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PREFACE

It must be acknowledged that the essays presented here do not constitute a systematic account of any sort but represent occasional forays. Some deal with matters that happened to evoke my interest, others grew out of a chance encounter with a text I deemed to be of particular value. Throughout, challenges of the work itself more than compensated the author's efforts.

Logic has always been of crucially important concern to philosophers. My own involvement with the history of logic goes back to my work on Leibniz in the 1950s (represented by Chapter 8 of the present book). Thereafter, during the 1960s, I devoted considerable effort to the contributions of the medieval logicians of the Arabic-using world (here represented in Chapters 2-6). Moreover, I have from time to time returned to the area to look at some aspects of the more recent scene, as Chapters 8-9 illustrate.

In some instances the present essays have been overtaken by subsequent events—events which in fact they helped to promote. This is true in particular in chapter 6's analysis of Arabic work regarding temporal modalities, which was instrumental in evoking the important contributions of Tony Street of Cambridge University.

I am very grateful to Estelle Burris for her patient and conscientious help in preparing the material for publication.

Nicholas Rescher Pittsburgh PA May 2006

Chapter 1

ON ARISTOTLE'S APODEICTIC SYLLOGISMS

1. INTRODUCTION

Virtually all modern modal logicians have been troubled by Aristotle's insistence that, given a valid *first figure* categorical syllogism (of the purely assertoric type, XXX, where X is to represent the actual and L is necessary) that take the format

Major Premiss $(P_{\rm M})$ <u>Minor Premiss $(P_{\rm m})$ </u> Conclusion

will have the corresponding modal syllogism (of type LXL)

Necessarily: $P_{\rm M}$ $\underline{P_{\rm m}}$ Necessarily: C

also be valid. The correspondingly *LLL* syllogism must, of course, also be valid *a fortiori*, while the corresponding *XLL* syllogism will—so Aristotle has it—be invalid. Despite extensive discussions of the problem, a convincing rationale for Aristotle's theory has yet to be provided.¹ The aim of the present discussion is to propose a suggestion along these lines.

The leading idea of the present proposal is that, given syllogistic terms α and β , it is possible to define yet another term [$\alpha\beta$] to represent the β -

¹ For an overview of the current position, together with references to the literature, see Storrs McCall, *Aristotle's Modal Syllogisms* (Amsterdam, 1963) and Nicholas Rescher, "Aristotle Theory of Modal Syllogisms and Its Interpretation," in *Essays* in *Philosophical Analysis* (Pittsburgh, 1969), pp. 33-60. For the general background of the Aristotelian syllogistic see Gunther Patzig, *Aristotle's Theory of the Syllogism* (Dordrecht, 1968).

species of α . As will be seen below, these bracketed terms represent a version of Aristotle's process of *ecthesis* ("selecting out" a part of the range of a syllogistic term). The $[\alpha\beta]$'s are specifically those α 's which must be β relative to the hypothesis that they are α 's (by conditional or relative necessitation). Thus they might, for example, be those humans (α) that must be female (β), as some certainty must be. The essential point regarding this special term, one that is central for our present purposes, is that it is such as to validate the inference:

 $\frac{A\alpha\beta}{LA\ \alpha[\alpha\beta]}$

Intuitively, if all α 's are β 's, then all α 's must be such that they are necessity $[\alpha\beta]$'s, where this is the α subspecies of the β 's. (Thus if All mice are rodents, then All mice are necessarily members of the mouse subspecies of rodents.) Correspondingly, we would also have the inference:

 $\frac{I\alpha\beta}{LI\alpha[\alpha\beta]}$

Thus if Some dogs are pomeranians, then Some dogs [viz. pomeranians] are necessarily members of the pomeranian subspecies of dogs.

Such "bracketed terms", as we shall call them provide the materials out of which our interpretation of Aristotle's apodeictic syllogisms will be constructed. Once terms of this type are introduced, it becomes an interesting and significant result that the apodeictic sector of the Aristotelian modal syllogistic follows *in toto* as a natural consequence.

