Second Bwanakare

Non-Extensive Entropy Econometrics for Low Frequency Series

National Accounts-Based Inverse Problems

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Managing Editor: Konrad Sarzyński

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To my wife Rose and my daughter Ozane for their many sacrifices

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Summary

This book provides a new and robust power-law (PL)-based, non-extensive entropy econometrics approach to the economic modelling of ill-behaved inverse problems. Particular attention is paid to national account-based general equilibrium models known for their relative complexity.

In theoretical terms, the approach generalizes Gibbs-Shannon-Golan entropy models, which are useful for describing ergodic phenomena. In essence, this entropy econometrics approach constitutes a junction of two distinct concepts: Jayne's maximum entropy principle and the Bayesian generalized method of moments. Rival econometric techniques are not conceptually adapted to solving complex inverse problems or are seriously limited when it comes to practical implementation.

In recent years, PL-based Tsallis entropy has been applied in many fields. Its popularity can be attributed to its ability to more accurately describe heavy tail, non-ergodic phenomena. However, the link between PL and economic phenomena has been neglected—probably because the Gaussian family of laws are globally sufficient for time (or space) aggregated data and easy to use and interpret. Recent literature shows that the amplitude and frequency of macroeconomic fluctuations do not substantially diverge from many extreme events, natural or human-related, once explained at the same time or space-scale by PL. In particular, in the real world, socioeconomic rare events may, through long-range correlation processes, have higher impact than more frequent events could. Because of this and based on existing literature, this monograph proposes an econometric extension called *Non-extensive Entropy Econometrics* or, using a less technical expression, *Superstar-Generalised Econometrics*.

Recent developments in information-theoretic built upon Tsallis non-additive statistics are powerful enough to put established econometric theory in question and suggest new approaches. As will be discussed throughout this book, long-range correlation and observed time invariant scale structure of high frequency series may still be conserved—in some classes of non-linear models—through a process of time (or space) aggregation of statistical data. In such a case, the non-extensive entropy econometrics approach generally provides higher parameter estimator efficiency over existing competitive econometrics procedures. Next, when aggregated data converge to the Gaussian attractor, as generally happens, outputs from Gibbs-Shannon entropy coincide with those derived through Tsallis entropy. In general, when the model involved displays less complexity (with a well-behaved data matrix) and remains closer to Gaussian law, computed outputs by both entropy econometrics approaches should coincide or approximate those derived through most classical econometric approaches. Thus, the proposed non-ergodic approach could at least be as good as the existing estimation techniques. On empirical grounds, it helps in ensuring stability of the estimated parameters and in solving some classes of, up to now, intractable non-linear PL-related models. Furthermore, the approach remains one of the most appro-

priate for solving all classes of inverse problems, whether deterministic or dynamic. It is a more general approach. Finally, this approach helps us better assess—thanks to the Tsallis-q parameter—the interconnection level (complexity) between economic systems described by the model.

Consequently, this book aims at providing a new paradigm for econometric modelling through non-extensive (cross) entropy information-theoretic. Reaching this goal requires some intermediary results obtained through a synthesis of the existing, sometimes sparse literature. There are, then, methodological issues to address. Among these is the application of non-extensive entropy to low frequency time series. This constitutes a new challenge and must be clarified. Next, generalizing Gibbs-Kullback-Leibler information divergence to the Tsallis non-ergodic econometric model with different constraining moment formulations in both classes of entropy model will require special attention since we are not aware of any publications on the subject. Another important intermediary result of this work will be the proposition of a new theorem linking PL and macroeconomics on both the supply and demand sides. Its demonstration will provide new keys for carrying out further Tsallis entropy econometric modelling. Finally, we will provide an *ad hoc* statistical inference corresponding to the new modelling approach presented here.

The first part of the monograph presents basic targets and principal hypotheses.

In the second part, we present definitions and quantitative properties of statistical theory of information. Progressively, a link between the statistical theory of information and the generalized ill-posed inverse problem is established. After having shown the properties of the Shannon-Jaynes maximum entropy principle in detail, techniques for solving ill-behaved problems, from the Moore-Penrose generalized inverse problem to non-extensive entropy, are compared. Intrinsic relationships between both forms of Shannon-Jaynes[1] and Tsallis entropies are also shown. After having presented Kullback-Leibler information divergence, a generalization of this concept to non-extensive entropy is developed. A general linear non-extensive entropy econometric model is then introduced. It will play an important role for models to be developed in subsequent chapters. Next, an inferential formalism for parameter confidence interval area is proposed. This part is concluded with an applications example: the estimation of a Tsallis entropy econometrics model using the case of labour demand anticipation with a time series, error-correction model. Its outputs are compared with those of other approaches through Monte-Carlo simulations.

