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Aircraft Control Allocation

Aerospace Series
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AIRCRAFT CONTROL ALLOCATION

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Dedication

For Craig Steidle, Bob Hanley, and John Foster. Thanks guys.

Series Preface

The field of aerospace is multi-disciplinary and wide ranging, covering a large variety of products, disciplines and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce innovative and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed onto others working in the industry and also researchers, teachers and the student body in universities.

The *Aerospace Series* aims to be a practical, topical and relevant series of books aimed at people working in the aerospace industry, including engineering professionals and operators, engineers in academia, and allied professions such commercial and legal executives. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation and support of aircraft, as well as topics such as infrastructure operations and current advances in research and technology.

Modern aircraft are designed with multiple control surfaces, and possibly other control effectors e.g. thrust vectoring, and therefore problems can arise as to how to combine these control devices in an optimal manner, particularly in low-speed flight regimes where the aerodynamic surfaces lose their effectiveness.

This book, *Aircraft Control Allocation*, provides a detailed explanation of some selected topics relating to the aircraft control allocation problem. After providing some background material in aircraft control and control laws, a number of approaches that can be used to solve the control allocation problem are illustrated and the influence that they have on control law design discussed. Of particular note is the chapter describing some of the lessons learnt whilst designing the X-35 Flight Control System.

Peter Belobaba, Jonathan Cooper and Alan Seabridge

Glossary

$\dot{( )}$ Dot over quantity: the derivative with respect to time of the contents of the parentheses ( ).

$\hat{( )}$ Hat over quantity: the contents of the parentheses ( ) are approximate.

α Angle-of-attack: the aerodynamic angle between the projection of the relative wind onto the airplane's plane of symmetry and a suitably defined body fixed x-axis.

β Sideslip angle: The aerodynamic angle between the velocity vector and the airplane's plane of symmetry.

$\ell_1, \ell_2, \ell_\infty$ Vector norms: ℓ_2 is the square root of the sum of the squares of the entries in the vector. It appears everywhere. ℓ_1 is the sum of the absolute values of the entries and ℓ_∞ is the greatest absolute value. ℓ_1 and ℓ_∞ frequently appear in linear programming problems.

Ω Either:

1. Every combination of control effector deflections that are admissible; in other words, that are within the limits of travel or deflection.
2. A normally diagonal matrix used to specify the dynamics in a dynamic-inversion control law.

Φ The effects, usually body-axis moments, moment coefficients, or angular accelerations, of every combination of control effector deflections in Ω, q.v. (sense 1). Sometimes called the AMS, for 'attainable moment set' or subset.

ϕ Bank angle: one of three angles that define a 3-2-1 (z-y-x) rotation from inertial to body-fixed reference frames.

Π Either:

1. (Primarily) A subset of the attainable moments (Φ) consisting of all the moments that are generated by a particular control allocation method.
2. A plane surface that arises in Banks' method of allocation for the three-moment problem.

ψ Heading angle: one of three angles that define a 3-2-1 (z-y-x) rotation from inertial to body-fixed reference frames.

Θ A subset of Ω: all admissible controls that a particular control-allocation method can return as solutions to a control-allocation problem.

θ Pitch attitude: one of three angles that define a 3-2-1 (z-y-x) rotation from inertial to body-fixed reference frames.

B One of the matrices of the linearized equations of motion: *A* is the system matrix, *B* is control effectiveness matrix, and *C* is the output matrix.

C_{x_y} The non-dimensional stability or control derivative of x with respect to y: it is the non-dimensional form of X_y.

Comp Complementary: a superscript to certain dynamic responses.

Cont Controllable: a superscript to certain dynamic responses.

d, *des* Desired: a subscript to a dynamic response, or any other quantity.

F_B Body-fixed reference frames. The origin is at the airplane's center of mass. The axes x_B and z_B lie in the airplane's plane of symmetry. y_B completes the right-hand system. Once defined, a body-fixed reference system's orientation with respect to the body does not change. Two frequently used body-fixed reference frames are the principal axes and the stability-axis system.

F_H Local-horizontal reference frame. The axes x_H, y_H, and z_H are oriented north, east, and down, respectively. The earth is flat.

F_W Wind-axis system. The axis x_W lies in the direction of flight, opposite the relative wind. z_W is in the plane of symmetry, oriented downward.

g Either:

1. Acceleration of gravity, or
2. The non-dimensional units of load factor *n*, q.v.

I With subscripts; moment of inertia.

