
Robust Industrial Control Systems: Optimal Design Approach for Polynomial Systems

Michael J. Grimble

University of Strathclyde, UK



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To my dear family
Wendy
Claire, Mark and Callum
Andrew and Christie

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