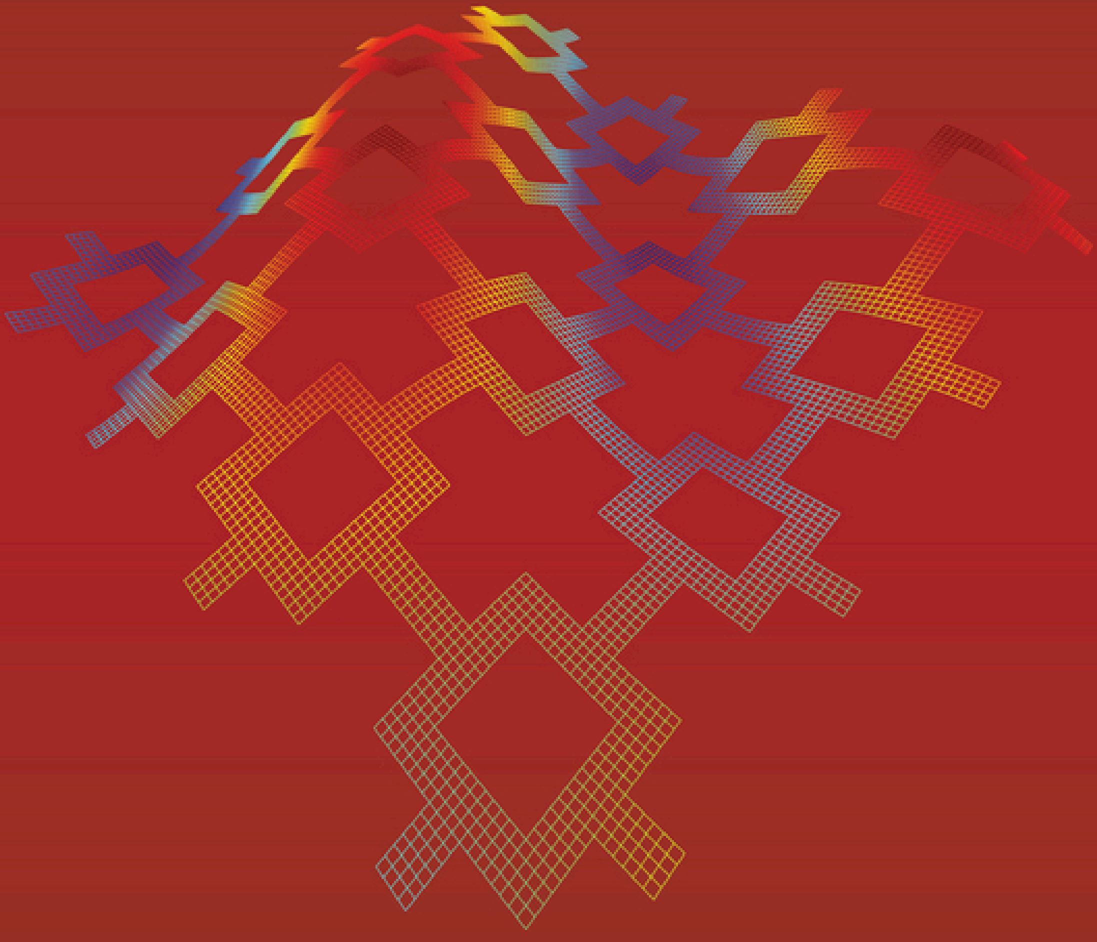
DYNAMICS OF LATTICE MATERIALS



Editors
A. Srikantha Phani | Mahmoud I. Hussein

WILEY

Dynamics of Lattice Materials

Dynamics of Lattice Materials

Edited by

A. Srikantha Phani

University of British Columbia, Canada

Mahmoud I. Hussein

University of Colorado Boulder, USA

WILEY

This edition first published 2017

Registered Office(s)
John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

Editorial Office
The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

Library of Congress Cataloging-in-Publication Data

Names: Phani, A. Srikantha, editor. | Hussein, Mahmoud I., editor.
Title: Dynamics of lattice materials / [edited by] A. Srikantha Phani, Mahmoud I. Hussein.
Description: Chichester, West Sussex, United Kingdom : John Wiley & Sons, Inc., 2017. | Includes bibliographical references and index.
Identifiers: LCCN 2016042860 | ISBN 9781118729595 (cloth) | ISBN 9781118729571 (epub) | ISBN 9781118729564 (Adobe PDF)
Subjects: LCSH: Lattice dynamics.
Classification: LCC QC176.8.L3 D85 2017 | DDC 530.4/11–dc23 LC record available at https://lccn.loc.gov/2016042860

Cover design by Wiley
Cover image: Courtesy of the author

Set in 10/12pt Warnock by SPi Global, Chennai, India

10 9 8 7 6 5 4 3 2 1

To Ananya and Krishna
To Alaa and Ismail

Contents

List of Contributors *xiii*
Foreword *xv*
Preface *xxv*

1 Introduction to Lattice Materials *1*
A. Srikantha Phani and Mahmoud I. Hussein
1.1 Introduction *1*
1.2 Lattice Materials and Structures *2*
1.2.1 Material versus Structure *3*
1.2.2 Motivation *3*
1.2.3 Classification of Lattices and Maxwell's Rule *4*
1.2.4 Manufacturing Methods *6*
1.2.5 Applications *7*
1.3 Overview of Chapters *8*
Acknowledgment *10*
References *10*

2 Elastostatics of Lattice Materials *19*
D. Pasini and S. Arabnejad
2.1 Introduction *19*
2.2 The RVE *21*
2.3 Surface Average Approach *22*
2.4 Volume Average Approach *25*
2.5 Force-based Approach *25*
2.6 Asymptotic Homogenization Method *26*
2.7 Generalized Continuum Theory *29*
2.8 Homogenization via Bloch Wave Analysis and the Cauchy–Born Hypothesis *32*
2.9 Multiscale Matrix-based Computational Technique *34*
2.10 Homogenization based on the Equation of Motion *36*
2.11 Case Study: Property Predictions for a Hexagonal Lattice *38*
2.12 Conclusions *42*
References *43*

3 **Elastodynamics of Lattice Materials** *53*
A. Srikantha Phani
3.1 Introduction *53*
3.2 One-dimensional Lattices *55*
3.2.1 Bloch's Theorem *57*
3.2.2 Application of Bloch's Theorem *59*
3.2.3 Dispersion Curves and Unit-cell Resonances *59*
3.2.4 Continuous Lattices: Local Resonance and sub-Bragg Band Gaps *61*
3.2.5 Dispersion Curves of a Beam Lattice *62*
3.2.6 Receptance Method *64*
3.2.7 Synopsis of 1D Lattices *67*
3.3 Two-dimensional Lattice Materials *67*
3.3.1 Application of Bloch's Theorem to 2D Lattices *67*
3.3.2 Discrete Square Lattice *70*
3.4 Lattice Materials *72*
3.4.1 Finite Element Modelling of the Unit Cell *75*
3.4.2 Band Structure of Lattice Topologies *77*
3.4.3 Directionality of Wave Propagation *84*
3.5 Tunneling and Evanescent Waves *85*
3.6 Concluding Remarks *87*
3.7 Acknowledgments *87*
References *87*

