

WILEY SERIES IN PROBABILITY AND STATISTICS

Statistical Control by Monitoring and Adjustment

SECOND EDITION



George E. P. Box

Alberto Luceño

María del Carmen Paniagua-Quiñones

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WILEY SERIES IN PROBABILITY AND STATISTICS

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Statistical Control by Monitoring and Adjustment

Second Edition

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Contents

Preface

xi

1 Introduction and Revision of Some Statistical Ideas

1

- 1.1 Necessity for Process Control, 1
- 1.2 SPC and EPC, 1
- 1.3 Process Monitoring Without a Model, 3
- 1.4 Detecting a Signal in Noise, 4
- 1.5 Measurement Data, 4
- 1.6 Two Important Characteristics of a Probability Distribution, 5
- 1.7 Normal Distribution, 6
- 1.8 Normal Distribution Defined by μ and σ , 6
- 1.9 Probabilities Associated with Normal Distribution, 7
- 1.10 Estimating Mean and Standard Deviation from Data, 8
- 1.11 Combining Estimates of σ^2 , 9
- 1.12 Data on Frequencies (Events): Poisson Distribution, 10
- 1.13 Normal Approximation to Poisson Distribution, 12
- 1.14 Data on Proportion Defective: Binomial Distribution, 12
- 1.15 Normal Approximation to Binomial Distribution, 14
- Appendix 1A: Central Limit Effect, 15
- Problems, 17

2 Standard Control Charts Under Ideal Conditions As a First Approximation

21

- 2.1 Control Charts for Process Monitoring, 21
- 2.2 Control Chart for Measurement (Variables) Data, 22
- 2.3 Shewhart Charts for Sample Average and Range, 24
- 2.4 Shewhart Chart for Sample Range, 26
- 2.5 Process Monitoring With Control Charts for Frequencies, 29
- 2.6 Data on Frequencies (Counts): Poisson Distribution, 30

- 2.7 Common Causes and Special Causes, 34
- 2.8 For What Kinds of Data Has the c Chart Been Used?, 36
- 2.9 Quality Control Charts for Proportions: p Chart, 37
- 2.10 EWMA Chart, 40
- 2.11 Process Monitoring Using Cumulative Sums, 46
- 2.12 Specification Limits, Target Accuracy, and Process Capability, 53
- 2.13 How Successful Process Monitoring can Improve Quality, 56
Problems, 57

3 What Can Go Wrong and What Can We Do About It? 61

- 3.1 Introduction, 61
- 3.2 Measurement Charts, 64
- 3.3 Need for Time Series Models, 65
- 3.4 Types of Variation, 65
- 3.5 Nonstationary Noise, 66
- 3.6 Values for constants, 71
- 3.7 Frequencies and Proportions, 74
- 3.8 Illustration, 76
- 3.9 Robustness of EWMA, 78
Appendix 3A: Alternative Forms of Relationships for EWMA's, 79
Questions, 79

4 Introduction to Forecasting and Process Dynamics 81

- 4.1 Forecasting with an EWMA, 81
- 4.2 Forecasting Sales of Dingles, 82
- 4.3 Pete's Rule, 85
- 4.4 Effect of Changing Discount Factor, 86
- 4.5 Estimating Best Discount Factor, 87
- 4.6 Standard Deviation of Forecast Errors and Probability Limits for Forecasts, 88
- 4.7 What to Do If You Do Not Have Enough Data to Estimate θ , 89
- 4.8 Introduction to Process Dynamics and Transfer Function, 89
- 4.9 Dynamic Systems and Transfer Functions, 90
- 4.10 Difference Equations to Represent Dynamic Relations, 90
- 4.11 Representing Dynamics of Industrial Process, 96
- 4.12 Transfer Function Models Using Difference Equations, 97

4.13	Stable and Unstable Systems, 98 Problems, 100	
5	Nonstationary Time Series Models for Process Disturbances	103
5.1	Reprise, 103	
5.2	Stationary Time Series Model in Which Successive Values Are Correlated, 104	
5.3	Major Effects of Statistical Dependence: Illustration, 105	
5.4	Random Walk, 106	
5.5	How to Test a Forecasting Method, 107	
5.6	Qualification of EWMA As a Forecast, 107	
5.7	Understanding Time Series Behavior with Variogram, 110	
5.8	Sticky Innovation Generating Model for Nonstationary Noise, 113	
5.9	Robustness of EWMA for Signal Extraction, 118	
5.10	Signal Extraction for Disturbance Model Due to Barnard, 118 Questions, 122 Problems, 122	
6	Repeated-Feedback Adjustment	125
6.1	Introduction to Discrete-Feedback Control, 125	
6.2	Inadequacy of NIID Models and Other Stationary Models for Control: Reiteration, 125	
6.3	Three Approaches to Repeated-Feedback Adjustment that Lead to Identical Conclusions, 126	
6.4	Some History, 130	
6.5	Adjustment Chart, 132	
6.6	Insensitivity to Choice of G , 134	
6.7	Compromise Value for G , 135	
6.8	Using Smaller Value of G to Reduce Adjustment Variance σ_x^2 , 136 Appendix 6A: Robustness of Integral Control, 137 Appendix 6B: Effect on Adjustment of Choosing G Different from λ_0 : Obtaining Equation (6.12), 139 Appendix 6C: Average Reduction in Mean-Square Error Due to Adjustment for Observations Generated by IMA Model, 140 Questions, 140 Problems, 140	

