

Nicholas Rescher
Studies in Quantitative Philosophizing

For Patrick Grim
In cordial friendship

Nicholas Rescher

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Quantitative Philosophizing

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Preface

Mathematics is so powerful and useful a thought-tool that virtually from the start of the subject philosophers have occasionally throughout the years been tempted to use it in the course of their work—with varying but almost always interesting results.

The present book brings together several case studies, dealing with relevant facets of the work of some of philosophy's all-time greats. The subject-matter topic being addressed differs significantly, but in each case there is an attempt to apply mathematical methods and perspectives to the solution of a key philosophical issue in a way that throws instructive light upon it.

On this basis it emerges that the question “Are mathematical methods useful in philosophy?” finds a suggestive response in the fact that over two millennia key figures in the history of the subject have indeed thought so. And they have substantiated this view not so much by abstract argumentation on the basis of general principles, but by making this point through actual practice.

Plato is reported as insisting that the good philosopher must be competent in mathematics. And as these studies show that some of the most accomplished philosophers since his day proceeded in their own work in a way that indicates emphatic agreement.

The first three chapters (on Plato, Aristotle, and Ockham) appear here for the first time. The final three have previously appeared in article form. (Detailed references are given in the footnotes.)

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Nicholas Rescher
Pittsburgh, Pennsylvania
October 2009

Chapter 1

ON THE EPISTEMOLOGY OF PLATO'S DIVIDED LINE

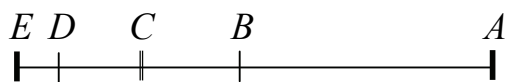
1. THE DIVIDED LINE AND ITS DIVISIONS

The principal contentions of this discussion are four: (1) that no interpretation of Books VI and VII of the *Republic* should be deemed adequate that fails to integrate its philosophical content with the mathematical detail Plato uses in his description of man's cognitive situation, but that some plausible account must be given of why and how it is that those quantitative relations—the proportionalities and not mere analogies of the Divided Line—should hold; (2) that for a variety of reasons over 100 years of commentary has failed to meet this demand in a plausible way, a (3) that to remedy this situation it is instructive to take the Line's narrative as literally as possible, and then look to its emplacement within the larger issues in Plato's epistemology; and (4) that rather than dealing with different sorts of *objects*, the Line discussion deals with different modes (grades) of knowing (or, better, *cognition*).

* * *

In Book VI of his classic dialogue, *The Republic*, Plato contemplated four factors at issue in inquiry and cognition: ideals or ideas (such as perfect beauty, justice, or goodness);¹ mathematical idealizations (such as triangles, circles, or spheres); mundane, visible objects made by nature or man; and mere images, such as shadows and reflections. For abbreviative convenience we shall refer to these Platonic types as ideas (or forms), mathematical, sensibles, and images, respectively.

With this classification in view, Plato proceeded to envision our knowledge about the world in terms of an arrangement whose situation stands as follows:



In setting this out he proceeded as follows:

Suppose you take a line $[EA]$, cut it into two unequal parts [at C] to represent, in proportion, the worlds of things seen $[EC]$ and that of things thought $[CA]$, and then cut each part in the same proportion [at D and B]. Your two parts in the world of things seen $[ED$ and $DC]$ will differ in degree of clearness and dimness, and one part $[ED]$ will contain mere [sensory] images such as, first of all shadows, then reflections in water then surfaces which are of a close texture, smooth and shiny, and everything of that kind, if you understand.²

The realm of ideas is generated and organized under the aegis of a supreme agency, the Idea of the Good. In the lead-up to the discussion of the Divided Line in book VI of the *Republic*, Plato (or, rather, his protagonist Socrates) acknowledges (506d–e) his incapacity to expound the Idea of the Good itself, instead stressing its role in accounting for certain consequences, its “offspring” (*ekgonos*) and the “highest studies” (*mathêmata megista*, 504A) that provide a pathway towards it. And this path, so he maintains, can be illustrated by means of that diagrammatic line.

