

Wolfgang Witteveen

Computational efficient flexible multibody
dynamics with nonlinear loads and many
modes

Professorial Dissertation

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Computational efficient flexible
multibody dynamics with nonlinear
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Kurzfassung

Motivation: Die Berücksichtigung von Finite Elemente (FE) Strukturen in der Mehrkörpersimulation (MKS) hat sich zu einer etablierten Methode entwickelt, sofern die Anzahl der Krafteinleitungspunkte moderat bleibt. In den letzten Jahren gibt es Bestrebungen auch verteilte Lasten, wie sie zum Beispiel durch den Kontakt zweier Körper entstehen, in der MKS zu berücksichtigen. Bei fein vernetzten FE Strukturen ergeben sich somit sehr viele mögliche Krafteinleitungspunkte. In so einem Fall versagen die herkömmlichen Methoden und führen zu exorbitant hohen Rechenzeiten. Im letzten Jahrzehnt wurden Ansätze zur reduzierten Berechnung der Verformungen in einem verteilten Krafteinleitungsgebiet vorgestellt. Dabei kommen spezielle Ansatzvektoren zum Einsatz, die hier „*lokale Moden*“ genannt werden. Diese lokalen Moden führen zu einer Reduktion der involvierten Gleichungen um mehrere Größenordnungen. Allerdings werden bei sehr großen potentiellen Krafteinleitungsgebieten immer noch sehr viele lokale Moden benötigt, beispielsweise mehrere tausend. Da jeder lokale Mode zu einer Differentialgleichung führt, ist mit üblichen Methoden eine schnelle numerische Zeitintegration nicht möglich. In dieser Arbeit werden zwei methodische Weiterentwicklungen vorgeschlagen, um derartige Systeme schnell und genau in der Zeit zu integrieren.

Ergebnisse: Der erste Vorschlag zur Effizienzsteigerung betrifft die numerische Zeitintegration mit vielen lokalen Moden. Derartige Moden zeichnen sich durch eine hohe Steifigkeit aus. Die entsprechenden Differentialgleichungen werden deshalb separat von den nicht-steifen Gleichungen integriert. Dabei kommt auch eine optimalere Integrationsmethode zum Einsatz. Der zweite Vorschlag zur Effizienzsteigerung betrifft die Berechnung der verteilten Kräfte. Es war bisher üblich, diese auf Basis von FE Knotengrößen zu berechnen. Damit ergeben sich zwangsläufig viele Gleichungen. Als deutlich effizientere Alternative wird eine allgemeine Methode zum Aufbau eines reduzierten Gleichungssystems vorgeschlagen. Man spricht dann von Hyperreduktion.

Nutzen: Mit Hilfe der vorgeschlagenen Methoden ist es beispielsweise möglich, den Kontakt zwischen zwei flexiblen Körpern in der MKS schnell und genau zu berücksichtigen. Verformungen und Spannungen haben die gleiche Qualität wie die FE Methode. Die Rechenzeiten sind so niedrig, dass Probleme, die bisher als wirtschaftlich nicht rechenbar eingestuft wurden, mit geringen Rechenzeiten simuliert werden können.

Schlagwörter: Flexible Mehrkörperdynamik, Floating Frame of Reference Formulation, Numerische Zeitintegration, Reduktion, Hyper-Reduktion, Kontaktmechanik

Abstract

Motivation: The consideration of Finite Element (FE) structures in multibody simulation (MBS) has become an established method, especially when the number of force application points remains moderate. In recent years, a trend can be observed in which distributed loads are considered as well, such as those arising from the contact of two elastic bodies. For finely-meshed FE structures, this results in a large number of possible force application points. In such a case, conventional methods fail, leading to exorbitantly high computation times. In the last decade, approaches for the reduced computation of deformations inside distributed load application areas were introduced. Special approach vectors are used, called ‘local modes’ here. These local modes lead to a reduction in the involved equations by several orders of magnitude. However, for very large potential load application areas, a large number of local modes is still required—for example, several thousand. Since each local mode leads to a differential equation, a fast numerical time integration is not possible with common methods. In this work, two methodological improvements are proposed for a fast and accurate time integration of such systems.

Results: The first proposal to increase efficiency concerns numerical time integration with many local modes. Such modes are characterized by a high stiffness. It is proposed to integrate the corresponding differential equations separately from the non-stiff ones. In addition, a more efficient integration method is suggested as well. The second proposal concerns the computation of the distributed forces. Till now, it was common practice to compute them on the basis of FE node quantities, which results in many equations. As a much more efficient alternative, a general method for the construction of a reduced system of equations is proposed. This is called hyper-reduction.

