Photonic Waveguides

Theory and Applications

Azzedine Boudrioua

Series Editor
Pierre-Noël Favennec

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First published in France in 2006 by Hermes Science/Lavoisier entitled *Optique intégrée : théorie et applications* © LAVOISIER, 2006
First published in Great Britain and the United States in 2009 by ISTE Ltd and John Wiley & Sons, Inc.

ISTE Ltd
27-37 St George's Road
London SW19 4 EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

Library of Congress Cataloging-in-Publication Data

Boudrioua, Azzedine.
[Optique intégrée. English]
Photonic waveguides : theory and applications / Azzedine Boudrioua.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-84821-027-1
1. Integrated optics. 2. Photonics. I. Title.
TA1660.B68 2009
621.36'93--dc22

2009005091

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN: 978-1-84821-027-1

Printed and bound in Great Britain by CPI/Antony Rowe, Chippenham and Eastbourne.

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Foreword

The various properties of light are the basis of many applications in sectors as varied as biology, metrology and telecommunications. Control of the propagation of light is not easy to achieve; photons, like electrons, do not easily obey and cunning must be used to obtain the desired effect. As with electrons, we need to carry out optical signal processing for data transmission and treatment. For this it is important to control the photons at different levels: their generation and detection, as well as their propagation in the matter.

In recent decades, optics has escaped from the tradition which used the propagation in open space, by using optical fibers and integrated optical components. The use of optical fibers has allowed control of the propagation of spatially confined light, without notable losses of power at long distances. The use of photon properties opens the door for many devices. Treatment of the visible signal is designated to integrated optical devices due to the use of photon properties. As for electronic integrated circuits, there will be on the same "chip" optical functions either of emission or of photo-detection with their electronic monitoring circuit and all the optical connectors between components. These photonic integrated circuits must allow good fabrication reproducibility and, moreover, are adapted to collective fabrication in great numbers.

Waveguides of integrated optics constitute one of the most important elements in the building of all-optical technology. Thanks to the work undertaken in parallel by theorists, Maxwell's equations, technologists and experimenters, the fabrication and characterization of integrated optics components has taken great steps towards really miniaturized integrated optics and there is more to come. To improve the performances of integrated optics in the treatment of the signals as well as in the optical switching systems, it is necessary to modify and create new two-dimensional or three-dimensional guiding structures controlled on a micrometric or even on a nano-metric scale.

This book by Azzedine Boudrioua is interested particularly in waveguides of integrated optics. Certain works cover one particular aspect, theory or characterization. This book covers all the aspects of waveguides, from theoretical descriptions to fabrication and applications. This book on integrated optics gives the state of the art of this area of optics, developed a few decades ago but still under development, especially with the appearance of photonic crystals. The work is not only limited to the linear aspect of waveguides; indeed a chapter is dedicated to non-linear effects in waveguides, and another focuses on electro-optical effects and the devices using these effects. The book ends with a chapter on photonic crystals. This chapter, written as an introduction to these new components, provides the reader with an understanding of components controlled on a nano-metric scale, which belong to the field of the nanotechnologies.

Boudrioua's objective was to produce an explanatory, accessible work for researchers and students that would provide a good starting point for those interested in acquiring knowledge in the field of integrated optics. From my point of view, the present work filled this objective perfectly. To my knowledge there are few works covering all of the subjects described in this work at the time of publication, and there is no doubt that it will become a well used reference.

Frédérique de Fornel
Director of research at CNRS
University of Bourgogne

Acknowledgments

The existence of this book is a happy corollary of a profitable exchange of ideas with Mrs Frederique de Fornel (Director of Research at the CNRS, University of Dijon) at the time of a visit to her very friendly company between Valence and Dijon in November 2003, after having taken part in the National Days of Guided Optics (JNOG 2003). I would like to express here my gratitude for and recognition of the encouragement and support that she gave me.

I also testify my gratitude to Mr Pierre Noël Favennec, who followed the development of this project with much patience. His availability, kindness, assistance and encouragement touched me greatly. He did everything to make this project become reality.

It is sometimes providential meetings that positively influence the path of a person. It was thus here with the meetings with Mrs Frederique de Fornel and Mr Pierre Noël Favennec.

This work would probably never have been conceived without the contribution of my colleague and friend Régis Kremer (Lecturer at the University of Metz) with whom I have shared all these years of research. A tremendous recognition to Régis; he never failed in his support and his encouragement especially in the difficult moments spent together. My most cordial thanks also go to his wife Sylvie Kremer (Lecturer at the University of Metz) for the second reading of the manuscript. These same thanks are extended to Ahmed Brara (Doctor of Mechanics of Materials and Director of Research) who helped me to make this manuscript readable. The many discussions with my friend Ahmed were of great importance to me.

For clarity, the preparation of this book required the utilization of various PhD theses manuscripts which I have had the opportunity to supervise these past years.

For this reason, all PhD students who have allowed the use of passages or figures from their manuscripts are also warmly thanked.

