Ordinary Differential Equations

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Preface

PURPOSE AND PREREQUISITES

This book is intended for students of science, engineering, and mathematics, as a textbook for a first course in differential equations, typically in the student's third or fourth semester. It is expected that the student has completed a differential and integral calculus sequence, but prior knowledge of linear algebra is not a prerequisite, and that material is provided here when it is needed.

TO THE INSTRUCTOR

The following points are to describe and explain some of the pedagogical decisions and approaches adopted in this text.

1. **Transition to higher-order equations.** The passage from first-order equations to equations of higher order is, we believe, often made more difficult for the student than necessary. Typically, the discussion of higher-order equations begins with the citing of an existence-and-uniqueness theorem and the introduction of linear independence and Wronskians, none of which is needed for the important case of equations with constant coefficients, which is indeed the first case to be studied. Consequently, second-order equations tend to look like a "new subject" to the student. Instead, we focus immediately on second-order equations with constant coefficients and derive their general solution in only a few pages (Theorem 2.2.1 of Section 2.2), using only results obtained in Chapter 1 for first-order equations. Proof of Theorem 2.2.1 is elementary, relying only on the factoring of the differential operator and the known solution of first-order equations with constant coefficients. The latter is not put forward as a solution method, but only to prove the theorem, and we are careful to caution the student that factorization cannot be expected to be useful for nonconstant-coefficient equations.

   The advantage of this approach is that the general solution of $y'' + p_1 y' + p_2 y = 0$ (in which the p_j's are constants) is obtained quickly and easily, without first introducing an existence-and-uniqueness theorem, linear independence, or Wronskians. The remainder of Sections 2.2 and 2.3 is devoted to familiarizing the student with the various solution forms: (a) the real exponentials and hyperbolic functions and (b) the complex exponentials and the circular functions. With that done, linear independence, Wronskians, existence, uniqueness, and general solution are introduced next, in Section 2.4, at which point the discussion can then be more readily grasped by the student, by virtue of the already completed discussion of the constant-coefficient case in Sections 2.2 and 2.3.