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The Mechanics of Meaning

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The Mechanics of Meaning

Propositional Content
and the Logical Space
of Wittgenstein's *Tractatus*

by

David Hyder

To my parents

Preface

Wittgenstein's *Tractatus Logico-philosophicus* still enjoys its reputation as one of the most obscure works in the philosophical canon. Yet despite its obscurity, or perhaps indeed because of it, it continues to be taught in the classroom. One reason for this is undoubtedly the openness of the book: most people can find something there that interests them, and the text is so spare that there is plenty of room to lodge one's own interests. So in deciding to publish a full-length study, one courts disaster. After all, the joy of the book, at least for those who do enjoy it, lies in doing the interpretive work oneself, and someone who presumes to tell us what it all means is obviously something of a spoilsport. Nevertheless, my aim in the following pages has been to give a concrete interpretation of a major part of Wittgenstein's early work. Few would be so foolish as to claim to give an exhaustive reading of the *Tractatus*, and I do not imagine I have. But I do propose one that applies to the book as a whole. In other words, I hope that the reader will gain insight into the meanings of terms and passages of the text whose central importance has never been in question, even though their meaning has remained opaque. And I advocate a particular view of the main theme of the book, that is to say of the problems that motivate it, which has received little attention in the literature. My method has not been, however, to engage in a close reading of the *Tractatus* itself, rather I have concentrated much of my effort on earlier sources: Wittgenstein's letters and notebooks, Russell's *Theory of Knowledge*, Helmholtz's "Facts in Perception", Hertz's *Mechanics*. My aim has been to reconstruct as fully as possible the set of problems that Wittgenstein took himself to be working on in the years 1912-1918, in order to explain the solutions we see him give in the *Tractatus*.

I am aware that this approach is vulnerable to a serious objection: Why should we not take the *Tractatus* as a self-contained philosophical work, and interpret, so to speak, *within* the text? How can one hope to determine what an elementary fact is, for instance, by looking to sources outside the text itself? To this I would reply first that should the reading I am proposing directly conflict with a passage in the *Tractatus*, then I would of course have to revise it. But if such objections are directed against the use of such external sources generally, I can only reply that esoteric readings of Wittgenstein do not get us very far. First, sticking to the one text is a good classroom exercise, but it cannot set methodological bounds on scholarly interpretation. Second, although it is hard

when reading Wittgenstein to hook up what he says to other lines within the philosophical tradition, I think that we must make an effort to do so. If we fail to, we end up with a philosophy that cannot be applied. We need more exoteric work, and that means scholarship that establishes systematic links to other philosophers and philosophies. That is what I have tried to do.

It is in part for these reasons that I have adopted two conventions in the text that some may find inconvenient. The first is to retain, for the most part, the notation of *Principia Mathematica* when citing or commenting on Russell's and Wittgenstein's work. Explanations of the notation I do use in my text (and it is not that much) are given on p. XIII, and the reader is referred to *Principia Mathematica* itself should more detail be required. This approach seemed preferable because it may help others avoid a problem I encountered when I began research in this area, namely the difficulty of understanding all that strange notation in *Principia Mathematica,* and in the *Notebooks 1914-16.* My hope is that readers of my comments will immediately be able to understand the relevant passages in the *Tractatus*, in Russell's and Whitehead's works, and in the *Notebooks* should they decide turn to the originals. A further reason is that both Russell's notation and Wittgenstein's changes to it are not mere additions to their properly philosophical work. The two go hand in hand, and transliteration into modern notation is not always possible without distorting the original sense. This is particularly evident when one considers, for instance, the role of free and bound variables in *Principia Mathematica*, but it also proves to be the case when we look at Wittgenstein's "copula-theory".