The third and fourth parts of the book—and, to a certain extent, the fifth part—are closely related to each other since a social accounting matrix can be seen as a kind of input-output transaction matrix generalization. The separation of these two

1 Here we prefer to shorten the name of this form of entropy. Scientists who have contributed to this form of entropy are many and cannot all be mentioned. It could be 'succinctly' named "Gibbs-Shannon- Jaynes-Kullback-Leibler Golan entropy econometrics."

parts has avoided highly horizontal and vertical subdivisions in the book, thereby preserving the clarity of the study. These two parts provide economic applications of statistical theory presented through Part II. Part III focuses on updating and forecasting national accounts tables. In particular, a new efficient approach to forecast input-output tables—or their extended forms—is set forth. The RAS approach is presented as a competing technique with empirical application and comments. To show one of the possible fields of entropy model implementation, we provide an ecological model to be solved as an inverse problem.

After having proposed a theorem linking PL distribution and the macroeconomic aggregative structure of national accounts, the problem of balancing a social accounting matrix (SAM) in the context of non-ergodicity is posed and solved in Part IV. The example presented deals with the actual problems of updating a SAM in real-world conditions.

In Part V, a computable general equilibrium (CGE) model is presented as a national account-related model. Two important concepts are discussed in the context of optimum property that both of them convey: the maximum entropy principle and the Pareto-optimum. Next, we open a short, epistemological discussion on two competitive and frequently confused estimation approaches, the Bayesian approach and the maximum entropy principal. An approach using non-extensive relative entropy for parameter estimation in the case of a constant elasticity of substitution (CES) function is proposed through the presentation of the CGE model.

To show the extensions of the standard national accounts table and to go beyond the general equilibrium framework, an environmentally extended social accounting matrix and a subsequent theoretical model displaying externalities are presented in Part VI. Finally, a carbon tax and double dividend theory model is presented and its social welfare impact is derived as well.

The last part of the book concludes with the principal findings and proposes areas for further investigation and research.

Two examples are provided in Annex C and D. The first concerns the use of GAMS as a platform for economic programming. The second presents some hints for solving inverse problems in the context of the proposed model.

To enable readers to better understand the results in the different chapters, they are accompanied by detailed examples or case studies and summarizing comments. As such, this book can be an ideal reference for students and researchers in many disciplines (infometrics, econometrics, statistics, national accounting, optimal control, etc.) interested in becoming familiar with approaches that reflect the most recent developments in statistical theory of information and their application for stochastic inverse problem modelling. Last but not least, the discussion in this book is limited to technical issues; it does not cover the philosophical implications of non-extensive entropy, whether general or within the discipline of economics.

PART I: **Generalities and Scope of the Book**

1 Generalities

1.1 Information-Theoretic Maximum Entropy Principle and Inverse Problem

1.1.1 Information-Theoretic Maximum Entropy Principle

According to recent literature (Golan, Judge, & Miller, 1996; Golan, 2008), the information-theoretic maximum entropy principle is a coincident junction of two lines of research: inferential statistics and statistical thermodynamics.

The first line of research emerged in the beginning of the 18th century through the work of Bernoulli (Jaynes, 1957; Halmos & Savage, 1949; Bayes, 1763; and Laplace, 1774). They developed the Principle of Sufficient Reason, which consists of determining the state of the system on the basis of limited information (moments) from a subsystem. This principle was later extended in the last century by Jeffreys (1946), Cox (1946), and Jaynes (1957b) to the principle of "not telling more than you know," thus suggesting the necessity of avoiding additional hypotheses imposed merely to simplify the problem to be solved. The purpose of all of the above authors' research was to retrieve characteristics of a general population on the basis of limited information from a possibly non-representative sample of that population, out of risky or non-convenient hypotheses.