2. THE TECHNICAL RESULT

The notation and terminology here used will be that of McCall's *Aris-totle's Modal Syllogisms*, except for the additional primitive use of termbracketing and replacing McCall's rule of substitution, p. 37, by: (i") Rule of Substitution of terms for variables, where this does not involve identifying terms.²

An axiomatization of the assertoric moods *XXX*—and correspondingly of the apodeictic moods *LLL*—in line with the above revisions will be assumed.

In order to extend this basis to include all the apodeictic moods, we adopt the following axiomatic rules with respect to bracketed terms:

Group 1: *Modal Inferences of Type X to L*

I. *C Aab LAa*[*ab*] II. *C Iab LIa*[*ab*]

Group 2: Modal Inferences of Type L to L

I. C LAab LA[ca]bII. C LEab LE[ca]b

These four rules together with the laws of conversion and of modal conversion suffice to yield all the apodeictic moods. To show that all the valid apodeictic moods are desirable on this basis, we shall prove Fitch-style all of those of the first figure:

Barbara LXL	1	<i>LAbc</i> hyp
2	<u>Aab</u>	hyp
3	LAa[ab]	2, 1
4	LA[ab]c	1, III
5	LAab	2, 4, Barbara <i>LLL</i>
Celarent LXL	1	<i>LEbc</i> hyp
2	Aab	hyp
3	LAa[ab]	2, 1
4	LE[ab]c	1, IV
5	LEac	3, 4, Celarent <i>LLL</i>

² We shall not attempt to formalize (i') rigorously but the intent of (i") is that (say) l[ab]c, lbc, l[ab][cd], and (even) l[ab][ba] or I[ab][bb] be regarded as substitution instances of *Iab*, but not *Iaa* or I[ab][ab].

Darii <i>LXL</i>	1	LAbc	hyp
	2	Iab	hyp
	3	LIa[ab]	2, II
	4	LA[ab]c	1, III
	5	Llac	3, 4, Darii <i>LLL</i>
Ferio LXL	1	LEbc	hyp
Ferio LXL	$\begin{array}{c c}1\\2\end{array}$	LEbc Iab	hyp hyp
Ferio <i>LXL</i>	$\begin{array}{c c}1\\2\\3\end{array}$		• 1
Ferio <i>LXL</i>	I	Iab	hyp
Ferio <i>LXL</i>	3	Iab LIa[ab]	hyp 2, II

It should be noted that all the derivations follow a perfectly uniform plan, viz., (1) the use of bracketed terms to obtain (using I/II) a modalization from the assertoric minor premiss, in view of which (2) the bracketed term at issue in this minor can be subsumed as a special case under the apodeictic major (using III/IV).³

major premiss: minor premiss:: general rule: special case

When we take note of this line of thought we see why Aristotle taught that the major premiss of a modal syllogism can strengthen the modality of the conclusion above that of the minor premiss. For a rule that is necessarily (say) applicable to all of a group will be necessarily applicable to any sub-group, pretty much regardless of how this sub-group is constituted. On this view, the necessary properties of a genus must necessarily characterize even a contingently differentiated species. If all elms are necessarily deciduous, and all trees in my yard are elms, then all trees in my yard are necessarily deciduous (even though it is not necessary that the trees in my yard be elms). The "special case" subsumption at issue here can be viewed as a mode of application of the *dictum de omni et nullo* (*ibid.*, pp. 54-55).