The second convention I have adopted is that of always quoting Wittgenstein in German, and providing translations in the footnotes. My thinking here is similar: Wittgenstein wrote in German, and the German text is the primary source. Translation into English is not always possible without distorting the original sense. As a rule, I quote other sources in the original language, and provide translations in the body text. Fragmentary passages, or passages quoted at length elsewhere in the text are, however, quoted in translation. In almost all cases, the translations are my own. I often render words that connect to the sciences with greater emphasis on that meaning: *Abbildung* as mapping, *Mannigfaltigkeit* as manifold, *etc.* This last term, which is central to my interpretation, is particularly nettlesome, since its two meanings can only be rendered in English by the distinct terms "multiplicity" and "manifold". It does not help that these two meanings are not always truly distinct in German. My translations are often less elegant than the established ones, and indeed I intend mine as glosses, not as improvements on the latter, which often better capture the natural sense of an expression. The only exceptions are those passages from Wittgenstein's notebooks where the agreement between my translation and Anscombe's was so close that is was pointless to preserve the slight differences. In these cases, I have simply used her translation.

Table of Contents

Notation

Dot-notation and modern equivalents

$p \supset p . \supset . p \lor \sim p$ $(p \supset p) \supset (p \lor \sim p)$

$T_0(a).T_1(F).\varepsilon_1(a,F): \supset .Fa \lor \sim Fa$ $(T_0(a) \& T_1(F) \& \varepsilon_1(a,F)) \supset (Fa \lor \sim Fa)$

Notation from Principia used by
Russell and Wittgenstein in 1912-1916

$T_0\hat{x}$	The function formed from a particular proposition, such as T_0a.
$xT_0\hat{x}$	The class satisfying $T_0\hat{x}$, *i.e.* $\{x : T_0x\}$
T_0x	An arbitrary value of $T_0\hat{x}$.
$(x).T_0x$	The assertion of all significant values of $T_0\hat{x}$.
$(\exists x,y)\varepsilon_1(x,y)$	Wittgenstein's "epsilon-copula": it is the function defining the class of all pairs of elementary predicates and individuals that form existing facts.
$\varepsilon_1(a,F)$	When F and a are related by the copula, it follows that Fa, and conversely.
$J(S,a,R,b)$	Russell's "judgment-relation" in its early and late versions. In the early version, the components of a possible fact $R(a,b)$ are related to a subject S in the judgment-relation J. The later version adds the "form" γ, which corresponds to Wittgenstein's "copula".
$J(S,a,R,b,\gamma)$	
$x \sim \mathit{et}'y$	The function "is not a member of $t'y$", where $t'y$ is the type of y.

1. Introduction

Readers of the later Wittgenstein will be familiar with a regress argument of the following form. Suppose that in order to utter a meaningful sentence, I must have mastered rules governing the use of the words in that sentence. Suppose furthermore that these rules are something I have learned, either by being told what they are, or by being shown the objects to which the words refer. In the latter case, I will have derived rules for using the words from knowledge of their referents, for instance from knowing what *kinds* of things they are. In the former one, I will have done so by drawing on my mastery of other words, namely those used to express the rules themselves. Either way, Wittgenstein argues, we run into problems: every ostensive definition, every statement of a rule can be misunderstood. The misuse of words can never wholly be forestalled in this manner, from which Wittgenstein concludes that learning how to use words correctly is not the same as learning how to distinguish between different kinds of things, nor is it like memorising a book of rules. Knowledge, if it is to be stated, must be expressed in a language whose words have meanings. But if the meanings of words always depend on further knowledge, we could never get started with the business of speaking meaningfully. I will refer to such a regress as a "sense-truth regress" in what follows.

Such arguments are targeted against a particular conception of intentionality. On this view, intentions mark the division between meaningful and meaningless speech—only if I have an intention when uttering a phrase can I be said to have meant something, to have asserted something that counts as right or wrong. Wittgenstein challenges us to explain what this "having an intention" consists in. He argues that so long as we think of intentions as involving a peculiar kind of linguistic symbol (a rule or a sample), we shall beg the question, for the doubt concerning the correct application of *these* symbols will simply recur. What distinguishes meaningful from meaningless language cannot itself be open to doubt concerning its meaning. In short, it cannot itself be language, in so far as language is something that, by its very nature, can be misunderstood. The space of meaning is bounded at its periphery neither by true statements concerning the make-up of the world, such as those made in traditional metaphysical theories, nor by rules that are themselves expressed in language, even if these are imagined to be the private mental languages of idea-theories.