The second line of research is represented, amongst others, by Maxwell (1871), Boltzmann (1871), Cauchy (1855), Weierstrass (1886), Lévy and Gibbs (Gibbs, 1902), Shannon (1948), Jaynes (1957, 1957b), Rényi (1961), Bregman (1967), Mandelbrot (1967), Tsallis (1988). Its main objective was to provide mathematical formalism to statistical modelling of physical information related to natural phenomena. Thanks to the celebrated work of Tsallis (1988), on non-extensive thermodynamics[2], this second line elegantly extended its multidisciplinary applications to "auto-organized systems" and to the social sciences, particularly in financial fields.

The ascent and development of the post-war information theory-based, maximum entropy proposed by Shannon (1948) can be viewed as a major step toward the rapid extension of the discipline. Less than a decade was needed to develop the information-theoretic principles of statistical inference, inverse problem solution methodology based on Gibbs-Shannon maximum entropy, and its generalizations by Kullback and Leibler (1951), Kullback (1959) and Jaynes (1957b). The above authors developed, in particular, fundamental notions in statistics, such as sufficiency and efficiency

2 Currently, this theory—undoubtedly the best—generalizes Boltzmann-Gibbs statistics for describing the case of anomalous systems characterized by non-ergodicity or metastable states. It thus better fits dynamic correlation of complex systems and can be better explained (e.g., Douglas, 2006), amongst many others.

(Halmos & Savage, 1979), a generalization of Cramer-Rao inequality (e.g., Kullback, 1959) and the introduction of a general linear model as a consistency restriction (Heckelei et al., 2008) through Bayesian philosophy. Thus, it became possible to unify heterogeneous statistical procedures via the concepts of information theory. Lindley (2008), on the other hand, had provided the interpretation that a statistical sample could be viewed as a noisy channel (Shannon's terminology) that conveys a message about a parameter (or a set of parameters) with a certain prior distribution. This new interpretation extended application of Shannon's ideas to statistical theory by referring to the information in a statistical sample rather than in a message.

Over the last two decades the literature concerned with applying entropy in social science has grown considerably and disserves closer attention. On one side, Shannon-Jaynes-Kullback-Leibler-based approaches are currently used for modelling economic phenomena competitively with classical econometrics. A new paradigm in econometrical modelling is taking place and finds its roots in the influential work of Golan, Judge, and Miller (1996). The present monograph constitutes an illustration of this.

As mentioned above, this approach is particularly useful in the case of solving inverse problems or ill-behaved matrices when we try to estimate parameters of an econometric model on the basis of insufficient information from an observed sample, and this estimation may concern the behaviour of an individual element within the system.

Insufficient information implies that we are trying to solve an ill-posed problem, which plausibly can arise in the following cases:

— data from sampling design are not sufficient and/or complete due to technical or financial limitations—*small area official statistics* could illustrate this situation;

— non-stationary or non-co-integrating variables are resulting from bad model specification;

— data from the statistical sample are linearly dependent or collinear for various reasons;

— Gaussian properties of random disturbance are put into question due to, amongst many others things[3], systematic errors from the survey process;

— the model is not linear and *approximate* linearization remains the last possibility;

— aggregated (in time or space) data observations hide a very complex system represented, for instance, by a PL distribution, and multi-fractal properties of the system may exist.

3 It is not excluded that distribution law may be erroneously applied since, for instance, randomness is dependent on the experimental setup or the sophistication of the apparatus involved in measuring the phenomenon (Smith, 2001).

