The Mathematics of Derivatives

Tools for Designing Numerical Algorithms

ROBERT L. NAVIN

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This book is dedicated with love to Thais Roda Noya.

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Preface

This book is based on a course intended to quickly teach the basics of derivative mathematics to skilled software designers with no knowledge of financial derivatives. I created these notes for my firm—while still a small startup trading-analytics software company—Real Time Risk Systems LLC. We started the business to build a flexible, fast, real-time pricing-and-risk software application for large derivatives trading operations that deal in a variety of products. A significant design feature of the application was to allow complicated models of derivatives to be "plugged-in" without necessarily writing these models into the initial software package.

In the course of developing and using the course from which this book is derived, it became apparent to me that it would be useful to other industry practitioners for a similar purpose: the education and training of programmers (and even trainee quantitative analysts). The professional programmers, of course, have no need of becoming professional "quants." They do, however, need a basic and broad level of understanding of the mathematical formalism of derivatives as quickly as possible. They also need to cover a broad spectrum of material in enough detail to offer a solid grounding and without a lot of mathematical rigor.

I wanted students who took my course—and those who now read this book—to have the ability to talk sensibly to a quantitative analyst and to understand what quants have to say as well. This book is therefore economically designed for the ground it covers in order to save time and focus on the essentials—and without requiring graduate-level mathematics. It is not a rigorous academic text. Nevertheless, all the basics are available, condensed, and in one place. I view this book as more of an engineering textbook than a thorough mathematics treatise.

Acknowledgments

None of the material here is original although most of the formulae have been rederived or written down from memory and are surely stamped with only my own idiosyncrasies. The significant exception is section 8.1.1., which had such a simple and elegant treatment in the original paper by O. Cheyette that there was no simplification that I could supply. There are many excellent textbooks available, including John C. Hull's *Options, Futures and Other Derivatives*, 6th ed., Jonathan E. Ingersoll's *Theory of Financial Decision Making*, and Darrell Duffie's *Dynamic Asset Pricing Theory*. The majority of the material covered here is also covered in those books. Again, the point of coverage of this book is a quick overview for nonquants. For more in-depth and rigorous coverage, I strongly recommend Hull, Ingersoll, Duffie, and many of the other comprehensive derivatives texts.

That said, I acknowledge the great indebtedness that I have to the people from whom I have learned financial derivatives, namely Gunnar Klinkhammer and Barry Ryan at CMS in Los Angeles, who taught me risk-neutral pricing; and Scott Waltz, Albert Sizook, Bill Cherney, Amitabha Sen, and Guillermo Bubliek at Swiss Bank/O'Connor in Chicago from whom Jeff Miller and I learned (and discussed with each other) the elements of finance theory and application, including numerical techniques. The work of Jean-Philippe Bouchaud, which came to me from my discussions with Jeff Miller, also had a significant impact on my understanding of risk-neutral pricing and hedging strategies. I would like to thank Rosie Rush for early copyediting and helping me to put this work together for publication. Many thanks to Pamela Van Giessen at John Wiley and Sons for her help and support for this project.

I would like to thank my colleagues Chris Leon and Nikolai Avteniev for being the first canaries in the mineshaft on whom I tried out the lecture course from which these notes arose. I would also like to thank Rosie Rush for early copyediting and helping me to put this work together for publication. Many thanks to Pamela Van Giessen and Jennifer MacDonald at John Wiley and Sons for their help and support for this project.