This line of argument is evident at a number of points in the *Philosophical Investigations*, where it is taken to show that there must be rules that are somehow followed blindly, without further interpretation. But it also motivates much of the theory of language and logic advanced in the *Tractatus*. Indeed, it is present *in nuce* in Wittgenstein's earliest philosophical writings, where it is marshalled against Russell's "theory of judgment". This theory was explicitly targeted against theories of propositional judgment like those of Meinong and Frege, which relied on intentional objects or senses in order to explain how the judging subject could be directed toward a determinate state of affairs. Such intentional objects allow one to finesse the problem of explaining how judgments retain a definite meaning when they are false, in other words when the intended state of affairs does not obtain. On Russell's view, however, such intentional entities were dangerously idealistic, and he aimed to eliminate them by means of his judgment-theory. Thus the problem of accounting for false judgments was a central concern of this theory. And Wittgenstein, during his first stay at Cambridge with Russell, seized on just this difficulty. He argued that on Russell's approach one could not distinguish *false* judgments from nonsense without assuming that other judgments were true. In his view, Russell's theory of judgment made meaning dependent on truth, and thus it had to be wrong.

In letters from the period 1912-13, in the "Notes on Logic" and the "Notes Dictated to Moore" of the same period, and finally in the *Tractatus* itself, Wittgenstein insists that theories of propositional judgment must show that it is "impossible to judge nonsense". And he quickly comes to the conclusion that the impossibility of nonsense judgments can be secured only if we postulate a logical language whose very structure prevents the formation of nonsensical propositional signs. He does not explain in the *Tractatus* why this is the only alternative, although we do find there one specific application of a general regress argument to the case of non-denoting names. This argument is implicit in the principle that the significance of a proposition (which for Wittgenstein meant its bivalence) cannot depend on the truth of another.[1] For if all propositions depended on the truth of others for their meaning, it would follow that the entire structure of meaningful language depended on the contingent truth of some set of propositions, which seems absurd. Alternatively, there might be some propositions which were necessarily true, from whose truth the meanings of the rest depended. But then they would not be bivalent, and would therefore not count as significant. The connection of this principle to Wittgenstein's

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[1]   See *Tractatus* 2.0211, and the "Notes Dictated to Moore". In L. Wittgenstein. *Notebooks 1914-1916*. 2nd edition. eds. G. H. von Wright and G. E. M. Anscombe. Oxford: Basil Blackwell. 1979, p. 117. In the following, references to the *Tractatus* will be made only with the number of the relevant section. References to the "Prototractatus" will be prefaced with the abbreviation "PT".

claim that it is, or ought to be, impossible to judge nonsense is clear enough. If the same act of judgment sometimes counts as significant and sometimes as nonsensical, this must be because in the second case certain conditions on significance are not fulfilled. So we might seek to guard against this eventuality by postulating that these conditions be met, meaning that the judgment act would count as legitimate only under certain constraints. Wittgenstein contended that the notion of such postulates, of constraints on the scope of judgment, is incoherent. Propositions and judgment do not and cannot depend on the truth of other propositions for their significance.

Although these two postulates (that one cannot judge nonsense, and that no proposition depends on another for its sense) are explicitly stated in the *Tractatus*, their link to another central thesis of the book, namely the doctrine of showing and saying, is not immediately obvious. Wittgenstein suggests that the logical syntax of language cannot be "said", although it is "shown" by the propositions of logic. This logical syntax somehow points at the shared structure of language and world, and is in consequence a condition for the existence of significant propositions. Thus it follows immediately that logical propositions do not make true statements in Wittgenstein's usual sense: if they did, then we would clearly have a case where the truth of some propositions determined the sense of others, and Wittgenstein will not allow this. If a sense-truth regress can be avoided, it is by denying that such boundary conditions on meaning can ever be false, or perversely enough, that they are ever really true.

