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Dieter Röss

Learning and Teaching Mathematics using Simulations

Plus 2000 Examples from Physics

De Gruyter

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Preface

The idea of writing this *digital book* was born during discussions among a circle of friends¹ of the following questions:

Author

- (a) Why are physics and mathematics so unpopular at School?
- (b) Why are there not more school leavers that are eager to study natural sciences and technology?
- (c) Why do the large majority of first year students dismiss the very good subject-related and professional career opportunities in these professions?

Already, in the final years of school, mathematics and physics are considered to be *hard subjects*. Universities grudgingly accept that the mathematical knowledge of many school leavers is insufficient for taking up subject studies and needs to be improved by bridging courses.

A shockingly large number of students already fail in the first semesters of university. This will have serious consequences for the future welfare of our society, as we urgently require a sufficient number of well qualified young professionals in scientific and technical jobs to succeed the current generation of scientists and engineers.

It is easy to understand why the younger generation choose those *soft subjects* at university for which they feel better equipped and where they see better chances of success. The fact that the monetary concerns of finding a job later are not considered to be crucial in subject choice can actually be considered as a likeable attitude in students.

Why is it that mathematics and physics are considered to be so *difficult*? In fact, these should benefit from not being *rote learning subjects*: if one has understood a specific physics or mathematics problem within its context, one can forget the small details, since they can be reconstructed from the larger context.

It is obvious that, in our schools, one often does not manage to achieve this state of understanding and insight into the mathematical structure and laws of nature; the instruction therefore cannot provide the wonderful experience of having *understood* something. Thus, physics can indeed become a *cumbersome* subject full of incomprehensible and disconnected formulas and tedious calculations, and mathematics an art of *computation* that is build on memorization, and which increases in complexity from

¹ Leading members of the *Deutschen Physikalischen Gesellschaft* (DPG; with 57 000 members, the largest society of its kind world wide), of the *Wilhelm and Else Heraeus Foundation* (WEH-Stiftung), and individual colleagues from the physics community, among them in particular Prof. Dr. Siegfried Grossmann and Prof. Dr. Werner Martienssen.

the times tables up to integration, while the fundamental ideas and deeper connections never become clear to the learner.

As the PISA study showed in 2003, this dilemma has developed in recent decades to such an extent that the level of mathematics and physics at German schools has declined from an earlier assumed top position to a “measured” weak, mediocre level. Similar declines in the standards of mathematics and physics have been reported in other countries.

What is the reason for this problem? We think that one of the most important reasons can be found in the **subject-specific education of future teachers at the universities!** Teacher training has often been treated as a stripped down appendix to the education of scientists. However, school teachers determine, in their respective subjects, the quality of education and the interests of the next generation! Their very important role in society as *multipliers* has been neglected in relation to the education of future researchers representing a given subject. The resulting lack of recognition for students preparing to become teachers has certainly contributed to the lack of young teachers available to fill open positions in physics and mathematics.

Two developments in the immediate past have worsened this situation and made it clear that a turnaround is necessary:

- Educational policy has given, for good reason, more prominence to didactics and pedagogical studies, but has limited the duration of studies very rigidly. This, however, has meant that not much time is left for studying the subject matter.
- Students experienced the Bologna process in Europe as a transformation of the traditional freedom of academic study into stricter control, of a kind experienced at the schools that they had just fled. Overloaded syllabuses, and nearly continuous inspection of study progress with early crucial examinations led to early selection and a high failure rate.

The attempt to serve “old wine in new skins”, that is, to cram the traditional degree programs and the subject matter, which has grown due to scientific progress, into a shorter bachelor degree, has led to partial chaos and a general unhappiness with study conditions.

In 2005 *Siegfried Grossman* entered into discussions with the author with the conclusion that it is a fundamental mistake to mix the subject-specific education of teachers with that of researchers.² They demanded specially developed *sui generis* curricula for studies preparing for the teaching professions, which are directed at the future teaching job and that take into account the available time, which in Germany is limited by trainee teachers having to study two different subjects.

2 S. Grossmann, D. Röss: “Thesen zum Lehramtsstudium Physik – Plädoyer für eine eigenständige Lehrerbildung”, Physik Journal© 2005 4 (2005) Nr. 10, page 49

Understanding the subject and connections between different areas should be paramount, as opposed to detailed knowledge and specialized skills. In 2006, the DPG produced a careful analysis and documentation and thus made *sui generis* studies a general demand of colleagues involved in the DPG.