4 **Wave Propagation in Damped Lattice Materials** *93*
Dimitri Krattiger, A. Srikantha Phani and Mahmoud I. Hussein
4.1 Introduction *93*
4.2 One-dimensional Mass–Spring–Damper Model *95*
4.2.1 1D Model Description *95*
4.2.2 Free-wave Solution *96*
State-space Wave Calculation *97*
Bloch–Rayleigh Perturbation Method *97*
4.2.3 Driven-wave Solution *98*
4.2.4 1D Damped Band Structures *98*
4.3 Two-dimensional Plate–Plate Lattice Model *99*
4.3.1 2D Model Description *99*
4.3.2 Extension of Driven-wave Calculations to 2D Domains *100*
4.3.3 2D Damped Band Structures *101*
References *104*

5 **Wave Propagation in Nonlinear Lattice Materials** *107*
Kevin L. Manktelow, Massimo Ruzzene and Michael J. Leamy
5.1 Overview *107*
5.2 Weakly Nonlinear Dispersion Analysis *108*
5.3 Application to a 1D Monoatomic Chain *114*
5.3.1 Overview *114*
5.3.2 Model Description and Nonlinear Governing Equation *114*

5.3.3 Single-wave Dispersion Analysis *115*
5.3.4 Multi-wave Dispersion Analysis *116*
Case 1. General Wave–Wave Interactions *117*
Case 2. Long-wavelength Limit Wave–Wave Interactions *119*
5.3.5 Numerical Verification and Discussion *122*
5.4 Application to a 2D Monoatomic Lattice *123*
5.4.1 Overview *123*
5.4.2 Model Description and Nonlinear Governing Equation *124*
5.4.3 Multiple-scale Perturbation Analysis *125*
5.4.4 Analysis of Predicted Dispersion Shifts *127*
5.4.5 Numerical Simulation Validation Cases *129*
Analysis Method *130*
Orthogonal and Oblique Interaction *131*
5.4.6 Application: Amplitude-tunable Focusing *133*
Summary *134*
Acknowledgements *135*
References *135*

6 Stability of Lattice Materials *139*
Filippo Casadei, Pai Wang and Katia Bertoldi
6.1 Introduction *139*
6.2 Geometry, Material, and Loading Conditions *140*
6.3 Stability of Finite-sized Specimens *141*
6.4 Stability of Infinite Periodic Specimens *142*
6.4.1 Microscopic Instability *142*
6.5 Post-buckling Analysis *145*
6.6 Effect of Buckling and Large Deformation on the Propagation Of Elastic Waves *146*
6.7 Conclusions *150*
References *151*

7 Impact and Blast Response of Lattice Materials *155*
Matthew Smith, Wesley J. Cantwell and Zhongwei Guan
7.1 Introduction *155*
7.2 Literature Review *155*
7.2.1 Dynamic Response of Cellular Structures *155*
7.2.2 Shock- and Blast-loading Responses of Cellular Structures *157*
7.2.3 Dynamic Indentation Performance of Cellular Structures *158*
7.3 Manufacturing Process *159*
7.3.1 The Selective Laser Melting Technique *159*
7.3.2 Sandwich Panel Manufacture *160*
7.4 Dynamic and Blast Loading of Lattice Materials *161*
7.4.1 Experimental Method – Drop-hammer Impact Tests *161*
7.4.2 Experimental Method – Blast Tests on Lattice Cubes *162*
7.4.3 Experimental Method – Blast Tests on Composite-lattice Sandwich Structures *163*

7.5 Results and Discussion *165*
7.5.1 Drop-hammer Impact Tests *165*
7.5.2 Blast Tests on the Lattice Structures *167*
7.5.3 Blast Tests on the Sandwich Panels *170*
Concluding Remarks *173*
Acknowledgements *174*
References *174*

8 Pentamode Lattice Structures *179*
Andrew N. Norris
8.1 Introduction *179*
8.2 Pentamode Materials *183*
8.2.1 General Properties *183*
8.2.2 Small Rigidity and Poisson's Ratio of a PM *185*
8.2.3 Wave Motion in a PM *186*
8.3 Lattice Models for PM *187*
8.3.1 Effective PM Properties of 2D and 3D Lattices *187*
8.3.2 Transversely Isotropic PM Lattice *188*
Effective Moduli: 2D *190*
8.4 Quasi-static Pentamode Properties of a Lattice in 2D and 3D *192*
8.4.1 General Formulation with Rigidity *192*
8.4.2 Pentamode Limit *194*
8.4.3 Two-dimensional Results for Finite Rigidity *195*
8.5 Conclusion *195*
Acknowledgements *196*
References *196*

9 Modal Reduction of Lattice Material Models *199*
Dimitri Krattiger and Mahmoud I. Hussein
9.1 Introduction *199*
9.2 Plate Model *200*
9.2.1 Mindlin–Reissner Plate Finite Elements *200*
9.2.2 Bloch Boundary Conditions *202*
9.2.3 Example Model *203*
9.3 Reduced Bloch Mode Expansion *204*
9.3.1 RBME Formulation *204*
9.3.2 RBME Example *205*
9.3.3 RBME Additional Considerations *207*
9.4 Bloch Mode Synthesis *208*
9.4.1 BMS Formulation *208*
9.4.2 BMS Example *210*
9.4.3 BMS Additional Considerations *210*
9.5 Comparison of RBME and BMS *212*
9.5.1 Model Size *212*
9.5.2 Computational Efficiency *213*
9.5.3 Ease of Implementation *214*
References *214*

10 Topology Optimization of Lattice Materials *217*
Osama R. Bilal and Mahmoud I. Hussein
10.1 Introduction *217*
10.2 Unit-cell Optimization *218*
10.2.1 Parametric, Shape, and Topology Optimization *218*
10.2.2 Selection of Studies from the Literature *218*
10.2.3 Design Search Space *219*
10.3 Plate-based Lattice Material Unit Cell *220*
10.3.1 Equation of Motion and FE Model *221*
10.3.2 Mathematical Formulation *222*
10.4 Genetic Algorithm *223*
10.4.1 Objective Function *223*
10.4.2 Fitness Function *224*
10.4.3 Selection *224*
10.4.4 Reproduction *224*
10.4.5 Initialization and Termination *225*
10.4.6 Implementation *225*
10.5 Appendix *226*
References *228*

11 Dynamics of Locally Resonant and Inertially Amplified Lattice Materials *233*
Cetin Yilmaz and Gregory M. Hulbert
11.1 Introduction *233*
11.2 Locally Resonant Lattice Materials *234*
11.2.1 1D Locally Resonant Lattices *234*
11.2.2 2D Locally Resonant Lattices *241*
11.2.3 3D Locally Resonant Lattices *243*
11.3 Inertially Amplified Lattice Materials *246*
11.3.1 1D Inertially Amplified Lattices *246*
11.3.2 2D Inertially Amplified Lattices *248*
11.3.3 3D Inertially Amplified Lattices *253*
11.4 Conclusions *255*
References *256*

12 Dynamics of Nanolattices: Polymer-Nanometal Lattices *259*
Craig A. Steeves, Glenn D. Hibbard, Manan Arya, and Ante T. Lausic
12.1 Introduction *259*
12.2 Fabrication *259*
12.2.1 Case Study *262*
12.3 Lattice Dynamics *263*
12.3.1 Lattice Properties *264*
Geometries of 3D Lattices *264*
Effective Material Properties of Nanometal-coated Polymer Lattices *265*
12.3.2 Finite-element Model *266*
Displacement Field *266*
Kinetic Energy *268*

