

Handbook of Monte Carlo Methods

Dirk P. Kroese

University of Queensland

Thomas Taimre

University of Queensland

Zdravko I. Botev

Université de Montréal



A JOHN WILEY & SONS, INC., PUBLICATION

This page intentionally left blank

Handbook of Monte Carlo Methods

WILEY SERIES IN PROBABILITY AND STATISTICS

Established by WALTER A. SHEWHART and SAMUEL S. WILKS

Editors: *David J. Balding, Noel A. C. Cressie, Garrett M. Fitzmaurice,
Iain M. Johnstone, Geert Molenberghs, David W. Scott, Adrian F. M. Smith,
Ruey S. Tsay, Sanford Weisberg*

Editors Emeriti: *Vic Barnett, J. Stuart Hunter, Joseph B. Kadane, Jozef L. Teugels*

A complete list of the titles in this series appears at the end of this volume.

Handbook of Monte Carlo Methods

Dirk P. Kroese

University of Queensland

Thomas Taimre

University of Queensland

Zdravko I. Botev

Université de Montréal



A JOHN WILEY & SONS, INC., PUBLICATION

Copyright © 2011 by John Wiley & Sons, Inc. All rights reserved

Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permission>.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Kroese, Dirk P.

Handbook for Monte Carlo methods / Dirk P. Kroese, Thomas Taimre, Zdravko I. Botev.

p. cm. — (Wiley series in probability and statistics ; 706)

Includes index.

ISBN 978-0-470-17793-8 (hardback)

1. Monte Carlo method. I. Taimre, Thomas, 1983– II. Botev, Zdravko I., 1982– III. Title.

QA298.K76 2011

518'.282—dc22

2010042348

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

To Lesley

— DPK

To Aita and Ilmar

— TT

To my parents, Maya and Ivan

— ZIB

This page intentionally left blank

CONTENTS

Preface	xvii
Acknowledgments	xix
1 Uniform Random Number Generation	1
1.1 Random Numbers	1
1.1.1 Properties of a Good Random Number Generator	2
1.1.2 Choosing a Good Random Number Generator	3
1.2 Generators Based on Linear Recurrences	4
1.2.1 Linear Congruential Generators	4
1.2.2 Multiple-Recursive Generators	5
1.2.3 Matrix Congruential Generators	6
1.2.4 Modulo 2 Linear Generators	6
1.3 Combined Generators	8
1.4 Other Generators	10
1.5 Tests for Random Number Generators	11
1.5.1 Spectral Test	12
1.5.2 Empirical Tests	14
References	21

2	Quasirandom Number Generation	25
2.1	Multidimensional Integration	25
2.2	Van der Corput and Digital Sequences	27
2.3	Halton Sequences	29
2.4	Faure Sequences	31
2.5	Sobol' Sequences	33
2.6	Lattice Methods	36
2.7	Randomization and Scrambling	38
	References	40
3	Random Variable Generation	43
3.1	Generic Algorithms Based on Common Transformations	44
3.1.1	Inverse-Transform Method	45
3.1.2	Other Transformation Methods	47
3.1.3	Table Lookup Method	55
3.1.4	Alias Method	56
3.1.5	Acceptance-Rejection Method	59
3.1.6	Ratio of Uniforms Method	66
3.2	Generation Methods for Multivariate Random Variables	67
3.2.1	Copulas	68
3.3	Generation Methods for Various Random Objects	70
3.3.1	Generating Order Statistics	70
3.3.2	Generating Uniform Random Vectors in a Simplex	71
3.3.3	Generating Random Vectors Uniformly Distributed in a Unit Hyperball and Hypersphere	74
3.3.4	Generating Random Vectors Uniformly Distributed in a Hyperellipsoid	75
3.3.5	Uniform Sampling on a Curve	75
3.3.6	Uniform Sampling on a Surface	76
3.3.7	Generating Random Permutations	79
3.3.8	Exact Sampling From a Conditional Bernoulli Distribution	80
	References	83
4	Probability Distributions	85
4.1	Discrete Distributions	85
4.1.1	Bernoulli Distribution	85
4.1.2	Binomial Distribution	86
4.1.3	Geometric Distribution	91
4.1.4	Hypergeometric Distribution	93
4.1.5	Negative Binomial Distribution	94