DPG-Memo

In order to realize this vision, it would be counterproductive to base our actions in relation to schools and students on past conditions or wishful thinking. We need to accept today's conditions, as well as the technical possibilities, in a positive spirit. The gymnasiums (German secondary schools that lead to the "Abitur", their final examination, allowing entrance to universities) should no longer be institutions for the elite, but should, in future, lead half of all children to the "Abitur". Access to a high level education could be similarly improved in many other educational systems around the world.

Our children and grandchildren are growing up in world with many stimulations and diversions, but have media skills that neither their parents nor grandparents had, for example their knowledge of, and playful dexterity with, the media and technological devices. This *digital book* is the attempt to put the abovementioned studies on a foundation that makes use of these skills and dexterity.

In this book, an important subset of the mathematical foundations is embedded in a systematically evolving text and presented with the help of numerical simulations, and is visualized in numerous ways. The PC takes care of the often tedious calculations. Thus, the user can concentrate on understanding the subject matter, the context and the algorithms used.

Since all the simulations are interactive and can, in many cases, also be used for scenarios that are totally different from those given, the students are thus given a quasi "*experimental*" access to mathematics. We make use of the fact that a visual impression is more intensive and permanent than a heard or read one, and that experience based on one's own actions results in deeper understanding than the mere reception of someone else's knowledge. In addition, playfulness is given free range to visually experience and grasp the intellectual stimulation and aesthetic beauty of mathematical structures.

The book provides colleagues in physics and mathematics with a thesaurus of simulations for the development of their own curricula. In addition to textbooks, this thesaurus gives physics students the possibility of a deepened understanding of fundamental mathematical notions and physical phenomena. Future teachers can, during their own training, experience the potential of modern media for the realization of interactive lessons in mathematics. Interested high school students can attempt a light-hearted introduction to a higher level of mathematics; they will probably have less trouble with the techniques used than some older people.

For the simulations, the package *Easy Java Simulation* (EJS) is used, which provides a simple fast-tracked introduction to the development of simulations in *Java*. The files produced with EJS are very transparent, and can be easily changed and

reused as building blocks for one's own developments. The author considers EJS to be a prime candidate to become the standard program for didactically oriented simulations.

The authors of *EJS*, *Francesco Esquembre* and *Wolfgang Christian*, have allowed me to supplement this text, which is primarily an introduction to mathematics, with more than 2000 physics-based simulations, for which I owe them many thanks. *Francesco Esquembre* has also assisted me personally in numerous ways with the creation of the mathematical simulations. I also thank *Eugene Butikov* for allowing me to include his wonderful cosmological simulations.

I want to give many thanks to *Siegfried Grossmann* for the dedication and care that he has applied to critically reading the text and simulations, and for the many valuable hints, which have contributed to the final version. *Ernst Dreisigacker*, the general manager of WEH-foundation, has supported me with the careful correction of details and with lively discussions.

Over the last three years I have had many involved discussions with *Werner Martienssen* about a book with a similar goal, i.e. to assist in reforming and improving the physics education of future teachers, and which is due to be published soon.³ The idea to write this *digital introduction to mathematics* came up during these discussions.

The staff members at De Gruyters have done a great job in the production of this complicated publication. My special thanks go to Dr. von Friedeburg and to Ulrike Swientek for their personal engagement and permanent encouragement. Katherine Thomasset was a great partner in the final translation of the German original into English.

I want to thank my wife Doris for the loving understanding with which she tolerated my absentmindedness while this work was written. I promise improvement!

16 May 2011

Dieter Röss

3 "Physik im 21. Jahrhundert: Essays zum Stand der Physik". Editors Werner Martienssen and Dieter Röss, Springer Berlin 2011

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Guide to simulation technique

Please use the deeply structured table of contents for the mathematical text, for the e-book in addition the search function of the Acrobat Reader.

The following index is intended as a door opener to the simulation technique used and to the mathematical simulations. In the e-book the specific pages can be directly addressed with links.

The number of a simulation in the index corresponds to its order of appearance in the text.