Plato’s Socrates then goes on to explain that in moving along a line from the mundane to the ideal we confront the following situation:

In the first part $[EC]$ the soul in its search is compelled to use the images of the things being imitated [that lie in DC] ... In the second part $[CA]$, the soul passes from an assumption to a first principle free from assumption, without the help of images which the other part $[EC]$ uses, and makes its path of enquiry amongst idealizations themselves by means of them alone. (510B)

Plato correspondingly distinguished between the visible “things of the eye” (things seen, *horata*) and the intelligible “things of the mind” (things thought, *noêta*). Preeminent in the later category are the “ideas” or “forms” (*ideai*) that provide the model or prototype (*paradeigma*) conformity to which constitutes things as the kind of thing they are. Yet not these ideas alone, but also the mathematical idealizations have a paramount role in the realm of intelligibles:

When geometers use visible figures and discuss about them, they are not thinking of these that they can see but rather the ideas that these resemble; a square *in itself* is what they speak of, and a diameter *in itself*, not the one they are drawing ... What they seek is to see those ideas which can be seen only by the mind. (510D)

Plato accordingly divided his line of cognition into two parts, respectively representing the intelligible and the visible realms, and then divides each of these into two parts into a higher and lower, each dealing with a correlative sort of cognition, as follows:

I. “Intelligibles”

1. Higher: ideas (*AB*)
2. Lower: mathematical (*BC*)

II. “Visibles”

1. Higher: sensibles (*CD*)
2. Lower: images (*DE*)

The cognitive landscape is mutually dualistic, contemplating two realms, the changeable and the unchangeable. However, the overall epistemology is quadratic, contemplating higher and lower modes of knowledge with respect to either category. Accordingly, as Plato saw it, what is pivotal with each of these four cognitive capacities in their relation to spatio-temporal issues can be indicated on the lines of Display 1.³

The four modes of cognition at issue thus differ in standing and status. At the top of the scale stand the Ideas—the timeless ultimates of Platonic concern. As G. W. Leibniz was to put it:

The Platonists were not far wrong in recognizing four kinds of cognition of the mind ... conjecture, experience, demonstration, and [finally] pure intuition which looks into the connections of truth by a single act of the mind and belongs to God in all things but is given to us in simple matters only.⁴

At the very bottom of the scale stand the “images” (*eikones*) at issue in suppositions based on the fleeting and superficial seemings of things: “shadows, reflections in pools and hard, smooth and polished surfaces, and everything of that sort” (510A).⁵ The formal deliberations of ratiocination and the concrete observations that ground our convictions about the world’s objects fall in between.

As regards the mathematical, there is an instructive passage in a critique of Plato in Aristotle’s *Metaphysics*.

Display 1

HOW CAPACITY CONCERNS DIFFER

Capacity	Mode of Cognition	Concerns	Temporal Aspect	(Mundane Spatio-Physical Aspect)	
<i>aisthesis</i>	<i>eikasia</i> (supposition conjecture or imagination)	Images (<i>eikones</i>)	Fleeting	Present	} Sensible Domain
<i>aisthesi</i>	<i>pistis</i> (observation-based conviction or belief)	Sensibles (<i>aisthêta</i>)	Transitory	Present	
<i>logos</i>	<i>logos dianoia</i> (rationcination or discursive thought)	Mathematicals (<i>mathêmatika</i>)	Unchanging	Representable*	} Intelligible Domain
<i>nous</i>	<i>epistêmê</i> (rational insight or reason)	Ideas (<i>ideai</i>)	Timeless	Absent	

*NOTE: What is here called *mathematicals* may encompass symbolically mediated thought in general. While physical objects such as diagrams and counters (“calculi”) can represent mathematical, the physical world’s objects only “participate” in ideals and cannot *represent* them. Participation reaches across a wider gap than does representation.

Besides the Sensibles (*aisthêta*) and the Forms (*ideai*) he says that there are mathematical (*mathêmatika*). These, so he says, are intermediate (*metaxa*) differing from the Sensibles in being eternal and immutable and from the Forms in that there are many like instances whereas the form itself is in each case unique. (*Metaphysics* 987b 14–18).

We thus have it that an individual Idea/Form is a single unique unit, despite there being a plurality of concrete particulars that participate in it. But a geometrical shape, for example a circle, has many abstract representations (differing in diameter, say), which are not concrete—though admitting of concrete participants in their turn.⁶

In summarizing the Divided Line discussion, the *Republic* stipulates that one should:

Accept the four response-capacities (*pathêmata*) of the soul as corresponding to those four sectors: rational insight (*noêsis*) as the highest, ratiocination (*dianoia*) as the second, conviction (*pistis*) as the third, and supposition (*eikasia*) as the last; and arrange them proportionately, considering that they involve clarity (*saphêneia*) to the extent that the objects involve actual truth (*alêtheia*). (511E)

