

Making Everything Easier!™

Revised Edition

Quantum Physics

FOR
DUMMIES®

Learn to:

- Make sense of complex terms and concepts, from eigenvalues to oscillators
- Tackle tough equations by working through numerous examples
- Understand the latest research in the field
- Prepare for graduate or professional exams

Steven Holzner

*Author, Physics For Dummies and
Physics Workbook For Dummies*



***Quantum
Physics***
FOR
DUMMIES®
REVISED EDITION

by Steven Holzner



WILEY

John Wiley & Sons, Inc.

Quantum Physics For Dummies®, Revised Edition

Published by
John Wiley & Sons, Inc.
111 River St.
Hoboken, NJ 07030-5774
www.wiley.com

Copyright © 2013 by John Wiley & Sons, Inc., Hoboken, New Jersey

Published by John Wiley & Sons, Inc., Hoboken, New Jersey

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-646-8600. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, 201-748-6011, fax 201-748-6008, or online at <http://www.wiley.com/go/permissions>.

Trademarks: Wiley, the Wiley Publishing logo, For Dummies, the Dummies Man logo, A Reference for the Rest of Us!, The Dummies Way, Dummies Daily, The Fun and Easy Way, Dummies.com and related trade dress are trademarks or registered trademarks of John Wiley & Sons, Inc. and/or its affiliates in the United States and other countries, and may not be used without written permission. All other trademarks are the property of their respective owners. John Wiley & Sons, Inc., is not associated with any product or vendor mentioned in this book.

LIMIT OF LIABILITY/DISCLAIMER OF WARRANTY: THE PUBLISHER AND THE AUTHOR MAKE NO REPRESENTATIONS OR WARRANTIES WITH RESPECT TO THE ACCURACY OR COMPLETENESS OF THE CONTENTS OF THIS WORK AND SPECIFICALLY DISCLAIM ALL WARRANTIES, INCLUDING WITHOUT LIMITATION WARRANTIES OF FITNESS FOR A PARTICULAR PURPOSE. NO WARRANTY MAY BE CREATED OR EXTENDED BY SALES OR PROMOTIONAL MATERIALS. THE ADVICE AND STRATEGIES CONTAINED HEREIN MAY NOT BE SUITABLE FOR EVERY SITUATION. THIS WORK IS SOLD WITH THE UNDERSTANDING THAT THE PUBLISHER IS NOT ENGAGED IN RENDERING LEGAL, ACCOUNTING, OR OTHER PROFESSIONAL SERVICES. IF PROFESSIONAL ASSISTANCE IS REQUIRED, THE SERVICES OF A COMPETENT PROFESSIONAL PERSON SHOULD BE SOUGHT. NEITHER THE PUBLISHER NOR THE AUTHOR SHALL BE LIABLE FOR DAMAGES ARISING HEREFROM. THE FACT THAT AN ORGANIZATION OR WEBSITE IS REFERRED TO IN THIS WORK AS A CITATION AND/OR A POTENTIAL SOURCE OF FURTHER INFORMATION DOES NOT MEAN THAT THE AUTHOR OR THE PUBLISHER ENDORSES THE INFORMATION THE ORGANIZATION OR WEBSITE MAY PROVIDE OR RECOMMENDATIONS IT MAY MAKE. FURTHER, READERS SHOULD BE AWARE THAT INTERNET WEBSITES LISTED IN THIS WORK MAY HAVE CHANGED OR DISAPPEARED BETWEEN WHEN THIS WORK WAS WRITTEN AND WHEN IT IS READ.

For general information on our other products and services, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002.

For technical support, please visit www.wiley.com/techsupport.

Wiley publishes in a variety of print and electronic formats and by print-on-demand. Some material included with standard print versions of this book may not be included in e-books or in print-on-demand. If this book refers to media such as a CD or DVD that is not included in the version you purchased, you may download this material at <http://booksupport.wiley.com>. For more information about Wiley products, visit www.wiley.com.

Library of Congress Control Number: 2012945682

ISBN 978-1-118-46082-5 (pbk); 978-1-118-46083-2 (ebk); ISBN 978-1-118-46086-3 (ebk); 978-1-118-84608-7 (ebk)

Manufactured in the United States of America

10 9 8 7 6 5 4 3 2 1



About the Author

Steven Holzner is an award-winning author of technical and science books (like *Physics For Dummies* and *Differential Equations For Dummies*). He graduated from MIT and did his PhD in physics at Cornell University, where he was on the teaching faculty for 10 years. He's also been on the faculty of MIT. Steve also teaches corporate groups around the country.