Azzedine BOUDRIOUA

Introduction

For ten years, optical telecommunications have had spectacular success, thanks to the explosion of the Internet. This spectacular development is the fruit of a great research and development effort in the field of guided optics, which led to the improvement of the performances of optical fibers. Appearing at the same time was the need to develop optical and optoelectronic components with a planar technology, able to generate, detect, modulate or commutate light, using waveguiding structures. This field of investigation is called integrated optics.

The research and developments undertaken in these two fields (guided and integrated optics) made it possible to provide on the market, optoelectronic components of any kind at low cost. Consequently, other applications in various fields were also developed.

As a matter of fact, today the use of optics includes strategic fields like space, military fields and also fields in everyday life like data storage (CD and DVD), medicine and unsuspected sectors like the car industry.

In a competitive way, the advent of nano-photonics is pushing the limits of photonic system miniaturization to scales lower than the wavelength. Ultimately, the 20^{th} century was the century of electronics, and the 21^{st} century will probably be the century of photonics. The basic idea supporting the use of the photon rather than the electron comes from the very high optical frequencies (200 THz), which allow a very broad bandwidth and offer an unequaled data transmission capacity.

Although optics is a very old science, its major improvements were made during the last quarter of the century. The first work on optics is from the School of Alexandria, Euclid (325-265 BC). However, the reform of optics was undertaken by the Muslim scientists of the medieval period with, at their head Al-Kindi (801-873) and especially Ibn Al-Haytham known under the name of Alhazen (965-1040). This famous scientist truly created the foundation of modern optics with his experimental approach to the propagation of light. He, indeed, introduced experimentation into physics and provided the basis for understanding the luminous phenomena and the control of light propagation (reflection and refraction).

The heritage of this eminent scientist was transmitted to us through his major book “Kitab Al-Manazir” (*The Book of Vision*) which was translated into Latin and distributed throughout western countries at the beginning of the 13th century. This book was used as a reference until the 17th century and influenced work on optics of the majority of renaissance period scientists. The first philosopher who studied and diffused Ibn Al-Hyatham’s work was his enthusiastic disciple Roger Bacon (1214-1292). He was aware of the importance of the Muslim heritage in the fields of science and philosophy. The science historian Gerard SIMON wrote:

> *Roger Bacon was the first to know the Optics of Alhazen (Ibn Al-Haytham) very well... he contributed to its diffusion and he particularly built on it his own work on optics, the Perspective and Multiplication Specierum* (towards 1260-1265)... *he thus accurately followed the analysis of the role of light, the description of the eye, theory of perception and the study of reflection and refraction formulated by Alhazen.* [Adding:] *Kepler renews optics (Paralipomena AD Vitellionem) around 1604 thanks to reading Alhazen and Witelo.*

D.C. Lindgerg emphasizes that Roger Bacon and Johannes Kepler were without any doubt the best disciples of Alhazen (*Optics & Photonics News*, 35 (2003)).

If the revolution of the concepts relating to light sometimes took several centuries, the explosion of telecommunications in the 1980s allowed optics to become a major technology in our everyday life.

Over the last few decades, the approach based on fundamental research and the development of new concepts has been transformed into research and development for new optical products in order to fulfill the increased demand of integrated optoelectronic components in particular for optical telecommunications.

Thus, optics have progressed and moved through four generations: conventional optics, micro-optics, integrated optics and more recently nano-optics (nano-photonics). From optical components of laboratory dimensions (meter and centimeter), research was directed towards micro-optics, particularly with the advent of optical fiber and laser diodes which made it possible to miniaturize photonic systems. Thereafter, integrated optics introduced the concept of integrated optical circuits by similarity to the integrated circuits in micro-electronics. This technology made it possible in many cases to be released from the limitations imposed by the use of light for signal processing.

The concept of "integrated optics" was introduced for the first time by S.E. Miller in 1960 from Bell Laboratories (USA). The approach suggested by Miller consisted of creating on the same substrate, passive and active components for light generation and treatment. The basic element of this type of circuit is the waveguide.

Finally, in the continuity of the idea suggested for the first time by the physicist R. Feynman in 1959, who spoke about the concept "Smaller, Faster, Cheaper", for which the emergent idea was the possibility of handling matter on an atomic and molecular scale in order to conceive and produce sub-micrometric components and systems, there thus appeared the concept of nanotechnology, which became a new challenge for scientific research around the world. In this nano-scale world, the photon is also building its own realm. Thus, nano-photonics actually make it possible to develop new optical components for light generation and treatment based on new paradigms (such as photonic crystals).

Progress in the previously mentioned research fields is incontestably determined by the fabrication and characterization of structures making it possible to manipulate the photon. Among them is the optical waveguide which constitutes the basic element of any integrated optical circuit. In optics, the waveguide plays the same role as the electric conductor (wire) for electronics.