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1 Introduction

1.1 Goal and structure of the digital book

This book is available in two versions: a printed one and an electronic one. All users can access the simulations at http://mathesim.degruyter.de/jws_en/. If you have chosen the eBook, or are accessing it through your library, the PDF is linked to the simulation files via the internet. The eBook also gives you, via the “supplemental material” button, access to download the complete software package (> 700 MB) for offline-use; the PDF in that package is linked to all simulations via hyperlinks. Buyers of the print version who would like to download and install the software to their local system can obtain access by contacting info@degruyter.com for registration. The directories of Figures 1.1 and 1.2 correspond to the download solution. It illustrates selected mathematical methods that are important for the presentation and the understanding of the contexts of physics.

The foundations of these mathematical methods are introduced by example. The programmability and computational power of the PC is used to visualize these methods, undertake calculations, change parameters, present connections interactively and present computation processes via interactive simulation and animation, in an interesting manner. In addition, playfulness is given free range. This presentation is also intended to make the beauty and aesthetics of mathematics visible.

The material provided in this book allows the user to penetrate mathematical structures and tools in an **experimental** manner. In particular, topics have been chosen that are difficult to imagine in an abstract manner, such as complex numbers, infinite sequences, transitions to the limit, fields, solutions of differential equations and so on.

All individual simulations contain extensive descriptions and suggestions for experiments. The user can always interactively engage with the simulations, and in many cases pre-programmed functions can be edited or new ones can be introduced. After some initial training in the *EJS (Easy Java Simulation)* program the user can open all files, change them and develop them further.

With one exception, Java programs have been used that were either created from scratch or taken from the freely available internet projects *Open Source Physics (OSP)* and *EJS*.

Our own mathematical simulations were created with the *EJS*-program that has been developed by *Francisco Esquembre*. Due to its graphical user interface, this program immensely simplifies the development and modification of simulations in com-

OSP

EJS

parison to “classical programming” in Java. This program and its documentation are contained in this book, but are also freely available on the internet.

We will, however, abstain from explaining the mathematical and computational techniques in systematic detail. The in-depth study of the mathematical and numerical methods will be left to specialist textbooks⁴.

The pictures contained in the the pdf file mostly show screen shots of the respective simulation. When in the caption of such a picture *simulation* is clicked at for the first time a small context menu appears that asks, as a security measure against viruses, whether this file should be opened. You may confirm this and mark a check box, to avoid this dialog in future. The simulation will then be started immediately after clicking on simulation.

Where the reader might wish to learn more about the topic, *links* to internet pages have been inserted next to the text. They often point to *Wikipedia*-pages, from where further navigation is easy. These links are resolved in the outlined text boxes on the margin. All simulations can be accessed individually at http://mathesim.degruyter.de/jws_en/. Buyers of the printed book can also ask for download of the complete file package with interlinked text at info@degruyter.com for operation at their PC.

The appendix in Chapter 11 contains a short introduction to the EJS program and an extensive collection of simulations from all areas of physics, which have mostly been created using this tool. In order for these simulations to run on your computer, the *Java Runtime Environment (JRE)* must be installed, which you can download for free from the *SUN*-homepage in the latest version, using the link given at the border.

It is advisable to follow the suggestion to install the JRE into `\Programs\JAVA\`. For newer EJS simulations with *3D-Rendering* you can download the *Java 3D* program from the same page.

Sun

1.2 Directories

The book consists of 3 units:

- The continuous **interlinked book text *e-Exmat*** as a *PDF* file of around 30 MB.
- A **secondary directory *workspace*** containing a directory tree, which is ordered according to topics and authors. It contains more than 2000 simulation files of around 650 MB size, of which around 1000 are executable *jar* files⁵ that can be activated from inside the text. The *launcher* files among them also link to many

⁴ For example: *Mathematischer Einführungskurs für die Physik* 9. Auflage, Siegfried Grossmann (Teubner 2008) ISBN 3-519-33074-1; *Open Source Physics – A User's Guide With Examples* Wolfgang Christian (Pearson 2006) ISBN-10: 080537759X and ISBN-13: 9780805377590; *Mathematische Grundlagen für das Lehramtsstudium Physik*, Franz Embacher (Vieweg + Teubner 2008), ISBN 978-3-8348-0619-2.

⁵ *jar* files can be executed by themselves.