Author's Acknowledgments

I'd particularly like to thank the people at Wiley: Tracy Boggier, Tim Gallan, and Danielle Voirol.

Dedication

To Nancy, of course!

Publisher's Acknowledgments

We're proud of this book; please send us your comments through our Dummies online registration form located at www.dummies.com/register/.

Some of the people who helped bring this book to market include the following:

Acquisitions, Editorial, and Media Development

Senior Project Editor: Tim Gallan

Acquisitions Editor: Tracy Boggier

Senior Copy Editor: Danielle Voirol

Assistant Editor: Erin Calligan Mooney

Technical Editors: Dan Funch Wohns,
Lev Kaplan, Ph.D.

Editorial Manager: Michelle Hacker

Editorial Assistants: Joe Niesen,
Jennette ElNaggar, David Lutton

Cartoons: Rich Tennant
(www.the5thwave.com)

Cover Photo: © MEHAU KULYK / Getty Images

Composition Services

Project Coordinator: Erin Smith

Layout and Graphics: Carl Byers,
Carrie A. Cesavice, Shawn Frazier,
Nikki Gately, Melissa Smith,
Christine Williams

Proofreaders: Joni Heredia, Shannon Ramsey

Indexer: Broccoli Information Management

Publishing and Editorial for Consumer Dummies

Kathleen Nebenhaus, Vice President and Executive Publisher

David Palmer, Associate Publisher

Kristin Ferguson-Wagstaffe, Product Development Director

Publishing for Technology Dummies

Andy Cummings, Vice President and Publisher

Composition Services

Gerry Fahey, Vice President of Production Services

Debbie Stailey, Director of Composition Services

Contents at a Glance

<i>Introduction</i>	<i>1</i>
<i>Part I: Small World, Huh? Essential Quantum Physics</i>	<i>7</i>
Chapter 1: Discoveries and Essential Quantum Physics	9
Chapter 2: Entering the Matrix: Welcome to State Vectors.....	23
<i>Part II: Bound and Undetermined: Handling</i>	
<i>Particles in Bound States</i>	<i>55</i>
Chapter 3: Getting Stuck in Energy Wells	57
Chapter 4: Back and Forth with Harmonic Oscillators.....	91
<i>Part III: Turning to Angular Momentum and Spin</i>	<i>125</i>
Chapter 5: Working with Angular Momentum on the Quantum Level.....	127
Chapter 6: Getting Dizzy with Spin.....	157
<i>Part IV: Multiple Dimensions: Going 3D</i>	
<i>with Quantum Physics</i>	<i>167</i>
Chapter 7: Rectangular Coordinates: Solving Problems in Three Dimensions.....	169
Chapter 8: Solving Problems in Three Dimensions: Spherical Coordinates	189
Chapter 9: Understanding Hydrogen Atoms	205
Chapter 10: Handling Many Identical Particles	231
<i>Part V: Group Dynamics: Introducing</i>	
<i>Multiple Particles</i>	<i>253</i>
Chapter 11: Giving Systems a Push: Perturbation Theory	255
Chapter 12: Wham-Blam! Scattering Theory	275
<i>Part VI: The Part of Tens</i>	<i>293</i>
Chapter 13: Ten Quantum Physics Tutorials	295
Chapter 14: Ten Quantum Physics Triumphs	299
<i>Glossary</i>	<i>303</i>
<i>Index</i>	<i>309</i>

Table of Contents

Introduction 1

About This Book	1
Conventions Used in This Book.....	2
Foolish Assumptions.....	2
How This Book Is Organized	2
Part I: Small World, Huh? Essential Quantum Physics.....	3
Part II: Bound and Undetermined: Handling Particles in Bound States.....	3
Part III: Turning to Angular Momentum and Spin.....	3
Part IV: Multiple Dimensions: Going 3D with Quantum Physics.....	3
Part V: Group Dynamics: Introducing Multiple Particles.....	4
Part VI: The Part of Tens.....	4
Icons Used in This Book	4
Where to Go from Here.....	5

