# DESIGN AND ANALYSIS OF COMPOSITE STRUCTURES WITH APPLICATIONS TO AEROSPACE STRUCTURES

**Christos Kassapoglou** Delft University of Technology, The Netherlands



# DESIGN AND ANALYSIS OF COMPOSITE STRUCTURES

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## Contents

About the Author		ix
Serie	s Preface	X
Prefa	ace	xi
1	Applications of Advanced Composites in Aircraft Structures	1
Refe	rences	7
2	Cost of Composites: a Qualitative Discussion	9
2.1	Recurring Cost	10
2.2	Nonrecurring Cost	18
2.3	Technology Selection	20
2.4	Summary and Conclusions	27
Exer	cises	30
Refe	rences	30
3	Review of Classical Laminated Plate Theory	33
3.1	Composite Materials: Definitions, Symbols and Terminology	33
3.2	Constitutive Equations in Three Dimensions	35
	3.2.1 Tensor Transformations	37
3.3	Constitutive Equations in Two Dimensions: Plane Stress	39
Exer	cises	52
Refe	rences	53
4	Review of Laminate Strength and Failure Criteria	55
4.1	Maximum Stress Failure Theory	57
4.2	Maximum Strain Failure Theory	58
4.3	Tsai–Hill Failure Theory	58
4.4	Tsai–Wu Failure Theory	59
4.5	Other Failure Theories	59
Refe	rences	60
5	Composite Structural Components and Mathematical Formulation	63
5.1	Overview of Composite Airframe	63
	5.1.1 The Structural Design Process: The Analyst's Perspective	64

	5.1.2	Basic Design Concept and Process/Material Considerations		
		for Aircraft Parts	69	
	5.1.3	Sources of Uncertainty: Applied Loads, Usage and Material Scatter	72	
	5.1.4	Environmental Effects	75	
	5.1.5	Effect of Damage	76	
	5.1.6	Design Values and Allowables	78	
	5.1.7	Additional Considerations of the Design Process	81	
5.2	Govern	ing Equations	82	
	5.2.1	Equilibrium Equations	82	
	5.2.2	Stress–Strain Equations	84	
	5.2.3	Strain-Displacement Equations	85	
	5.2.4	von Karman Anisotropic Plate Equations for Large Deflections	86	
5.3	Reduct	ions of Governing Equations: Applications to Specific Problems	91	
	5.3.1	Composite Plate Under Localized in-Plane Load	92	
	5.3.2	Composite Plate Under Out-of-Plane Point Load	103	
5.4	Energy	Methods	106	
	5.4.1	Energy Expressions for Composite Plates	107	
Exerc	cises		113	
Refe	rences		116	
6	Buckli	ng of Composite Plates	119	
61	Bucklin	ng of Rectangular Composite Plate under Biaxial Loading	119	
6.2	Bucklin	ng of Rectangular Composite Plate under Uniaxial Compression	122	
0.2	6.2.1	Uniaxial Compression. Three Sides Simply Supported. One Side Free	124	
6.3	Bucklin	ng of Rectangular Composite Plate under Shear	127	
6.4	Buckli	ng of Long Rectangular Composite Plates under Shear	129	
6.5	Buckli	Composite Plates under Combined Loads		
6.6	Design	Equations for Different Boundary Conditions and Load		
	Combi	nations	138	
Exerc	cises		141	
Refe	rences		143	
7	Post-B	uckling	145	
7.1	Post-B	uckling Analysis of Composite Panels under Compression	149	
	7.1.1	Application: Post-Buckled Panel Under Compression	157	
7.2	Post-B	uckling Analysis of Composite Plates under Shear	159	
	7.2.1	Post-buckling of Stiffened Composite Panels under Shear	163	
	7.2.2	Post-buckling of Stiffened Composite Panels under Combined		
		Uniaxial and Shear Loading	171	
Exerc	cises		174	
Refe	rences		177	
8	Design	and Analysis of Composite Beams	179	
8.1	Cross-s	section Definition Based on Design Guidelines	179	
8.2	Cross-s	sectional Properties	182	
8.3	Colum	n Buckling	188	

