THE ELLIPSE

A HISTORICAL AND MATHEMATICAL JOURNEY

Arthur Mazer

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PREFACE

Two of my passions are history and math. Historians often consider mathematics separate or at best tangential to their own discipline, while, by contrast, historians snuggle up with philosophy and the arts in an intimate embrace. Try this experiment: go to the library and randomly select a history book on ancient Greece. The book will describe the geopolitical landscape in which Greek culture emerged, the incessant feuding between the city states, the wars with Persia, the Peloponnesian war, and the Macedonian conquest. Also included in the book will be a section on the influential Greek philosophers and philosophical schools. And equally likely is an analysis of the artwork that provides a reflection of the times. Most likely, there is no reference to mathematical and scientific achievements, and the rare book that does mention mathematics and science is very stingy in its offerings. The reader is left to conclude that philosophical ideals are the drivers of historical change, the evolution of which can be seen in the arts. Mathematical and scientific achievements are mere outcomes of the philosophical drivers and not worth mentioning in a book on history.

There is of course the opposite argument in which one exchanges the positions of the mathematician with that of the philosophers. That is, mathematics and science are the drivers of historical evolution and in Darwinian fashion philosophies and political entities that promote scientific excellence flourish, while those that do not fade away. This latter argument provides the perspective for this book.

The seventeenth century was the bridge between the sixteenth century's counterreformation and the eighteenth century's enlightenment. It was the mathematicians who built that bridge as their efforts to settle the geocentric versus heliocentric debate over the universal order resulted in Newton's and Leibniz' invention of calculus along with Newton's laws of motion. The mathematicians concluded the debate with their demonstration that the planets revolve around the sun along elliptic pathways. In a broader context, the outcome of the argument was a scientific breakthrough that altered European philosophies so that their nations could utilize their newly found scientific prowess. *The Ellipse* relates the story from the beginnings of the geocentric versus heliocentric debate to its conclusion.

The impact of the debate is sufficient to warrant a retelling of the story. But this is not only a story of tremendous political, philosophical, and not to mention scientific and mathematical consequences, it is also one heck of a story that rivals any Hollywood production. Were we not taken in by Humphrey Bogart and Katherine Hepburn's dedication to a seemingly impossible mission in *The African Queen*? Johannes Kepler launched himself on a mission impossible that he pursued with fierce dedication as it consumed 8 years of his life. Were we not enthralled by Abigail Breslin as her fresh honesty disarmed the pretentious organizers of the Sunshine Pageant in *Little Miss Sunshine*? In the face of the Inquisition as they condemned Bruno to death at the pyre, Bruno exposed the hypocrisy of his sentencers stating, "You give this sentence with more fear than I receive it." Such are the elements of this story that it is not only significant but also compelling.

Those somewhat familiar with this story might launch a protest. Given its centrality to man's development, this story has been picked over by many outstanding individuals. The result is that there are already many fine accessible books on the topic, such as Arthur Koestler's *The Sleepwalkers*. What does *The Ellipse* offer? There are two offerings. First, the premise above that mathematics and science are the drivers of historical evolution directs the historical narrative. There is a true exchange of the roles of philosophers and mathematicians from what is evident in the standard historical literature. As with standard history books, this book describes the geopolitical environment. But philosophers are given a scant role, while mathematicians assume the center stage. Second, this is predominantly a math book with a specific objective. The objective is to take the reader through all of the mathematics necessary to derive the ellipse as the shape of a planet's path about the sun. The historical narrative accompanies the mathematics providing background music.

Throughout the book, the ellipse remains the goal, but it receives little attention until the very last mathematical section. Most of the book sets the stage, and the mathematical props of geometry, algebra, trigonometry, and calculus are put in place. Presenting these topics allows for the participation of a wide audience. Basic topics are available for those who may not as of yet had an introduction to one or more of the foundational subjects. And for those who have allowed their mathematical knowledge to dissipate due to lack of practice over several years, a review of the topics allows for a reacquaintance. Finally, for those who are well versed and find the exercise of deriving the ellipse trivial, enjoy the accompanying narrative.