This progress also requires important work regarding the materials and technology to be used. Similar to the development of electronics, the engineering of materials took several decades to develop adequate materials to carry out reliable and effective optoelectronic components. For example, lithium niobate ($LiNbO_3$) is a major dielectric material. It has been used for many years for the fabrication of optoelectronic components for optical signal processing. The use of this material in the form of optical waveguides made it possible in many cases to be released from the limitations related to the use of bulk crystals.

The objective of this book is to provide researchers and students undertaking studies at a Master's level with a teaching aid to understand the basis of integrated optics. This book is a synthesis of theoretical approaches and experimental techniques necessary for the study of the guiding structures. It is based in particular on the research tasks undertaken in this field by the author for about 15 years.

The originality of this book comes from the fact that the ideal models are often accompanied by the experimental tools and their setting to characterize the studied phenomenon. The marriage of the theory and the experiment make the comprehension of the physical phenomena simple and didactic.

The structure of this book is organized into six chapters. Chapter 1 gives the theory of optical waveguides, particularly reporting the study of planar and channel waveguides.

In Chapter 2, the principles of waveguide fabrication techniques are discussed and a review of materials for integrated optics is also reported.

Chapter 3 describes the experimental techniques used for the characterization of guiding structures. The technique of prism coupling – m-line spectroscopy – is described and discussed from theoretical and experimental points of view. The second part of this chapter is devoted to optical losses within the guides, with on the one hand, the presentation of the physical origin of losses and on the other hand, experimental techniques to measure these losses.

The non-linear optical effects in waveguides are covered in Chapter 4. This chapter focuses on second order phenomena and more specifically the second harmonic generation of light.

Chapter 5 is dedicated to the electro-optic effect in waveguides. This chapter covers the electro-optic modulation and its applications in the field of optical telecommunications.

Chapters 4 and 5 present the two theoretical and experimental aspects. The various devices used for the non-linear optical characterization and electro-optics of waveguides are also discussed further.

Finally, Chapter 6 is designed like an introduction to photonic crystals. The photonic crystals are a great part of nano-photonics, which takes an increasingly important place in photonic technologies. This new approach to manipulate the photon will probably provide the ideal solution for allowing integrated optics to make an important technological leap. This chapter is written as an introduction to this field and is far from exhaustive.

Optics in four generations

Technology	Conventional Optics	Micro-Optics	Integrated Optics	Nano-optic
Components	Laser Lenses Mirrors, etc.	LED, laser diodes, fiber optics, micro-lenses	**Integrated optical circuit lasers and monomode fibers**	Integrated optical circuit, optical diode and transistor logic circuits
Alignment Propagation Scale of contacts Scale of devices	Important Beam (~ 1cm) 1 cm 1 m²	Important (difficult) multimode (~ 1mm) 1 mm 10 cm²	**Not necessary Waveguide (~ µm) 1 µm ~ cm²**	Not necessary Photonic crystals < µm ~ cm²

C **Structuring $\chi^{(1)}$ and $\chi^{(2)}$?!**

C **Manipulating the "photon" and functionality!**

Figure 1. *Summary of the evolution of optics*

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Chapter 1

Optical Waveguide Theory

Optical waveguides are structures with three layers controlling light confinement and propagation in a well defined direction inside the central layer (Figure 1.1).

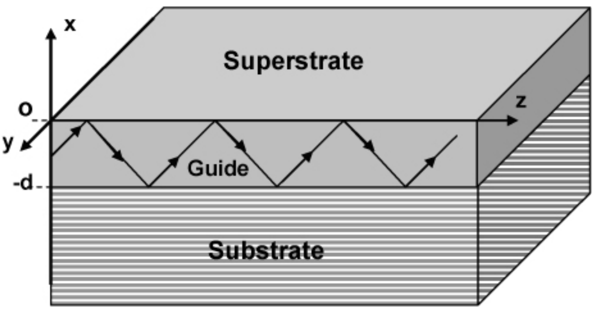

Figure 1.1. *Planar optical waveguide*

Light confinement is carried out by successive total reflections on the two interface guides – substrate and guide – superstrate.

Light propagation is governed by an interference phenomenon which occurs inside the guide between two waves; one of them undergoes two successive total reflections. For a better understanding of the guided wave propagation, we will recall the main principles of these two phenomena, total reflection and interference, inside a transparent plate with parallel faces.

1.1. Principles of optics

1.1.1. *Total reflection phenomenon*

Let us consider an interface separating two mediums 1 and 2, which are dielectric, lossless, homogenous and isotropic with refractive indices n_1 and n_2, respectively. An electromagnetic wave propagates from 1 to 2 with an angle of incidence θi related to the normal of the interface (Figure 1.2).

