



Biostatistical Methods

The Assessment of Relative Risks

Second Edition

JOHN M. LACHIN



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Biostatistical Methods

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Second Edition

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Summary: "This book focuses on the comparison, contrast, and assessment of risks on the basis of clinical investigations. It develops basic concepts as well as deriving biostatistical methods through both the application of classical mathematical statistical tools and more modern likelihood-based theories. The first half of the book presents methods for the analysis of single and multiple 2x2 tables for cross-sectional, prospective, and retrospective (case-control) sampling, with and without matching using fixed and two-stage random effects models. The text then moves on to present a more modern likelihood- or model-based approach, which includes unconditional and conditional logistic regression; the analysis of count data and the Poisson regression model; the analysis of event time data, including the proportional hazards and multiplicative intensity models; and elements of categorical data analysis (expanded in this edition). SAS subroutines are both showcased in the text and embellished online by way of a dedicated author website. The book contains a technical, but accessible appendix that presents the core mathematical statistical theory used for the development of classical and modern statistical methods"—Provided by publisher.

Includes bibliographical references and index.

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To my family

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Contents

Preface	xix
Preface to First Edition	xxi
1 Biostatistics and Biomedical Science	1
1.1 Statistics and the Scientific Method	1
1.2 Biostatistics	2
1.3 Natural History of Disease Progression	3
1.4 Types of Biomedical Studies	5
1.5 Studies of Diabetic Nephropathy	7
2 Relative Risk Estimates and Tests for Independent Groups	13
2.1 Probability as a Measure of Risk	14
2.1.1 Prevalence and Incidence	14
2.1.2 Binomial Distribution and Large Sample Approximations	14
2.1.3 Asymmetric Confidence Limits	16
2.1.4 Case of Zero Events	19
2.2 Measures of Differential or Relative Risk	19
2.3 Large Sample Distribution	23
2.3.1 Risk Difference	23
2.3.2 Relative Risk	24

2.3.3	Odds Ratio	26
2.4	Sampling Models: Likelihoods	28
2.4.1	Unconditional Product Binomial Likelihood	28
2.4.2	Conditional Hypergeometric Likelihood	28
2.4.3	Maximum Likelihood Estimates	30
2.4.4	Asymptotically Unbiased Estimates	30
2.5	Exact Inference	32
2.5.1	Confidence Limits	32
2.5.2	Fisher-Irwin Exact Test	33
2.6	Large Sample Inferences	36
2.6.1	General Considerations	36
2.6.2	Unconditional Test	38
2.6.3	Conditional Mantel-Haenszel Test	39
2.6.4	Cochran's Test	40
2.6.5	Likelihood Ratio Test	42
2.6.6	Test-Based Confidence Limits	42
2.6.7	Continuity Correction	43
2.6.8	Establishing Equivalence or Noninferiority	45
2.7	SAS PROC FREQ	48
2.8	Other Measures of Differential Risk	52
2.8.1	Attributable Risk Fraction	52
2.8.2	Population Attributable Risk	53
2.8.3	Number Needed to Treat	56
2.9	Polychotomous and Ordinal Data	56
2.9.1	Multinomial Distribution and Large Sample Approximation	56
2.9.2	Pearson Chi-Square Test	57
2.9.3	Pearson Goodness-of-Fit Test	59
2.9.4	Logits	60
2.10	Two Independent Groups with Polychotomous Response	61
2.10.1	Large Sample Test of Proportions	61
2.10.2	The Pearson Contingency Chi-Square Test	62
2.10.3	Odds Ratios	63
2.10.4	Rank Tests: Cochran-Mantel-Haenszel Mean Scores Test	63
2.11	Multiple Independent Groups	67
2.11.1	The Pearson Test	67
2.11.2	Measures of Association	69
2.11.3	Logits	69
2.11.4	Multiple Tests	69