³ This substantiates the idea of Rescher *op. cit.* (pp. 53-55) that a leading intuition of Aristotle's apodeictic syllogistic is that of a special case falling under a necessary rule: In short, Aristotle espouses the validity of Barbara *LXL* not on grounds of abstract formal logic, but on grounds of *applied* logic, on *epistemological* grounds. What he has in mind is the application of modal syllogisms within the framework of a theory of scientific inference along the lines of his own conceptions. We must recognize that it is Aristotle's concept that in truly scientific reasoning the relationship of major to minor premiss is governed by the proposition:

The adequacy of any formalization of Aristotle's theory of modal syllogisms depends not only on having the right theorems but also on lacking the wrong ones (which is where Lukasiewicz fails). An important test case is that the theory accepts Barbara *LXL* but omits Barbara *XLL*. We are safe on the first count; how do we fare on the second? Let us attempt to prove Barbara *XLL*:

Barbara
$$LXL$$
 1 Abc hyp
2 $LAab$ hyp
•
•
n $LAac$?

Clearly *LAac* is unavailable without the introduction of bracketed terms. Applying rule *I* to premiss *L* will yield *LAb*[*bc*]. This together with premiss 2 gives us *LAa*[*bc*]—*by* Barbara *LLL*. But now we are unable to proceed further; we simply cannot infer *LAac* from *LAa*[*bc*].⁴ Since this is in fact our only method of attack, Barbara *XLL* cannot be proven.

The remaining first-figure syllogisms will also be blocked for the type *XLL*. Take Celarent first:

Celarent XLL 1 Ebc
2
$$LAab$$

•
n $LEac$

This is blocked because there is no way of obtaining an L-qualified proposition from an E-premiss (or any negative premiss).

Next consider Darii:

⁴ If all α 's are necessarily β 's-that-in-fact-are- γ 's, it does not follow that all α 's are necessarily γ 's.

Darii XLL 1
$$Abc$$

2 $LIab$
3 $Lab[bc]$ 1, I
4 $LIa[bc]$ 2, 3, Darii LLL
•
•
n $LIac$

But this inference cannot be accomplished because we cannot infer *LIac* from LIa[bc].⁵

Finally take Ferio:

This inference *too* is blocked because there is no way of obtaining an *L*-qualified proposition from an E-premiss (or any negative premiss).

It might be noted that the four first figure *XLL* syllogisms are blocked by three principles:

(1) Disallowing the inference of any L-qualified proposition from a

C LAa[bc] LAac C LIa[bc] LIac C Aab A[ca]b C Eab E[ca] b

⁵ It deserves note that we cannot without serious consequences postulate the nonmodal counterpart of IV, viz., (IV) C *Eab E[ca]* b (together with the obvious moral principle that I- *CafJ* yields I- *CLaLfJ*). *For* IV entails C *l*[ca] *b lab*, whose modalized version is *CLl*[ca] *b LIab* or equivalently C *LIa[bc] LIac*. And just this principle must be excluded if Darii *XLL* is *to* be blocked. It is thus indicated that the assertoric counterparts *of* III and IV must be rejected, so that these represent specifically apodeictic modes *of* inference. In summary, by contrast with the acceptable theses I-IV, the following four theses should thus be rejected:

negative premiss.

(2) Disallowing the inference of *LAac* from *LAa[bc]*.

(3) Disallowing the inference of Llac from Lla[bc].⁶

These last two principles amount to: Disallowing the elimination of a bracketed term from an affirmative premiss.

Thus if appropriate restrictions (of a rather plausible sort) are postulated for inferences involving bracketed terms, none of the apodeictic syllogisms Aristotle regards as illicit will be forthcoming.

If the machinery developed thus far is acceptable from an Aristotelian point of view, we can perhaps explain Aristotle's silence regarding the validity of $LA\alpha\alpha$. If we are to reject $CA\alpha\beta LA\alpha\beta$ (which one must certainly reject), then given our machinery, we are committed to rejecting $LA\alpha\alpha$.⁷ This may be seen as follows:

1	Aab	hypothesis
2	LAa[ab]	1, I
3	LAbb	by the thesis at issue
4	LA[b]b	3, III
5	LAab	2, 4, Barbara LLL

This serves to motivate omission of $LA\alpha\alpha$. We can only explain the lack of an explicit rejection by saying that if one must reject *LAaa*, one might well prefer doing so quietly. (Though if one is enough of an essentialist, it would seem not incongruous to take the view that among all the α 's some should be α 's of necessity but others merely by accident, so that $LA\alpha\alpha$ would not be acceptable.)⁸ Although the Aristotelian modal syllogistic

⁶ Restrictions (2) and (3) are clearly plausible. If all or some α 's are γ 's, that does *not* mean they must necessarily be members of the γ -species of β 's.