8.4	Beam of	on an Elastic Foundation under Compression	189
8.5	Crippli	ng	194
	8.5.1	One-Edge-Free (OEF) Crippling	196
	8.5.2	No-Edge-Free (NEF) Crippling	200
	8.5.3	Crippling under Bending Loads	202
	8.5.4	Crippling of Closed-Section Beams	207
86	Import	ance of Radius Regions at Flange Intersections	207
87	Inter-ri	vet Buckling of Stiffener Flanges	210
8.8	Applic	ation: Analysis of Stiffeners in a Stiffened Panel under Compression	210
Everc	ises	aton. Analysis of Surferens in a Surferend Farer under Compression	213
Dofor	ancas		210
Kelei	cilces		
9	Skin-S	tiffened Structure	223
9.1	Smeari	ng of Stiffness Properties (Equivalent Stiffness)	223
	9.1.1	Equivalent Membrane Stiffnesses	223
	912	Equivalent Bending Stiffnesses	225
92	Failure	Modes of a Stiffened Panel	223
.2	9 2 1	Local Ruckling (Retween Stiffeners) Versus Overall Panel Ruckling	227
	2.2.1	(the Panel Breaker Condition)	228
	022	Skin_Stiffener Senaration	220
03	Δ dditic	anal Considerations for Stiffened Panels	250
).5	031	'Pinching' of Skin	251
	0.3.1	Co Curing Versus Ronding Versus Eastening	251
Evere	9.3.2	Co-Curing versus bonding versus rustening	251
Dafar	1505		255
Kelei	ences		238
10	Sandw	ich Structure	259
10.1	Sandwi	ich Bending Stiffnesses	260
10.2	Bucklin	ng of Sandwich Structure	262
	10.2.1	Buckling of Sandwich Under Compression	262
	10.2.2	Buckling of Sandwich Under Shear	264
	10.2.3	Buckling of Sandwich Under Combined Loading	265
10.3	Sandwi	ich Wrinkling	265
	10.3.1	Sandwich Wrinkling Under Compression	265
	10.3.2	Sandwich Wrinkling Under Shear	276
10.4	10.3.3 San Jari	Sanawich Wrinkling Under Combined Loads	270
10.4	Sandwi		278
	10.4.1 10.4.2	Sandwich Crimping Under Compression Sandwich Crimping Under Shear	278
10.5	Sandwi	ich Intracellular Buckling (Dimpling) under Compression	278
10.6	Attachi	ing Sandwich Structures	279
	10.6.1	Core Ramp-Down Regions	280
	10.6.2	Alternatives to Core Ramp-Down	282
Exerc	ises	-	284
Refer	ences		288

11	Good Design Practices and Design 'Rules of Thumb'	289
11.1	Lay up/Stacking Sequence-related	289
11.2	Loading and Performance-related	290
11.3	Guidelines Related to Environmental Sensitivity and	
	Manufacturing Constraints	292
11.4	Configuration and Layout-related	292
Exercises		294
Refer	295	
Index	K	297

### About the Author

Christos Kassapoglou received his BS degree in Aeronautics and Astronautics and two MS degrees (Aeronautics and Astronautics and Mechanical Engineering) all from Massachusetts Institute of Technology. Since 1984 he has worked in industry, first at Beech Aircraft on the all-composite Starship I and then at Sikorsky Aircraft in the Structures Research Group specializing on analysis of composite structures of the all-composite Comanche and other helicopters, and leading internally funded research and programs funded by NASA and the US Army. Since 2001 he has been consulting with various companies in the US on applications of composite structures on airplanes and helicopters. He joined the faculty of the Aerospace Engineering Department of the Delft University of Technology (Aerospace Structures) in 2007 as an Associate Professor. His interests include fatigue and damage tolerance of composites, analysis of sandwich structures, design and optimization for cost and weight, and technology optimization. He has over 40 journal papers and 3 issued or pending patents on related subjects. He is a member of AIAA, AHS, and SAMPE.

### Series Preface

The field of aerospace is wide ranging and covers a variety of products, disciplines and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce exciting and technologically challenging products. A wealth of knowledge is contained by practitioners and professionals in the aerospace fields that is of benefit to other practitioners in the industry, and to those entering the industry from University.

The Aerospace Series aims to be a practical and topical series of books aimed at engineering professionals, operators, users and allied professions such as commercial and legal executives in the aerospace industry. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation and support of aircraft as well as topics such as infrastructure operations, and developments in research and technology. The intention is to provide a source of relevant information that will be of interest and benefit to all those people working in aerospace.