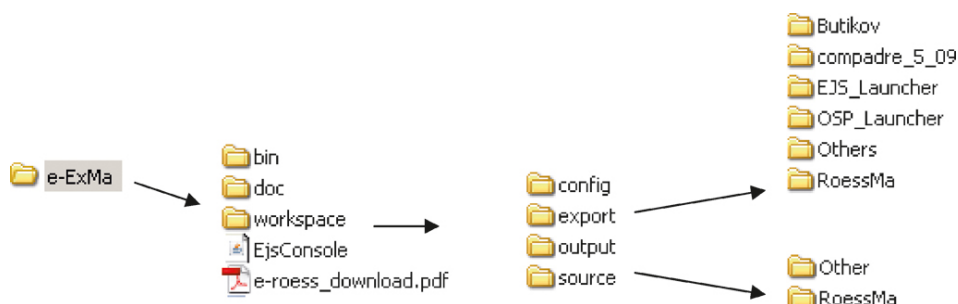


Figure 1.1. Main directory tree.

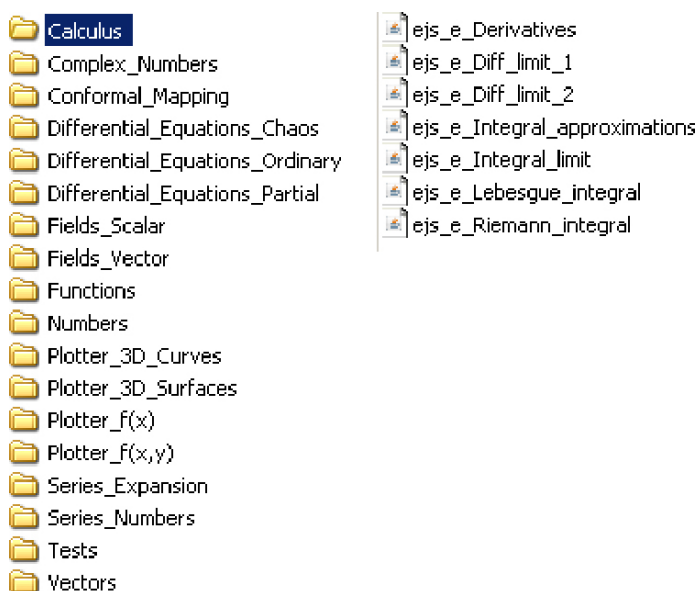


Figure 1.2. Directory tree of mathematical simulations.

secondary files for the individual simulations. The remainder are saved as *xml* files.⁶

- The **EJS console** for opening the *xml* files and working on the *jar* files, supplemented by documentation on the EJS program. The console does not have to be installed. It is contained in the program and can be started directly.

Console

⁶ *xml* is an abbreviation for *Extensible Markup Language*. For our purposes *xml* files are text files that contain the instruction for the simulations. They cannot be executed by themselves, but are opened from the EJS console, from where the corresponding *jar* file is easily created via assembling the required Java library components. To view or directly change the *xml* file one can, for example, open it with *notepad*. An introduction to *EJS* is given in the appendix.

The main directory **e-ExMa**, which can be copied to an arbitrary place on the hard disk, contains the text file **e-Roess_download.pdf** the **Ejs console** and the directory **workspace**, with all simulation files. In **doc** are some documentation files for EJS and in **bin**, some configuration files and library files of the console. *Workspace* contains as sub-directories **export** for all executable jar files and **source** for the xml files that are meant to be opened from the console. In **output**, html files are saved while working with the console.

Export is divided into **RoessMa** for the mathematical simulations of the ongoing text, **Butikov** and **compadre** for the physics simulation files of the appendix. **Other** in the directory **source** contains further physics simulations in *xml*- and *jar*-format.

It is advisable to create links for the pdf text file and the *console* on the screen (in Windows: *Desktop*), in order to find these files quickly without searching for them. Please take care not to change the deeper directory structures, otherwise some hyperlinks will not work.

As long as the simulations are accessed from the text, you do not have worry about the directory structure, since it is saved in the hyperlinks. However, as soon as you want to edit a simulation from the console, you are asked for the location of the file.

The directories **RoessMa** in **export** and **source** are structured into directories in the same way according to topics. This is illustrated in Figure 1.2 for the example of the sub-directory **Calculus** with 6 individual simulations. The initially empty directory **Tests** is intended to save the data for your own experiments. This setup prevents the original files from being overwritten by mistake.

1.3 Usage and technical conventions

Most simulations are interactive. The user has several alternative ways of intervening, although not necessarily in parallel.

Individual points or elements of the graphical presentation can be “pulled” with the *mouse* and thus parameters can be changed. In this case, the mouse pointer changes into a hand symbol when it is positioned on the element.