4.1.6	Phase-Type Distribution (Discrete Case)	96
4.1.7	Poisson Distribution	98
4.1.8	Uniform Distribution (Discrete Case)	101
4.2	Continuous Distributions	102
4.2.1	Beta Distribution	102
4.2.2	Cauchy Distribution	106
4.2.3	Exponential Distribution	108
4.2.4	F Distribution	109
4.2.5	Fréchet Distribution	111
4.2.6	Gamma Distribution	112
4.2.7	Gumbel Distribution	116
4.2.8	Laplace Distribution	118
4.2.9	Logistic Distribution	119
4.2.10	Log-Normal Distribution	120
4.2.11	Normal Distribution	122
4.2.12	Pareto Distribution	125
4.2.13	Phase-Type Distribution (Continuous Case)	126
4.2.14	Stable Distribution	129
4.2.15	Student's t Distribution	131
4.2.16	Uniform Distribution (Continuous Case)	134
4.2.17	Wald Distribution	135
4.2.18	Weibull Distribution	137
4.3	Multivariate Distributions	138
4.3.1	Dirichlet Distribution	139
4.3.2	Multinomial Distribution	141
4.3.3	Multivariate Normal Distribution	143
4.3.4	Multivariate Student's t Distribution	147
4.3.5	Wishart Distribution	148
	References	150
5	Random Process Generation	153
5.1	Gaussian Processes	154
5.1.1	Markovian Gaussian Processes	159
5.1.2	Stationary Gaussian Processes and the FFT	160
5.2	Markov Chains	162
5.3	Markov Jump Processes	166
5.4	Poisson Processes	170
5.4.1	Compound Poisson Process	174
5.5	Wiener Process and Brownian Motion	177
5.6	Stochastic Differential Equations and Diffusion Processes	183
5.6.1	Euler's Method	185
5.6.2	Milstein's Method	187

5.6.3	Implicit Euler	188
5.6.4	Exact Methods	189
5.6.5	Error and Accuracy	191
5.7	Brownian Bridge	193
5.8	Geometric Brownian Motion	196
5.9	Ornstein–Uhlenbeck Process	198
5.10	Reflected Brownian Motion	200
5.11	Fractional Brownian Motion	203
5.12	Random Fields	206
5.13	Lévy Processes	208
5.13.1	Increasing Lévy Processes	211
5.13.2	Generating Lévy Processes	214
5.14	Time Series	219
	References	222
6	Markov Chain Monte Carlo	225
6.1	Metropolis–Hastings Algorithm	226
6.1.1	Independence Sampler	227
6.1.2	Random Walk Sampler	230
6.2	Gibbs Sampler	233
6.3	Specialized Samplers	240
6.3.1	Hit-and-Run Sampler	240
6.3.2	Shake-and-Bake Sampler	251
6.3.3	Metropolis–Gibbs Hybrids	256
6.3.4	Multiple-Try Metropolis–Hastings	257
6.3.5	Auxiliary Variable Methods	259
6.3.6	Reversible Jump Sampler	269
6.4	Implementation Issues	273
6.5	Perfect Sampling	274
	References	276
7	Discrete Event Simulation	281
7.1	Simulation Models	281
7.2	Discrete Event Systems	283
7.3	Event-Oriented Approach	285
7.4	More Examples of Discrete Event Simulation	289
7.4.1	Inventory System	289
7.4.2	Tandem Queue	293
7.4.3	Repairman Problem	296
	References	300