⁷ In consequence of this rejection it would no longer be necessary to introduce the above-mentioned restriction on McCall's rule of substitution.

⁸ Previous attempts to formalize Aristotle's modal syllogic (specifically those of Lukasiewicz and McCall) also explicitly reject *LA/X/X*. See Jan Lukasiewicz, *Aristotle's Syllogistic*, 2nd edition (Oxford, 1957), p. 190, and Storrs McCall, *op. cit.*, p. 50.

must reject the thesis $LA\alpha\alpha$, the cognate thesis $LA[\alpha\alpha]$ is readily demonstrable:

1

Actually, although a strict proof does not seem available, it would appear that $LA[\alpha\alpha][\alpha\alpha]$ —and indeed even $LA[\alpha\beta][\alpha\beta]$ —could well be viewed as acceptable theses.

It is worthwhile to point out that the system, suggested here is consistent. We define a function h inductively as follows: (a) if α is a variable, $h(\alpha) = \alpha$, (b) $h([\alpha\beta]) = h(\beta)$, (c) $h(A\alpha\beta) = Ah(\alpha) h(\beta)$, (d) $h(I \alpha\beta) = Ih(\alpha) h(\beta)$, (e) $h(N\alpha) = Nh(\alpha)$, (f) $h(L\alpha) = h(\alpha)$ and (g) $h(C\alpha\beta) = Ch(\alpha) h(\beta)$. Clearly, if α is a theorem, $h(\alpha)$ is a theorem of the assertoric theory of the syllogism. So, our system is consistent if the assertoric theory is. But the latter is consistent.⁹

3. ECTHESIS

Aristotle does not give proofs for Baroco *LLL* and Bocardo *LLL* but merely outlines how they are to proceed (*An. pr.*, i. 8, 30a6). Both are to be proven by ecthesis.

We propose to construe this process—which Aristotle leaves somewhat mysterious—along the following lines:

(1) Nonmodal ecthesis

$$\frac{I\alpha\beta}{(\exists\gamma) [K a \gamma \alpha A \gamma \beta]} \qquad \qquad \frac{O\alpha\beta}{(\exists\gamma) [K a \gamma \alpha E \gamma \beta]}$$

(2) Modal ecthesis

$$\frac{LI\alpha\beta}{(\exists \gamma) [KLA[a\gamma]\alpha \ LA[\alpha\gamma]\beta]} \qquad \qquad \frac{LO\alpha\beta}{(\exists \gamma) [KLA[a\gamma]\alpha \ LE[\alpha\gamma]\beta]}$$

⁹ See, for example, J. C. Shepherdson's "On The Interpretation of Aristotelian Syllogistic," *Journal of Symbolic Logic*, vol. 21 (1956), pp. 137-147.

Ecthesis thus conceived, is a process for inferring universal propositions from particulars.¹⁰ Its central feature in the modal case is its recourse to bracketed terms as introduced above. (It might be noted that the inferences in (1) and (2) are to be reversible into corresponding inverse forms.) Thus our construal of nonmodal ecthesis coincides with that of Patzig.¹¹ Aristotle's observations at *An. pr.*, i. 6, 28a 22-26, are simply a *statement* of the inverse form of the affirmative case of nonmodal ecthesis, rather than representing—as W. D. Ross complains—an attempt at "merely proving one third-figure syllogism by means of another which is no more obviously valid."¹²