Benefit: Using the proposed methods, it is efficiently possible to consider contact between two flexible bodies in the multibody simulation. Deformations and stresses have the same quality as when the FE method is used. The short computation times enable the simulation of problems that were previously considered to be economically non-computable.

Keywords: Flexible Multibody Simulation, Floating Frame of Reference Formulation, Numerical Time Integration, Hyper Reduction, Reduction, Contact Mechanics

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List of abbreviation

BT	Balanced Truncation
CG	Controllability Gramian
CMS	Component Mode Synthesis
DEIM	Discrete Empirical Interpolation Method
dof	Degree of Freedom(s)
EoM	Equations of Motion
FDM	Finite Difference Method
FE	Finite Element
FEM	Finite Element Method
FF	Floating Frame
FFRF	Floating Frame of Reference Formulation
FFR	Floating Frame of Reference
HR	Hyper-Reduction
IF	Inertial Frame
IR	Inertia Relief
LTI	Linear Time Invariant
MBS	Multibody System
MM	Moment Matching
MOR	Model Order Reduction
OG	Observability Gramian
POD	Proper Orthogonal Decomposition
POM	Proper Orthogonal Mode
SHR	Semi-Hyper-Reduction
TVD	Trial Vector Derivatives

1 Introduction

Multi-body system (MBS) simulation has become an established numerical tool in engineering science and in industry for predicting the motion of a system of solid bodies. These bodies can be rigid or flexible and interact with each other and the environment via forces and constraints. An established method for describing a deformable solid body is the so-called Floating Frame of Reference Formulation (FFRF). Here, deformations are described with the help of trial functions in a co-moving coordinate system. For the common case that the elastic body is modelled with the Finite Element Method (FEM), trial vectors are used instead of trial functions. These are often also called modes. In general, this is not correct. However, this term has become so common that the expressions ‘trial functions’, ‘trial vectors’, and ‘modes’ are also used synonymously in this work. With the help of such modes, the set of mathematical equations is reduced from the number of Finite Element (FE) nodal degrees of freedom (dof) to the number of modes. This is called model order reduction (MOR) or substructuring; it enables numerical time integration with reasonable computation times even for complex-shaped and flexible bodies. Common suggestions for proper modes focus on the correct representation of global deformations, as well as local deformations at single concentrated force application points. A typical example of such a situation is the multi-body simulation of a vehicle with an FE model of the car body. In this case, at all points where the chassis and drivetrain are connected to the car body, such concentrated force application points are required. It is obvious that the number of these points remains manageable. Since for each dof of these force application points, a trial vector is necessary, the total number of trial vectors results from the sum of these node dof and the number of vibration modes considered. This number is also the size of the system of differential equations for describing the flexible deformation. A typical order of magnitude is 10 to 50, at most 100 to perhaps 500, enabling efficient time integration. Summary publications in respect of the history, modelling, and numerical time integration of such flexible multibody systems can be found, for example, in Schielen (1997, 2005), Shabana (1997), Wasfy and Noor (2003), Wittenbrug (2008), Bauchau (2011) and Shabana (2013). In recent years, there has been a trend to consider not only concentrated forces, but also widely distributed loads with trial vectors. These loads are mostly dependent in a nonlinear way on the state of the elastic body, and possibly also on another elastic body. A highly relevant example would be contact forces acting between flexible bodies. Witteveen and Irschik (2009), Witteveen and Pichler (2014), G rardin and Rixen (2015), and Pichler, Witteveen, and Fischer (2017a and 2017b) propose specific trial vectors to represent small-

sliding contact in joints. In the case of finite sliding contact, Sherif (2012) and Sherif, Witteveen, Holl, Irschik, and Mayrhofer (2013) propose suitable trial vectors. An application for elastohydrodynamic contact between piston and cylinder is reported by Koller, Witteveen, Pichler, and Fischer (2020). The latter’s examples deal with surface loads whereas, for example, Tiso, Jansen, and Abdalla (2011) report on an application to distributed internal forces, specifically geometric nonlinearities, see section 2.2.2 for more literature on that.