Display 2

DESIGNATION FOR THE PLATONIC CAPACITIES

epitêmê/logos/pistis/eikasia

rational insight//ratiocination//conviction//supposition (Rescher 2009)

intuition//demonstration//belief//conjecture (Whewell 1860)

intelligence//thinking//belief//imagining (Cornford 1945)

reason//understanding//belief//imagination (Wedberg, 1955)

reason//understanding//belief//conjecture (Rouse 1956)

intelligence//understanding//faith//conjecture (Malcolm 1962)

intelligence//thinking//belief//illusion (Cross and Woosley 1964)

intelligence//thought//conviction//conjecture (Robinson 1984)

understanding//thought//confidence//imagination (Fine 1990)

understanding//thought//belief//imagination (Grube 1974)

intellect//thought//trust//fancy (Denyer 2007)

As Display 2 indicates, Plato's translators have used a wide variety of terms for rendering the four Platonic faculties. While I believe my own translations come closest to what Plato has in view, I think that the time has passed for every discussant to introduce his own terminology. And so while I myself believe that the best nomenclature would be:

Rational Insight//Ratiocination//Conviction//Supposition

nevertheless, in the interests of impartiality, I think that we can live with the majority-rules reading of:

Intellect//Thought//Belief//Imagination

On this basis, every polled interpreter gets to have *something* their own way excepting—alas!—myself. Still, for the present I shall sink my own preferences in deference to the common good.

Be the issue of terminology as it may, the fact remains that a definite four-rung ladder is at issue here, which conjointly characterizes both a type of knowing and a grade of knowledge. In ascending order these four are: superficial inspection (*eikasia*), observation (*pistis*), mathematically informed understanding (*dianoia*), and rational insight (*epistêmê*). Here mind-managed *dianoia*, formal reasoning based on mathematics and logic, is seen as a more powerful cognitive instrumentality than anything that the senses have to offer us. But at the very top of the scale stands *epistêmê*, the authentic rational knowledge characterized by Plato as unerring (*anmarêton*: 477A), access to which is possible through dialectical reasoning alone. And what renders *dianoia*/mathematics inferior to *noêsis*/ideatics is that mathematical reasoning still relies on images (diagrams) and hypotheses while the methods of dialectic involve no such “contaminating” compromises with an inferior resource.

Those four Platonic capacities are not different stages of learning, let alone “stages of mental development.” Nor do they address different kinds of existents of variantly inferior and superior nature, but rather different and variantly meritorious modes of cognition regarding existence: they deal not in degrees of reality but differently adequate degrees of insight into reality. And in just this way one recent interpreter speaks very sensibly of “*les quatre degrés de connaissance*.”⁷ What we have here is, in effect, four grades of knowing: superficial inspecting, close examination, quantitative measurement, and synoptic analysis. They represent different modes of knowing that offer increasingly more accurate insight into the nature of True Reality.⁸ Accordingly, the question “Does the Divided Line discussion deal with process (modes of cognition) or with product (objects of cognition): does it deal with ontology or with epistemology?” has to be answered by accentuating the latter. On the perspective at issue here, the crux lies in different modes of knowing, all addressed to one selfsame object, Reality, but dealing with it in different cognitive ways having very different degrees of clarity and adequacy.⁹

Along just these lines, Henry Jackson wrote “Now if the object of the inferior intellectual method is to the object of the superior as an image or reflection is to the thing itself ... it would seem that the objects of the two sorts of intellectual methods are not distinct existence, but the same existences viewed [differently—] in the one case indirectly and in the other

case directly.”¹⁰ To be sure, Stocks 1911 maintained that Plato subscribed “an old assumption, prevailed among the Greeks, [namely] that differences of apprehension must be due to differences of the apprehended.”¹¹ There is, however, no reason to saddle Plato with the idea that different capacities *must* deal with different sorts of objects, but only that they *can* do so. In specific, those “higher” capacities need not deal with a higher class of objects: it is just that they can do so on occasion. The key point, as I see it, is not so much—or not saliently—that different sorts of things—different kinds of existing things are at issue here, but rather that we deal with different features of one single kind of thing—Reality—which can figure in cognition with very varying degrees of illumination. The issue, in sum, is a matter of dealing with things differently rather than one of dealing with different things. (See Display 3.)