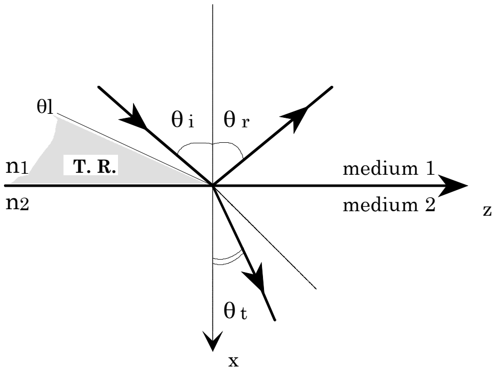

Figure 1.2. *Reflection on an interface (medium 1/medium 2)*

The electric field of the incident wave is given by:

$$\vec{E}_i = \vec{E}_{i0} \exp i(n_1 \vec{k}_i \vec{r} - \omega t) \quad [1.1]$$

with: $\vec{k}_i \vec{r} = k\left(x \cos \theta_i + z \sin \theta_i\right)$ [1.2]

$\vec{k} = k\vec{u} = \frac{2\pi}{\lambda}\vec{u}$ is the wave vector in the vacuum (λ: wavelength in the vacuum) and E_{i0} is the incident wave amplitude. The electric fields of the reflected and transmitted waves can be written:

$$\vec{E}_r = \vec{E}_{r0} \exp i(n_1 \vec{k}_r \vec{r} - \omega t) = \vec{E}_{r0} \exp i\left(n_1 k\left(- x \cos \theta_r + z \sin \theta_r\right) - \omega t\right) \quad [1.3]$$

$$\vec{E}_t = \vec{E}_{t0} \exp i(n_1 \vec{k}_t \vec{r} - \omega t) = \vec{E}_{t0} \exp i\left(n_1 k\left(x \cos \theta_t + z \sin \theta_t\right) - \omega t\right) \quad [1.4]$$

In addition, refraction law is given by:

$$n_1 \sin\theta_i = n_2 \sin\theta_t \qquad [1.5]$$

Therefore, $\sin\theta_i = \dfrac{n_2}{n_1}\sin\theta_t$ [1.6]

In the case of $n_1 > n_2$, there is an incident angle θ_l, as:

$$\sin\theta_l = \frac{n_2}{n_1} \qquad [1.7]$$

For $\theta_i > \theta_l$, the incident wave is totally reflected into medium 1 (total reflection) and the angle θ_t of the transmitted wave is complex [Bor 1999, War 1988]:

$$\begin{aligned}\cos\theta_t &= \left(1-\sin^2\theta_t\right)^{1/2} = \left(1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i\right)^{1/2} \\ &= \frac{i}{n_2}\left(n_1^2\sin^2\theta_i - n_2^2\right)^{1/2} = i\chi\end{aligned} \qquad [1.8]$$

From [1.4], the transmitted wave can be written as:

$$\exp j\left(\vec{k}_t\vec{r} - \omega.t\right) = \exp j\left(k_{zt}z - \omega t\right)\exp\left(-\chi.x\right) \qquad [1.9]$$

This wave propagates in the Oz direction with an amplitude exponentially decreasing in the Ox direction. This is called an evanescent wave. Also, according to Fresnel's formulae, the considered wave undergoes a phase shift compared to the incident wave, given by [Bor 1999]:

$$\Phi_{TE} = 2artg\left[\frac{n_1^2\sin^2\theta_i - n_2^2}{n_1^2 - n_1^2\sin^2\theta_i}\right]^{1/2} \quad \text{(TE incident wave)} \qquad [1.10]$$

$$\Phi_{TM} = 2artg\left[\left(\frac{n_1}{n_2}\right)\left(\frac{n_1^2\sin^2\theta_i - n_2^2}{n_1^2 - n_1^2\sin^2\theta_i}\right)\right]^{1/2} \quad \text{(TM incident wave)} \qquad [1.11]$$

Relations [1.9] and [1.10] will be used throughout this chapter in order to study the propagation of guided waves. Note that evanescent waves have been experimentally investigated and they are currently utilized in the field of integrated optics. A similar phenomenon appears at the interface between a dielectric and a metallic layer generating, under specific conditions, a surface plasmon [Rae 1997].

1.1.2. *Parallel-face plate*

Let us consider a transparent plate with parallel faces (Figure 1.3), with refractive index n and a thickness d, placed in air (index = 1). We will focus on the calculation of the difference of the optical path (δ) between the first two rays transmitted throughout the plate (the same approach can be applied for the first two reflected rays).

$$\delta = [SOABCR_2] - [SOAHR_1] \qquad [1.12]$$

We can easily show that δ is given by:

$$\delta = 2nd \cos\theta \qquad [1.13]$$

where θ is the propagation angle within the plate.