Let us examine the argument for Baroco *LLL* as $Ross^{13}$ presents it. According to Ross (p. 317) the proof goes as follows: assume that all *B* is necessarily *A* and that some C is necessarily not *A*. Take some species of C (say D) which is necessarily not *A*. Then all *B* is necessarily *A*, all D is necessarily not *A*, therefore all D is necessarily not *B* (by Camestres *LLL*). Therefore some C is necessarily not *B*. The reasoning may be formulated as follows:

1	LAba	hyp
2	LOca	hyp
3	$(\exists d) LE[cd]a$	ecthesis on 2
4	$(\exists d) LE[cd]b$	1, 3, Camestres LLL
5	LOcb	4, inverse ecthesis

Next, consider the argument for Bocardo *LLL*. Ross (*ibid.*) construes the argument as follows: assume that some C is necessarily not A and that all C is necessarily B. Take a species of C (say D) which is necessarily not A.

L

¹⁰ The inverse inferences (closely akin to Darapti and Felapton) are, of course, also valid, so that we are, in effect, dealing with equivalences.

¹¹ Cf. Gunther Patzig, Aristotle's Theory of the Syllogism (New York, 1968), pp. 156-168. In support of his interpretation of nonmodal ecthesis, Patzig cites Anal. Pr., i.28, 43b43-Ha2 and Ha9-11, which appears to be a statement of the equivalence of the premisses and their respective conclusions in (1).

¹² W. D. Ross, *Aristotle's Prior and Posterior Analyties* (Oxford, 1949), p. 32.

¹³ W. D. Ross, *ibid*.

Then all D is necessarily not A, all D is necessarily B, therefore some B is necessarily not A (by Felapton *LLL*). The reasoning also is readily formalized as follows:

1	LOca LAcb	hyp
2	LAcb	hyp
3	$(\exists d)$ <i>LE</i> [<i>cd</i>] <i>a</i>	ecthesis on 1
4	$(\exists d) LE[dc]a$	ecthesis on 1 3 (supposing $E[\alpha\beta]\gamma$ yields $E[\beta\alpha]\gamma$)
5	$(\forall d) LA[cd]b$	2, III
6		4, 5, Felapton <i>LLL</i>

The use of bracketed terms to explicate ecthesis along the lines outlined above thus provides a simple way to systematize the Aristotelian justification of certain apodeictic syllogisms.

4. CONCLUSION

The use of bracketed terms in connection with modal and ecthesis involving reasonings—is analogous in one significant respect: In both cases their introduction allows us "to do the impossible" in Aristotelian logic—albeit in a perfectly legitimate way. In the one case we move from an assertoric to an apodeictic proposition:

 $\frac{A\alpha\beta}{LA\alpha[\alpha\beta]}$

In the other case we move from a particular to a universal proposition:

In both cases the bracketing operator enables us to "select" from among all the α 's those which—given that a certain relationship holds between the α 's and β 's—bear a yet more stringent relation to the β 's than the α 's in general do.

The just indicated argument paradigm

 $\frac{LI\alpha\beta}{(\exists \gamma)LA[\alpha \gamma]\beta}$

deserves further comment. It is crucial that the particularized relation the premiss lays down between a and P (their I-linkage) is necessary, otherwise the conclusion would clearly not be forthcoming. Thus perception—which can establish particular linkages *de facto* but not necessarily—cannot provide scientific knowledge.¹⁴ Chance conjunctions in general cannot in the very nature of things be subject to demonstrations of necessity.¹⁵

That nonmodal ecthesis is a logically warranted (indeed virtually trivial) process can be seen along the following lines

- 1. Assume by way of hypothesis that: Some *a* is *b*.
- 2. Let X_1, X_2, \ldots be specifically those *a*'s that are b's and let us designate the group of these X_1 , the "*a*'s at issue", as X.
- 3. Then all these *X*'s are *a*'s (by definition of *X*) and moreover all *X*'s are *b*'s, and conversely (for the same reason).