So if we grant Wittgenstein his two postulates, and we accept the claim that logical propositions reflect the essential structure of language, we can infer that logical propositions cannot be true propositions, at least in Wittgenstein's strict sense. But this is to interpret entirely within the context of the *Tractatus*'s own, obscure doctrines. What is missing here, and I would suggest that this is so in the literature generally, is a unified, positive explanation of these postulates. Why did Wittgenstein first claim that one's theory of propositional judgment must make it impossible to judge nonsense? What sorts of supplementary conditions on judgment were he (and Russell) considering? And why exactly do these constraints get demoted (or elevated, depending on your point of view) to the status of the unsayable? By giving specific historical and systematic answers to these questions, we can arrive at a better understanding not only of the *Tractatus*, but of Wittgenstein's philosophy in general. For the regress argument I derived above from the *Investigations* is anticipated in the *Tractatus* not just in its general outlines, but in many of its details. Seeing how this is so allows one to see that the function of the *Tractatus*'s logical space closely parallels that of a language-game in the later work.

The connection between these various Tractarian theses is more readily grasped in modern terminology. As I just observed, Wittgenstein's first work in this area took the form of a criticism of Russell. He argued that on Russell's

theory of propositional judgment, judgments were guaranteed to be meaningful only under certain conditions. For instance, the objects involved in the judgment had to exist, and, furthermore, they had to belong to the appropriate ontological types. However, such constraints could be expressed only metalinguistically if one was to avoid either contradictions or mere redundancy. Suppose that in order to be sure that my judgment that "Your hat is brown" is meaningful, I must eliminate the possibility that your hat does not exist. I may try to do this by means of a constraint such as "there is an x such that x = your hat", an expression that Russell took to mean "your hat exists". In fact, if this constraint is to achieve anything, it must mean something like "'your hat' denotes a". For otherwise the constraint might be meaningless for exactly the same reason that "Your hat is brown" threatened to be so. If you don't have a hat, then "your hat is brown" and "your hat exists" are equally meaningless, because the phrase "your hat" lacks a denotation in both cases. On the other hand, if you *do* have one, the constraint is redundant. Adding significance-constraints on judgments at the object level therefore achieves nothing. On the other hand, adding them at the meta-level leads to a regress. Suppose that to ensure that I speak meaningfully in an object language, I must first have fixed the syntax and semantics of that language at the meta-level. Then the difficulties will simply recur, for all the judgments at the meta-level will be open to the same ambiguities. In the meta-constraint "'your hat' denotes a", we have made use of the terms "a" and "'your hat'". What is to ensure that these terms have denotations?[2]

The first main interpretational strand in this book will therefore be to explain how Wittgenstein arrived at his diagnosis of this problem in Russell's theory of judgment, and how he thought he resolved it in the *Tractatus* by introducing the notion of a "logical space" of elementary facts and propositions. I maintain that any adequate interpretation of Wittgenstein's early work must take this route, for the simple reason that Russell's theory was the point of origin of Wittgenstein's own work on a propositional theory.[3] Still, such an analysis gives us

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[2]   For the purposes of my exposition here, I am using the example of non-denoting names, which is the one sort of constraint on significance that has received substantial attention in the literature. The bulk of my analysis in this book, however, concerns other sorts of constraints (on the ontological type membership of objects, on the existence of forms), which have rarely been discussed, and which are, in my view, of greater significance for our understanding of Wittgenstein's philosophy, both early and late. The regress argument gets its bite from the problems generated by such constraints, and it is they, far more than indestructibility of names, which still play an active role in the *Investigations*.

[3]   I do not mean that such a genetic analysis is always required of interpretative work. However, the obscurity of Wittgenstein's text demands that we provide as detailed an account of the problems Wittgenstein sought to solve as we do of the answers. Above all, researchers did not know of the existence of Russell's *Theory of Knowledge* manuscript until the 1970's, and it has only been available in print only since 1984. So we are obliged, in my