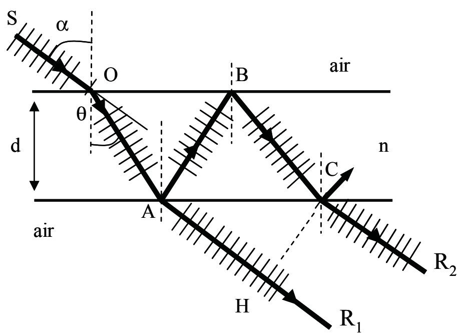

Figure 1.3. *Interference between two rays transmitted by a parallel face plate with a thickness d and a refractive index n*

In these conditions the parallel-face plate introduces a phase shift between the two rays R_1 and R_2 given by the following relation:

$$\varphi = 2ndk\cos\theta = \frac{4\pi}{\lambda} nd\cos\theta \qquad [1.14]$$

The latter is at the origin of the interference between the two rays R_1 and R_2. The transmitted rays are parallel, thus the interference phenomenon is located at infinity. However, we can observe the interference fringes in the Fresnel's field on a screen placed at the focal distance of a convergent lens [Bor 1999, War 1988, Rae 1997, Sal 1991].

To summarize, when a plane wave propagates, two kinds of phase mismatches can be considered: the first is due to total reflection, and the second could be due to a difference in the optical path.

1.2. Guided wave study

1.2.1. *General description*

Let us consider the case of three mediums with a central layer of refractive index n, surrounded by two layers of indices n_s and n_a called the substrate and superstrate respectively.

Light confinement in such a structure (waveguide) is based on a total reflection (TR) phenomenon on the interfaces ($n-n_s$) and ($n-n_a$). Thus, two critical angles θ_{c1} and θ_{c2} can be defined as:

$$\sin\theta_{c1} = \frac{n_a}{n}$$
$$\sin\theta_{c2} = \frac{n_s}{n} \qquad [1.15]$$

if $n > n_s > n_a$, then $\theta_{c1} < \theta_{c2}$. We can distinguish three situations related to the propagation angle θ:

– $\theta < \theta_{c1}$: air modes (if $n_a = 1$)

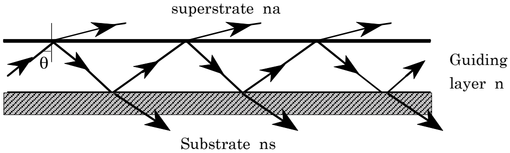

Figure 1.4. *Air modes*

In this case, the total reflection conditions are not satisfied on the two interfaces, and there is no confinement of the light. Light propagates as substrate-superstrate radiation modes. We will reconsider the concept of modes in the continuation of this chapter.

– $\theta_{c1} < \theta < \theta_{c2}$: substrate modes

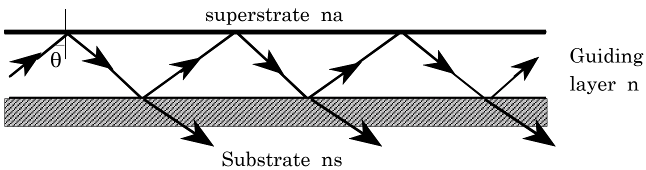

Figure 1.5. *Substrate modes*

The total reflection of the light is carried out only on the interface guide-superstrate (air). A part of the light is refracted through the interface guide-substrate. These are substrate modes.

– $\theta > \theta_{c2}$: guided modes

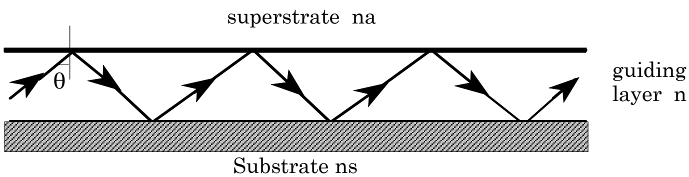

Figure 1.6. *Guided modes*

In this situation, light is in total reflection on the two interfaces, guide-superstrate and guide-substrate, remaining confined between them. These are well confined guided modes. Radiation and substrate modes are called leaky modes, and have been discussed by several authors [Mar 1969, DIN 1983]. They can be used to characterize the optical properties of planar waveguides.

In the rest of this chapter, we will be interested only in the guided modes which theoretically allow light propagation without loss in the guiding layer, and play a very important role in integrated optics.

So far we have only considered structures with a guiding zone of a constant refractive index. We will, however, also be interested in waveguides having a central zone whose index varies. In addition, the index profile of the guiding layer (n) enables us to distinguish two types of waveguides:

– the step index waveguide, where the refractive index remains constant throughout the guiding layer depth x (Figure 1.7a), thus we can write: n = constant; and

– the graded index waveguide, where the refractive index varies throughout the guiding layer depth x (Figure 1.7b). Therefore, we can write: n = n(x).