Part 1: Small World, Huh? Essential Quantum Physics..... 7

Chapter 1: Discoveries and Essential Quantum Physics 9

Being Discrete: The Trouble with Black-Body Radiation	10
First attempt: Wien's Formula.....	12
Second attempt: Rayleigh-Jeans Law	12
An intuitive (quantum) leap: Max Planck's spectrum.....	12
The First Pieces: Seeing Light as Particles	13
Solving the photoelectric effect.....	14
Scattering light off electrons: The Compton effect.....	16
Proof positron? Dirac and pair production	17
A Dual Identity: Looking at Particles as Waves	18
You Can't Know Everything (But You Can Figure the Odds).....	20
The Heisenberg uncertainty principle	20
Rolling the dice: Quantum physics and probability.....	21

Chapter 2: Entering the Matrix: Welcome to State Vectors 23

Creating Your Own Vectors in Hilbert Space.....	24
Making Life Easier with Dirac Notation	26
Abbreviating state vectors as kets	27
Writing the Hermitian conjugate as a bra.....	28
Multiplying bras and kets: A probability of 1.....	29



Covering all your bases: Bras and kets as basis-less state vectors	30
Understanding some relationships using kets	30
Grooving with Operators	31
Hello, operator: How operators work	31
I expected that: Finding expectation values	33
Looking at linear operators	34
Going Hermitian with Hermitian Operators and Adjoins	35
Forward and Backward: Finding the Commutator	36
Commuting	37
Finding anti-Hermitian operators	37
Starting from Scratch and Ending Up with Heisenberg	38
Eigenvectors and Eigenvalues: They're Naturally Eigentastic!	42
Understanding how they work	45
Finding eigenvectors and eigenvalues	46
Preparing for the Inversion: Simplifying with Unitary Operators	49
Comparing Matrix and Continuous Representations	51
Going continuous with calculus	52
Doing the wave	52

Part II: Bound and Undetermined: Handling Particles in Bound States..... **55**

Chapter 3: Getting Stuck in Energy Wells **57**

Looking into a Square Well	57
Trapping Particles in Potential Wells	59
Binding particles in potential wells	60
Escaping from potential wells	60
Trapping Particles in Infinite Square Potential Wells	61
Finding a wave-function equation	61
Determining the energy levels	62
Normalizing the wave function	64
Adding time dependence to wave functions	65
Shifting to symmetric square well potentials	67
Limited Potential: Taking a Look at Particles and Potential Steps	68
Assuming the particle has plenty of energy	69
Assuming the particle doesn't have enough energy	74
Hitting the Wall: Particles and Potential Barriers	78
Getting through potential barriers when $E > V_0$	79
Getting through potential barriers, even when $E < V_0$	81
Particles Unbound: Solving the Schrödinger Equation for Free Particles	85
Getting a physical particle with a wave packet	87
Going through a Gaussian example	88

Chapter 4: Back and Forth with Harmonic Oscillators 91

Grappling with the Harmonic Oscillator Hamiltonians	91
Going classical with harmonic oscillation.....	92
Understanding total energy in quantum oscillation.....	93
Creation and Annihilation: Introducing the Harmonic Oscillator	
Operators.....	94
Mind your p's and q's: Getting the energy state equations.....	94
Finding the Eigenstates.....	96
Using a and $a^\dagger$ directly.....	97
Finding the harmonic oscillator energy eigenstates	99
Putting in some numbers	106
Looking at Harmonic Oscillator Operators as Matrices.....	108
A Jolt of Java: Using Code to Solve the Schrödinger Equation	
Numerically.....	114
Making your approximations	114
Building the actual code	116
Running the code	123

Part III: Turning to Angular Momentum and Spin..... 125**Chapter 5: Working with Angular Momentum
on the Quantum Level 127**

Ringing the Operators: Round and Round with Angular Momentum.....	128
Finding Commutators of L_x , L_y , and L_z	130
Creating the Angular Momentum Eigenstates	131
Finding the Angular Momentum Eigenvalues	133
Deriving eigenstate equations with $\beta_{\max}$ and $\beta_{\min}$	133
Getting rotational energy of a diatomic molecule	136
Finding the Eigenvalues of the Raising and Lowering Operators.....	138
Interpreting Angular Momentum with Matrices.....	139
Rounding It Out: Switching to the Spherical Coordinate System.....	146
The eigenfunctions of L_z in spherical coordinates	149
The eigenfunctions of L^2 in spherical coordinates.....	150