In her illuminating 1990 paper, Gail Fine contests what she calls the *two-world theory* according to which there is the world of sense and the world of intelligible forms, the first accessible only to mere belief but the second accessible to actual knowledge. The present approach takes this rejection one step further. It rejects not only the idea that different cognitive faculties address different “worlds” but also that they address different *or* dimensions of Reality. Instead, those different faculties address one object (Reality as it were) but with a very different yield in point of informative adequacy—though what even the lowest and most imperfect of them provides is not *entirely* useless. And the rationale of this view of the matter is in the final analysis that it best and most smoothly accommodates the comparabilities on which the entire Divided Line discussion is predicated.

And this view of the matter is nowise contradicted by the discussion at the end of Book V where Plato stresses the different powers (*dunameis*) and different missions or functions of the former facilities. The contention *heterô ara heteron ti dunanenê hekatera autôn pephuken ti*. (*Republic*, 478) has indeed been translated: “Each of them, since it has a different power, is related to a different object” (Shorey-Loeb). But it would actually be more helpful—and more accurate—here to read *product* rather than *object*, seeing that this would alternate the suggestion that some distinctive type of thing is at issue.

Viewed from this angle, the discussion of the cognitive faculties at the end of Book 5 is seen to hold that they deal with different takes on the real, and so not with different *kinds* of existents but with different ways of gaining a cognitive grip on what exists. It is certainly possible to argue for that variant interpretation, but the governing analogy of clarity of vision and il-

lumination militates against this. The person who sees clearly, the person who sees poorly, and the person who is near-blind do not see different objects but rather all see rather differently and take what is seen to have very different features—only some few of which are authentic.

Display 3

PLATO'S VIEW OF COGNITIVE PROCESSES AND THEIR OBJECTS

<i>Cognitive Resource or Capacity</i>	<i>Process of Cognition</i>	<i>Resources of Cognition</i>
I. KNOWING (<i>nous</i> or <i>gnôsis</i>)	I. INSIGHT (<i>noêsis</i>)	I. INTELLIGIBLE THOUGHTS (<i>noêta</i>)
1. Rational insight (<i>epistêmê</i>)	1. Intuitive grasps (<i>epistasis</i>)	1. Ideals and ideas, "Forms" (<i>ideai, gnôsta</i>)
2. Ratiocination (<i>dianoia</i>)	2. Formal reasoning (<i>dianoêsis</i>)	2. Mathematical Conceptions (<i>mathêmata</i>)
II. OPINING (<i>doxa</i>) [SENSING]	II. SENSORY APPREHENSIONS or (<i>aisthêta</i>)	II. SENSE JUDGMENTS (<i>doxasta</i>)
1. Conviction (<i>pistis</i>)	1. Observation (<i>horasis</i>) and more generally perception (<i>aesthesia</i>)	1. Observed Features (<i>horata</i>)
2. Conjecture and seeming (<i>eikasia</i>)	2. Imaging (<i>hêmioîsis</i>)	2. Casual Appearances or "Images" (<i>phantasmata</i> or <i>eikona</i>)

And so, notwithstanding the inclination of interpreters to have it that Plato holds that different faculties address different *sorts* or *kinds* or *classes* (Wedberg 1955, p. 108) of objects, the prospect is not only open but actually inviting of seeing what is at issue is a matter of different features or aspects of reality, differentiated with regard to the extent of the accuracy, authenticity of the information being furnished. So that, for example, those "mere appearances" do not reflect a clear grasp of a murky (or shadowy) object, but rather the confused, fuzzy product of a poor vision of reality.¹²

It is a salient feature of the Divided Line narrative that a certain proportionality obtains uniformly throughout these divisions, as represented by the dual proportions:

$$I : II :: I_1 : I_2 :: II_1 : II_2$$

Thus overall, all of the following ratios (proportions) are all to be identical.

- opinion : knowledge ($EC : CA$)
- mathematical idealizations : ideal realities ($CB : BA$)
- appearances : perceptions ($ED : DC$)

Operative throughout is the crucial contrast between deep understanding (*gnôsis*) and mere superficial belief (*doxa*).

The resultant situation is encapsulated in the line elaboration of Display 4. Scholars have worried—and of course disagreed—about whether the line is horizontal or vertical or diagonal.¹³ But this worry overlooks the clear lesson of Greek geometry that the orientation of a diagram just does not matter when the internal relations of a figure is at issue.

Against this background, the present discussion will implement a certain definite perspective and procedure. It proposes to take the Divided Line narrative seriously as it stands literally and not more than minimally figurative or metaphorical. And it then asks where this leads in regard to the larger issues of Plato's epistemology. So where most discussants have asked what Plato's epistemology means for the Divided Line, the present discussion proposes to reverse this interpretative strategy.