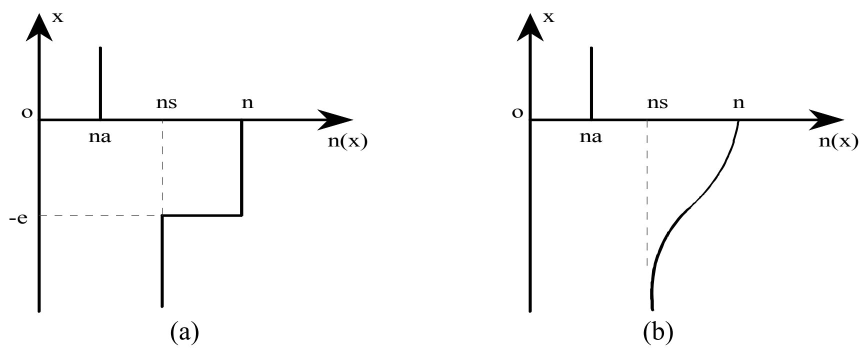

Figure 1.7. *(a) Step index waveguide, (b) graded index waveguide*

1.2.2. *Step index planar waveguide*

1.2.2.1. *Guided modes dispersion equation*

1.2.2.1.1. Optic-ray approach

A planar waveguide is characterized by parallel planar boundaries with respect to one direction (x) and is infinite in extent in the other directions (y and z). The refractive indices of the superstrate, the guiding layer and the substrate are n_a, n and n_s respectively.

Let us consider a plane wave propagating in the Oz direction. In the optic-ray approach, light propagation is carried out by the superposition of several plane waves being propagated in zigzags, between the two interfaces (n, n_a) and (n, n_s), in the Oz direction (Figure 1.8). The light ray is defined as the direction of the optical energy flux (direction of the Poynting vector).

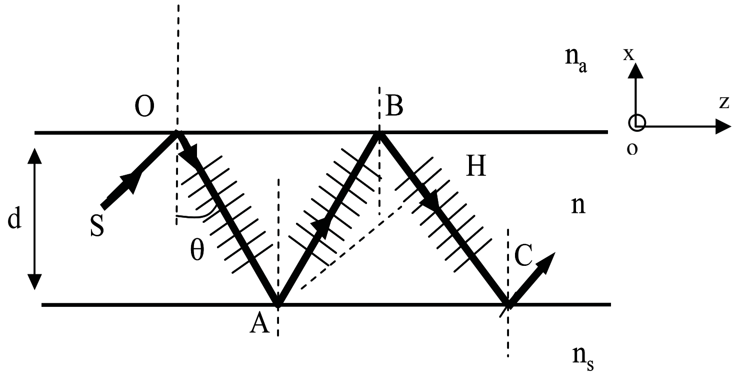

Figure 1.8. *Propagation in zigzags within a planar waveguide*

This situation is similar to that of the parallel-face plate presented in the previous section. Phase mismatch for such a wave between the points A and H (see Figure 1.8) is composed of three terms: phase mismatch $\Delta\phi$, due to the difference in optical paths; and the two phase mismatches, which are due to the total reflection on the two interfaces, $\Phi_{(n,na)}$ and $\Phi_{(n,ns)}$, with:

$$\Delta\phi = 2ndk\cos\theta = \frac{4\pi}{\lambda}nd\cos\theta \qquad [1.16]$$

and $\Phi_{(n,na)}$ and $\Phi_{(n,ns)}$ given by expressions [1.10] and [1.11].

In order to maintain light propagation within the guiding layer, it is important that light undergoes constructive interference. For that, the total phase mismatch should be a multiple of 2π. Therefore, we can write the following guided mode dispersion equation:

$$2ndk\cos\theta - \Phi_{(n,n_a)} - \Phi_{(n,n_s)} = 2m\pi \qquad [1.17]$$

m: integer ≥ 0
d: thickness of the guiding layer
k: wave number (2π/λ)

Equation [1.17] can also be written as:

$$2kd\sqrt{\left(n^2 - N_m^2\right)} = \Phi_{(n,n_a)} + \Phi_{(n,n_s)} + 2m\pi$$
$$N_m = n\sin\theta_m \qquad [1.18]$$

$$\Phi_{\left(n,n_j\right)} = arctg\left[\left(\frac{n}{n_j}\right)^{2\rho}\left(\frac{\left(N_m^2 - n_j^2\right)}{\left(n^2 - N_m^2\right)}\right)^{1/2}\right] \qquad [1.19]$$

$$\rho = \begin{cases} 0 & \text{TE polarization} \\ 1 & \text{TM polarization} \end{cases} \qquad [1.20]$$

On the one hand, equation [1.18] indicates that light propagates within a medium of a refractive index $n\sin\theta$ called the effective index, N_m. On the other hand, equation [1.18] imposes discrete values of θ related to different values of m. These values determine the set of guided modes of the structure.

Relation [1.18] is called the guided mode dispersion equation. It represents the condition to be satisfied in order to confine light within the guiding layer. It is a resonant condition indicating that the wave phase in A and H is the same modulus 2π [Nis 1989, Tie 1970, Tie 1977, Mur 1999, Mar 1991, Yar 1973].

However, it is clear that the optic-ray approach does not allow the determination of the guided mode electric field distribution. For that, it is necessary to consider the Maxwell's equation approach.

1.2.2.1.2. Maxwell's equation approach

In this section, we will develop the theoretical study of light propagation within a planar waveguide starting from Maxwell's equations. This study will lead us to the guided mode dispersion equation as well as the distribution of the electromagnetic field in the guide.