Using the traditional econometrical approaches in one or more of the above cases—without additional simplifying hypotheses—could lead to various estimation problems owing to the nonexistence of a bounded solution or the instability of estimates. Consequently, outputs from traditional econometrical approaches will display, at best, poor informative parameters. In the literature, there are other well-known techniques to cope with inverse problems or ill-conditioned data. Among them, two popular techniques deserve our attention: the bi-proportional RAS approach (and its variants), particularly used for updating or forecasting input/output matrices (Parikh, 1979) and the Moore-Penrose pseudo-inverse technique, useful for inverting irregular matrices (e.g., Green, 2003, p. 833). In spite of their popularity, both techniques present serious drawbacks in empirical investigations. In fact, the RAS techniques, in spite of their divergence information nature, remain less adapted to solving stochastic problems or to optimizing the information criterion function under a larger number of different prior constraining data. Since Moore-Penrose generalized inverse ensures a minimum distance (Y-BX) only when the matrix B has full rank, it will not reflect an optimal solution in other cases. Golan et al. (1996) have clearly shown higher efficiency of Shannon maximum entropy econometrics over the above cited methods in recovering unknown information when data or model design is poorly conditioned. The suggested superiority stands on the fact that it combines and generalizes maximum entropy philosophy (as in the second law of thermodynamics) and statistical theory of information attributes as a Bayesian information processing rule. As demonstrated convincingly by Golan (1996, 2006), Shannon entropy econometrics formalism may generalize least squares (LS) and the maximum likelihood (ML) approaches and belongs to the class of Bayesian method of moments (BMOM). It is worthwhile to point out that in the coming chapters many cases of cross-entropy (or minimum entropy) formalism will be used in place of maximum entropy. This is because, in this study, many problems to be treated involve information measuring in the context of the Kullback-Leibler framework.

This monograph does not intend to treat the case of high frequency series for which a rich literature already exists. We invite readers interested in the case of high frequency series to see, for instance, J.W. Kantelhardt (2008) for testing for the existence of fractal or multi-fractal properties, suggesting the case of a PL distribution.

1.2 Motivation of the Work

1.2.1 Frequent Limitations of Shannon-Gibbs Maximum Entropy Econometrics

In spite of a growing interest in the research community, some incisive critics have come forward to address Shannon-based entropy econometrics (e.g., Heckelei et al., 2008). According to some authors, generalized maximum entropy (GME) or cross-entropy (GCE) econometrical techniques face at least three difficulties. The first is related to the specification and interpretation of prior information, imposed via the use of discrete support points, and assigning prior probabilities to them. The authors argue that there are complications that result from the combination of priors and their interaction with the criterion of maximum entropy or minimum cross-entropy in determining the final estimated *a posteriori* probabilities on the support space. The second group of criticisms questions the sense of the entropy objective function once combined with the prior and data information. The last problem, according to the same authors, refers to computational difficulties owing to the mathematical complexity of the model with an unnecessarily large number of parameters or variables.

Concerning the first criticism, the problem—selecting a prior support space and prior probabilities on it—exists since estimation outputs seem to be extremely sensitive to initial conditions. However, when there is a theory or some knowledge about the space on which parameters are supposed to be staying, the problem becomes tractable. In particular, when we have to estimate parameters in the form of ratios, the performance of entropy formalism is high. To this counterargument, it is worthwhile to add that GME or GCE formalism constitutes an approach based on the Bayesian efficient processing rule and, as such, prior values are not fixed constraints of the model; they combine and adapt with respect to other sets of information (e.g., consistency function) added to the model to update a new parameter level in the entropy criterion function.

The second problem concerns questioning the sense or interpretability of output probabilities from the maximum entropy criterion function once combined with real world probability-related restrictions. One cannot comment on this problem without making reference to the important contribution of Jaynes (1957, 1957b), who proposed a way to estimate unknown probabilities of a discrete system in the presence of less data point observations than parameters to be estimated through the celebrated example of Jaynes dice. Given a set of all possible ways of distribution resulting from all micro-elements of a system, Jaynes proposed using the one that generates the most "uncertain"[4] distribution. To understand this problem, the question becomes a matter of combining philosophical interpretation of the maximum entropy principle with that of Jaynes' formulation in the context of Shannon entropy. Depending on the type of entropy[5] considered, output estimates will have slightly different meaning. However, all interpretations refer to parameter values that assure a long-run, steady-

4 Here we are in the realm of the second law of thermodynamics, which stipulates, in terms of entropy, that natural equilibrium of any set of events is reached once disorder inside them becomes optimal. This results from their property of having equal (ergodic system) odds to occur. In that state, we reach the maximum uncertainty about which event should occur in the next trial.

5 Later, for comparison, properties of the most well-known types of entropy in the literature will be presented.

state equilibrium of the system (relations defined by the model) with respect to data and other knowledge at hand, usually in the form of moments and/or normalization conditions. Owing to maximum entropy alone, the more consistent moments are or the more other *a priori* information binds, the more output probabilities will differ from those in a uniform distribution. Considering the above, interpretation of the maximum entropy model is far removed from interpretation of the classical model, especially in the case of the econometric linear model where estimates mean a change in the endogenous variable due to unitary change in an explicative variable, that is, in *ceteris paribus* conditions.