2.11.5	Rank and Correlation Tests	73
2.11.6	The Cochran-Armitage Test for Trend	74
2.11.7	Exact Tests	76
2.12	Problems	76
3	Sample Size, Power, and Efficiency	85
3.1	Estimation Precision	86
3.2	Power of Z-Tests	87
3.2.1	Type I and II Errors and Power	87
3.2.2	Power and Sample Size	90
3.3	Test for Two Proportions	92
3.3.1	Power of the Z-Test	93
3.3.2	Relative Risk and Odds Ratio	95
3.3.3	Equivalence	96
3.3.4	Noninferiority	98
3.4	Power of Chi-Square Tests	99
3.4.1	Noncentral Chi-Square Distribution	99
3.4.2	Pearson Chi-Square Tests	100
3.4.3	The Mean Score (Rank) Test	102
3.4.4	The Cochran-Armitage Test of Trend	104
3.5	SAS PROC POWER	106
3.5.1	Test for Two Proportions	106
3.5.2	Wilcoxon Mann-Whitney Test	107
3.6	Efficiency	108
3.6.1	Pitman Efficiency	108
3.6.2	Asymptotic Relative Efficiency	110
3.6.3	Estimation Efficiency	111
3.6.4	Stratified Versus Unstratified Analysis of Risk Differences	112
3.7	Problems	115
4	Stratified-Adjusted Analysis for Independent Groups	119
4.1	Introduction	119
4.2	Mantel-Haenszel Test and Cochran's Test	121
4.2.1	Conditional Within-Strata Analysis	121
4.2.2	Marginal Unadjusted Analysis	121
4.2.3	Mantel-Haenszel Test	123
4.2.4	Cochran's Test	125
4.3	Stratified-Adjusted Estimators	126

4.3.1	Mantel-Haenszel Estimates	126
4.3.2	Test-Based Confidence Limits	127
4.3.3	Large Sample Variance of the Log Odds Ratio	128
4.3.4	Maximum Likelihood Estimate of the Common Odds Ratio	130
4.3.5	Minimum Variance Linear Estimators	131
4.3.6	MVLE Versus Mantel-Haenszel Estimates	134
4.3.7	SAS PROC FREQ	135
4.4	Nature of Covariate Adjustment	136
4.4.1	Confounding and Effect Modification	137
4.4.2	Stratification Adjustment and Regression Adjustment	138
4.4.3	When Does Adjustment Matter?	140
4.5	Multivariate Tests of Hypotheses	145
4.5.1	Multivariate Null Hypothesis	145
4.5.2	Omnibus Test	146
4.5.3	Multiple Tests	148
4.5.4	Partitioning of the Omnibus Alternative Hypothesis	149
4.6	Tests of Homogeneity	150
4.6.1	Contrast Test of Homogeneity	151
4.6.2	Cochran's Test of Homogeneity	153
4.6.3	Zelen's Test	154
4.6.4	Breslow-Day Test for Odds Ratios	155
4.6.5	Tarone Test for Odds Ratios	156
4.7	Efficient Tests of No Partial Association	156
4.7.1	Restricted Alternative Hypothesis of Association	156
4.7.2	Radhakrishna Family of Efficient Tests of Association	158
4.8	Asymptotic Relative Efficiency of Competing Tests	163
4.8.1	Family of Tests	163
4.8.2	Asymptotic Relative Efficiency	165
4.9	Maximin-Efficient Robust Tests	169
4.9.1	Maximin Efficiency	169
4.9.2	Gastwirth Scale Robust Test	170
4.9.3	Wei-Lachin Test of Stochastic Ordering	171
4.9.4	Comparison of Weighted Tests	174
4.10	Random Effects Model	175
4.10.1	Measurement Error Model	175
4.10.2	Stratified-Adjusted Estimates from Multiple 2×2 Tables	177
4.11	Power and Sample Size for Tests of Association	183
4.11.1	Power Function of the Radhakrishna Family	184

4.11.2	Power and Sample Size for Cochran's Test	186
4.12	Polychotomous and Ordinal Data	188
4.12.1	Cochran-Mantel-Haenszel Tests	188
4.12.2	Stratified-Adjusted Estimates	189
4.12.3	Vector Test of Homogeneity	191
4.12.4	Stratified Mean Scores Estimate and Test	191
4.12.5	Stratified Cochran-Armitage Test of Trend	192
4.13	Problems	193
5	Case-Control and Matched Studies	201
5.1	Unmatched Case-Control (Retrospective) Sampling	201
5.1.1	Odds Ratio	202
5.1.2	Relative Risk	204
5.1.3	Attributable Risk	205
5.2	Matching	206
5.2.1	Frequency Matching	207
5.2.2	Matched Pairs Design: Cross-Sectional or Prospective	208
5.3	Tests of Association for Matched Pairs	211
5.3.1	Exact Test	211
5.3.2	McNemar's Large Sample Test	212
5.3.3	SAS PROC FREQ	213
5.4	Measures of Association for Matched Pairs	214
5.4.1	Conditional Odds Ratio	214
5.4.2	Confidence Limits for the Odds Ratio	215
5.4.3	Conditional Large Sample Test and Confidence Limits	217
5.4.4	Mantel-Haenszel Analysis	217
5.4.5	Relative Risk for Matched Pairs	218
5.4.6	Attributable Risk for Matched Pairs	219
5.5	Pair-Matched Retrospective Study	220
5.5.1	Conditional Odds Ratio	221
5.5.2	Relative Risks from Matched Retrospective Studies	222
5.6	Power Function of McNemar's Test	223
5.6.1	Unconditional Power Function	223
5.6.2	Conditional Power Function	224
5.6.3	Other Approaches	225
5.6.4	Matching Efficiency	226
5.7	Stratified Analysis of Pair-Matched Tables	227
5.7.1	Pair and Member Stratification	227