Strain Potential Energy 269
Collected Equation of Motion 270
12.3.3 Floquet–Bloch Principles 271
Generalized Forces in Bloch Analysis 272
Reduced Equation of Motion 274
12.3.4 Dispersion Curves for the Octet Lattice 275
12.3.5 Lattice Tuning 277
Bandgap Placement 277
Lattice Optimization 277
12.4 Conclusions 278
12.5 Appendix: Shape Functions for a Timoshenko Beam with Six Nodal Degrees of Freedom 279
References 280

Index 283

List of Contributors

M. Arya
University of Toronto
Ontario
Canada

S. Arabnejad
McGill University
Montreal
Quebec
Canada

K. Bertoldi
Harvard University
Cambridge
Massachusetts
USA

O.R. Bilal
University of Colorado Boulder
Colorado
USA

W.J. Cantwell
Khalifa University of Science
Technology and Research
Abu Dhabi
UAE

F. Casadei
Harvard University
Cambridge
Massachusetts
USA

Z.W. Guan
University of Liverpool
UK

G. Hibbard
University of Toronto
Ontario
Canada

G.M. Hulbert
University of Michigan
Ann Arbor
USA

M.I. Hussein
University of Colorado Boulder
Colorado
USA

D. Krattiger
University of Colorado Boulder
Colorado
USA

A.T. Lausic
University of Toronto
Ontario
Canada

M.J. Leamy
Georgia Institute of Technology
Atlanta
USA

K. Manktelow
Georgia Institute of Technology
Atlanta
USA

A.N. Norris
Rutgers University
Piscataway
New Jersey
USA

D. Pasini
McGill University
Montreal
Quebec
Canada

M. Ruzzene
Georgia Institute of Technology
Atlanta
USA

M. Smith
University of Sheffield
Rotherham
UK

A.S. Phani
University of British Columbia
Vancouver
Canada

C.A. Steeves
University of Toronto
Ontario
Canada

C. Yilmaz
Bogazici University
Istanbul
Turkey

P. Wang
Harvard University
Cambridge
Massachusetts
USA

Foreword

When Srikantha Phani and Mahmoud Hussein asked me if I would write a foreword to their book, I was at first a bit hesitant as I was very busy working on a new book (*Extending the Theory of Composites to Other Areas of Science*, now submitted for publication with chapters coauthored by Maxence Cassier, Ornella Mattei, Mordehai Milgrom, Aaron Welters and myself). But then, when I saw the high quality of the chapters submitted by various people, I was happy to agree.

In 1928 the doctoral thesis of Felix Bloch established the quantum theory of solids, using Bloch waves to describe the electrons. Following this, in 1931–1932, Alan Herries Wilson explained how energy bands of electrons can make a material a conductor, a semiconductor or an insulator. Subsequently there was a tremendous effort directed towards calculating the electronic properties of crystals by calculating their band structure; that is, through solving Schrödinger's equation in a periodic system. So it is rather surprising that it took until the late 1980s for similar calculations to be done for wave equations in man-made periodic structures (with the exception of the layered materials that Lord Rayleigh in 1887 had shown exhibited a band gap). Subsequently there was exponentially growing interest in the subject, as illustrated by the graph in the extensive "Photonic and sonic band gap and metamaterial bibliography" of Jonathan Dowling [1], which he maintained until 2008. Now there seems to be a similar migration of ideas from people who have studied topological insulators in the context of the quantum 2D Hall effect to the study of similar effects in man-made periodic structures where there is some time-symmetry breaking. In the context of elasticity this time-symmetry breaking can be achieved with gyroscopic metamaterials [2] and, most significantly, waves can only travel in one direction around the boundary.

This book sheds light on the dynamics of lattice materials from different perspectives. As I read through it, connections with other work (sometimes mine) came to mind. I suspect this is probably a reflection of my background, as the writer (or writers) may have been exposed to different schools of thought than myself, but I believe the cross-pollination of ideas is always beneficial to the advancement of science. Therefore I hope the collection of remarks I have made here will lead the reader (if they have the time to explore the references I have given) on some excursions of the mind that complement those provided by the authors of the individual chapters.

Chapter 1 provides the setting for the book, giving a brief but excellent introduction to lattice materials. Maxwell's rule for determining the stiffness of a structure is discussed and, as the authors mention, Maxwell realized this is only a necessary condition for a structure to be stiff. The exact condition is nontrivial to determine, but in a 2D structure one can play the "pebble game" to resolve the question [3]. Maxwell's counting rule has been generalized for periodic lattices [4]. Perhaps there is a generalization (maybe an obvious one) of the "pebble game" to periodic 2D lattices, but I have not fully explored the literature. The possible motions of kinematically indeterminant periodic arrays of rigid rods with flexible joints are of considerable interest to me, and the case in which the macroscopic motions are affine is described in the literature [5, 6], and references therein.

Pasini and Arabnejad's Chapter 2 provides an excellent survey of homogenization methods for the elastostatics of lattice materials. This is still very much an active area of research. It is an important one, because not only do the homogenized equations govern the macroscopic response but also, as emphasized by Pasini and Arabnejad, the solution of the so-called cell-problem (that is needed to calculate the effective moduli) can provide useful estimates of the maximum fields in the material, which are helpful in knowing if plastic yielding or cracking might occur. While Pasini and Arabnejad's review concentrates on periodic lattice materials, it is worth mentioning that, curiously, for random composites the justification of successive terms in the asymptotic expansions, such as their Eq. (2.12), requires successively higher dimensions of space [7, and references therein]; while 2D and 3D composites are those of practical interest, one may of course think of composites in higher dimensions too. Also, it is important to remember that with high-contrast linear elastic materials one can theoretically achieve almost any homogenized response compatible with the natural constraint of positivity of the elastic energy [8]: non-local interactions in the homogenized equations can be achieved with dumbbell shaped inclusions where the diameter of the bar is so small that it does not couple with the surrounding medium except in the near vicinity of the bar. These results are only in the framework of linear elasticity, because such bars can easily buckle when the dumbbell is under compression. Some beautiful examples of exotic elastic behavior, which go beyond that of Cosserat theory, are given by Seppecher, Alibert, and Dell Isola [9].

Chapter 3, by Phani, gives a great introduction to the elastodynamics of lattice materials. I especially like their use of simple mass-spring models. My coauthors and I find mass-spring models, with the addition of rigid elements, to be very helpful in explaining concepts such as negative effective mass, anisotropic mass density, and (when the springs have some viscous damping) complex effective mass density [10, 11]. In fact it is possible (with the framework of linear elasticity) to give a complete characterization of the possible dynamic responses of multiterminal mass-spring networks [12]. The presentation by Phani of the deformation modes associated with the branches in the dispersion diagram in Figures 3.13–3.17 is beautiful, and sheds a lot more light on the behavior than is contained in dispersion curves, which frequently is all most scientists present. Also, I would mention that a dramatic illustration of the directionality of wave propagation is in phonon focussing [13]. If at low temperatures one heats a crystal from below by directing a laser at a point on the surface, then the distribution of heat on the top surface (as seen by the height of liquid helium on the surface that, due to the fountain effect, flows towards the heat) has amazing patterns, due to caustics in the "slowness" surface associated with the direction of elastic wave propagation in crystals that is governed simply by the elasticity tensor of the crystal. The elastic waves carry the heat (phonons). It is worth remarking that, subsequent to pioneering work by Bensoussan, Lions, and Papanicolaou in Chapter 4 of their book [14], there has been a resurgence of interest in high-frequency homogenization at stationary points in the dispersion diagram, which may be local minima or maxima, or even saddle points [15–21]. The wave is a modulated Bloch wave and modulation satisfies appropriate effective equations. The most interesting effects occur when one has a saddle point: then the effective equation is hyperbolic and there are associated characteristic directions. One may also employ homogenization techniques for travelling waves at other points in the dispersion diagram [22–26].