7	Periodic Adjustment	143
7.1	Introduction, 143	
7.2	Periodic Adjustment, 143	
7.3	Starting Scheme for Periodic Adjustment, 146	
7.4	Numerical Calculations for Bounded Adjustment, 146	
7.5	Simple Device for Facilitating Bounded Adjustment, 150	
7.6	Bounded Adjustment Seen as Process of Tracking, 153	
7.7	Combination of Adjustment and Monitoring, 153	
7.8	Bounded Adjustment for Series not Generated by IMA Model, 155	
	Problems, 160	
8	Control of Process with Inertia	163
8.1	Adjustment Depending on Last Two Output Errors, 163	
8.2	Minimum Mean-Square Error Control of Process With First-Order Dynamics, 167	
8.3	Schemes with Constrained Adjustment, 169	
8.4	PI Schemes with Constrained Adjustment, 170	
8.5	Optimal and Near-Optimal Constrained PI Schemes: Choice of P , 171	
8.6	Choice of G For $P = 0$ and $P = -0.25$, 172	
8.7	PI Schemes for Process With Dead Time, 178	
8.8	Process Monitoring and Process Adjustment, 181	
8.9	Feedback Adjustment Applied to Process in Perfect State of Control, 182	
8.10	Using Shewhart Chart to Adjust Unstable Process, 182	
8.11	Feedforward Control, 183	
	Appendix 8A: Equivalence of Equations for PI Control, 184	
	Appendix 8B: Effect of Errors in Adjustment, 184	
	Appendix 8C: Choices for G and P to Attain Optimal Constrained PI Control for Various Values of λ_0 and δ_0 with $d_0 = 0$ and $d_0 = 1$, 185	
	Questions, 191	
	Problems, 191	
9	Explicit Consideration of Monetary Cost	193
9.1	Introduction, 193	
9.2	How Often Should You Take Data?, 197	
9.3	Choosing Adjustment Schemes Directly in Terms of Costs, 203	

Appendix 9A: Functions $h(L/\lambda\sigma_a)$ and $q(L/\lambda\sigma_a)$ in Table 9.1, 205

Appendix 9B: Calculation of Minimum-Cost Schemes, 205 Problems, 207

10 Cuscore Charts: Looking for Signals in Noise 209

10.1 Introduction, 209

10.2 How Are Cuscore Statistics Obtained?, 216

10.3 Efficient Monitoring Charts, 219

10.4 Useful Method for Obtaining Detector When Looking for Signal in Noise Not Necessarily White Noise, 221

10.5 Looking for Single Spike, 223

10.6 Some Time Series Examples, 224

Appendix 10A: Likelihood, Fisher's Efficient Score, and Cuscore Statistics, 227

Appendix 10B: Useful Procedure for Obtaining Appropriate Cuscore Statistic, 230

Appendix 10C: Detector Series for IMA Model, 231 Problems, 231

11 Monitoring an Operating Feedback System 235

11.1 Looking for Spike in Disturbance z_t Subjected to Integral Control, 235

11.2 Looking for Exponential Signal in Disturbance Subject to Integral Control, 237

11.3 Monitoring Process with Inertia Represented by First-Order Dynamics, 238

11.4 Reconstructing Disturbance Pattern, 240

Appendix 11A: Derivation of Equation (11.3), 240

Appendix 11B: Derivation of Equation (11.10), 242

Appendix 11C: Derivation of Equation (11.14), 243

12 Brief Review of Time Series Analysis 245

12.1 Serial Dependence: Autocorrelation Function and Variogram, 245

12.2 Relation of Autocorrelation Function and Variogram, 246

12.3 Some Time Series Models, 247

12.4 Stationary Models, 247

12.5 Autoregressive Moving-Average Models, 250

12.6 Nonstationary Models, 253

12.7	IMA [or ARIMA(0, 1, 1)] Model, 253	
12.8	Modeling Time Series Data, 255	
12.9	Model Identification, Model Fitting, and Diagnostic Checking, 256	
12.10	Forecasting, 261	
12.11	Estimation with Closed-Loop Data, 266	
12.12	Conclusion, 269	
	Appendix 12A: Other Tools for Identification of Time Series Models, 269	
	Appendix 12B: Estimation of Time Series Parameters, 270	
Solutions to Exercises and Problems		273
References and Further Reading		307
Appendix Three Time Series		321
Index		327

