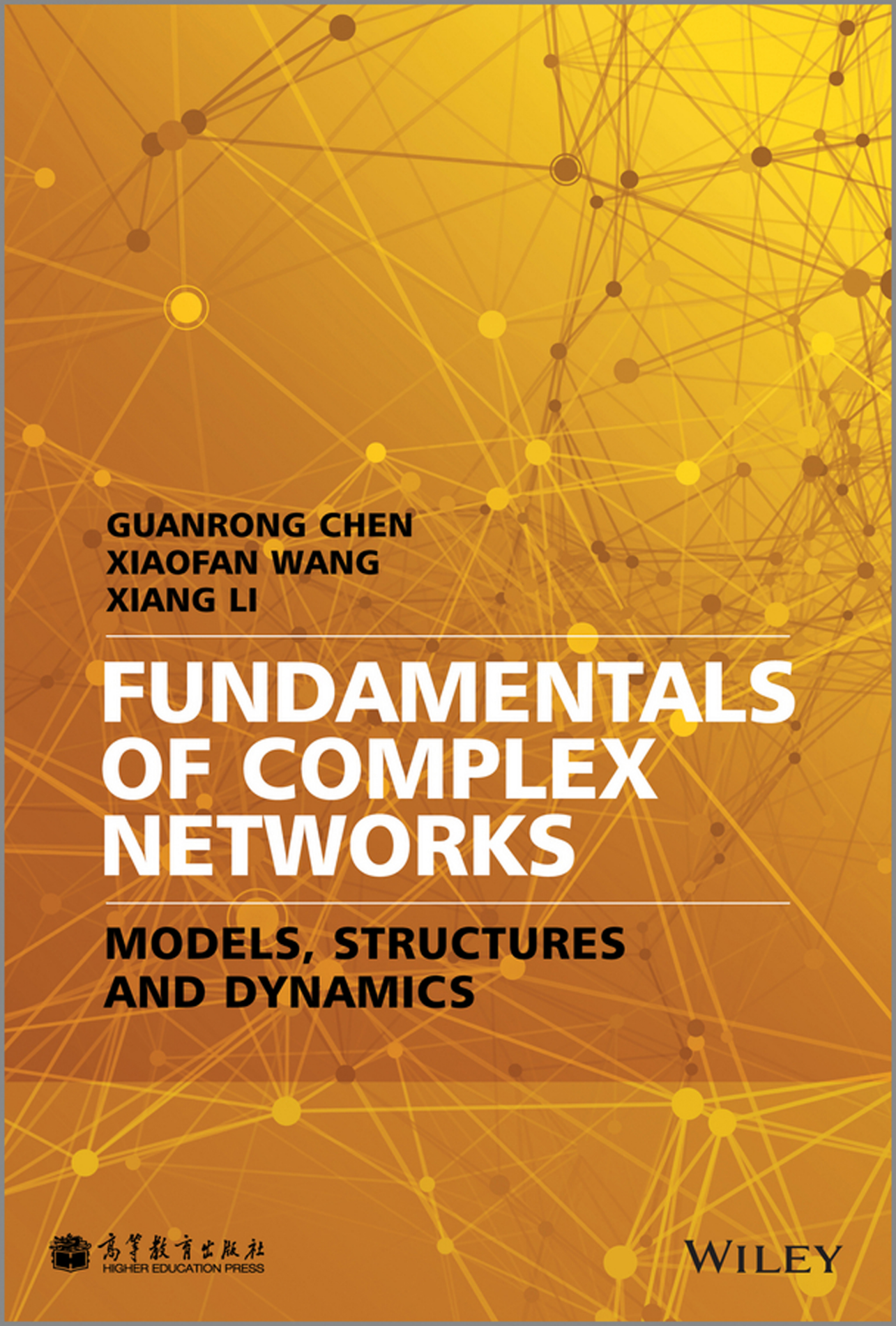




**GUANRONG CHEN
XIAOFAN WANG
XIANG LI**

FUNDAMENTALS OF COMPLEX NETWORKS

**MODELS, STRUCTURES
AND DYNAMICS**

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MODELS, STRUCTURES AND DYNAMICS

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Preface

The extensive study of complex networks is pervading sciences and engineering today, from physical, technological, biological, to social sciences. Their impacts on engineering and technology, in particular, are prominent and their influence is deemed to be far-reaching. Familiar complex networks include the Internet, the World Wide Web, wireless communication networks, biological neural networks, power grids, social relation and scientific cooperation networks, and so on. Research on fundamental properties and dynamical behaviors of various complex networks has recently become overwhelming.

The field of complex networks is indeed developing so fast and so wide that most newcomers typically find it quite difficult to know where to start their learning and research on the subject. Although there are some well-written textbooks and research monographs that can be adopted for studies by newcomers, these references are generally too advanced or too broad for those readers to comprehend, especially in a relatively short period of time; they are not easily used as textbooks for a short course on the subject either. Driven by such teaching and learning demands, this book has been designed to serve as a concise textbook for newcomers to the field. It is written as a one-semester introductory text for upper-division undergraduate or first-year graduate students in natural science, mathematics and engineering, or as an edited volume for self-study, or as a handy reference for research.

The book title retains the adjective “complex” to reflect the historical perspective and to emphasize the nature of the subject, which is in line with the common phrases of complex systems and complex dynamics alike, and therefore should not be seen as redundant.

The style of writing in this book is intended to be informal, emphasizing basic ideas and methodologies with elementary and sometimes heuristic mathematical arguments, easily readable by anyone having minimal knowledge of calculus, linear algebra and ordinary differential equations. In this regard, and to be self-contained, a preliminary chapter on graph theory, probability and statistics as well as dynamical systems is included after the first overview chapter. The book is divided into two parts: Part I *Fundamental Theory* is a detailed text consisting of three chapters, presenting background information and basic materials needed to learn the subject, with a variety of exercises for illustrating fundamental concepts and familiarizing related modeling and analysis techniques. Part II *Applications – Selected Topics* contains several selected application-oriented topics, which are all independent of each other, in the sense that one can choose any chapter to teach or to learn individually without referring to the contents of the other chapters in this part. Of course, the current arrangement of chapters would also be a logical ordering if one decided to read through this whole portion of the book. The last chapter of this part provides only outlines of several emerging topics which are believed important and promising, with sufficient numbers of key references provided for interested readers’ future studies.

This book is a slightly modified and extended version of the same authors’ earlier version of the book entitled *Introduction to Complex Networks: Models, Structures and Dynamics*, published by the Higher Education Press, Beijing in 2012. The basic materials of this book have been used in teaching a post-graduate course on *Complex Networks: Modeling, Dynamics and Control* at the City University of Hong Kong since 2007, received valuable feedback from students which has made the present version better suited to a text for both teaching and learning.