5.7.2	Stratified Mantel-Haenszel Analysis	228
5.7.3	MVLE	229
5.7.4	Tests of Homogeneity and Association	229
5.7.5	Random Effects Model Analysis	232
5.8	Multiple Matching: Mantel-Haenszel Analysis	232
5.9	Matched Polychotomous Data	234
5.9.1	McNemar's Test	234
5.9.2	Bowker's Test of Symmetry	234
5.9.3	Marginal Homogeneity and Quasi-symmetry	235
5.10	Kappa Index of Agreement	235
5.10.1	Duplicate Gradings, Binary Characteristic	235
5.10.2	Duplicate Gradings, Polychotomous or Ordinal Characteristic	237
5.10.3	Multiple Gradings, Intraclass Correlation	239
5.11	Problems	239
6	Applications of Maximum Likelihood and Efficient Scores	247
6.1	Binomial	247
6.2	2×2 Table: Product Binomial (Unconditionally)	249
6.2.1	MLEs And Their Asymptotic Distribution	249
6.2.2	Logit Model	250
6.2.3	Tests of Significance	254
6.3	2×2 Table, Conditionally	257
6.4	Score-Based Estimate	258
6.5	Stratified Score Analysis of Independent 2×2 Tables	260
6.5.1	Conditional Mantel-Haenszel Test and the Score Estimate	260
6.5.2	Unconditional Cochran Test as a $C(\alpha)$ Test	261
6.6	Matched Pairs	263
6.6.1	Unconditional Logit Model	263
6.6.2	Conditional Logit Model	265
6.6.3	Conditional Likelihood Ratio Test	268
6.6.4	Conditional Score Test	268
6.6.5	Matched Case-Control Study	268
6.7	Iterative Maximum Likelihood	269
6.7.1	Newton-Raphson (or Newton's Method)	269
6.7.2	Fisher Scoring (Method of Scoring)	270
6.8	Problems	275

7	Logistic Regression Models	283
7.1	Unconditional Logistic Regression Model	283
7.1.1	General Logistic Regression Model	283
7.1.2	Logistic Regression and Binomial Logit Regression	286
7.1.3	SAS Procedures	288
7.1.4	Stratified 2×2 Tables	291
7.1.5	Family of Binomial Regression Models	293
7.2	Interpretation of the Logistic Regression Model	294
7.2.1	Model Coefficients and Odds Ratios	294
7.2.2	Class Effects in PROC LOGISTIC	300
7.2.3	Partial Regression Coefficients	302
7.2.4	Model Building: Stepwise Procedures	304
7.2.5	Disproportionate Sampling	307
7.2.6	Unmatched Case-Control Study	308
7.3	Tests of Significance	309
7.3.1	Likelihood Ratio Tests	309
7.3.2	Efficient Scores Test	310
7.3.3	Wald Tests	312
7.3.4	SAS PROC GENMOD	314
7.3.5	Robust Inferences	317
7.3.6	Power and Sample Size	321
7.4	Interactions	325
7.4.1	Qualitative–Qualitative Covariate Interaction	326
7.4.2	Interactions with a Quantitative Covariate	330
7.5	Measures of the Strength of Association	333
7.5.1	Squared Error Loss	333
7.5.2	Entropy Loss	334
7.6	Conditional Logistic Regression Model for Matched Sets	337
7.6.1	Conditional Logistic Model	337
7.6.2	Matched Retrospective Study	340
7.6.3	Fitting the General Conditional Logistic Regression Model	341
7.6.4	Allowing for Clinic Effects in a Randomized Trial	341
7.6.5	Robust Inference	345
7.6.6	Explained Variation	348
7.6.7	Power and Sample Size	348
7.7	Models for Polychotomous or Ordinal Data	352
7.7.1	Multinomial Logistic Model	352
7.7.2	Proportional Odds Model	357