.1 illustrates a single vehicle example which is infeasible.

The tour in Figure 2.1 is a minimum-length tour constructed with all required vertices visited. However, the solution is infeasible since there is an uncovered target. We note the UGV could sense the target while in transit. The CTP model is modified to allow coverage of such targets by the en route UGV.

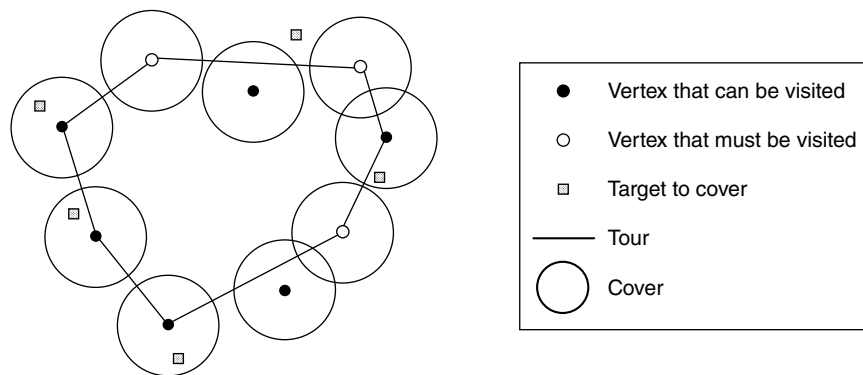


Figure 2.1 Possible solution for a CTP. *Source:* Tan, Huang Teng; Hill, Raymond R., “The In-Transit Vigilant Covering Tour Problem for Routing Unmanned Ground Vehicles,” *MORS Journal*, Vol. 18, No. 4, 2013, John Wiley and Sons, LTD.

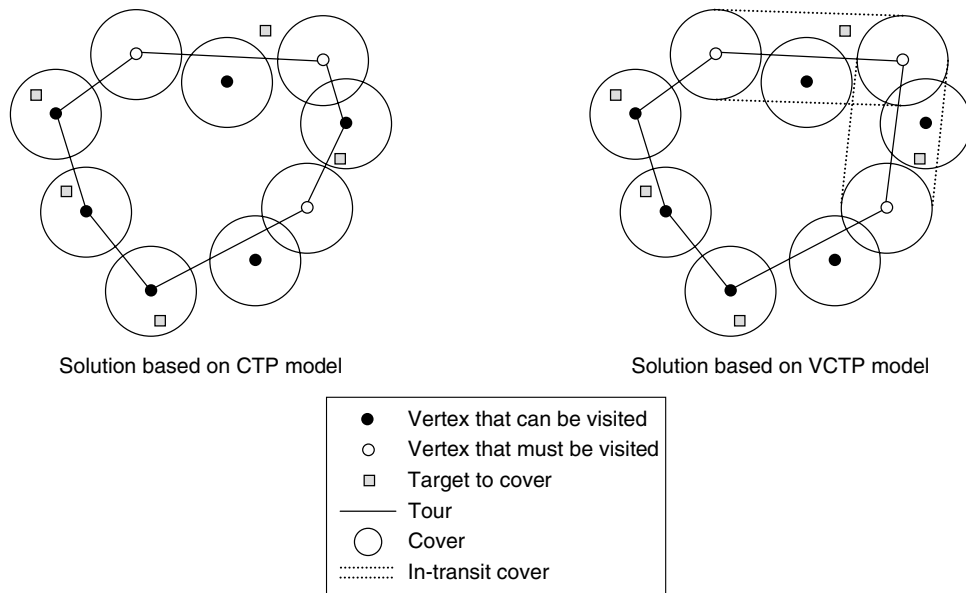


Figure 2.2 Optimal solution for a CTP and a VCTP. *Source:* Tan, Huang Teng; Hill, Raymond R., “The In-Transit Vigilant Covering Tour Problem for Routing Unmanned Ground Vehicles,” *MORS Journal*, Vol. 18, No. 4, 2013, John Wiley and Sons, LTD.

Considering coverage *during transit* is a logical assumption for a base security defense problem, that is, a UGV can cover a potential adversarial spot during its movement between checkpoints. Thus, while a UGV is traveling along the route and transiting between checkpoints, it could pass within some fixed proximity distance and detect (or cover) the adversarial spot. Figure 2.2 compares the infeasible result in Figure 2.1 with a solution based on the VCTP.

There are three distinct differences between the CTP and VCTP models. First, note that the vertex not covered in the CTP is now covered in the VCTP during a route transition with no change of route required. Thus, the revised model effectively increases the amount of coverage as both the traveled vertices and edges provide coverage. Second, we can shorten the tour length, as one of the visited vertices is not required in the VCTP tour since a tour edge provides the requisite coverage. Lastly, the solution based on the VCTP model is feasible.

As illustrated in Figure 2.2, the coverage of a target by a vertex visit changes to coverage by an edge transit, thus if the same vertices are considered, the solution of the VCTP model will involve an equal or lesser number of vertices compared to the CTP model. Thus, the optimal tour length of the VCTP model provides a lower bound for the CTP. Since target coverage is by both edge and/or vertex, the VCTP provides an upper bound for the CTP on targets covered.

2.5 Mathematical Formulation

The mathematical development of the VCTP is presented in this section. The basic Vehicle Routing Problem (VRP) model [9] was used as a basis for the VCTP model. We utilize the two-index vehicle flow formulation in the single vehicle variant of the VCTP model [10]; it is extended into the three-index vehicle flow formulation for the mVCTP. The two-index vehicle flow formulation uses $O(n^2)$ binary variables a_{ij} and $O(n)$ binary variables b_i , where a_{ij} and b_i are defined as:

$$a_{ij} = \begin{cases} 1, & \text{edge}(v_i, v_j) \text{ is part of the tour} \\ 0, & \text{otherwise} \end{cases}$$

$$b_i = \begin{cases} 1, & \text{vertex } v_i \text{ is part of the tour} \\ 0, & \text{otherwise} \end{cases}$$

An important component of the VCTP lies in the introduction of two pre-processed matrices, α_{ij}^k and β_i^k , which are defined as the in-transit edge coverage and vertex coverage matrices, respectively.

For the in-transit edge coverage matrix α_{ij}^k , for target, k , an i by j matrix is formulated to determine if edge (v_i, v_j) can provide coverage of a target. Figure 2.3 illustrates in-transit vigilant coverage of target w_k by edge (v_i, v_j) as it lies within the pre-determined perpendicular distance c from the edge.

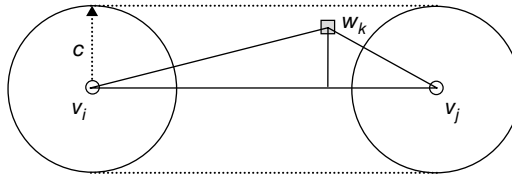


Figure 2.3 In-transit vigilant coverage by edge (v_i, v_j) on target w_k . *Source:* Tan, Huang Teng; Hill, Raymond R., “The In-Transit Vigilant Covering Tour Problem for Routing Unmanned Ground Vehicles,” *MORS Journal*, Vol. 18, No. 4, 2013, John Wiley and Sons, LTD.