Chapter 4, by Krattiger, Phani, and Hussein examines wave propagation in damped lattice materials, both for passive waves and driven waves. One rarely sees dispersion diagrams with damping, but of course for many materials damping is a significant factor. Their dispersion diagrams with driven waves (Figures 4.2 and 4.4) have an interesting and complex structure. It is interesting that some periodic materials with damping can have trivial dispersion relations, with a dispersion diagram equivalent to that of a homogeneous damped material [27, 28]: this happens when the moduli are analytic functions, not of the frequency, but of the complex variable $x_1 + ix_2$, where $i = \sqrt{-1}$ and for a 2D material x_1 and x_2 are the Cartesian spatial coordinates. Closely related materials were discovered by Horsley, Artoni, and La Rocca, who realized they would not reflect radiation incident from one side, whatever the angle of incidence [29].

In Chapter 5 by Manktelow, Ruzzene, and Leamy we encounter the exciting topic of wave propagation in nonlinear lattice materials. The study of nonlinear effect in composites is largely a wide-open area of research: there are so many interesting and novel directions that could be explored, and it is a certainty that surprises await. One surprise we found is as follows [30]. When one mixes linear conducting composites in fixed proportions, if one wants to maximize the current in the direction of the electric field then it is best to layer the materials with the layer boundaries parallel to the applied field; by contrast, in some nonlinear materials we found that the maximum current sometimes occurs when the layer interfaces are normal to the applied field. Manktelow, Ruzzene, and Leamy talk about higher harmonic generation in nonlinear materials. Anyone who has used an inexpensive green laser may be interested to know that the green light comes from frequency doubling the infrared light from a neodymium-ion oscillator as it passes through a nonlinear crystal, and this can pose a danger if the conversion is faulty because the infrared light can easily damage eyes [31].

Chapter 6 by Casadei, Wang and Bertoldi also deals with nonlinearity, but in the context of buckling creating a pattern transformation that can be used to tune the propagation of elastic waves. This is fantastic work, and in an entirely new direction. Buckling instabilities are well known in Bertoldi's group: they created the Buckliball a structured sphere that remains approximately spherical, but much reduced in size, as it buckles [32]. Much remains to be explored in this area: one especially significant result that I have found is that materials that combine a stable phase with an unstable one could have a stiffness greater than diamond in dynamic bending experiments [33]. It had been hoped that one could get stiffnesses dramatically higher than that of the components in stable static materials too [34], but this was ruled out when it was realized that the well-known elastic variational principles still hold even when some of the components are in isolation unstable (that is, they have negative elastic moduli) [35] .

I found interesting the work in Chapter 7 of Smith, Cantwell, and Guan on the impact and blast response of lattice materials. A feature of their experiments is that the stress has a plateau as the lattice structure is crumpled. This is exactly what one needs if the aim is to minimize the maximum force felt by an object colliding with the structure, subject to the constraint that the object should decelerate over a fixed distance. We recently encountered similar questions when trying to determine the optimal non-linear rope for a falling climber [36]. The answer turned out to be a "rope" with a stress plateau, like a shape memory wire (and with a big hysterisis loop to absorb the energy). It is pretty amazing to see the progress that has been made recently with impact-resistant composites: a good example is the composite metal foam of Afsaneh Rabiei, which literally obliterates bullets [37].

Pentamode materials, as discussed by Norris in Chapter 8, are a class of materials close to my heart. When we invented them, back in 1995 [38], we never dreamed they would actually be made, but that is exactly what the group of Martin Wegener did, in an amazing feat of 3D lithography [39]. Their lattice structure is similar to diamond, with a stiff double-cone structure replacing each carbon bond. This structure ensures that the tips of four double-cone structures meet at each vertex. This is the essential feature: treating the double-cone structures as struts, the tension in one determines uniquely the tension in the other three. This is simply balance of forces. Thus the structure as a whole can essentially only support one stress, but that stress can be any desired symmetric matrix if the pentamode lattice structure is appropriately tailored. Water is a bit like a pentamode, but unlike water, which can only support a hydrostatic stress, pentamodes can support any desired stress matrix, in other words, a desired mixture of shear and compression. They are the building blocks for constructing any desired elasticity matrix C_* that is positive definite. Elasticity tensors of 3D materials are actually fourth-order tensors, specifically linear maps on the space of symmetric matrices, but using a basis on the 6D space of symmetric matrices, they can be represented by a 6-by-6 matrix as is common in engineering notation. Expressing C_* in terms of its eigenvectors and eigenvalues,

$$C_* = \sum_{i=1}^{6} \lambda_i \mathrm{v}_i \otimes \mathrm{v}_i. \tag{1}$$

The idea, roughly speaking, is to find six pentamode structures, each supporting a stress represented by the vector v_i, $i = 1, 2, \dots, 6$. The stiffness of the material and the necks of the junction regions at the vertices need to be adjusted so each pentamode structure has an effective elasticity tensor close to

$$C_*^{(i)} = \lambda_i \mathrm{v}_i \otimes \mathrm{v}_i. \tag{2}$$

Then one successively superimposes all these six pentamode structures, with their lattice structures being offset to avoid collisions. Additionally, one may need to deform the structures appropriately to avoid these collisions [38], and when one does this it is necessary to readjust the stiffness of the material in the structure to maintain the value of λ_i. Then the remaining void in the structure is replaced by an extremely compliant material. Its presence is just needed for technical reasons, to ensure that the assumptions of homogenization theory are valid so that the elastic properties can be described by an effective tensor. But it is so compliant that essentially the effective elasticity tensor is just a sum of the effective elasticity tensors of the six pentamodes; in other words, the elastic interaction between the six pentamodes is neglible. In this way we arrive at a material with (approximately) the desired elasticity tensor C_*. Now, Andrew Norris and the group of Martin Wegener have become the leading experts on pentamodes and their 2D equivalents, which strictly speaking should be called bimodes. One important observation that Norris makes (see his Eq. (8.5)) is that if a pentamode is macroscopically inhomogeneous then the stress field it supports should be divergence-free in the absence of body forces such as gravitational forces. The new and important ingredient in the chapter of Norris is the analytic inclusion of bending effects, to better analyse the elements of the effective elasticity tensor.

Chapter 9, by Krattinger and Hussein, uses a reduced number of modes in a Bloch mode expansion to treat the vibration of plates within a frequency range of interest. Expanding on the ideas of structural mechanics, where one splits a structure into substructures, conducts a modal analysis on each of these, and then links the modes through interface boundary conditions, they develop a similar procedure at the unit-cell level for very efficiently calculating the band structure, which they call "Bloch mode synthesis." I very much like the word "platonic crystal" [40] – crafted after the terms photonic crystals, phononic crystals, and plasmonic crystals – which Ross McPhedran coined for such studies of the propagation of flexural waves through plates with periodic structure. The term has caught on in Australia, France, New Zealand and the UK (where Ross is a frequent visitor) but not yet in the U.S.