only half of the picture, because Wittgenstein went on to give this result a quite particular twist. He took it to show that our meaningful use of language presupposed that both experience and language had spatial structures, and, furthermore, that these structures could not be meaningfully described, that they could perhaps be shown, but certainly not said. In adopting this position, he drew on a neo-Kantian tradition in German-language philosophy of science that ascribed a central role to the concept of a manifold both in theories of perception as well as in the philosophy of science. It should be emphasised that Russell was aware of much work in this tradition, as a glance at his *Essay on the Foundations of Geometry* will confirm. But Russell was ultimately hostile to any argument that assumed the existence of *a priori* structures of experience. For instance we see him arguing, in his 1913 *Theory of Knowledge*, that Kant's understanding of space has "suffered a series of shattering blows", with the result that "the space of actual experience is appropriated by psychology, the space of geometry is appropriated by logic, and the space of physics is left halting between them in the humbled garb of a working hypothesis".[4] Russell's logico-epistemological project at this point aimed to show how scientific notions like that of space were logically constructed by the subject, instead of being inherent phenomenal structures of an idealist consciousness. Furthermore, the truth of these notions when applied to experience was to consist in their corresponding to objective states of affairs, and not to *a priori* conditions of experience. His theory of judgment, in turn, was to explain how logic could emerge in a world consisting only of subjects, objects, relations and forms—an atomised and monistic world bereft of ideas or representations, let alone structures of experience. So Wittgenstein's rejection of Russell's approach, which was influenced by two distinct forms of neo-Kantianism (that of Frege, and that of scientific epistemologists such as Helmholtz, Hertz and Boltzmann) was not just a rejection of specific theses within the theory of propositional judgment. Consciously or not, Wittgenstein was reasserting the Kantian doctrines that there cannot be logically determinate judgments without intuitive structures of experience, and, more strongly, that logic is not—as Russell maintained—the most general form of natural science, but is rather the by-product of our structures of cognitive representation.

The second strand of my interpretation concerns this neo-Kantian tradition of "scientific epistemology". By examining the philosophical writings of Hermann von Helmholtz and Heinrich Hertz, I show how Kant's *a priori* intuitions of space and time were extended in their work into what I call "manifold-

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opinion, to fill in some basic historiographical gaps at this point if we are to make interpretative progress in the future.

[4] B. Russell. *Theory of Knowledge. The 1913 Manuscript. The Collected Papers of Bertrand Russell.* vol. 7. ed. E.R. Eames and K. Blackwell. London: Allen & Unwin. 1984, p. 22.

theories". These are theories whose fundamental structures are derived from the theory of manifolds that developed in the second half of the nineteenth century, beginning with the work of Riemann and Graßmann. I will argue that Wittgenstein's notion of a "logical space" is one more instance of such a theory. The *Tractatus*'s theory of language supposes that both elementary propositions and the elementary facts to which they refer (when true) are organised in isomorphic structures. Each elementary proposition points to what Wittgenstein calls a "logical place" in the space of elementary facts. The dimensions of these manifolds correspond to sets of intersubstitutable objects and names, so that the symbol that results when one of these names is replaced by a variable selects a cut through the field of elementary propositions. Significant propositions in the strict sense always assert something about connections between points (logical places) in the space of elementary propositions, and can therefore be true or false depending on whether these connections obtain.[5] In contrast, logical propositions pick out invariant structural properties of the space itself, and are for this reason always true. They stand in the same relation to the logical space as geometrical propositions do to Kant's pure intuition of space. They are true *a priori*, in that they pick out invariant properties of the fundamental structure of experience. But for this very reason, Wittgenstein argues, they are devoid of empirical content, and cannot be viewed as making significant assertions in a strict sense.

In the opening chapters I keep these two strands distinct. Chapter 2 deals with Helmholtz's manifold-theory of perception and its relation to later German-language theories of knowledge and science. Chapter 3 gives a detailed account of Russell's theory of judgment and of the problems it raised. By separating these two themes of the *Tractatus*, I want to emphasise that Wittgenstein's reasons for moving to such a spatial model lay within the philosophy of logic. Although he came to interpret his theory more and more from the point of view of such manifold-theories (both phenomenological and physical), this should not blind us to the central connection to the philosophy of logic and the theory of intentional judgment. Briefly put, Wittgenstein transcendentally deduces the existence of a logical space, and the premises of this deduction are found in the theory of meaning. In Chapters 4 and 5, I describe the programme of research that culminated in the *Tractatus*'s theory of logical space. Finally, in Chapter 6, the background in scientific epistemology is connected with the logicist one, and I move from there to a more general account of the philosophy of science in the *Tractatus*. The Conclusion considers the significance of the reading I propose for our understanding of Wittgenstein's later work. In order to give the reader a sense of how these quite diverse topics condense in what

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[5]   In the case of an elementary proposition on its own, what is asserted is only that the elementary fact that belongs at that place in fact obtains.

became the *Tractatus*'s theory of language, I will give a brief overview of my discussion in the following pages, beginning with some background on Russell's theory of judgment.

