

# Biomedical Calculations

Principles and Practice

**Richard F. Burton**

*Institute of Biomedical and Life Sciences  
University of Glasgow, UK*



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# Preface

This is a guide to quantitative thinking in the biomedical sciences for students and professionals. Many students are unsure about such basic concepts as  $10^{-3}$ ,  $x^{1/2}$ ,  $\log x$ , reciprocal, percentage, microlitre, millimoles per litre, order of magnitude and calories. Other people who understand these well may yet falter in applying them – whether to straightforward situations like preparing drug solutions or to more interesting quantitative problems. The book is intended to help all such people. Its backbone is one simple, but neglected idea that can transform one's abilities. However, because no such idea can suffice if one lacks basic concepts like those I have just listed, I try to provide these in a way that will not alienate those who already have them. The biomedical examples may prove interesting in themselves, for some are not to be found in other textbooks.

I do not write books without some personal inner drive. (Moreover, echoing Spike Milligan, 'I vowed never to write another [science] book – and this is it'.) I am not innately clever at solving quantitative problems myself, but a few years ago I came upon the simple key, or guiding principle, to what I like to call 'calculating science'. I do not recall when the penny dropped, but there is nothing like revelations late in life for creating zealots! What I realized was that, if I paid more attention to the units involved (e.g. grams per litre,  $\text{kg m s}^{-2}$  etc.), answers to calculations could sometimes emerge automatically with little need for further reasoning – and my mistakes were fewer. The benefits were most obvious when a weary brain was applied to unfamiliar aspects of biology. Some readers will think I discovered the obvious. Indeed page four of the physics textbook I used at school implies in one paragraph much of what struck me only later as a revelation. However, that otherwise excellent book gave the matter too little emphasis and did not include units in the working of its subsequent calculations. Many recent physics textbooks also fail to do so. So what is really an ancient principle is too often neglected. Regardless of what schoolbooks may say about it and teachers teach,

there are certainly many university students, and indeed professional scientists, who have not acquired this key to confident quantitative thinking.

A good name for this idea is 'unit analysis'. I spell it out in Chapter 1 and hope that its full nature and power will be further revealed throughout the book. So far I am emphasizing the value of unit analysis in calculations, but it may also be applied in contexts where no numbers are involved, as when one is trying to make sense of formulae and equations. In that context, readers familiar with 'dimensional analysis' will see that the two ideas are closely related. Dimensional analysis, which is outlined in Chapter 1, can be more elegant, but is often less helpful in biological contexts. For consistency it is just unit analysis that I use here.

My first proper research, which was about the shapes of leaves and long ago, would have proceeded faster if I had understood dimensional or unit analysis. Now the challenge of writing this book has led me into a sideline of research (anthropometry) that I had no reason to contemplate otherwise. Either technique of analysis can prove invaluable as one tries to understand relationships amongst biological variables, and anthropometry has given me some useful illustrations of this.

Unit analysis calls for an understanding of units and, following a unit theme, I look successively at length, area, volume, mass, moles and equivalents, combining some of these as units for density and concentration. Time is added later, allowing treatment of speed, acceleration and rates of flow. Subsequent chapters add force and pressure, and then energy, work, power and temperature. Other topics are integrated into this progression according to the following guiding principles.

- The mathematics should be simple, involving nothing more advanced than exponents, logarithms and very basic algebra, and help is offered even with these. Topics requiring logarithms are confined to the final chapters.
- Formulae are not just to be applied, but, where possible, to be understood.
- Plotting and interpreting graphs are necessary skills.
- There are topics of enough general importance that they need to be included, such as dosage calculation, renal clearance, gas mixtures, buffering and acid–base balance. Not every reader will require these, but I try to make more general points in dealing with them. As I like to illustrate, a particular quantitative approach – a way of thinking or form of calculation – may apply in varied contexts.

- Not all the biological topics need to be as exhaustively treated here as they are in standard courses and textbooks – so long as sufficient background is provided.
- Desirable as it may be to work with a consistent set of units (e.g. SI), one should be able to cope happily with the variety of units that are inevitably encountered in the real world. Consistency is therefore unhelpful in this book.
- The reader should be offered problems for practice, preferably problems having useful or informative answers.

There is no one ideal way of selecting and ordering my topics and of balancing levels of difficulty for a diverse readership, but I trust that my choices prove helpful and interesting.

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