Chapter 10 by Bilal and Hussein deals with topology optimization of lattice materials. Their pixel-based designs remind me very much of the digital metamaterials of my colleague Rajesh Menon (also produced by topology optimization, but in the context of electromagnetism rather than elasticity), which have been incredibly successful, for example resulting in the world's smallest polarization beam-splitter [41]. The field of topology optimization has seen some amazing achievements, producing stuctures with fascinating and sometimes unexpected geometries that optimize performance in some respect. In particular, the group of Ole Sigmund in Denmark is well known for mastering this art, and recently they have used it for acoustic design [42]; the next wave of symphony halls will probably use the technique in their designs.

Chapter 11 presents work by Yilmaz and Hulbert on the dynamics of locally resonant and inertially amplified lattice materials. Nano-sized silver and gold metal spheres, that are resonant to light account for the beautiful colors of the Roman Lycurgus cup [43], and many stained glass windows gain their colors from such local resonances [44]. Resonant arrays of metallic split rings may lead to artifical magnetism [45], with the effective magnetic permeability taking negative values in appropriate frequency ranges [46]. Low-frequency spectral gaps were noticed by Zhikov [47, 48]. Negative effective mass densities, due to local resonances, were discovered in 2000 [49], although it was not until later that the experiments were correctly interpreted [50]. In periodic arrays of split cylinders, negative magnetic permeability can be related to the negative effective mass density in antiplane vibrations, due to the fact that both are governed by the Helmholtz equation [51]. The generation of band gaps through inertial amplification is nicely explained through essentially 1D models by Yilmaz and Hulbert in Section 11.3.1: the key aspect is that small macroscopic movements cause large amplitude movements of the internal masses. They then explore both 2D and 3D lattices. One would suspect that nonlinear effects could be very important in these models, even for quite small amplitudes of vibrations, although I do not know whether this has been explored.

Chapter 12 by Steeves, Hibbard, Arya, and Lausic provides an absolutely superb introduction to 3D printing, with a step-by-step explanation of the processes involved, highlighting the advantages of metal-coated polymer structures. In Figure 12.2 the improvement of adding a metal coating does not look particularly dramatic, until you realize there is a different scale (on the right-hand side of the graph), so in fact the improvement is about an order of magnitude in the tensile stress of the structure can support. Estimates for the elastic properties are obtained and the problem of optimizing the band gap to be as wide as possible, and at the desired frequencies, is discussed. There has been a lot of numerical work on optimizing band gaps. What I find most interesting is that it is possible to derive upper bounds on the width of band gaps that are sharp when the contrast between phases is low [52].

That ends my foreword, and now I hope the reader will go on and thoroughly enjoy the book.

Graeme. W. Milton
Salt Lake City, Utah

References

1 J. Dowling, "Photonic and sonic band-gap and metamaterial bibliography," nd. URL: http://www.phys.lsu.edu/~jdowling/pbgbib.html.

2 L. M. Nash, D. Kleckner, A. Read, V. Vitelli, A. M. Turner, and W. T. M. Irvine, "Topological mechanics of gyroscopic metamaterials," *Proceedings of the National Academy of Sciences*, vol. 112, no. 47, pp. 14495–14500, 2015.

3 D. J. Jacobs and B. Hendrickson, "An algorithm for two-dimensional rigidity percolation: The pebble game," *Journal of Computational Physics*, vol. 137, no. 2, pp. 346–365, 1997.

4 S. D. Guest and P. W. Fowler, "Symmetry-extended counting rules for periodic frameworks," *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 372, no. 2008, p. 20120029, 2013.

5 G. W. Milton, "Complete characterization of the macroscopic deformations of periodic unimode metamaterials of rigid bars and pivots," *Journal of the Mechanics and Physics of Solids*, vol. 61, no. 7, pp. 1543–1560, 2013.

6 G. W. Milton, "Adaptable nonlinear bimode metamaterials using rigid bars, pivots, and actuators," *Journal of the Mechanics and Physics of Solids*, vol. 61, no. 7, pp. 1561–1568, 2013.

7 Y. Gu, "High order correctors and two-scale expansions in stochastic homogenization," *arxiv.org*, pp. 1–28, 2016.

8 M. Camar-Eddine and P. Seppecher, "Determination of the closure of the set of elasticity functionals," *Archive for Rational Mechanics and Analysis*, vol. 170, no. 3, pp. 211–245, 2003.

9 P. Seppecher, J.-J. Alibert, and F. D. Isola, "Linear elastic trusses leading to continua with exotic mechanical interactions," *Journal of Physics: Conference Series*, vol. 319, no. 1, p. 012018, 2011.

10 G. W. Milton, M. Briane, and J. R. Willis, "On cloaking for elasticity and physical equations with a transformation invariant form," *New Journal of Physics*, vol. 8, no. 10, p. 248, 2006.

11 G. W. Milton and J. R. Willis, "On modifications of Newton's second law and linear continuum elastodynamics," *Proceedings of the Royal Society A: Mathematical, Physical, & Engineering Sciences*, vol. 463, no. 2079, pp. 855–880, 2007.

12 F. Guevara Vasquez, G. W. Milton, and D. Onofrei, "Complete characterization and synthesis of the response function of elastodynamic networks," *Journal of Elasticity*, vol. 102, no. 1, pp. 31–54, 2011.

13 B. Taylor, H. J. Maris, and C. Elbaum, "Phono. focusing in solids," *Physical Review Letters*, vol. 23, no. 8, pp. 416–419, 1969.

14 A. Bensoussan, J.-L. Lions, and G. C. Papanicolaou, *Asymptotic Analysis for Periodic Structures*, vol. 5 of Studies in Mathematics and its Applications. Amsterdam: North-Holland Publishing Co., 1978.

15 M. S. Birman and T. A. Suslina, "Homogenization of a multidimensional periodic elliptic operator in a neighborhood of the edge of an internal gap," *Journal of Mathematical Sciences (New York, NY)*, vol. 136, no. 2, pp. 3682–3690, 2006.

16 R. V. Craster, J. Kaplunov, and A. V. Pichugin, "High frequency homogenization for periodic media," *Proceedings of the Royal Society A: Mathematical, Physical, & Engineering Sciences*, vol. 466, no. 2120, pp. 2341–2362, 2010.

17 M. A. Hoefer and M. I. Weinstein, "Defect modes and homogenization of periodic Schrödinger operators," *SIAM Journal on Mathematical Analysis*, vol. 43, no. 2, pp. 971–996, 2011.

18 T. Antonakakis and R. V. Craster, "High frequency asymptotics for microstructured thin elastic plates and platonics," *Proceedings of the Royal Society A: Mathematical, Physical, & Engineering Sciences*, vol. 468, no. 2141, pp. 1408–1427, 2012.

19 T. Antonakakis, R. V. Craster, and S. Guenneau, "Asymptotics for metamaterials and photonic crystals," *Proceedings of the Royal Society of London. Series A*, vol. 469, no. 2152, p. 20120533, 2013.

20 T. Antonakakis, R. V. Craster, and S. Guenneau, "Homogenization for elastic photonic crystals and metamaterials," *Journal of the Mechanics and Physics of Solids*, vol. 71, pp. 84–96, 2014.