Numerical values of different parameters can be entered into *number fields*. However, this change only becomes active if the enter key has been pressed and the text field, which turned yellow when entering text, becomes clear again. If the text field turns red, a mistake with the input has occurred (often a comma has been used instead of a full stop as decimal point; the correct format is, for example, 12.3 instead of 12,3).

From a *list of options*, given functions or parameter values can be selected with the mouse.

With *sliding controls*, individual parameters can be changed continuously or in steps.

Functions that are displayed in a *text field* can be changed or rewritten from scratch. Again, the changed function is submitted with enter.

When formulas are written in printed text, we often use short hand notation, by unspoken convention:

- for items that are ambiguous and that can be misinterpreted as text, like ab for a times b or $\sin a$ for $\sin(a)$;
- for items that can be misunderstood by software as formatting characters for text, such as x^2 for $x*x$ or $x^{\wedge}2$;
- for those special characters that cannot be interpreted by programs, such as \dot{y} for $\frac{dy}{dt}$ for derivatives with respect to time.

For the input for numerical programs such as *EXCEL/VBA*, *Java*, *VBA* or *Mathematics* the notation must be unambiguous.

The fundamental rule is: all parts of the formula must be entered directly via the keyboard without the use of special characters. Combined characters must be mapped to an equivalent number of keyboard characters, in order for these to be correctly identified by the program. (Example: y' as a derivative combined from two keyboard characters; a unique text like “derivative with respect to t ” might also be interpreted by a program). In particular, the following notations have to be noted:

- Addition and subtraction: $a + b$, $a - b$;
- Multiplication: $a*b$;
- Do not omit brackets: $a*\sin(b)$;
- Division: a/b ; $(a + b)/(c + d)$;
- Power: $a^{\wedge}b$;
- Exponential function: $\exp(a)$;

Many simulations use a *parser* to translate the formulas entered as text into *Java* format. In this case, the following notation is permissible, which can also be used recursively.

Parser

$\operatorname{atanh}(x)$	$\operatorname{ceil}(x)$	$\cos(x)$	$\cosh(x)$	$\exp(x)$	$\operatorname{frac}(x)$
$\operatorname{floor}(x)$	$\operatorname{int}(x)$	$\ln(x)$	$\log(x)$	$\operatorname{random}(x)$	$\operatorname{round}(x)$
$\operatorname{abs}(x)$	$\operatorname{acos}(x)$	$\operatorname{acosh}(x)$	$\operatorname{asin}(x)$	$\operatorname{asinh}(x)$	$\operatorname{atanh}(x)$
$\operatorname{sign}(x)$	$\sin(x)$	$\sinh(x)$	$\operatorname{sqr}(x)$	$\operatorname{sqrt}(x)$	$\operatorname{step}(x)$
$\tan(x)$	$\tanh(x)$	$\operatorname{atan2}(x, y)$	$\max(x, y)$	$\min(x, y)$	$\operatorname{mod}(x, y)$

Here we have $\operatorname{acos} = \text{arc cosine}$, $\cosh = \text{hyperbolic cosine}$.

The important expression $\operatorname{atan2}(x, y)$ prevents the ambiguities of the arctan by automatically yielding the correct angle in the second and third quadrant; here x and y are the sides of the triangle involved and x is opposite to the angle.

$step(x)$ is a very interesting function in practice. It switches at $x = 0$ from 0 to 1. If one wants to superimpose the function $f(x)$ to the function $g(x)$ from $x = x_1$ onwards, then this can be written as $g(x) + f(x)*step(x - x_1)$. For some simulations the *Math* package is used together with Java for calculations. In this case, the functions are prepended with *Math* as follows: *Math.cos(x)*.

Further details about the functions and terminology used in Java can be found from many sources on the internet, for example via searching for *Java & Math*. Just try the link at the border.

Math

1.4 Example of a simulation: *The Möbius band*

As an example of the possibilities of interactive simulations as they will be used in the following chapters, Figure 1.3a shows a rotating *Möbius band* in three dimensional projection. Among the closed bands in space the Möbius band is characterized by the fact that it makes half a twist, and thus, during one circulation, both sides are covered by a traveller; it has “only one surface”. In the picture of the simulation, one sees the formulas for the three spatial coordinates with the variables p and q , which contain two parameters a and b , which can be changed with sliders. The slider for a changes the number of half twists, while the other one changes the height of the band. If a non-integer number is chosen for the number of half twists a , the band can be cut, and rejoined with another number. If this number is even, one obtains normal bands with 2 surfaces. If this number is odd, one obtains a Möbius band that has additional twists.