The use of composite materials for aerospace structures has increased dramatically in the last three decades. The attractive strength-to-weight ratios, improved fatigue and corrosion resistance, and ability to tailor the geometry and fibre orientations, combined with recent advances in fabrication, have made composites a very attractive option for aerospace applications from both a technical and financial viewpoint. This has been tempered by problems associated with damage tolerance and detection, damage repair, environmental degradation and assembly joints. The anisotropic nature of composites also dramatically increases the number of variables that need to be considered in the design of any aerospace structure.

This book, *Design and Analysis of Composite Structures: With Application to Aerospace Structures*, provides a methodology of various analysis approaches that can be used for the preliminary design of aerospace structures without having to resort to finite elements. Representative types of composite structure are described, along with techniques to define the geometry and lay-up stacking sequence required to withstand the applied loads. The value of such a set of tools is to enable rapid initial trade-off preliminary design studies to be made, before using a detailed Finite Element analysis on the finalized design configurations.

Allan Seabridge, Roy Langton, Jonathan Cooper and Peter Belobaba

### Preface

This book is a compilation of analysis and design methods for structural components made of advanced composites. The term 'advanced composites' is used here somewhat loosely and refers to materials consisting of a high-performance fiber (graphite, glass, Kevlar<sup>®</sup>, etc) embedded in a polymeric matrix (epoxy, bismaleimide, PEEK etc). The material in this book is the product of lecture notes used in graduate-level classes in Advanced Composites Design and Optimization courses taught at the Delft University of Technology.

The book is aimed at fourth year undergraduate or graduate level students and starting engineering professionals in the composites industry. The reader is expected to be familiar with classical laminated-plate theory (CLPT) and first ply failure criteria. Also, some awareness of energy methods, and Rayleigh–Ritz approaches will make following some of the solution methods easier. In addition, basic applied mathematics knowledge such as Fourier series, simple solutions of partial differential equations, and calculus of variations are subjects that the reader should have some familiarity with.

A series of attractive properties of composites such as high stiffness and strength-to-weight ratios, reduced sensitivity to cyclic loads, improved corrosion resistance, and, above all, the ability to tailor the configuration (geometry and stacking sequence) to specific loading conditions for optimum performance has made them a prime candidate material for use in aerospace applications. In addition, the advent of automated fabrication methods such as advanced fiber/tow placement, automated tape laying, filament winding, etc. has made it possible to produce complex components at costs competitive with if not lower than metallic counterparts. This increase in the use of composites has brought to the forefront the need for reliable analysis and design methods that can assist engineers in implementing composites in aerospace structures. This book is a small contribution towards fulfilling that need.

The objective is to provide methodology and analysis approaches that can be used in preliminary design. The emphasis is on methods that do not use finite elements or other computationally expensive approaches in order to allow the rapid generation of alternative designs that can be traded against each other. This will provide insight in how different design variables and parameters of a problem affect the result.

The approach to preliminary design and analysis may differ according to the application and the persons involved. It combines a series of attributes such as experience, intuition, inspiration and thorough knowledge of the basics. Of these, intuition and inspiration cannot be captured in the pages of a book or itemized in a series of steps. For the first attribute, experience, an attempt can be made to collect previous best practices which can serve as guidelines for future work. Only the last attribute, knowledge of the basics, can be formulated in such a way that the reader can learn and understand them and then apply them to his/her own applications. And doing that is neither easy nor guaranteed to be exhaustive. The wide variety of applications and the peculiarities that each may require in the approach, preclude any complete and in-depth presentation of the material. It is only hoped that the material presented here will serve as a starting point for most types of design and analysis problems.

Given these difficulties, the material covered in this book is an attempt to show representative types of composite structure and some of the approaches that may be used in determining the geometry and stacking sequences that meet applied loads without failure. It should be emphasized that not all methods presented here are equally accurate nor do they have the same range of applicability. Every effort has been made to present, along with each approach, its limitations. There are many more methods than the ones presented here and they vary in accuracy and range of applicability. Additional references are given where some of these methods can be found.

These methods cannot replace thorough finite element analyses which, when properly set up, will be more accurate than most of the methods presented here. Unfortunately, the complexity of some of the problems and the current (and foreseeable) computational efficiency in implementing finite element solutions precludes their extensive use during preliminary design or, even, early phases of the detailed design. There is not enough time to trade hundreds or thousands of designs in an optimization effort to determine the 'best' design if the analysis method is based on detailed finite elements. On the other hand, once the design configuration has been finalized or a couple of configurations have been downselected using simpler, more efficient approaches, detailed finite elements can and should be used to provide accurate predictions for the performance, point to areas where revisions of the design are necessary, and, eventually, provide supporting analysis for the certification effort of a product.