8	Statistical Analysis of Simulation Data	301
8.1	Simulation Data	301
8.1.1	Data Visualization	302
8.1.2	Data Summarization	303
8.2	Estimation of Performance Measures for Independent Data	305
8.2.1	Delta Method	308
8.3	Estimation of Steady-State Performance Measures	309
8.3.1	Covariance Method	309
8.3.2	Batch Means Method	311
8.3.3	Regenerative Method	313
8.4	Empirical Cdf	316
8.5	Kernel Density Estimation	319
8.5.1	Least Squares Cross Validation	321
8.5.2	Plug-in Bandwidth Selection	326
8.6	Resampling and the Bootstrap Method	331
8.7	Goodness of Fit	333
8.7.1	Graphical Procedures	334
8.7.2	Kolmogorov–Smirnov Test	336
8.7.3	Anderson–Darling Test	339
8.7.4	χ^2 Tests	340
	References	343
9	Variance Reduction	347
9.1	Variance Reduction Example	348
9.2	Antithetic Random Variables	349
9.3	Control Variables	351
9.4	Conditional Monte Carlo	354
9.5	Stratified Sampling	356
9.6	Latin Hypercube Sampling	360
9.7	Importance Sampling	362
9.7.1	Minimum-Variance Density	363
9.7.2	Variance Minimization Method	364
9.7.3	Cross-Entropy Method	366
9.7.4	Weighted Importance Sampling	368
9.7.5	Sequential Importance Sampling	369
9.7.6	Response Surface Estimation via Importance Sampling	373
9.8	Quasi Monte Carlo	376
	References	379

10	Rare-Event Simulation	381
10.1	Efficiency of Estimators	382
10.2	Importance Sampling Methods for Light Tails	385
10.2.1	Estimation of Stopping Time Probabilities	386
10.2.2	Estimation of Overflow Probabilities	389
10.2.3	Estimation For Compound Poisson Sums	391
10.3	Conditioning Methods for Heavy Tails	393
10.3.1	Estimation for Compound Sums	394
10.3.2	Sum of Nonidentically Distributed Random Variables	396
10.4	State-Dependent Importance Sampling	398
10.5	Cross-Entropy Method for Rare-Event Simulation	404
10.6	Splitting Method	409
	References	416
11	Estimation of Derivatives	421
11.1	Gradient Estimation	421
11.2	Finite Difference Method	423
11.3	Infinitesimal Perturbation Analysis	426
11.4	Score Function Method	428
11.4.1	Score Function Method With Importance Sampling	430
11.5	Weak Derivatives	433
11.6	Sensitivity Analysis for Regenerative Processes	435
	References	438
12	Randomized Optimization	441
12.1	Stochastic Approximation	441
12.2	Stochastic Counterpart Method	446
12.3	Simulated Annealing	449
12.4	Evolutionary Algorithms	452
12.4.1	Genetic Algorithms	452
12.4.2	Differential Evolution	454
12.4.3	Estimation of Distribution Algorithms	456
12.5	Cross-Entropy Method for Optimization	457
12.6	Other Randomized Optimization Techniques	460
	References	461
13	Cross-Entropy Method	463
13.1	Cross-Entropy Method	463
13.2	Cross-Entropy Method for Estimation	464
13.3	Cross-Entropy Method for Optimization	468
13.3.1	Combinatorial Optimization	469

13.3.2	Continuous Optimization	471
13.3.3	Constrained Optimization	473
13.3.4	Noisy Optimization	476
	References	477
14	Particle Methods	481
14.1	Sequential Monte Carlo	482
14.2	Particle Splitting	485
14.3	Splitting for Static Rare-Event Probability Estimation	486
14.4	Adaptive Splitting Algorithm	493
14.5	Estimation of Multidimensional Integrals	495
14.6	Combinatorial Optimization via Splitting	504
14.6.1	Knapsack Problem	505
14.6.2	Traveling Salesman Problem	506
14.6.3	Quadratic Assignment Problem	508
14.7	Markov Chain Monte Carlo With Splitting	509
	References	517
15	Applications to Finance	521
15.1	Standard Model	521
15.2	Pricing via Monte Carlo Simulation	526
15.3	Sensitivities	538
15.3.1	Pathwise Derivative Estimation	540
15.3.2	Score Function Method	542
	References	546
16	Applications to Network Reliability	549
16.1	Network Reliability	549
16.2	Evolution Model for a Static Network	551
16.3	Conditional Monte Carlo	554
16.3.1	Leap-Evolve Algorithm	560
16.4	Importance Sampling for Network Reliability	562
16.4.1	Importance Sampling Using Bounds	562
16.4.2	Importance Sampling With Conditional Monte Carlo	565
16.5	Splitting Method	567
16.5.1	Acceleration Using Bounds	573
	References	574
17	Applications to Differential Equations	577
17.1	Connections Between Stochastic and Partial Differential Equations	577