The formulas for the three space coordinates, as well as the time dependent animation component, can be edited, i.e. they can be changed. Using the same simulation, arbitrary animated surfaces in space can be visualized. The ability to edit opens a wide training field for the advanced understanding of functions that describe three and four dimensional processes. Figure 1.3b shows two examples from the simulation of Figure 1.3a. On the left a simple band with a full twist, and on the right a Möbius band with one and a half twists, were calculated.

The text pages of the simulation contain extensive descriptions, hints for many alternatives of the 3D-projection, and suggestions for experiments. Figure 1.4 shows the description window that appears next to the simulation when it is opened. In this example, it contains 4 pages:

Introduction with a description of the simulation and its controls;

Visualization with hints about the possibilities of 3D projection;

Functions for discussions of the mathematical formalism;

Experiments with suggestions for experiments that make sense.

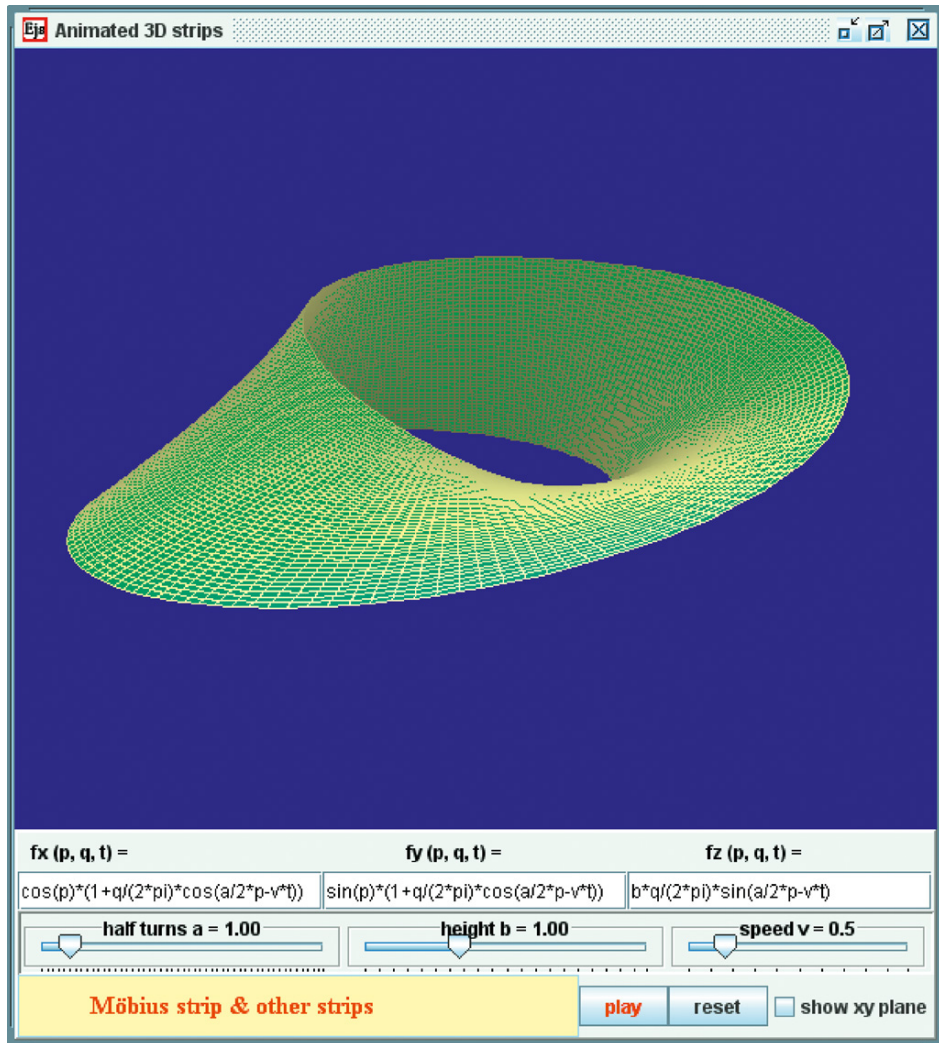


Figure 1.3a. Simulation. The figure shows a simple Möbius band in perspective 3D-projection. The three function boxes contain the parameter representations for the space coordinates. The variables p and q vary in the range $-\pi$ to π . The parameter a that determines the number of twists (here $1/2$) can be changed with a slider, as well as the parameter b that controls the height of the band. The z -component can be periodically modulated for $v > 0$ with the angular velocity v (play button) in order to create the impression of a rotating band. Using a check box, the xy -symmetry plane can be shown or suppressed.