21 L. Ceresoli, R. Abdeddaim, T. Antonakakis, B. Maling, M. Chmiaa, P. Sabouroux, G. Tayeb, S. Enoch, R. V. Craster, and S. Guenneau, "Dynamic effective anisotropy: Asymptotics, simulations and microwave experiments with dielectric fibres," *Physical Review B: Condensed Matter and Materials Physics*, vol. 92, no. 17, p. 174307, 2015.

22 G. Allaire, M. Palombaro, and J. Rauch, "Diffractive behaviour of the wave equation in periodic media: weak convergence analysis," *Annali di Mathematica Pura ed Applicata. Series IV*, vol. 188, no. 4, pp. 561–590, 2009.

23 M. Brassart and M. Lenczner, "A two-scale model for the periodic homogenization of the wave equation," *Journal de Mathématiques Pures et Appliquées*, vol. 93, no. 5, pp. 474–517, 2010.

24 G. Allaire, M. Palombaro, and J. Rauch, "Diffractive geometric optics for Bloch waves," *Archive for Rational Mechanics and Analysis*, vol. 202, no. 2, pp. 373–426, 2011.

25 G. Allaire, M. Palombaro, and J. Rauch, "Diffraction of Bloch wave packets for Maxwell's equations," *Communications in Contemporary Mathematics*, vol. 15, no. 06, p. 1350040, 2013.

26 D. Harutyunyan, R. V. Craster, and G. W. Milton, "High frequency homogenization for travelling waves in periodic media," *Proceedings of the Royal Society A: Mathematical, Physical, & Engineering Sciences*, vol. 472, no. 2191, p. 20160066, 2016.

27 G. W. Milton, "Exact band structure for the scalar wave equation with periodic complex moduli," *Physica. B, Condensed Matter*, vol. 338, no. 1–4, pp. 186–189, 2003.

28 G. W. Milton, "The exact photonic band structure for a class of media with periodic complex moduli," *Methods and Applications of Analysis*, vol. 11, no. 3, pp. 413–422, 2004.

29 S. A. R. Horsley, M. Artoni, and G. C. La Rocca, "Spatial Kramers–Kronig relations and the reflection of waves," *Nature Photonics*, vol. 9, pp. 436–439, 2015.

30 G. W. Milton and S. K. Serkov, "Bounding the current in nonlinear conducting composites," *Journal of the Mechanics and Physics of Solids*, vol. 48, no. 6/7, pp. 1295–1324, 2000.

31 C. W. C. Jemellie Galang, Alessandro Restelli and E. Hagley, "The dangerous dark companion of bright green lasers," *SPIE Newsroom, 10 January*, 2011. URL: http://spie.org/newsroom/3328-the-dangerous-dark-companion-of-bright-green-lasers.

32 Bertoldi Group webpage. URL: http://bertoldi.seas.harvard.edu/pages/buckliball-buckling-induced-encapsulation.

33 T. Jaglinski, D. Kochmann, D. Stone, and R. S. Lakes, "Composite materials with viscoelastic stiffness greater than diamond," *Science*, vol. 315, no. 5812, pp. 620–622, 2007.

34 R. S. Lakes and W. J. Drugan, "Dramatically stiffer elastic composite materials due to a negative stiffness phase?," *Journal of the Mechanics and Physics of Solids*, vol. 50, no. 5, pp. 979–1009, 2002.

35 D. M. Kochmann and G. W. Milton, "Rigorous bounds on the effective moduli of composites and inhomogeneous bodies with negative-stiffness phases," *Journal of the Mechanics and Physics of Solids*, vol. 71, pp. 46–63, 2014.

36 D. Harutyunyan, G. W. Milton, T. J. Dick, and J. Boyer, "On ideal dynamic climbing ropes," *Journal of Sports Engineering and Technology*, 2016.

37 M. Shipman, "Metal foam obliterates bullets – and that's just the beginning," *NC State News, 5 April*, 2016. URL: https://news.ncsu.edu/2016/04/metal-foam-tough-2016/.

38 G. W. Milton and A. V. Cherkaev, "Which elasticity tensors are realizable?," *ASME Journal of Engineering Materials and Technology*, vol. 117, no. 4, pp. 483–493, 1995.

39 M. Kadic, T. Bückmann, N. Stenger, M. Thiel, and M. Wegener, "On the practicability of pentamode mechanical metamaterials," *Applied Physics Letters*, vol. 100, no. 19, p. 191901, 2012.

40 Wikipedia article, "Platonic crystal." URL: https://en.wikipedia.org/wiki/Platonic_crystal.

41 B. Shen, P. Wang, R. Polson, and R. Menon, "An integrated-nanophotonics polarization beamsplitter with $2.4 \times 2.4\,\mu\mathrm{m}^2$ footprint," *Nature Photonics*, vol. 9, no. 2, pp. 378–382, 2015.

42 R. E. Christiansen, O. Sigmund, and E. Fernandez-Grande, "Experimental validation of a topology optimized acoustic cavity," *The Journal of the Acoustical Society of America*, vol. 138, no. 6, pp. 3470–3474, 2015.

43 Wikipedia article, “Lycurgus cup.” URL: https://en.wikipedia.org/wiki/Lycurgus_Cup.

44 J. C. Maxwell Garnett, “Colours in metal glasses and in metallic films,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical, and Engineering Sciences*, vol. 203, no. 359–371, pp. 385–420, 1904.

45 S. A. Schelkunoff and H. T. Friis, *Antennas: The Theory and Practice*, pp. 584–585–. New York / London / Sydney, Australia: John Wiley and Sons, 1952.

46 J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, “Magnetism from conductors and enhanced nonlinear phenomena,” *IEEE transactions on microwave theory and techniques*, vol. 47, no. 11, pp. 2075–2084, 1999.

47 V. V. Zhikov, “On an extension and an application of the two-scale convergence method,” *Matematicheskii Sbornik*, vol. 191, no. 7, pp. 31–72, 2000.

48 V. V. Zhikov, “On spectrum gaps of some divergent elliptic operators with periodic coefficients,” *Algebra i Analiz*, vol. 16, no. 5, pp. 34–58, 2004.

49 Z. Liu, X. Zhang, Y. Mao, Y. Y. Zhu, Z. Yang, C. T. Chan, and P. Sheng, “Locally resonant sonic materials,” *Science*, vol. 289, no. 5485, pp. 1734–1736, 2000.

50 Z. Liu, C. T. Chan, and P. Sheng, “Analytic model of phononic crystals with local resonances,” *Physical Review B: Condensed Matter and Materials Physics*, vol. 71, no. 1, p. 014103, 2005.

51 A. B. Movchan and S. Guenneau, “Split-ring resonators and localized modes,” *Physical Review B: Condensed Matter and Materials Physics*, vol. 70, no. 12, p. 125116, 2004.

52 M. C. Rechtsman and S. Torquato, “Method for obtaining upper bounds on photonic band gaps,” *Physical Review B: Condensed Matter and Materials Physics*, vol. 80, no. 15, p. 155126, 2009.

Preface

A lattice material may be viewed as an enlarged and carefully tuned crystal, artificially constructed to function precisely as desired in engineering applications. It is formed from a spatially periodic network of interconnected rods, beams, plates or other slender structures. The ability to tailor the unit-cell architecture of a lattice material makes it possible to attain superior mechanical, elastodynamic and acoustic properties for numerous industrial applications – properties that may not be achievable using conventional materials. Naturally inspired by concepts from crystal physics, the methods and analysis techniques used in the study of lattice materials directly apply to periodic materials in general, including phononic crystals and elastic metamaterials that exhibit local resonances and/or other unique features.