1.1 Russell's Theory of Judgment

In the years after Whitehead and he completed *Principia Mathematica*, Russell turned his attention to epistemology, and to the unfinished theory of judgment that he had presented in the introduction to that book. As I suggested above, this theory was intended to eliminate propositions or intentions as independently subsisting entities, and to demonstrate instead how they were abstracted out of acts of judgment. Russell hoped thereby to construct a theory of intentionality within a monistic universe, in which the subject, the objects with which he was acquainted, and his cognitive acts would all have same ontological status. In this monistic world, propositional judgment consisted in an unmediated relation between a subject and the objects of his judgment—no thoughts, senses or ideas would intervene. The truth of a judgment would then be defined as a relation between such a "judgment-complex" and the fact (also a complex, in Russell's terminology) whose existence the judgment asserted.

I should emphasise that this project was not merely part of Russell's and Moore's anti-idealistic backlash—it was fundamentally connected to Russell's understanding of the logicist project, that is to say to the status that logical and mathematical propositions were to be given now that the analysis of *Principia Mathematica* was in place. By reducing propositional judgment to the plane of things, Russell aimed to give every logico-mathematical proposition a strictly objective interpretation. They would not refer to ideal entities, and there would be, in consequence, no epistemological mystery concerning the justification or objective significance of mathematics in the sciences. The truth of logical and mathematical propositions would consist quite simply in correspondence relations between (highly abstract) states of affairs, some of which would be the judgment-complexes, and others the facts to which they referred. For example, the mathematical theory of manifolds, although undoubtedly a theoretical construction of human mathematicians, would consist, on Russell's theory, in true propositions whose variables ranged over real objects. Like Husserl, Russell aims to explain how mathematical theories can be simultaneously human constructions (for the theory of manifolds is undoubtedly the product of mathematical investigations), and at the same time objectively true (for they can be referred to objective features of the world). But in sharp distinction to Husserl, Frege and other idealist philosophers of mathematics, Russell wanted to avoid concluding that because mathematics is an intellectual construct, its subject matter is in *any* sense the human mind.

Despite these long-term aims, Russell quickly bogged down in his efforts, not least because of Wittgenstein's attacks. I have already indicated the thrust of Wittgenstein's objections. On Russell's theory, he objected in the *Tractatus*, it would be possible to "judge nonsense", by which he meant that the theory did not ensure that the propositional content of a given judgment or assertion was uniquely correlated with a possible state of affairs. Its failure to do so meant that there was no principled distinction between judgments and propositions that could be true, and pseudo-judgments and pseudo-propositions that could never be so. In short, his objection reduced to insisting that to be termed "false", a judgment must be *possibly* true, for if this were not the case, there would be no distinction between sense and nonsense.[6] In a letter to Russell from the summer of 1913, Wittgenstein insisted that the significance of judgments in the sense just outlined must be secured "without the need for any other premises".

His notes from the years following, both written and dictated, repeatedly address the following dilemma: either the elements composing a (supposed) propositional judgment are so typed and ordered that they unambiguously describe a unique and possible state of affairs, or they are not. Suppose they are not—that is, suppose that we require additional premises (I shall call them "riders") to ensure that the propositional judgment has the right types of elements and an unambiguous structure. These premises would include assertions concerning the semantic categories of the elements of the propositional judgment, as well as assertions that the objects about which I judge exist, as in the example I gave above. But assertions are judgments, and in consequence the same ambiguities that were to be shored up by the riders on the original judgment will recur in the riders themselves. If I need to know that "Your hat is brown" is a legitimate judgment only if a hat is a physical object, and brown is a colour, then I also need to know that "a hat is a physical object" and "brown is a colour" are legitimate judgments. So it might seem that the reason I know that, for instance, "your brown is brown" is nonsense is that I know that "brown is a physical object" is false. But then we would have at least one propositional function that is significant for both hats and colours, namely "x is a physical object". If we assume that the categorical distinction between colours and their bearers is absolute, then this is a contradiction. If we allow that it is not absolute, then there is no longer any reason to regard "Your brown is brown" as nonsense. Lastly, if we maintain that it is absolute, but that there is no contradiction here because the function "x is a physical object" has a truth-value on