Thus between the "*a*'s at issue", viz., X_1, X_2, \ldots , and *b* we have inserted a "middle term" (X) in such a way that (1) All the "a's at issue" are X's (and conversely) and (2) All X's are b's. No doubt here, in the assertoric (non-modal) case, we have done this insertion in a logically trivial way.

But in the modal case when *Some a is necessarily* β the issue of inserting an intermediate X such that both *All the a's at issue are X* and *All X is necessarily* β is not trivial at all. For whereas the motivation of the first of the two inferences under consideration is essentially a matter of pure logic that of the second is at bottom not logical, but metaphysical. If *some* α 's are necessarily α 's, then—so the inference has it—there must be some α delimitative species, the $[\alpha\gamma]$'s, *all* of which are necessarily P's. If some metals are necessarily magnet-attracted then there must be a type of metal (e.g., iron) all of which is necessarily magnet attracted. The governing intuition operative here lies deep in the philosophy of nature: Whenever α 's are such that some of them *must* be β 's, then this fact is capable of *ration*-

¹⁴ Cf. Aal. Post., I 31.

¹⁵ *Ibid*.

alization, i.e., there must in principle be a *natural kind* of α 's that are necessarily (essentially, lawfully) β 's.

A precursor version of the principle of causality is at work here: If some "men exposed to a certain virus" are in (the naturally necessitated course of things) "men who contract a certain disease", but some are not, then there must be some *characteristic* present within the former group in virtue of which those of its members exhibiting this characteristic *must all* contract the disease if exposed to it. To explain that some α 's have to be β 's we must find a naturally constituted species of the α 's all the members of which are necessarily β 's.¹⁶ Thus given "Some α 's are of necessity β 's", it follows from the requisites of explanatory rationalization that for some species γ of the α 's we have "All γ 's are necessarily β 's." We come here to what is essentially not a principle of logic but a metaphysical principle of rationalization. At this precise juncture, the logic of the matter is applied rather than pure—fusing with the theory of scientific explanation presented in *Posterior Analytics*.

From this standpoint, then, the principle of modal ecthesis

$\frac{LI\alpha\beta}{(\exists\gamma)KLA[\alpha\gamma]} LA[\alpha\gamma]\beta$

is based upon metaphysical rather than strictly logical considerations. This principle underwrites the equivalence:

 $LI\alpha\beta$ if and only if $(\exists\gamma)LA[\alpha\gamma]\beta$

This, in effect, is a "generalization principle for necessary connection". It stipulates that whenever a necessary connection exists between two particular groups α and β the matter cannot rest there. There must besomehow, no matter how well concealed—a *universal* necessary relationship from which this particular case derives and in what it inheres. There can be no particular necessity as such: necessity, whenever encountered, is always a specific instance of a *universal* necessity. It is thus easy to see the

¹⁶ The idea is closely analogous with the "generalization principle" in modern ethics, i.e., the thesis that if some certain men are obligated (or entitled) to do something, then this must be so because they belong to a group *all* of whose members are obligated (or entitled) to do so.

basis for Aristotle's policy (in *Posterior Analytics* and elsewhere) of assimilating necessity to universality. This perspective highlights Aristotle's fundamental position that science, since it deals with the necessary, cannot but deal with the universal as well. The irreducibly particular—the accidental—lies wholly outside the sphere of scientific rationalization.

Insofar as this view of the matter has merit, it stresses the conclusion that the fundamental motivation of Aristotle's modal syllogistic is heavily indebted to metaphysical rather than strictly logical considerations. Be this as it may, it is, in any case, significant that by introducing such an ecthesis-related specification of terms, the apodeictic sector of Aristotle's modal syllogistic is capable of complete and straightforward systematization.¹⁷

¹⁷ This chapter was originally published in *The Review of Metaphysics*, vol. 24 (1971), pp. 178-84. It was written in collaboration with Zane Parks.