

SYSTEM IDENTIFICATION

A Frequency Domain Approach

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To all people of goodwill

Rik

To Annick, Ine, Maarten, and Sanne

Johan

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