Kine Kinematic: a superscript to certain dynamic responses.

L, C, D Lift, side force, and drag: wind-axis forces in the x_W-, y_W- and z_W-directions, respectively.

L, M, N Body-axis moments about the x_B axis (rolling), y_B axis (pitching), and z_B axis (yawing), respectively.

L Either:

1. Lift, or
2. Rolling moment, depending on context.

LD Lateral-directional, meaning all motions, accelerations, forces, and so on, that are not longitudinal, q.v. Sometimes *lat-dir*.

Long Longitudinal, meaning all motions, accelerations, forces, and so on, that take place in the airplane's plane of symmetry. Pitching moments, velocities, and accelerations are about the airplane's y_B-axis but the motion is in the x_B–z_B plane.

m The mass of the airplane.

n Load factor, the ratio of lift to weight, $n = L/W$. Measured in *g*s.

p, q, r Body-axis roll rate, pitch rate, and yaw rate, respectively.

P A generalized inverse of a matrix *B*: $BPB = B$ and $PBP = P$, with appropriate dimensions.

Ref Subscript, 'evaluated in reference conditions'.

$\mathbf{u}$ Vector of control effector variables.

$\mathbf{u}_{\min}, \mathbf{u}_{\max}$ Vector of control effector limits, minimum or maximum.

$\mathbf{u}_l, \mathbf{u}_u$ Vector of control effector limits, lower or upper. This notation seems preferred by linear programmers over $\mathbf{u}_{\min}, \mathbf{u}_{\max}$, q.v.

x_B, y_B, z_B Names of body-axes.

W A weighting matrix, generally diagonal and positive.

x_W, y_W, z_W Names of wind axes.

X_y Where X is a force or moment and y is a state or control, a dimensional derivative, $\partial X/\partial y$. It is the dimensional form of C_{x_y}, q.v. The definition does *not* include division by mass or moment of inertia. If y is a control effector the result is called a control derivative, otherwise it is called a stability derivative.

X, Y, Z Body-axis forces in the x-, y- and z-directions, respectively.

x, y, z Names of axes. With no subscripts usually taken to be body-axes.

ACTIVE Advanced Control Technology for Integrated Vehicles. A research F-15 with differential canards, axisymmetric thrust vectoring, and other novel features.

ADMIRE Aero-Data Model In a Research Environment, simulation code. See Appendix B.

Admissible Of a control effector or suite of control effectors, those deflections that are within the physical limits of employment.

AMS Attainable moment subset or set, Φ.

Angular accelerations See Objectives.

ARI Aileron-rudder interconnect. Normally used to reduce adverse yaw due to aileron deflection.

Attainable Of moments or accelerations; that which can be generated by some admissible combination of control effectors. The term may be applied globally, meaning there is some theoretical combination, or locally, to a particular control allocation method, meaning those combinations of control effectors that the method will generate using its rules.

Basic feasible solution Of linear programs, a basic solution to the equality constraints in a linear program that also solves the inequality constraints.

Basic solution Of linear programs, a solution to the l linear equality constraints of a linear program in 'standard form' with $k - l$ of the decision variables at their bound.

CAS Control augmentation system.

Control effectiveness A measure of the effect of utilizing a control effector, either moment, moment coefficient, or angular acceleration.

Control authority The aggregate effect of the effectiveness of all the control effectors in whatever combination.

Control power Angular acceleration per unit of control deflection.

CHR Cooper–Harper rating; sometimes HQR.

Constraint Of a control effector, a limiting position, usually imposed by the hardware. It may also refer to a limit on the rate of travel. In linear programming, a constraint may refer to the position limits, but also of an equality that must be satisfied. Thus $\mathbf{u} \leq \mathbf{u}_{\max}$ is an inequality constraint, and $B\mathbf{u} = \mathbf{m}_{des}$ is an equality constraint.

Control effector The devices that directly effect control by changing forces or moments, such as ailerons or rudders. When we say 'the controls' with no qualification, we usually mean the control effectors. The sign convention for conventional flapping control effectors follows a right-hand rule, with the thumb along the axis about which the effector is designed to generate moments, and the curled fingers denoting the positive deflection of the trailing edge.