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[6]   More precisely, in order to be either true or false in the strict sense, a proposition must be bivalent. In order to be bivalent, it must specify a possible state of affairs. In consequence, the unambiguous correlation of a proposition with a possible state of affairs is the basic requirement for meaningful judgment.

the argument "hat", but not on "brown", then our rider will do no work for us. As in the case of the existential constraint I outlined previously, this rider is either absurd or redundant. Wittgenstein concluded that, whatever account one was to give of propositional judgment, it could not be the case that the significance of propositional judgments depended on knowledge of the truth of riders such as these.[7] Once again, either they would lead to contradictions or they would fail in the job assigned them.

In making these objections, Wittgenstein was invoking a difficulty that Russell and Whitehead had recognised themselves in the late stages of their work on *Principia Mathematica*. Realising that functions straddling types (functions they fortunately rarely needed) would undermine the distinctions between the types, they replied that all those propositions in the book that used such functions should be regarded as statements concerning symbolism. Today, we would say that all such statements were expressed in a meta-language. In those propositions of the book where no such functions occur, but where there is what they called "typical ambiguity" (as in propositions containing only free variables), the appropriate matching of symbols was taken to be *implicitly* secured. Such expressions could traverse the hierarchy of types and functions precisely because they were not, strictly speaking, propositions, but only schemata thereof. Once one pinned such a schema down to a particular level in the hierarchy by fixing the type of one of the variables, the appropriate types of the other variables were supposed to be fixed implicitly by their syntactic connections in the schema. From a modern point of view, some propositions in *Principia Mathematica* are in the object-language, some are in the meta-language, and some are indeed mixed. For instance, the antecedent to a conditional containing type-restrictions is to be interpreted at the meta-level, whereas the consequent is to be interpreted at the object-level.

It seems, however, that Russell and Whitehead were not overly troubled by the peculiar status of the type-theory, nor by the various axioms (of reducibility, of infinity) which were also required to secure the significance of large parts of the work by guaranteeing that various objects and functions existed. Wittgen-

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[7] In order to forestall an obvious objection, I should emphasise that the argument I present here is a step in a regress argument. As in the *Investigations*, the point is not that we cannot have rules for making distinctions between kinds of things, nor statements asserting that certain things exist, nor that such rules cannot be used to narrow the scope of statements to a particular domain. The point is rather that such rules, in so far as they are themselves expressions (or, in the early work, judgments), cannot cap an interpretative regress. In the limit, there cannot be such rules. Thus in the example above, we might want to follow Frege in allowing that both "Your hat is brown" and "Your brown is brown" are significant because both "brown" and "hat" are proper-names, and such a category is more fundamental than those of physical objects or colours. But then the argument could be applied again with regard to the logical categories of proper-names and function-names. In Chapters 3 and 4 we will see why this difficulty was so troublesome for Russell's judgment-theory.

stein, by contrast, inveighed against all of these prerequisites for the signifi-
cance of the book in his letters to Russell in the period of 1912-1913. It is not
easy for us today to appreciate why these criticisms were so serious for Rus-
sell's project. But if we recall the dependencies I mentioned above—logical
concepts are defined in terms of ontology by means of a theory of propositional
judgment, and scientific and mathematical concepts are then developed within
logic—we will see that these were indeed grave problems for Russell. If we
need an axiom securing the existence of an infinite (or indeed of a finite) num-
ber of objects to be true in order for other propositions to be significant, then
parts of what Russell regarded as logic depend on empirical requirements not
just for their *truth* but for their very *meaning*. If the significant application of
every proposition in *Principia* depends on a meta-linguistic check that all its
names denote, and that they all denote things of the right types, then the meta-
language itself had better be in order. But a meta-language contains terms refer-
ring to both the object language and its referents, so that using one will push us
in the wrong direction. Russell wanted to reduce logical concepts to ontological
ones, so that truth, meaning and mathematics would be objective features of
reality, features inhering in the correspondence between judgment-complexes
and judged-complexes. If significant judgments cannot be formulated without
meta-linguistic constraints, the whole project will founder. According to Rus-
sell, the symbolism of *Principia*, indeed language in general, depends for its
meaning on the underlying intentional judgments. These are, in turn, objective
states of affairs. So it is absurd to assume that restrictions implemented in a
meta-language could in any sense be used to ensure that judgments be signifi-
cant. On the contrary, it is the significance of the judgments that grounds inten-
tionality, and thus the meanings of signs.