The last criticisms concern the burden arising from the computational and numerical process—a problem common to all complex, nonlinear systems. Thanks to recent developments of computer software, this problem is now less important.

In many empirical studies that attempt to solve inverse problems, the Shannon entropy-based approach is relatively efficient in recovering information. However, gaining in parameter precision requires good design of the prior. In particular, the point support space must fit into the space of the true population parameter values. As Golan et al. (1996) have shown, when prior design is weak, outputs of Shannon entropy econometrics will produce approximately the same parameter precision as traditional econometrical methods, such as LS or the ML, which means Shannon entropy could discount information not fitting the maximum entropy principle as expected.

The above criticisms of the Shannon entropy econometrics model remain relatively weak as has been shown through the preceding discussion.

According to us, the main drawback related to that form of model is due to the analytical function of constraining moments. In fact, as already suggested, long-range correlation and observed time invariant scale structure of high frequency series may still be conserved—in some classes of non-linear models—through a time—or space—aggregation process of statistical data. This raises the question of why this study proposes a new approach of Tsallis non-extensive entropy econometrics.

The next section provides a first answer by showing potential theoretical and then empirical drawbacks of the Shannon-Gibbs entropy model and potential advantages from the PL-related Tsallis non-extensive entropy approach.

1.2.2 Rationale of PL-Related Tsallis Entropy Econometrics and Low Frequency Series

This section presents the essence of the scientific contribution of this monograph to econometric modelling. For a few decades, PL has confirmed its central role in describing a large array of systems, natural and manmade. While most scientific fields have integrated this new element into their analytical approaches, econometrics and hence, economics globally, is still dwelling—probably for practical reasons—

on the Gaussian fundamentals. This study takes a step forward by introducing Tsallis non-extensive entropy to low frequency series econometric modelling. The potential advantages of this new approach will be presented, in particular, its capacity to analytically solve complex PL-related functions. Since any mathematical function form can be represented by a PL formulation, the importance of the proposed approach becomes clear. To be concrete, one of the complex nonlinear models is the fractionally integrated moving average (ARFIMA) model, which, to our knowledge, has remained non-tractable using traditional statistical instruments. An empirical application to solve such a class of models will be implemented at the end of Part V of this book.

According to several studies (Bottazzi et al., 2007), (Champernowne, 1953), (Gabaix, 2008), a large array of economic laws take the form of a PL, in particular macroeconomic scaling laws, distribution of income, wealth, size of cities and firms[6], and distribution of financial variables such as returns and trading volume. Ormerod and Mounfield (2012) underscore a PL distribution of business cycle duration. Stanley et al. (1998) have studied the dynamics of a general system composed of interacting units, each with a complex internal structure comprising many subunits, where the subunits grow in a multiplicative way over a period of twenty years. They found that this system followed a PL distribution. It is worthwhile to note the similarity of such a system with the internal mechanism of national accounts tables, such as a SAM, also composed of interacting economic sectors, each with a complex internal structure defined by firms exercising similar business. Ikeda and Souma (2008) have made an international comparison of labour productivity distribution for manufacturing and non-manufacturing firms. A PL distribution in terms of firms and sector productivity was found in US and Japanese data. Testing the Gibrat's law of proportionate effect, Fujiwara et al. (2004) have found, among others things, that the upper-tail of the distribution of firm size can be fitted with a PL (Pareto-Zipf law). The list of PL evidence here is limited to social science.

Since this study focuses on the immense potentiality of PL-related economic models, PL ubiquity in the social sciences will be underscored and a theorem showing the PL character of national accounts in its aggregate form will be presented.

In line with the rationale for the proposed methodology detailed below, the following from recent literature is evidence of entropy:

– Non-extensive entropy, as such, models the non-ergodic systems which compound Levy[7] instable phenomena[8] converging in the long range to the Gaussian basin of attraction. In the limiting case, non-extensive entropy converges to Shannon Gibbs entropy.