Control inceptor Cockpit devices that control, through direct linkage or a flight-control system or computer, the control effectors. Positive control inceptor deflections correspond to positive deflections of the effectors they are connected to, barring such things as aileron–rudder interconnects (ARI, q.v.).

Cycling Of a linear program, a condition in which a sequence of vertices is visited by a solver for which the objective function does not decrease, eventually returning to the starting point in the cycle. Cycling represents a failure to converge and must be addressed by choosing an exchange rule designed to prevent it.

Degenerate basic solution Of linear programs, a basic solution to a linear program in which one of the l decision variables in the basis is at its bound in addition to the non-basic variables.

Decision variables The set of unknown parameters being optimized in a linear program.

FBW Fly by wire. The pilot flies the computer, the computer flies the airplane.

FQ Flying qualities.

Ganged Said of mechanical devices that are linked so that they move in fixed relation to each other, such as ailerons.

HARV High angle-of-attack research vehicle.

HQ Handling qualities.

HQR Handling qualities rating.

Interior point method One of a family of numerical methods that seek to find the optimal solution to a linear program by moving through the interior of the feasible set.

Intersection Of two objects (q.v.), an object that is wholly contained in each of the two.

Lat-Dir Lateral-directional.

LEU, LED Leading-edge up, down. Terms used to describe the deflection of leading-edge control surfaces.

Linear programming A problem, or the method of solving that problem, of optimization of an objective subject to linear equality and inequality constraints. To the purpose of this book, a method of allocating controls subject to position constraints.

Moments See Objectives.

Moment coefficients See Objectives.

Object A generalization of any of the several polytopes that describe sets of admissible controls and attainable moments.

Objectives Those which control effectors are intended to generate. Originally control allocation sought to find the control effectors that generated specified moments, or moment coefficients. Subsequently researchers have tended toward using angular accelerations as the objectives. We will generally speak of the objectives as being moments.

Object notation A method of identifying objects (q.v.) using a 0 for a control at its lower limit, a 1 at its upper limit, and a 2 if it can be anywhere in between.

OBM On-board model. A set of aerodynamic data for an aircraft stored in the aircraft's flight control computer.

Over-actuated control system See Redundant controls.

Phase one/two program Phase one of a linear programming solver solves a modified problem in order to locate an initial feasible solution for the phase two solver that will optimize the original problem.

PIO Pilot-induced oscillation. There's a more politically correct term that removes the onus from the pilot.

PR Pilot rating; sometimes HQR, q.v.

Preferred Of a solution to the control allocation problem, a control effector configuration that is as close as possible to to one that is preferred. Minimum norm solutions are used as preferred solutions often.

Pseudo control A combination of control effectors intended to create a certain effect, such as the excitation of a particular dynamic response mode of the airplane.

Redundant controls Control effectors are seldom *redundant*, in the sense that the designer had no use for them in mind. The control effectors that are redundant in higher-speed flight may be critical in slow-speed flight. The term just means that there are more control effectors than objectives, q.v. As used in this book, it means there are more than three control effectors to generate the three moments or angular accelerations.

SAS Stability augmentation system.

Simplex An extension of a triangle (two-dimensional), or tetrahedron (three-dimensional), to an arbitrary number of dimensions. An n-dimensional simplex is defined by the convex hull of $n + 1$ vertices.

Simplex method Either:

1. (Dantzig) Algorithms based on Dantzig's original numerical algorithm for the solution of linear programs, introduced in 1947. The simplex method moves between neighboring vertices, basic solutions, of the feasible set, decreasing the cost function until the optimum is found.
2. (Nelder Mead) Also known as downhill simplex. Numerical solution algorithm that iterates an n-dimensional simplex to minimize n-dimensional, non-linear, unconstrained optimization problems. Heuristic rules at each step govern how to modify the simplex.

Slack variable Variables augmenting the decision variables in a linear program so that inequality constraints can be converted to equality constraints.

TEU, TED, TEL, TER   Trailing-edge up, down, left, right. Terms used to describe the deflection of flapping control surfaces.

Union   Of two objects (q.v.), the smallest object of which the two given objects are both members.

Warm start   A heuristic method for initializing a linear program solver given a pre-existing optimal solution to a similar problem.

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1

Introduction

The general theme of the book is to reproduce the research and insights that led the authors through their seminal studies into airplane control allocation. There is much research remaining to be done in the field of control allocation, and by following the thinking that preceded the fruitful directions taken by other researchers, new areas of inquiry will be opened.