Preface

In 1962, in discussing a paper that had been presented to the Royal Statistical Society, Professor J. H. Westcott said, “Speaking as a control engineer, I welcome this flirtation between control engineering and statistics. I doubt however they can yet be said to be ‘going steady.’” The reference is to the first of the series of articles by Box and Jenkins (1962, 1963, 1966, 1968, 1970), by Box and MacGregor (1974) and by MacGregor (1972, 1976). These addressed statisticians and quality professionals about some elementary ideas from engineering process control (EPC) which could supplement and improve statistical process control (SPC). Although at the time these initiatives did not excite much interest, there is now considerable activity in this area. The purpose of this book is to bring together these two kinds of process control and explain the less familiar ideas to a wider audience.

Shewhart charts are monitoring procedures for debugging the process and have two aspects:

1. Plotting and displaying developing process data
2. Drawing of limit lines to indicate when deviations from the norm are sufficiently large to justify the search for special causes

The first of these is very powerful and can account for many successes. However, the second assumes a fixed mean and sequential independence of process data. These approximations are frequently misleading.

This leads to the conclusion that process variation must be represented by *non-stationary* models that have no fixed mean. Analysis of industrial data confirms this. Such analysis frequently points to a particular nonstationary model called the *integrated moving average* (IMA). This model does not suffer from the deficiencies mentioned above. In particular, it requires that observations are measured *not* from a fixed mean but from a local adaptive average: the exponentially weighted moving average (EWMA).

The IMA model produces *two* component series. The first is a sequence of EWMA's which can estimate the current deviation from target. It shows when a serious deviation has occurred and calls for appropriate process adjustment. The second series is a sequence of residuals that can be used for process monitoring

and are plotted on a Shewhart chart that can suggest, find, and eliminate assignable causes.

So that the reader can quickly appreciate these important concepts, an example is given early in the book (Section 3.5.2) in which 200 observations from a nonstationary system are analyzed. The adjustment chart shows that only two adjustments are needed to remove major nonstationarity. Also the accompanying monitoring chart reveals a special cause midway through the series. (To fully understand such procedures and apply them to other examples require an elementary knowledge of a number of topics that do not appear in most discussions of SPC.) These include forecasting and process dynamics (Chapter 4), nonstationary time series (Chapter 5), discrete feedback control (Chapter 6), control by periodic adjustment (Chapter 7), control of a process with inertia (Chapter 8), minimizing cost of control (Chapter 9), efficient search for signals in noise using cuscore statistics (Chapter 10), monitoring an operating feedback system (Chapter 11), and review of time series analysis (Chapter 12). There are two preliminary chapters useful for revision or for those with no previous knowledge of SPC. Chapter 1 describes some necessary statistical tools and Chapter 2 standard SPC methods and how they would work if the assumptions were true. If you are familiar with the basic ideas of SPC, you can omit these two chapters.

Earlier drafts of the first edition of this book (under the title *Statistical Control by Monitoring and Feedback Adjustment*) were begun by one of the authors while a visiting fellow at the Institute of Advanced Study in the Behavioral Sciences at Stanford, a visit partially supported by a grant from the Guggenheim Foundation. The authors are also grateful for support from the National Science Foundation (Grant DMI-9414765), the Dirección General de Investigación Científica y Técnica of Spain (Grant PB95-0583), the Consejo Nacional de Ciencia y Tecnología of México (CONACYT, Grant 135617), and the University of Cantabria, Santander, Spain.

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CHAPTER 1

Introduction and Revision of Some Statistical Ideas

Now here, you see it takes all the running you can do, to keep in the same place.

Through the Looking Glass, LEWIS CARROLL

This chapter provides a quick revision of some basic ideas of statistics necessary for process control. These include dot diagrams, probability distributions, mean, variance, standard deviation, properties of averages, the central limit theorem, and the normal, Poisson, and binomial distributions.

1.1 NECESSITY FOR PROCESS CONTROL

If you have a house, you know that you must work hard to keep it habitable—the tiles on the roof, the paint on the walls, the washing machine, the refrigerator, the television, all need attention from time to time. A car, a friendship, and our own bodies must similarly be continually nurtured or they will not remain in shape very long. The same is true for industrial processes. If left to themselves, machines do not stay adjusted, components wear out, and managers and operators forget, miscommunicate, and change jobs. Thus a stable stationary state is an unnatural one and its approximate achievement requires a hard and continuous fight. Both process *monitoring* and process *adjustment* can help achieve this. Both are likely to be needed.

1.2 SPC AND EPC

Some 80 years ago Walter Shewhart introduced statistical process control (SPC) using charts. These quality control charts have been widely applied, especially in the parts industries, such as automobile manufacture, and have resulted in dramatic savings and important improvements in quality of products. Control charts are devices used to *monitor* quality characteristics and so keep them as close as possible

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