2. WHAT DO THOSE PROPORTIONS REPRESENT?

A helpful starting point for considering in Plato's account here is the idea of a relational comparison or analogy based on the pattern:

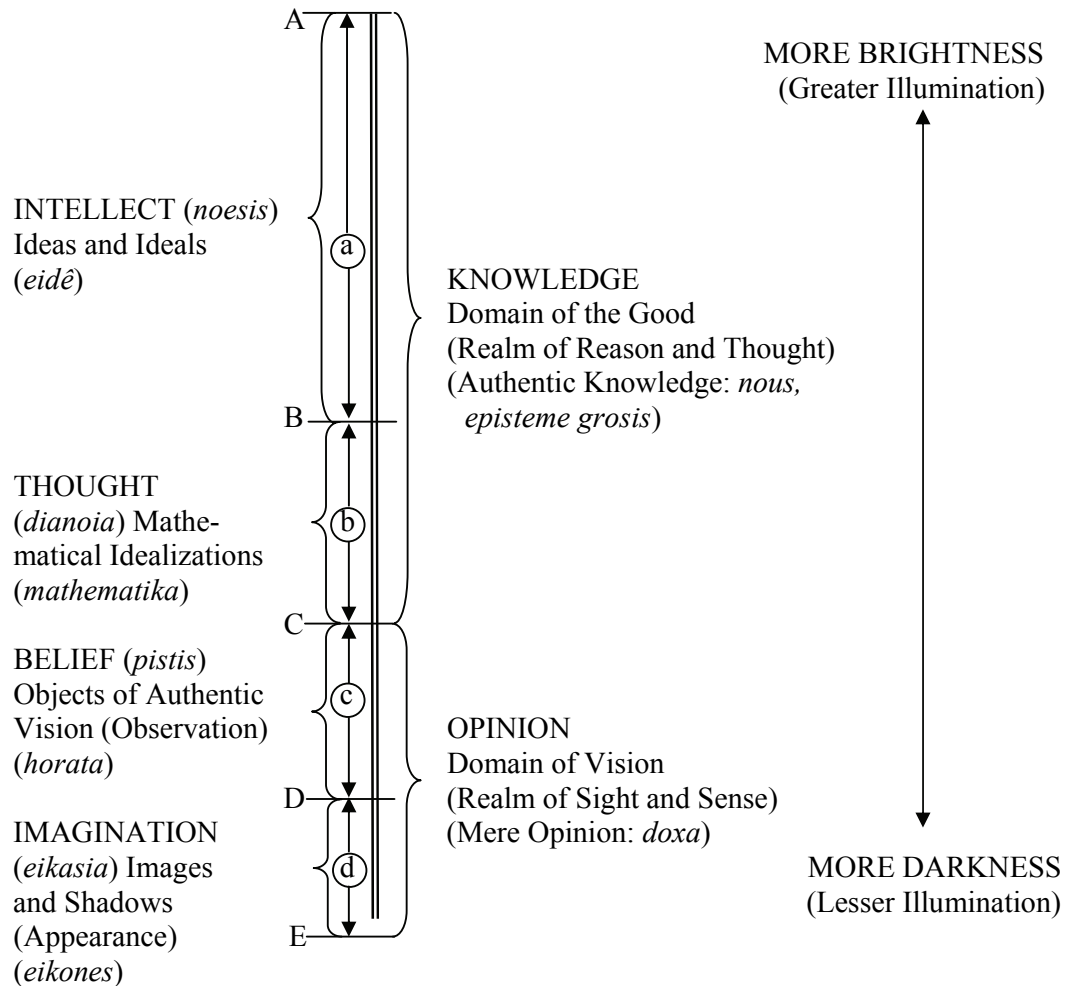
- Even as X is to Y in point of ϕ so also Z is to W in point of ϕ .

On this basis, for example, the “ship of state” analogy would emerge roughly as follows:

- Even as a ship's people (crew and passengers) live under the aegis of a directive power (the captain) that is ultimately responsible for their well-being, so also do the people of a country live under the aegis of

Display 4

PLATO'S DIVIDED LINE



a directive power (the government) that is ultimately responsible for their well-being.

What is at issue in all such cases is an analogizing proportionality of the format:

$$X : Y :: Z : W \quad \text{in point of } \phi$$