This trend towards accounting for spatially distributed effects using trial vectors has at least three consequences that are very unfavourable for efficient time integration:

- (1) Under certain circumstances, many additional modes are necessary to represent a local load effect occurring in a large area of potential contact. This is the case for many contact problems. Sherif (2012) provides an illustrative example where two rollers roll on top of each other. An accurate representation of the deformations on the entire roller surface because of the contact with the other roller, several thousand (!) additional trial vectors are necessary for each roller. The reduction of the number of equations from several hundred thousand nodal dof to several thousand modes per roller is considerable, but numerical time integration with reasonable computation times is still not possible with the current methods. Many additional trial vectors for the representation of local deformations, therefore, lead to many degrees of freedom in the numerical time integration.
- (2) Trial vectors representing local deformations have very high stiffness. Vibration modes representing global deformations, on the other hand, have very low stiffness. Hence, the resultant system of differential equations is very stiff, which is a challenge for numerical time integration.
- (3) Usually, the distributed forces (or stresses) have to be calculated on the basis of the FE nodal displacements. This is because the corresponding field equations are formulated in the FE nodal domain. This means that for each iteration step of the time integration, the state of all the FE nodes involved has to be reconstructed. The corresponding equations are then solved for all involved nodal dof. Finally, the nodal forces need to be projected back into the subspace of the trial vectors. In other words, the time integration is performed in a reduced space, but not the computation of the loads. Hence, the load computation becomes a bottleneck with respect to the simulation time.

1.1 Motivation

The previous outlined trend to describe distributed loads via trial vectors in the multi-body simulation leads to long computation times when conventional approaches are used. Though these computation times are still significantly shorter than those of the nonlinear FEM, the question of economic efficiency arises more and more with the increase in the number of modes.

1.2 Goal

The goal of this work is to introduce methods that lead to significant computational time savings without losing result quality, when many modes are required for the computation of distributed nonlinear loads.

1.3 Outline

This work can be roughly divided into three parts:

- I. In the first part (sections 2 and 3), the basics are recapitulated. Section 2 provides a brief overview of model reduction via projection. The derivation of the equations of motion of a flexible body in the FFRF is outlined in section 3. In section 4, special attention is paid to those parts of the equations of motion that hold the body's compliance.
- II. In the second part (section 4), a numerical time integration method is proposed that is CPU time-efficient and accurate, even in the presence of many modes.
- III. The third part (sections 5 to 7) is devoted to the optimized computation of nonlinear forces. After explaining the basic idea of using stress modes in section 5, two concrete methods for efficient load computation are proposed: Semi-hyper-reduction (section 6) and hyper-reduction (section 7).

2 Brief review on model reduction of finite element structures via projection

Model order reduction is a key issue when the dynamic of Finite Element (FE) models is investigated. It is concerned with the question whether a system with a large number of degrees of freedom (dof) n_{FE} can be accurately represented by a transformed system of the size n_f with $n_f \ll n_{FE}$. An introduction and general reviews on this topic can be found in Noor (1994), Meirovitch (1980), Zu (2004) and de Klerk, Rixen and Voormeeren (2008) for general reviews and introduction on this topic. Since the 1950s a lot of reduction methods have been proposed. The first publications dealt with linear systems. After some time, proposals for systems with nonlinear behavior followed. In many of those publications, the displacement or deformation dof at which forces or torques act play a special role. Therefore, from now these dof are named either input dof or interface dof.

2.1 Linear FE models

The content of this section has been published in a very similar way in Witteveen (2012).

2.1.1 Introduction

For linear elastic structures with a moderate number of input dof, mode based reduction methods, like Component Mode Synthesis (CMS) have been developed to very reliable standard tools. An overview of the family of CMS methods can be found in Craig (1987) and in Craig (2000). Another, more recently published review has been done by de Klerk, Rixen and Voormeeren (2008). All CMS variants have in common, that the reduction base is formed by vibration modes and static deflection shapes. The latter ones are often obtained by static loads acting on the interface dof. In the last years some effort has been made in order to adapt model reduction methods for mechanical structures which come originally from control engineering, namely moment matching (MM) and balanced truncation (BT). In contrast to CMS where the trial vectors have a physical meaning, the latter two methods are based on mathematical considerations. A more detailed introduction into MM which is also known as ‘Krylov subspace method’ can be found in Lehner and Eberhard (2006), Craig and Su (1991), Benner (2012) and Lohmann and Salimbarhrami (2004). For an introduction to BT the interested reader is referred to Meyer and Srinivasan (1996), Reis and Stykel (2008), Benner and Saak (2011) and Yan, Tan and Mc Gaugy (2008). The