The waveguide is formed by three dielectric mediums that are homogenous, isotropic, linear and lossless. The magnetic permeability is considered as constant μ_0. Light propagates in the Oz direction, and the structure is taken as infinite with invariant properties in the Oy direction. Solving this problem consists of finding out solutions for Maxwell's equations that satisfy the boundary conditions imposed by the structure.

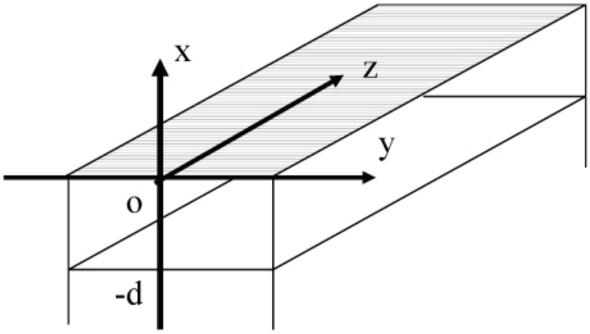

Figure 1.9. *Schematic planar optical waveguide*

The electromagnetic wave within every medium of such a structure is given by the following Maxwell's equations:

$$\vec{\nabla} \wedge \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$
$$\vec{\nabla} \wedge \vec{H} = \varepsilon_0 n^2 \frac{\partial \vec{E}}{\partial t} \qquad [1.21]$$

$$\vec{\nabla}\vec{E} = 0$$
$$\vec{\nabla}\vec{H} = 0 \qquad [1.22]$$
$$\vec{B} = \mu_0 \vec{H} \quad \text{and} \quad \vec{D} = \varepsilon \vec{E}$$

with:
- $\vec{E}$: electric field vector;
- $\vec{B}$: magnetic field vector;
- $\vec{H}$: magnetic induction vector ;
- $\vec{D}$: electric displacement vector.

The solutions take the form:

$$\vec{E} = \vec{E} \exp i(\omega t - \vec{k}\vec{r}) = \vec{E} \exp i(\omega t - \beta z)$$
$$\vec{H} = \vec{H} \exp i(\omega t - \vec{k}\vec{r}) = \vec{H} \exp i(\omega t - \beta z) \qquad [1.23]$$

As the structure is infinite in the Oy direction, the problem has no dependence on y ($\frac{\partial}{\partial y} = 0$).

An electromagnetic field can be considered as a sum of two polarized fields TE and TM, corresponding to a TE (Transverse Electric) wave with $\vec{E}$ parallel to Oy and a TM (Transverse Magnetic) wave where $\vec{H}$ is transverse. Then, from Maxwell's equations, it can be written that:

$$\begin{cases} \frac{\partial^2 E_y}{\partial x^2} + \left(k^2 n^2 - \beta^2\right) E_y = 0 \\ H_x = -\frac{\beta}{\omega\mu_0} E_y \\ H_z = -\frac{1}{i\omega\mu_0} \frac{\partial E_y}{\partial x} \end{cases} \qquad \text{TE modes} \qquad [1.24]$$

$$\begin{cases} \frac{\partial^2 H_y}{\partial x^2} + \left(k^2 n^2 - \beta^2\right) H_y = 0 \\ E_x = -\frac{\beta}{\omega\mu_0} H_y \\ E_z = -\frac{1}{i\omega\mu_0} \frac{\partial H_y}{\partial x} \end{cases} \qquad \text{TM modes} \qquad [1.25]$$

Note that the previous separation in TE and TM modes can be performed in the case of a planar optical waveguide with a refractive index n(x) which depends on one transverse coordinate only.

Generally speaking, we have to solve the following Helmholtz's equation:

$$\frac{\partial^2 F}{\partial x^2} + \left(k^2 n^2 - \beta^2\right) F = 0 \qquad [1.26]$$

with: $F = E$ or H depending on the light polarization.

It is necessary to solve the previous equation in the three mediums, n, n_s and n_a.

TE modes

Equation [1.26] can be written for the three layers as:

$$\frac{\partial^2 E_y}{\partial x^2} - q^2 E_y = 0 \quad \text{medium } n_a \qquad [1.27]$$

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0 \quad \text{medium n} \qquad [1.28]$$

$$\frac{\partial^2 E_y}{\partial x^2} - p^2 E_y = 0 \quad \text{medium } n_s \qquad [1.29]$$

$$\text{with: } \begin{array}{l} q^2 = \beta^2 - k^2 n_a^2 \\ h^2 = k^2 n^2 - \beta^2 \\ p^2 = \beta^2 - k^2 n_s^2 \end{array} \qquad [1.30]$$

β is the propagation constant given by: $\beta = kN = kn \sin\theta$ (N is the guided mode effective index).

