SHIFTING THE EARTH

The Mathematical Quest to Understand the Motion of the Universe

ARTHUR MAZER

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PREFACE

There is a comic of a skinny little kid who, while standing on a stool so that he can look at himself in a mirror, views the image of a ripping, six-packed, Shwartznegger-esque champ. It's all too true to life and, like the best of comics, begs a question. Suppose that the same skinny kid looks at the mirror 35 years later when he is a pudgy, balding man. What is the image that he will see? Will he continue to see the champ, or will he have come to accept who he is, but in doing so see more possibilities for himself than the little boy could have imagined? This question is the topic of many novels and movies. It is intriguing because there is no predestined path for the boy to follow and there is no definitive outcome.

Shifting the Earth relates the story of how humanity collectively dispossessed itself of its geocentric fabrication and accepted a less prestigious position in the universe. It was not preordained that humanity would toss out its self-image, pack up what remained, and move to a new heliocentric earth. Psychological barriers resisted the move. Vested interests battled to bound humanity on a geocentric earth, and while they did not prevail, they put up a good fight. One can even imagine a scenario where they might have prevailed. The mathematicians and astronomers who contributed to the accomplishment did not live in a vacuum and were very much affected by their cultural environment. *Shifting the Earth* describes of the interplay between the forces tugging at the contributors in different directions and how the contributors ultimately created a prevailing force of their own.

The question of a heliocentric versus geocentric structure was indisputably resolved in the seventeenth century, a century steeped in the controversies of the Counter-Reformation. The accomplishment was the conduit to the Enlightenment and the subsequent scientific–industrial revolution that has transformed our lives. The accomplishment has received short shrift within the historical literature where philosophy lies at the center of historical evolution. *Shifting the Earth* holds the perspective that since Newton proposed his laws of motion with the explicit intent of determining a planet's pathway, the scientific revolution has been the main driver of history. Philosophy has become a backseat passenger that must either adapt to the discoveries of the scientists, or be forcibly removed by another philosophy that wants to get on board. Starting with the ancient Greeks, *Shifting the Earth* examines this transition as it unfolds.

Since the investigation into the universal order began, mathematics has guided the investigators and has also been the language through which they convey their results. This is true for geocentric theories as well as the development of the heliocentric theory. Ptolemy's universe rests firmly on Euclid's mathematical foundation, which Ptolemy skillfully exploits to describe heaven's trajectories. Fifteen centuries later, Isaac Newton's universe rests firmly on Newtonian calculus and axioms of motion. In the interim, mathematics wielded in the hands of mathematicians and astronomers was the prodding rod shepherding humanity from a geocentric to a heliocentric world. Accordingly, mathematics is central to the story and occupies a central role in *Shifting the Earth*. Each chapter includes a mathematical treatment of significant contributions to both the geocentric and heliocentric theories.

This book encompasses two areas, history and mathematics, which our culture normally segregates. There is no delineated boundary between these areas. History subsumes mathematics, and mathematics influences history, so the segregation is as unnatural as separating water from broth. Both constitute the chicken soup, and with the same sentiment, I uphold that history and mathematics both constitute this story.

As a preview, let me provide an offering that highlights the interplay between the history and the mathematics. The preeminent European scientist of his day, Johannes Kepler, using methods that Archimedes developed two millennia before Kepler, asserts conservation of angular momentum in integral form, and uses this assertion to hunt down the ellipse. With his discovery of the ellipse as the planetary pathway, Kepler becomes Europe's most formidable advocate of a heliocentric system. Kepler is a celebrity throughout Europe, and the Jesuit community in Vienna woos Kepler, an excommunicated Lutheran, with the hope of converting him to Catholicism. During this same time, the Jesuits in Rome were leading an Inquisition against Galileo, charging him with heresy on the basis of his arguments in favor of a heliocentric universe.

The situation is pregnant with both mathematical and historical questions. What mathematics did Archimedes develop two millennia before Kepler that assisted Kepler with his discovery? If the mathematics had been in place for so long, why did it take two from the time of Archimedes to discover the true heliocentric system? Why did the Lutherans excommunicate Kepler? Why did the Jesuits woo Kepler, while at the same time persecute Galileo? Answers to these questions require a historical as well as mathematical response. Our story must address all of these questions, or the story will be incomplete.

Concerning the mathematics, I had a few choices to make. A first decision was to determine which material to include. The combined works of Copernicus, Kepler, Galileo, and Newton are well over 2000 pages. Add to that the works of Eudoxus, Aristarchus, Archimedes, Apollonius, Hipparchus, and Ptolemy, and one can conceive of over 3000 pages of material. Clearly, this had to be pared down. I applied subjective judgment weighing two objectives, historical significance, and direct significance, to the completion of a heliocentric theory. The result is that I've sheared the mathematical hedge into a shape that pleases me, recognizing that others would do so differently.