Apart from devoting quite a few pages to history, the presentation is unconventional in several respects. The style is informal with a focus on intuition as opposed to concrete proof. Additionally, the book includes topics that are not covered in a standard curriculum, that is, fractals, four-dimensional spheres, and constructing a pentagon. (I particularly want to provide supplementary material to teachers having students with a keen interest in mathematics.) Finally, I include linear algebra as a part of the chapter that addresses high school algebra. Normally, this material follows calculus. Nevertheless, calculus is not a prerequisite for linear algebra, and by keeping the presentation at an appropriate level, the ideas are accessible to a high school student. Once this tool is available, the scope of problems that one can address expands into new dimensions, literally.

There are prefaces in which the author claims their writing experience was filled with only joy and that the words came so naturally that the book nearly wrote itself. I am jealous for my experience has certainly been different. There were joyous moments, but difficulties visited me as well. The challenge of maintaining technical soundness within an informal writing style blanketed the project from its inception to the final word. Setting a balance between storytelling and mathematics has been equally confounding, as has been determining the information that I should park in these two zones. Fortunately, I have had the advice of many a good-natured friend to assist me with these challenges. I would like to acknowledge my high school geometry teacher, Joseph Triebsch, who first introduced me to Euclid and advised me to address

the above-mentioned challenges head on. Others who have assisted include Alejandro Aceves, Ted Gooley, David Halpern, and Tudor Ratiu. Their willingness to take time from their quite busy schedules and provide honest feedback is greatly appreciated. Should the reader judge that I have not adequately met the aforementioned challenges, it is not due to my not having been forewarned and equally not due to a lack of alternative approaches as suggested by my friends. The project did allow me to get in touch with old friends, all of whom I have not been in contact with for many years. This experience was filled with only joy and more than compensated for the difficulties that surfaced during the writing.

I must also acknowledge my family, Lijuan, Julius, and Amelia, for putting up with me. For over a year around the dinner table, they were absolutely cheery while listening to my discourses on *The Ellipse*. I still cannot discern whether they actually enjoyed my hijacking of the normal family conversation time and conversion of it to lecture sessions or were just indulging their clueless old man. Either way, I am lucky and in their debt.

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CHAPTER 1

INTRODUCTION

My first teaching job did not start out too smoothly. I would feverishly spend my evenings preparing material that I thought would excite the students. Then the next day I would watch the expression on my students' faces as they sat through my lecture. Their expressions were similar to that on my Uncle Moe's face when he once recalled an experience on the Bataan Death March. How could the lectures that I painstakingly prepared with the hope of instilling excitement have been as tortuous as the Bataan Death March? To find an answer to this question, I went to the source. I asked the students what was going wrong. After 16 years, with the exception of one suggestion, I have vague recollection of the students' feedback. After 16 years, with the exception of one individual, I cannot remember the faces behind any of the suggestions. Concerning the one individual, not only do I have clarity concerning her face and suggestion, but I also have perfect recollection of my response.

The individual suggested that I deliver the lectures in storylike fashion and have a story behind the mathematics that was being taught. My response that I kept to myself was "you have got to be joking." My feeling was that mathematics was the story; the story cannot be changed to something else to accommodate someone's lack of appreciation for the subject. This was one suggestion that I did not oblige. And while for the most part the other students responded positively to the changes that I did make, this student sat through the entire semester with her tortured expression intact.

It is difficult to recall the specifics of something that was said over 16 years ago, the contents of a normal conversation remain in the past while we move on. Despite my reaction, there must have been some meaning that resonated and continued doing so, otherwise I would have long ago forgotten the conversation. Now I see the student's suggestion as brilliant and right on target. By not taking her suggestion, I blew the chance to get more students excited by mathematics through compelling and human stories that are at the heart of mathematics. At the time, I just did not have the vision to see what she was getting at. After 16 years, I have once more given it some thought and this book is the resulting vision. This is a mathematical story and a true one at that.

The story follows man's pursuit of the ellipse. The ellipse is the shape of a planet's path as it orbits the sun. The ellipse is special because it is a demonstration of man's successful efforts to describe his natural environment using mathematics and this mathematical revelation paved the pathway from the Counter Reformation

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