INDUCTANCE

Loop and Partial

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IEEE
Celebrating 125 Years
of Engineering the Future

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This book is dedicated to the memory of my Father and my Mother

Oscar Paul

and

Louise Paul

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PREFACE

This book has been written to provide a thorough and complete discussion of virtually all aspects of inductance: both "loop" and "partial." There is considerable misunderstanding and misapplication of the important concepts of inductance. Undergraduate electrical engineering curricula generally discuss "loop" inductance only very briefly and only in one undergraduate course at the beginning of the junior year in a four-year curriculum. However, that curriculum is replete with the analysis of electric circuits containing the inductance symbol. In all those electric circuit analysis courses, the *values* of the inductors are given and are not derived from physical principles. Yet in the world of industry, the analyst must somehow obtain these values as well as construct inductors having the chosen values of inductance used in the circuit analysis. This book addresses that missing link: calculation of the values of the various physical constructions of inductors, both intentional and unintentional, from basic electromagnetic principles and laws.

In addition, today's high-speed digital systems as well as high-frequency analog systems are using increasingly higher spectral content signals. Numerous "unintended" inductances such as those of the interconnection leads are becoming increasingly important in determining whether these high-speed, high-frequency systems will function properly. This is generally classified as the "signal integrity" of those systems and is an increasingly important aspect of digital system design as clock and data speeds increase at a dramatic rate. Some ten years ago the effects of interconnects such as printed circuit board lands on the function of the modules that lands interconnect were not important and could be ignored. Today, it is critical that circuit models of these interconnects be included in any analysis of the overall system. The concept of "partial inductance" is the critical link in being able to model these interconnects. Partial inductance is not covered in any undergraduate electrical engineering course but is becoming increasingly important in digital system design. A substantial portion of this book is devoted to that topic.

One of the important contributions of this book is the detailed derivation of the loop and partial inductances of numerous configurations of current-carrying conductors. Although the derivations are sometimes tedious, there is nothing we can do about it because the results are dictated by the laws of electromagnetics, and these can be complicated. Unlike other textbooks, all the details regarding derivations for the inductance of inductors are given. Although these are simplified where possible, only so much simplification can be accepted if the reader is to have a clear and unambiguous view of how the result is obtained.

In Chapter 1 we discuss inductance and show important parallels between inductance and capacitance along with some historical details. All of the derivations of the inductance of various inductors first require that we obtain their magnetic fields. Chapter 2 is devoted to this task. The fundamental laws of Biot–Savart, Gauss, and Ampère are discussed, and numerous calculations of the magnetic fields are obtained from them. In addition, the vector magnetic potential method of computing the magnetic fields is also discussed, along with the method of images and energy stored in the magnetic field. In Chapter 3 we provide a complete explanation of how the inductance, which is computed for dc currents, can be used to characterize the effect of time-varying currents. Maxwell's equations for time-varying currents are discussed in detail. An iterative solution of them is given which shows why and when the inductor, derived for dc currents, can be used to characterize the effects of time-varying currents.

All aspects of the derivation of the "loop" inductance of various current-carrying loops are covered in Chapter 4. The flux linkage method, the vector magnetic potential method, and the Neumann integral for determining the "loop" inductance are used, and the "loop" inductances are calculated from all three methods. The proximity effect for closely spaced conductors is discussed along with the loop inductance of various transmission lines.

In Chapter 5 we provide details for computation of the "partial" inductances of wires. Both the self-partial inductance of wires and the mutual partial inductances between wires are derived. These generic results can then be used to "build" a model for other current-carrying structures. Chapter 6 contains all corresponding details about the derivation of the partial inductances of conductors of rectangular cross section, referred to as "lands." The concept of geometric mean distance as an aid to the calculation of partial inductances is discussed and derived for various structures.

The final chapter of the book, Chapter 7, provides a focus on when one should use "loop" inductance and when one should use "partial" inductance for determining the effect of current-carrying conductors. This chapter is meant to provide a simple discussion of this in order to focus the results of previous chapters. The chapter concludes with the solution of a problem involving coupling between two circuit loops using the "loop" inductance method and then using the "partial" inductance method. Both methods yield the same answer, as expected. This example clearly shows the advantages of using "partial" inductance to characterize "unintentional inductors" such as wires and lands.

With the present and increasing emphasis on high-speed digital systems and high-frequency analog systems, it is imperative that system designers develop an intimate understanding of the concepts and methods in this book. No longer can we rely on low-speed, low-frequency systems to keep us from needing to learn these new concepts and analysis skills.

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