Some highlights of composite applications from the 1950s to today are given in Chapter 1 with emphasis on nonmilitary applications. Recurring and nonrecurring cost issues that may affect design decisions are presented in Chapter 2 for specific fabrication processes. Chapter 3 provides a review of CLPT and Chapter 4 summarizes strength failure criteria for composite plates; these two chapters are meant as a quick refresher of some of the basic concepts and equations that will be used in subsequent chapters.

Chapter 5 presents the governing equations for anisotropic plates. It includes the von Karman large-deflection equations that are used later to generate simple solutions for postbuckled composite plates under compression. These are followed by a presentation of the types of composite parts found in aerospace structures and the design philosophy typically used to come up with a geometric shape. Design requirements and desired attributes are also discussed. This sets the stage for quantitative requirements that address uncertainties during the design and during service of a fielded structure. Uncertainties in applied loads, and variations in usage from one user to another are briefly discussed. A more detailed discussion about uncertainties in material performance (material scatter) leads to the introduction of statistically meaningful (A- and B-basis) design values or allowables. Finally, sensitivity to damage and environmental conditions is discussed and the use of knockdown factors for preliminary design is introduced.

Chapter 6 contains a discussion of buckling of composite plates. Plates are introduced first and beams follow (Chapter 8) because failure modes of beams such as crippling can

be introduced more easily as special cases of plate buckling and post-buckling. Buckling under compression is discussed first, followed by buckling under shear. Combined load cases are treated next and a table including different boundary conditions and load cases is provided.

Post-buckling under compression and shear is treated in Chapter 7. For applied compression, an approximate solution to the governing (von Karman) equations for large deflections of plates is presented. For applied shear, an approach that is a modification of the standard approach for metals undergoing diagonal tension is presented. A brief section follows suggesting how post-buckling under combined compression and shear could be treated.

Design and analysis of composite beams (stiffeners, stringers, panel breakers, etc.) are treated in Chapter 8. Calculation of equivalent membrane and bending stiffnesses for crosssections consisting of members with different layups are presented first. These can be used with standard beam design equations and some examples are given. Buckling of beams and beams on elastic foundations is discussed next. This does not differentiate between metals and composites. The standard equations for metals can be used with appropriate (re)definition of terms such as membrane and bending stiffness. The effect of different end-conditions is also discussed. Crippling, or collapse after very-short-wavelength buckling, is discussed in detail deriving design equations for inter-rivet buckling are presented.

The two constituents, plates and beams are brought together in Chapter 9 where stiffened panels are discussed. The concept of smeared stiffness is introduced and its applicability discussed briefly. Then, special design conditions such as the panel breaker condition and failure modes such as skin–stiffener separation are analyzed in detail, concluding with design guidelines for stiffened panels derived from the previous analyses.

Sandwich structure is treated in Chapter 10. Aspects of sandwich modeling, in particular the effect of transverse shear on buckling, are treated first. Various failure modes such as wrinkling, crimping, and intracellular buckling are then discussed with particular emphasis on wrinkling with and without waviness. Interaction equations are introduced for analyzing sandwich structure under combined loading. A brief discussion on attachments including ramp-downs and associated design guidelines close this chapter.

The final chapter, Chapter 11, summarizes design guidelines and rules presented throughout the previous chapters. It also includes some additional rules, presented for the first time in this book, that have been found to be useful in designing composite structures.

To facilitate material coverage and in order to avoid having to read some chapters that may be considered of lesser interest or not directly related to the reader's needs, certain concepts and equations are presented in more than one place. This is minimized to avoid repetition and is done in such a way that reader does not have to interrupt reading a certain chapter and go back to find the original concept or equation on which the current derivation is based.

Specific problems are worked out in detail as examples of applications throughout the book Representative exercises are given at the end of each chapter. These require the determination of geometry and/or stacking sequence for a specific structure not to fail under certain applied loads. Many of them are created in such a way that more than one answer is acceptable reflecting real-life situations. Depending on the assumptions made and design rules enforced, different but still acceptable designs can be created. Even though low weight is the primary objective of most of the exercises, situations where other issues are important and end up