The authors defend their geometrical approach to visualizing the problem as one that provides greater insight into the mechanisms of the methods of solution that exist or may be contemplated. This is particularly true when considering the processes of reconfiguring the controls following the identification of a failure.

It is emphasized that the primary interest of the authors and the focus of the book is airplanes. Thus, we stick to a relatively small number objectives in the allocation problem, corresponding primarily to the three rotational degrees-of-freedom of airplanes and secondarily to the linear degrees-of-freedom. We acknowledge that there are many other fields that have similar problems, and believe our research lays a sound basis for other researchers to modify our results to apply them to their particular interests.

With respect to rigorous mathematical proofs, none will be found here. The authors are not mathematicians, as will be readily confirmed by any real mathematician who picks up this book. We certainly never thought before embarking on this research that ‘null space’ and ‘airplane’ would ever be used in the same sentence. We typically began by sketching a two-dimensional figure on the blackboard, something that seemed ‘intuitively obvious’ to us, and wondering if that figure generalized to higher dimensions.

Most important results have been proved in other sources: the many technical papers, theses, and dissertations that arose from our research, or in textbooks, particularly books that deal with linear algebra. Many of these publications are presented in Appendix C. Here we will just make claims that we are pretty sure are true. For instance, rather than prove that convexity is preserved under the mappings we describe, we will just assert it and perhaps give compelling evidence of its truth.

1.1 Redundant Control Effectors

The origins of our research into airplane control allocation lay in earlier research into model-following and dynamic-inversion control laws. The nature of model-following and

Aircraft Control Allocation, First Edition. Wayne Durham, Kenneth A. Bordignon and Roger Beck.

Companion website: www.wiley.com/go/durham/aircraft_control_allocation

dynamic-inversion algorithms is such that one is required to find a vector of control effector deflections that yield a desired moment, force, or acceleration. With three moments and three controls, the answer for a linear problem is a trivial matrix inversion. The physical limits of the control effectors does not affect the solution since the solution is unique. That is, for an airplane with ganged ailerons, a rudder, and an elevator, the combination of these effectors that will generate a specific moment vector is unique; if one or more saturates then the problem is not in the math but in the hardware.

Early problems arose when considering an airplane whose horizontal tails were not ganged to generate pitching moments only, but that could be displaced differentially as well to generate rolling moments and, unintentionally, yawing moments. By considering the left and right horizontal tails as independent we now have four control effectors for the three components of the moment vector to be generated. The linearized control effectiveness matrix (to be defined in Eq. (2.20)) is no longer square, but has three rows and four columns.

Figure 1.1 depicts a variety of control effector types. The airplane is USAF S/N 71-0290, the F-15 ACTIVE (Advanced Control Technology for Integrated Vehicles). The canards and horizontal tails are all-moving surfaces. The two vertical stabilizers have hinged rudders at their trailing edges. The wings have trailing-edge ailerons and flaps. Finally, both engines have axisymmetric thrust vectoring capabilities. Each of these various control effectors is capable of independent action.

Redundant control effectors are employed to extend an airplane's performance envelope, typically in the low-speed regime. Thrust vectoring generates moments long after conventional

Figure 1.1 F-15 ACTIVE

flapping control surfaces have lost effectiveness at low dynamic pressure. Thrust vectoring enhances the dog-fighting potential of the F-22 Raptor, and permits maneuvers such as Pugachev's Cobra to be performed in other aircraft.

Clever control allocation is not needed in high-speed flight, where more than ample forces and moments can be generated with small effector deflections. In low-speed flight aerodynamic effectors lose effectiveness and must be combined with other effectors (aerodynamic or propulsive). However, if there are more control effectors than moments or accelerations to be generated, methods of allocating these controls are needed.

We are now faced with a 'wide' control effectiveness matrix: more columns than rows. As we will see there is a simple mathematical way to 'invert' such matrices. The real problem arises when the physical limitations of the control effectors are considered. In other words, control effectors have hard deflection limits that cannot be exceeded. When simple mathematical solutions to the problem are used, it is possible for one or more effectors to be *unnecessarily* commanded past its limits, meaning that it will saturate. When a control effector is saturated, the assumptions on which the flight control system was based are no longer valid.

1.2 Overview

We will begin by discussing aircraft flight dynamics and control. This will consist of a very brief overview of flight dynamics and its nomenclature, offered to provide the reader with explanations for some of the terms used subsequently.