Chapter 6: Getting Dizzy with Spin 157

The Stern-Gerlach Experiment and the Case of the Missing Spot.....	157
Getting Down and Dirty with Spin and Eigenstates.....	159
Halves and Integers: Saying Hello to Fermions and Bosons	160
Spin Operators: Running Around with Angular Momentum.....	161
Working with Spin $1/2$ and Pauli Matrices	162
Spin $1/2$ matrices	163
Pauli matrices	165

***Part IV: Multiple Dimensions: Going 3D with Quantum Physics* 167**

Chapter 7: Rectangular Coordinates: Solving Problems in Three Dimensions169

The Schrödinger Equation: Now in 3D!.....	169
Solving Three-Dimensional Free Particle Problems.....	172
The x, y, and z equations	173
Finding the total energy equation.....	174
Adding time dependence and getting a physical solution	175
Getting Squared Away with 3D Rectangular Potentials.....	177
Determining the energy levels	180
Normalizing the wave function	181
Using a cubic potential.....	183
Springing into 3D Harmonic Oscillators	184

Chapter 8: Solving Problems in Three Dimensions: Spherical Coordinates.....189

A New Angle: Choosing Spherical Coordinates	
Instead of Rectangular.....	190
Taking a Good Look at Central Potentials in 3D.....	192
Breaking down the Schrödinger equation.....	192
The angular part of $\psi(r, \theta, \phi)$	193
The radial part of $\psi(r, \theta, \phi)$	194
Handling Free Particles in 3D with Spherical Coordinates	195
The spherical Bessel and Neumann functions.....	196
The limits for small and large ρ	197
Handling the Spherical Square Well Potential	198
Inside the square well: $0 < r < a$	199
Outside the square well: $r > a$	200
Getting the Goods on Isotropic Harmonic Oscillators	201

Chapter 9: Understanding Hydrogen Atoms205

Coming to Terms: The Schrödinger Equation	
for the Hydrogen Atom	205
Simplifying and Splitting the Schrödinger Equation for Hydrogen.....	208
Solving for $\psi(R)$	210
Solving for $\psi(r)$	211
Solving the radial Schrödinger equation for small r	211
Solving the radial Schrödinger equation for large r	212
You got the power: Putting together the solution for	
the radial equation.....	212
Fixing $f(r)$ to keep it finite	215
Finding the allowed energies of the hydrogen atom.....	216
Getting the form of the radial solution of the Schrödinger	
equation.....	218
Some hydrogen wave functions	220

Calculating the Energy Degeneracy of the Hydrogen Atom.....	222
Quantum states: Adding a little spin	224
On the lines: Getting the orbitals.....	226
Hunting the Elusive Electron.....	228

Chapter 10: Handling Many Identical Particles231

Many-Particle Systems, Generally Speaking.....	232
Considering wave functions and Hamiltonians.....	232
A Nobel opportunity: Considering multi-electron atoms	233
A Super-Powerful Tool: Interchange Symmetry.....	235
Order matters: Swapping particles with the exchange operator	235
Classifying symmetric and antisymmetric wave functions	237
Floating Cars: Tackling Systems of Many Distinguishable Particles.....	239
Juggling Many Identical Particles	242
Losing identity.....	242
Symmetry and antisymmetry	244
Exchange degeneracy: The steady Hamiltonian.....	244
Name that composite: Grooving with the symmetrization postulate.....	245
Building Symmetric and Antisymmetric Wave Functions	246
Working with Identical Noninteracting Particles	247
Wave functions of two-particle systems	248
Wave functions of three-or-more-particle systems	249
It's Not Come One, Come All: The Pauli Exclusion Principle.....	250
Figuring out the Periodic Table	251

Part V: Group Dynamics: Introducing Multiple Particles..... 253

Chapter 11: Giving Systems a Push: Perturbation Theory255

Introducing Time-Independent Perturbation Theory	255
Working with Perturbations to Nondegenerate Hamiltonians	256
A little expansion: Perturbing the equations	257
Matching the coefficients of λ and simplifying	258
Finding the first-order corrections	259
Finding the second-order corrections	261
Perturbation Theory to the Test: Harmonic Oscillators	
in Electric Fields.....	262
Finding exact solutions	264
Applying perturbation theory	264
Working with Perturbations to Degenerate Hamiltonians	269
Testing Degenerate Perturbation Theory:	
Hydrogen in Electric Fields.....	271

Chapter 12: Wham-Blam! Scattering Theory275

Introducing Particle Scattering and Cross Sections.....	275
---	-----