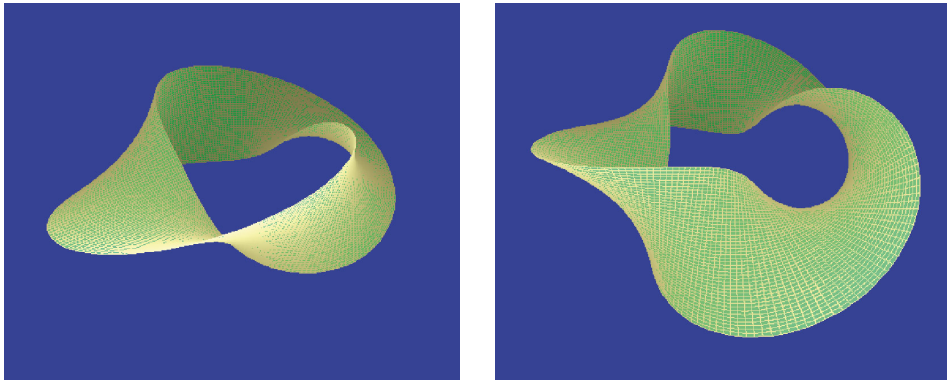


Figure 1.3b. Examples from the simulation in Figure 1.3a; on the left a simple loop with $a = 1$, on the right, a Möbius band with an additional twist ($a = 1.5$).

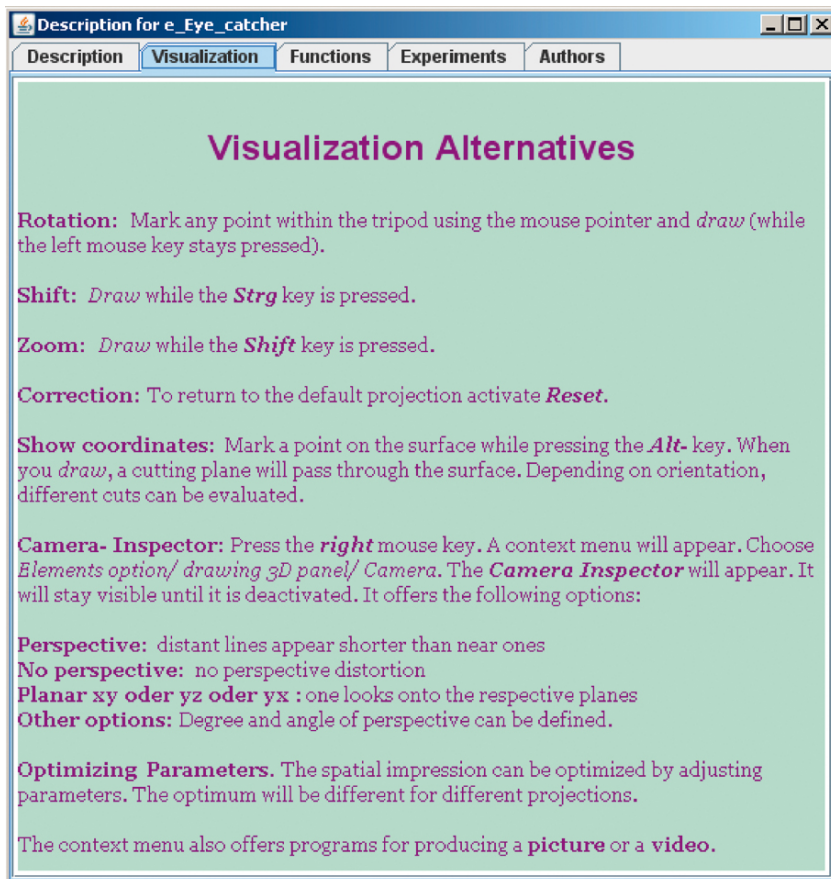


Figure 1.4. Description window of the simulation. Here it contains 4 pages, of which the visualization page is opened. Please test the possibilities after opening the simulation!