The guiding condition imposes the existence of a sinusoidal solution within the central layer n ($h^2 \geq 0$), with evanescent waves into mediums n_a and n_s (q^2 and $p^2 \geq 0$) [Nis 1989, Tie 1970, Tie 1977, Mur 1999, Mar 1991, Yar 1973]. So:

$$kn \geq \beta \geq \text{kns} \geq kn_a$$

The electric field E_y has the general form:

$$E_y(x) = \begin{cases} C\exp(-qx) & 0 \leq x \leq \infty \\ A\cos(hx) + B\sin(hx) & -d \leq x \leq 0 \\ D\exp[p(x+d)] & -\infty \leq x \leq -d \end{cases} \qquad [1.31]$$

where A, B, C and D are constants that can be determined by matching the boundary conditions which require the continuity of E_y and H_z. Note that for $\vec{D}$ and $\vec{B}$ the continuity conditions concern their normal components. Thus, at the interfaces (n, n_a) and (n, n_s), we can write:

$$\left.E_y\right|_n = \left.E_y\right|_{n_a}$$
$$\left.\frac{\partial E_y}{\partial x}\right|_n = \left.\frac{\partial E_y}{\partial x}\right|_{n_a} \qquad [1.32]$$
$$\left.E_y\right|_n = \left.E_y\right|_{n_s}$$
$$\left.\frac{\partial E_y}{\partial x}\right|_n = \left.\frac{\partial E_y}{\partial x}\right|_{n_s} \qquad [1.33]$$

These conditions allow us to write previous equations [1.31] in the following form:

$$E_y(x) = \begin{cases} C\exp(-qx) & 0 \leq x \leq \infty \\ C\left[\cos(hx) - (q/h)\sin(hx)\right] & -d \leq x \leq 0 \\ C\left[\cos(hd) + (q/h)\sin(hd)\right]\exp\left[p(x+d)\right] & -\infty \leq x \leq -d \end{cases} \qquad [1.34]$$

$\vec{H}$ can be determined from equations [1.23].

The equation system allows us to determine the electric field profile of each guided mode propagating within the structure. However, it is necessary to determine the constants C and β. For this we use the derivative continuity of the electromagnetic field in the guide. After simplification, we can find:

$$\tan(hd) = \frac{p+q}{h - \frac{pq}{h}} \qquad [1.35]$$

This relation can be transformed into:

$$dh = \arctan\left(\frac{q}{h}\right) + \arctan\left(\frac{p}{h}\right) + m\pi \qquad [1.36]$$

where m is an integer ≥ 0. This defines the guided mode order.

(For the previous calculation, we can use: $\tan(a+b) = \dfrac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$)

Equation [1.36] represents the TE guided mode dispersion equation. By replacing p, q and h in [1.35], we obtain equation [1.17] previously described during consideration of the optic-ray approach.

Finally, we need to determine the constant C in order to obtain a complete description of the guided modes travelling within the structure. For that, we use the normalization condition [Yar 1973]. Calculation shows that [Vin 2003]:

$$C_m = 2h_m \left[\frac{\omega\mu_0}{|\beta_m| \left(d + \frac{1}{q_m} + \frac{1}{p_m} \right) \left(h_m^2 + q_m^2 \right)} \right]^{1/2} \qquad [1.37]$$

As an example, the solutions of the problem are schematically displayed on the following figure.

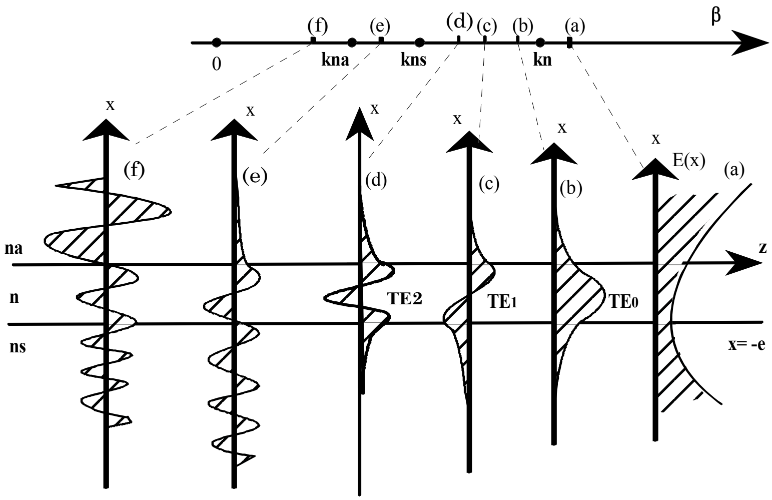

Figure 1.10. *Guided mode electric field distribution in a planar optical waveguide structure: (a) represents an unacceptable physical solution because $k^2n^2 - \beta^2 < 0$ in the three mediums; (b), (c) and (d) represent the electric field distribution of well guided modes ($TE_{0..}$); (e) represents a substrate mode – the optical energy is well confined at the interface (n,na) but it varies sinusoidally in the substrate; (f) shows the electric field distribution of a radiation mode, $k^2n^2 - \beta^2 > 0$ in the three mediums – this is an oscillatory solution in the three mediums (free extension outside the guide)*