In this book, we have sought to provide a comprehensive coverage of the emerging field of the dynamics of lattice materials. Co-written by a selection of leading researchers in the field, spanning three continents, the book gently introduces key concepts and fundamental theories in the discipline, while also boldly considering, often in considerable depth, the state of the art.

The topics covered include elastostatics (Chapter 2) and elastodynamics (Chapter 3), the effects of damping (Chapter 4), nonlinearity (Chapter 5), instabilities (Chapter 6) and impact loads (Chapter 7); exotic dynamics such as pentamodes (Chapter 8); model reduction (Chapter 9) and optimization (Chapter 10); metamaterial concepts including local resonance and inertial amplification (Chapter 11); and nano lattices (Chapter 12). Guided by an introductory chapter (Chapter 1), a systematic and unified synthesis of these topics pertaining to lattice materials is provided to help the reader consolidate concepts across the chapters.

The book is suitable for and accessible to graduate students and research scientists with backgrounds in dynamics, vibrations, and acoustics; mechanics and strength of materials; and condensed matter physics and materials science. It serves as a useful reference to researchers based in academia and practitioners in industrial research laboratories and design centers. It may also be used as a textbook for graduate courses on the mechanics of lattice materials, or a more focused course on wave propagation in periodic materials.

Many people have contributed to this book, directly or indirectly. First and foremost, colleagues and contributors to each chapter are acknowledged for their insightful presentations and diligent responses to requests from the editors. While credit for success goes to the contributing authors and their tireless efforts, the editors are responsible for any lingering typos or unintended omissions. We would like to extend our special thanks to Prof. Graeme Milton for his scholarly and insightful foreword. Credit also goes to the Wiley publishing team, especially to Paul Petralia for his initiative and sustained leadership, and to Nandhini Thandavamoorthy for her hard work and patience during the various stages of the evolution of this project. SP would like to acknowledge the funding for his research from the Natural Sciences and Engineering Research Council (NSERC) of Canada through its various programs, the assistance from his graduate students Behrooz Yousefzadeh, Lalitha Raghavan, Prateek Chopra and Ehsan Moosavimehr, and the support from his family members, particularly Ananya and Krishna. MIH, on his part, acknowledges funding for his research from several United States federal agencies, particularly the National Science Foundation, numerous seed grants from the University of Colorado Boulder, and the generous support provided through his H. Joseph Smead Faculty Fellowship. In addition to his student co-authors, Dimitri Krattiger and Osama Bilal, MIH is also grateful to current or former doctoral students Bruce Davis, Michael Frazier, Romik Khajehtourian, Clémence Bacquet, Hossein Honarvar, Alec Kucala, and Mary Bastawrous, and former postdoctoral fellow Lina Yang, for their assistance at the CU-Boulder Phononics Laboratory. Dimitri's efforts in creating the lattice image used on the front cover is much appreciated. Most of all, MIH is grateful to family members Alaa and Ismail (Jr.) in Boulder and Heba, Nahla, Iziz and Ismail (Sr.) in Cairo, Egypt.

The overarching goal of this book is to spur fundamental and applied research in the design, manufacturing, and utilization of lattice materials and structures across not only numerous existing applications, but also applications that are yet to be conceived.

February 2017

A. Srikantha Phani
Vancouver, British Columbia

Mahmoud I. Hussein
Boulder, Colorado

1

Introduction to Lattice Materials

A. Srikantha Phani[1] *and Mahmoud I. Hussein*[2]

[1] *Department of Mechanical Engineering, University of British Columbia, Vancouver, Canada*
[2] *Department of Aerospace Engineering Sciences, University of Colorado Boulder, USA*

1.1 Introduction

The word “lattice” implies a certain ordered pattern characterized by spatial periodicity, and hence symmetry. In crystalline solids, for example, atoms are arranged in a spatially periodic pattern or a lattice. Such a crystal lattice is specified by a unit cell and the associated basis vectors defining the directions of tessellation [1, 2]. Spatially repetitive patterns are not unique to atomic length scales. They appear over a wide range of length scales, spanning several disciplines and areas of application; see Figure 1.1 for a representative list. Carbon nanotubes [3] and single-layer graphene sheets [4] are periodic materials with nanoscale features. Microelectromechanical systems (MEMS) for radio frequency applications use microscale periodic architectures to form mechanical filters [5]. Biomedical implants such as cardiovascular stents are periodic cylindrical mesh structures [6, 7]. At macro and mega scales, periodic structural construction is widely used in composites in materials engineering [8, 9], turbomachinery in aerospace engineering [10, 11], and bridge and tower structures in civil engineering [12]. Aircraft surfaces typically use a skin-stinger configuration in the form of a uniform shell, reinforced at regular spatial intervals by identical stiffener/stingers. Similarly, rib-skin aircraft structural components, used in tails and fins, comprise two skins (plates) interconnected by ribs [13]. Interested readers are referred to the book by Gibson and Ashby [14] for further studies on lattice materials and the reviews by Mead and by Hussein et al. [15, 16] on the dynamics of periodic materials in general.

In a closely related research discipline, periodic materials are referred to as *phononic crystals* [17, 18], where strong analogies are drawn with their electromagnetic counterpart, photonic crystals. While there is a significant overlap between lattice materials and phononic crystals [19, 20], the former category is mostly associated with low-density construction and utilization in structural mechanics applications, whereas the latter is mostly connected to applications in applied physics, including filtering [21], waveguiding [22], sensing [23], imaging [24], and, more recently, vibrational energy harvesting [25], thermal transport management at the nanoscale [26], and control of wall-bounded flows [27]. Another class of artificial materials that possess unique

Dynamics of Lattice Materials, First Edition. Edited by A. Srikantha Phani and Mahmoud I. Hussein.

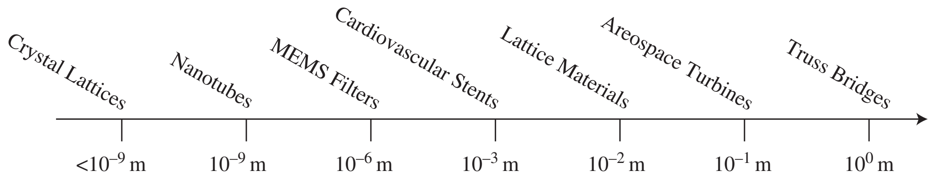


Figure 1.1 Periodic materials and structures across different length scales and disciplines. (MEMS: microelectromechanical systems.)