My subjective judgment is guided by the manner in which the discovery of the ellipse as the planetary pathway unfolds. Looking backward through time, Kepler discovered the pathway for the planet Mars using Tycho Brahe's observations. Both Copernicus and Ptolemy also present their models of Mars' motion, and there is a chain that links all three works. The presentation focuses on the planet Mars so that the links in the chain are made visible.

Just as there is no delineation between mathematics and history, there is no distinct historical line that marks the beginning or the end of this story. I chose to begin with Eudoxus, a contemporary of Plato and Aristotle who gives the first known systematic, mathematical model of the motion of the heavens in the ancient Western world. One could begin at an earlier time, looking back to the Babylonians, or a later time, with Ptolemy. The former was excluded because of a blur in our knowledge of the influence of pre-Eudoxian astronomy on Eudoxus. On the mathematical front, while Eudoxus' model is not a visible link in the Ptolemaic–Copernican–Keplerian chain, it initiates Western mathematical analysis of the universe's motions and as such it is that chain's invisible anchor. On the historical front, it was the Athenian culture that fostered not only Eudoxus but also Ptolemy. To delve into Ptolemy without giving some historical context is akin to a read of Abraham Lincoln's signature Gettysburg Address without an awareness of the issues causing the Civil War. I begin with Eudoxus and Athens because they leave their signature on all that follows.

As for the story's endpoint, the decision was more difficult. Had I written this book before the twentieth century, the distinct historical line marking the end would have been unmistakable. Newtonian reasoning was the final assault that deflated the geocentric myth and launched Europe into the Enlightenment. So successful were Newton's laws and so powerful were the mathematical tools he bequeathed that another myth displaced the geocentric myth; all would be understood by Newtonian reasoning. The twentieth century revealed the existence of a distinct historical Newtonian line but also that the universe is more than the physical manifestation of Newtonian reasoning. We now know that the universe behaves in ways that completely challenge our notions of what is reasonable. This realization is inherent in our story where our balding man looks at himself in the mirror and then outwardly toward the universe and sees previously unimagined possibilities in both. Einstein was the first to expose a bizarre universe and convince a mystified world of its reality. Einstein is where I choose to end the story.

While I believe that mathematics is central to the story, popular sentiment may disagree. Showing a flexible disposition, I accommodate popular sentiment by writing in a style that allows readers to engage the mathematics to their level of comfort. If you do not find the mathematics engaging, don't fret, skip over the mathematics and enjoy the narrative.

Let me finish this preface by addressing the reader who is interested in following some or all of the mathematical content. A decision point in presenting mathematical material concerns the presentation of works that predate modern mathematical concepts and notation. I had to decide whether to derive results using only the tools available to the original authors or make full use of modern mathematics. Clarity of exposition guided my decision. So that their bosses in the State Department can understand their reports, a China specialist writes in English. Similarly, I convey results using modern concepts and notation that are familiar to the present-day reader.

Another decision that is inherently bound with the choice of material, addresses the mathematical prerequisites. My efforts are aimed at reaching as broad an audience as possible, and that means keeping the prerequisites at a minimum. In all chapters except Chapters 2 and 4, I endeavor to present material at its simplest level. This means that whenever a conflict between presenting elementary mathematics or more technically advanced mathematics arises, I choose the path of an elementary exposition even if it is the less succinct. For example, I give a mathematical presentation of Newton's laws without the use of calculus. Indeed, with the exception of Chapters 2 and 4, the mathematical material is accessible to a high school student. Okay, I confess, I couldn't resist giving a calculus-based proof of the ellipse. With the exceptions of Chapter 2, Chapter 4, and the calculus-based section of Newtonian mechanics in Chapter 8, all of the material is accessible to a high school student.

Concerning Chapter 2, this material is somewhat tangential to the remainder of the book. Chapter 2 presents Eudoxus' universe. Eudoxus' work is all the more remarkable in that he accomplished his task prior to the development of foundational mathematics that I needed to describe it. With college-level mathematics, the model becomes tractable. Without college-level mathematics, I couldn't imagine how one gives a mathematical description, let alone how one solves the problem. I had two choices, not include the work or use higher-level mathematics, and for reasons described above, I chose the latter.

Chapter 4 presents work from Apollonius' *On Conics*. As with Eudoxus, this work is tangential to the remaining material and far ahead of its time. While this material could be presented at an elementary level without the use of calculus, in this one instance I chose to be succinct. Aside from succinctness, the use of calculus allows for an exploration into concepts that Apollonius hints at, but are now fully developed. The mathematical material of Chapters 2 and 4 is independent of the material in the rest the book. The reader without the prerequisites may skip these sections and follow everything else.

On a final note, I was quite pleased by the accessibility of special relativity. One can give a rather complete account of the mechanical side of special relativity and remain fully bounded within a standard high school mathematics curriculum. In fact, key findings of Einstein, those that assault our common sense, are more readily accessible than the fully intuitive works of Newton. It is my hope that *Shifting the Earth* is an agent of the assault.