In Chapter 3, I show how this sort of a sense-truth regress developed out of
Russell's theory of judgment. The "riders" required by such a theory are of
three sorts: riders on the types of the elements entering into Russell's judgment-
relation, riders securing the existence of what Russell called "logical forms",
and riders securing the existence of the objects involved in the judgment. I con-
centrate in my analysis on the first two of these, since the regress argument
ensuing from the second, which leads to the doctrine of the indestructibility of
objects, is well-known in the literature as a result of Anscombe's work.[8] In
Chapter 4, I reconstruct Wittgenstein's version of this theory (his "copula-
theory"), and show how his ultimate break with Russell led him to the theory
embodied in the *Tractatus*. On this view, the possibility of significant elemen-
tary propositions depends on the existence of two isomorphic spatial structures,
the one consisting of the field of elementary facts, and the second of the field of

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[8]   Above all in G.E.M. Anscombe. *An Introduction to Wittgenstein's »Tractatus«*. London:
      Hutchinson. 1959.

elementary propositional signs. This postulate, I argue, represents Wittgenstein's response to the sense-truth regress. Because the significance of elementary propositions cannot be secured by means of supplementary premises or riders, Wittgenstein assumed that the internal structure of these signs reflects those of the possible facts that they pick out. Only on this assumption, thought Wittgenstein, can we secure the distinction between significant propositions and nonsense without getting embroiled in a regress or in contradictions. These two parallel structures are what Wittgenstein came to call the "logical space".

It is this notion that links the properly logicist arguments of Wittgenstein's early work with those that are directly concerned with the philosophy of science, and which draw on the work of scientific epistemologists such as Helmholtz and Hertz. In Chapter 5, I discuss its role in the *Tractatus*'s theory of logic. The first part of this chapter concerns the theory of truth-functions. I then examine Wittgenstein's critique of Russell's and Frege's axiomatic demonstrations of logical propositions. This critique hinges on distinguishing propositions that are absolutely general and empirically true, in other words scientific principles, from those that are *logically* true. Both of his predecessors had failed, on Wittgenstein's view, to give a binding distinction between these two kinds of propositions, because their axiomatic method derived logical laws from higher ones without adequately justifying the latter. Russell had attempted to do so on ontological grounds, but this meant using a language that contravened (or presupposed) the very logical features of reality it was supposed to specify. Frege's approach avoided such difficulties by means of syntactic definitions and the introduction of both senses and truth-values. Wittgenstein rejected both of these approaches. On the *Tractatus*'s theory, the distinction between the two sorts of maximally general proposition is grounded in their differing relations to the logical space.

The treatment of truth-functional propositions is extended in the second half of Chapter 5 to cover quantification. Here I argue that the notion of a logical space first does real work for Wittgenstein only in the context of the theory of quantification. Wittgenstein conceives of quantified propositions as signs that select subsets of the manifold of elementary propositions on the basis of their common features. Thus logical propositions that contain quantifiers make statements about the inherent class structure, that is to say the internal properties of the logical space. These *a priori* classes are in turn the basis for contingent general propositions that are used as basic axioms in the various sciences. Logical propositions describe inherent, one might say topological properties of that space, whereas scientific laws are concerned with appearances within the space.

From a strictly logical point of view, Wittgenstein was already committed to this view once he rejected Russell's and his earlier reductive theories. His fundamental objection had been that these theories destroyed the propositional