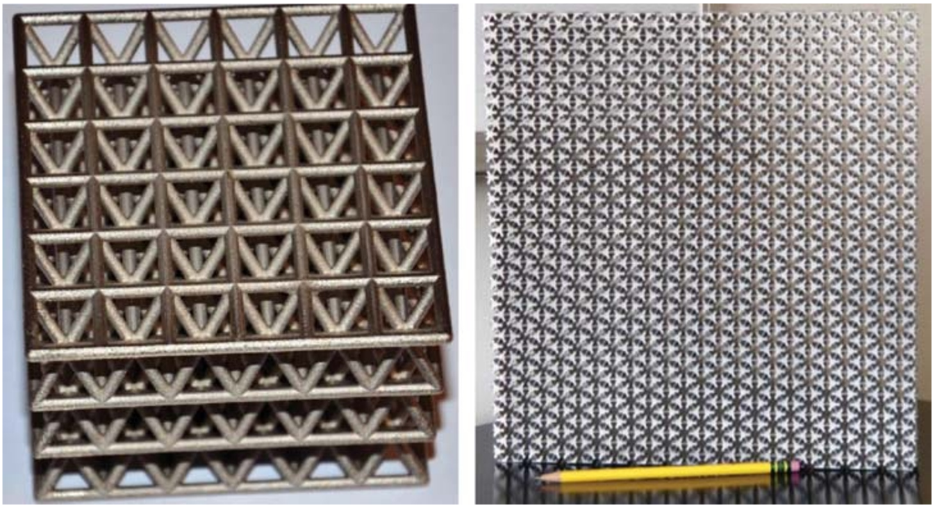
wave-propagation properties is referred to as *acoustic/elastic metamaterials* [28]. These are similar to phononic crystals, with the added feature of local resonators – small oscillating substructures integrally embedded within, or attached to the medium of the host material [29, 30]. However, unlike lattice materials and phononic crystals, periodicity is not a necessity for metamaterials. In addition to controlling sound and vibration, locally resonant "nanophononic metamaterials" have been shown to reduce thermal conductivity [31]. A recent book [32] and review article [16] provide historical background, the state of the art in the analysis and design of phononic crystals and metamaterials, together with their applications. In recent years, a new research community has formed around this discipline, now more broadly termed *phononics*, which incorporates the study and manipulation of "sound" waves in general and across the various spatial and temporal scales [33, 34].

The dynamic response of lattice materials, and structures, and by association phononic crystals and metamaterials, is the overarching theme of the book. We begin with a brief overview of periodic materials and structures, with emphasis on lattice materials, which are considered a new class of periodic materials. A formal classification is presented, followed by a discussion of manufacturing techniques and applications. A link to phononic crystals and acoustic/elastic metamaterials – also a new development in periodic materials – is presented when appropriate. We conclude this introductory chapter with an overview of the book.

1.2 Lattice Materials and Structures

A lattice material is defined as a spatially periodic network of structural elements, such as rods, beams, plates, or shells, whose constituent length scales are generally larger than the load-deformation length scales[1]; see Figure 1.2 for example. It possesses a spatially ordered pattern specified by a unit cell and associated tessellation directions (lattice basis vectors). The unit cell itself is an interconnected network of structural elements. Let us consider a network of flexural beams as an example. The material constituent of each beam can be a single homogeneous isotropic material (such as steel or aluminum) or a hierarchical anisotropic composite. Thus lattice materials, in the form of an interconnected spatially periodic network of composite beams, can be viewed as discrete multiscale materials with hierarchy. The ability to fabricate a spatially periodic network of beams using advanced manufacturing methods has spurred interest in lattice materials; see Fleck et al. [35] for a recent review. When viewed as a porous

1 This condition does not necessarily hold for lattice metamaterials where the size of unit-cell may be smaller than the deformation length scales.



(a) (b)

Figure 1.2 Lattice materials formed from a periodic network of beams: (a) ultralight nanometal truss hybrid lattice; (b) pentamode lattice.

solid, or a hybrid material (of fluid and metal) [36], the high-porosity limit yields a network of beams while the low-porosity limit leads to a continuum with pores. Most of the discussion and examples covered in this book are focused on material configurations at the high-porosity end of this range, although the ideas are usually relevant to low-porosity configurations as well.

1.2.1 Material versus Structure

A spatially periodic network of structural elements, such as beams, can be viewed both as a material and a structure for the following reasons. In engineering applications, employing a truss lattice of beams as a core in a sandwich panel, the length of each lattice beam is of the order of the thickness of the panel, and the thickness of each beam is typically an order of magnitude less. When the deformation processes of interest are at a length scale much larger than the individual beam length, a spatially periodic network of beams is termed a "lattice material" and has its own effective properties. At length scales of the order of the individual beam length, a spatially periodic network of beams behaves as a structure, such as a frame in a building or a truss in a bridge. Thus principles of structural mechanics can be applied to the design of lattice materials [37]. Another avenue for distinguishing between material and structure is in terms of the number unit cells, as well as the internal unit-cell symmetry. It is generally recognized that a for a finite system to exhibit material characteristics, at least a handful of unit cells are needed [38, 39]. In addition, a finite structure based on a repetition of a unit cell with symmetrical internal features is more likely to respond to dynamic loading in a manner consistent with the dispersion band structure of a material theoretically consisting of an infinite number of this unit cell [40, 41].

1.2.2 Motivation

The development of lattice materials is motivated by a desire to design multifunctional materials and structures that are not only light and stiff but also possess a desirable vibroacoustic response and thermal-transport properties, among other features. The need to overcome the limitations of metal foams [42, 43] has propelled the development of lattice materials, a process that has benefited from insights already acquired through studies of cellular solids [37, 44–48]. Similarly, accumulated research on the dynamics of periodic materials and structures (such as aircraft components and conventional composite materials) has provided a valuable knowledge base to build on for the study of wave-propagation characteristics in lattice materials. The following list provides an incomplete but indicative summary of efforts and motivations for current research in lattice materials.

1. Design lightweight and stiff/strong structures with optimal lattice core for multifunctional applications [49–52]. In this line of research, ongoing efforts aim to tailor the effective stiffness and strength of the truss lattice core to achieve high performance with the lowest possible density. The discovery of new unit-cell geometries using topology optimization and other computational methods is a promising avenue for further improvements [53, 54].
2. Advance mathematical modeling and analysis of complex lattice structures. This involves developing homogenization techniques for lattices [55, 56] and in-depth studies on the influence of damping [57–59] and nonlinearities [60–62] on the dispersive behavior of lattices.
3. Develop lattice unit-cell structures with tunable elastodynamic [63–65] and stability [66] properties.
4. Develop lattice-styled metamaterials based on periodic micro-architectures with extraordinary dynamic (acoustic and/or elastic) effective properties, not achievable using conventional materials [67, 68].
5. Create innovative nanostructured lattice materials based on periodic architectures for mechanical [50, 69, 70] and thermal [26, 31, 71] applications.

1.2.3 Classification of Lattices and Maxwell's Rule

Lattices can be classified based on their geometric or their mechanical deformation properties. Geometry-based classification is universally accepted in mathematics and solid-state physics. In 2D, planar lattices are classified into two categories: regular and semi-regular [72]. Regular lattices are obtained by tessellating a single, regular, polygonal unit cell to fill a plane. Here, a regular polygon is defined to be equiangular (all angles are equal) and equilateral (all lengths are equal). Square, triangle, and hexagon are the only plane-filling regular polygons, so there are only three regular planar lattices: square lattice, triangular lattice, and hexagonal lattice. In contrast to regular lattices, semi-regular lattices are obtained by tessellating a unit cell, containing more than one regular polygon, to fill a plane. There are only eight such semi-regular lattices; see Cundy and Rollett [72] for more detail. Kagome or triangular-hexagon lattice is a semi-regular lattice that is widely used in weaving baskets and in architectural construction. A detailed classification of 3D lattices and polyhedra can be found in the literature [72, 73].

Lattices can also be classified into bending- or stretching-dominated categories [37, 73] on the basis of their rigidity. A bending-dominated lattice responds to external loads by cell-wall bending, whereas a stretching-dominated lattice deforms predominantly by stretching. Bending-dominated lattices are less stiff and strong than