6 See (Bottazzi et al., 2007) for different standpoints on the subject.

7 Shlesinger, Zaslavsky, & Klafter, Strange Kinetics, 1993.

8 Shlesinger et al., Lévy Flights and Related Topics in Physics, 1995.

- PL-related Tsallis entropy should remain, even in the case of a low frequency series, a precious device for econometric modelling since the outputs provided by the exponential family law (e.g., the Gibbs-Shannon entropy approach) correspond to the Tsallis entropy limiting case when the Tsallis-q parameter equals unity.
- A number of complex phenomena involve long-range correlations which can be seen particularly when data are time scale-aggregated (Drożdż & Kwapień, 2012), (Rak et al., 2007). This is probably because of the interaction between the functional relationships describing the involved phenomena and the inheritance properties of a PL or because of their nonlinearity. Delimiting the threshold values for a PL transition towards the Gaussian structure (or to the exponential family law) as a function of the data frequency amplitude is difficult since each phenomenon may display its own rate of convergence—if any—towards the central theorem limit attractor.
- Systematic errors from statistical data collecting and processing may generate a kind of tail queue distribution. Thus, a systematic application of the Shannon-Gibbs entropy approach in the above cases—even on the basis of annual data—could be misleading. In the best case, it can lead to unstable solutions.
- On the other hand, since non-extensive Tsallis entropy generalizes the exponential family law (Nielsen & Nock, 2012), the Tsallis-q entropy methodology fits well with high or low frequency series.

In the class of a few types of entropy displaying higher-order entropy estimators able to generalize the Gaussian law, Tsallis non-extensive entropy has the valuable quality of concavity–and then stability—along the existence interval characterizing most real world phenomena. As far as the q-generalization of the Kullback-Leibler (K-L) relative entropy index is concerned, it conserves the same basic properties as the standard K-L entropy and can be used for the same purpose (Tsallis, 2009).

The above-enumerated points imply that in cases where the assumed Levy law complexity is not verified by empirical observation, outputs from the non-extensive entropy model converge with those derived from Shannon entropy. In other words, errors which involve taking a sample as if it were PL-driven has no consequence on outputs if the truth model belongs to the Gaussian basin of attraction. This explains why in most empirical applications—but by no means all—both forms of entropy provide similar results and the entropic Tsallis-q complexity parameter then tends to converge to unity, revealing the case of a normal distribution. Empirical examples will be presented at the end of this document, and the strength of Tsallis maximum entropy econometrics will be demonstrated in different contexts.

In summary, the following are entropy function regularities:

- The Tsallis entropy model generalizes the Shannon-Gibbs model, which constitutes a converging case of the former for the Tsallis-q parameter equal unity.

- The Shannon-Gibbs model fits natural or social phenomena displaying Gaussian properties.
- PL high frequency time (space) series scaling—aggregating—does not always lead to Gaussian low frequency time (space) series. Additionally, the rate of convergence from the PL to the Gaussian model, if any, varies according to the form of the function used.

Is it judicious to replace Shannon-Gibbs entropy modelling by Tsallis non-extensive entropy for empirical applications?

The answer is yes, and this is the motivation for this study. There are at least three expected advantages to introducing Tsallis non-extensive econometric modelling:

1. A data generating system characterized by a low—or no—convergence rate from PL to Gaussian distribution only becomes analytically tractable when using Tsallis entropy formalism. (This will be proven through an econometrical model with constant substitution elasticity and then considered as an inverse problem to be estimated later.)
2. The Tsallis entropy model displays higher stability than the Shannon-Gibbs, particularly when systematic errors affect statistical data.
3. The Tsallis-q parameter presents an expected advantage of monitoring complexity of systems by measuring how far a given random phenomenon is from the Gaussian benchmark. In addition to other advantages, this can help draw attention to the quality of collected data or the distribution involved.

The choice of national accounts-related models for testing the new approach of non-extensive entropy econometrics is motivated by the empirical inability of national systems of economic information to provide consistent data according to macroeconomic general equilibrium. As a result, national account tables are generally not balanced unless additional—often contradictory—assumptions are applied to balance them. However, following the principle of not adding (to a hypothetical truth) more than we know, it remains preferable to deal with an *unbalanced* national accounts table. Trying to balance such a table implies that we are faced with ill-behaved inverse problems. According to the existing literature, and as will be seen through this monograph, entropy formalism remains the best approach to solving such a category of complex problems. The superiority of Tsallis non-extensive entropy econometrics over other known econometrical or statistical procedures results from its capacity to generalize a large category of most known laws, including Gaussian distribution.