Next we will spend some time describing dynamic inversion control. This form of control lends itself naturally to the control allocation problem as we have posed it. We will briefly discuss 'conventional' control, and even more briefly mention model-following control. All three forms of control law determination have some need of control allocation.

After formulating the problem, we address the geometry of control allocation. We do this first considering two-moment problems. Two-moment problems have application in aircraft control, since often the lateral-directional problem (rolling and yawing) is treated separately from the longitudinal problem (pitching). Moreover, it is much easier to make figures on the page of two-dimensional objects than of three-dimensional ones.

The geometry of the three-moment problem is a natural extension of that of the two-moment problem, and it is discussed in detail. For each of the two- and three-moment problems a metric is offered that permits comparison of different control allocation methods for their effectiveness in solving the problem. This gives rise to the idea of a 'maximum set' of moments that can be generated using different control allocation schemes, and its importance is discussed.

A large section on solution methods follows. We explore all the allocation methods of which the authors have first-hand experience, and most are accompanied by numerical illustrations. One of the control allocation methods—linear programming—is briefly discussed. Because of the current interest among researchers in the subject of linear programming solutions, there is a separate section (Appendix A) that further explores linear programming in greater detail.

All the preceding has been based on a global problem: the total set of control deflections that yield the whole moment vector. Now we turn our attention to a local problem. Digital flight control computers solve the allocation problem scores or even hundreds of times a second. We look within one frame of the computer's operation and consider not just how far an effector can move, but how fast. This permits us to incorporate rate limits into the problem. This framewise allocation comes with a serious drawback sometimes called 'windup'. The remedy to windup is not hard and comes with some beneficial side effects.

Next we briefly explore control allocation and flight control system design. Example designs are given for a roll-rate command, pitch-rate command, and a sideslip controller. Finally, the consequences of using a non-optimal control allocation method are graphically displayed.

At the end of the text there is a chapter on some of the real-life applications of the previously described research. Lessons learned from the design of the X-35 control system are presented.

Throughout the book we occasionally make reference to simulation code. The MATLAB®/Simulink® based code is available at a companion website to this book, and it is fully explained in Appendix B. The simulation code offers different modules that implement the various control allocation methods described in the book. Readers are free to adapt and use this code to further explore the concepts of control allocation.

We feel that a common simulation source is essential to creating reproducible results. Many technical papers present simulation results with insufficient information about that simulation for the reader or reviewer to reproduce them. There are many assumptions inherent in one researcher's simulation code that cannot be conveyed in a brief paper but that may greatly affect the results one obtains.[1] The simulation code we provide came to us courtesy of the Swedish Defence Research Agency with their permissions. MATLAB®/Simulink® code is available in student and academic editions and should be very widely accessible.

The final appendix is an annotated bibliography, in which we clean out our files of control allocation and dynamic inversion papers and list them, along with their abstracts or other descriptive material. This appendix is a good place to look to see if anyone is pursuing interests related to yours. It is inevitable that some have been overlooked, either through our inattention or the fault of some search engine or other. Our loosely-enforced cut-off criterion for inclusion in this list was that the source be refereed. Conference papers were generally not included unless the material was unique and relevant.

Finally, we wish to emphasize that the content of this book reflects only topics with which the authors are personally familiar. It is not a survey of the control allocation literature. There are many very good and sound areas of control allocation research that we have not addressed, except as indicated in the annotated bibliography (Appendix C). There one will find works by Marc Bodson, Jim Buffington, Dave Doman, Dale Enns, Tony Page, and many others with whom the authors have had collegial exchanges.

In particular we appreciate a group familiarly known as 'Bull studs': John Bolling, Josh Durham, Michelle Glaze, Bob Grogan, Matt Hederstrom, Jeff Leedy, Bruce Munro, Mark Nelson, Tony Page, Kevin Scalera, and the others who toiled in the 'Sim Lab' to help make sense of all this. And lastly Fred Lutze, who pondered the various stages of our progress and occasionally said, 'You *could* do that, but it would be wrong.'

[1] For example, whether one begins the calculations in a given frame assuming that the last commanded controls have been achieved, or using the actual deflections that resulted. It can make a big difference. Most of what this statement means will be made clear in Chapter 7. See Bodson and Pohlchuck (1998) for some more insight.

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