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Index Number Theory and Price Statistics

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Im Namen der Herausgeber danke ich allen Wissenschaftlerinnen und Wissenschaftlern, die in diesem Zeitraum bereit waren, für die Jahrbücher für Nationalökonomie und Statistik Manuskripte zu begutachten. Mit ihrer Hilfe sind wir dem Ziel, eine möglichst schnelle Entscheidung über die Publikation der Einreichungen herbeizuführen, ziemlich nahe gekommen. Die Autoren konnten die detaillierten Verbesserungsvorschläge aufnehmen, und davon hat die Qualität der Manuskripte stark profitiert.

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Introduction to the Special Issue on Index Number Theory and Price Statistics

By Peter von der Lippe, Duisburg-Essen, and W. Erwin Diewert, Vancouver*

1 Introduction

Index theory as well as price statistics has undergone a remarkable change in the last twenty years. Some of the influential publications were the *System of National Accounts 1993*,[1] the famous Reports of the Boskin[2] and Schultze[3] Commissions, the ongoing creation of a number of Manuals and recommendations of best practice, created by international groups of experts and edited by international organisations such as the ILO, IMF, OECD, UNECE, Eurostat, or the World Bank. Thus we now have best practice Manuals for Productivity Measurement, Consumer Prices, Producer Prices, Export and Import Prices, the Measurement of Capital, the Measurement of Non-Market Production,[4] and additional Manuals or Handbooks for House Prices (or Residential Property Prices) and for International and Interregional Comparisons of Consumer Prices are in progress.

Such initiatives have not only contributed enormously to a fundamental reorientation of index number theory but also to the creation of new indices in official price statistics in a process which is from the outset based on an intense international cooperation. A most prominent project of this kind was to establish the Harmonised Index of Consumer Prices (HICP) in the European Union and there are a number of other new indices already existing or gradually emerging (e. g. price indices for labour costs, certain services and house prices). The interesting point is that new results in index number theory were explicitly taken into account in these efforts to improve official statistics in the last twenty or so years. Moreover we now not only experience more co-operation between the two spheres of activity which both are the subject of this special issue, that is index number theory and price statistics, but we may also note that the scope of index theory is now much broader than before. In former days, the focus of index number theory was very much confined to the determination of the appropriate formula for comparing prices (or quantities) related to two periods in time. However, we now encounter mathematical and conceptual studies related to many other economic measurement problems such as quality adjustment, productivity measurement, asset pricing, problems of aggregation and deflation, interspatial comparisons and other problems inspired by the practice of price statistics.

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* The authors thank Bert Balk for helpful comments. The financial support of the SSHRC of Canada is gratefully acknowledged.

1 See Eurostat (1993)

2 See Boskin et al. (1996).

3 See Schultze and Mackie (2002).

4 See Schreyer (2001), the ILO (2004), IMF (2004), IMF (2009) and Schreyer (2009, 2010).

It is appropriate to give an account of some of the impacts we owe to the above mentioned international activities because with this background material, it may be easier to understand why certain topics are addressed in this special issue.

The *System of National Accounts 1993* recommended a move to *chain indices* both in inflation measurement and the deflation of aggregates so that volumes could be derived from these deflated values. This turned out to be quite a fundamental change particularly when contrasted with the German price index tradition, which recommended fixed base indices. Furthermore, the Boskin Report strongly recommended that a consumer price index (CPI) should be patterned after the model of the (true) *cost-of-living index* (or better known as COLI) as it was developed in the economic approach to index number theory, as opposed to the fixed basket (and not chained) Laspeyres price index. The modified Laspeyres (1871) or Lowe (1823) index used to be the prevailing type of Consumer Price Index in large parts of Europe. For many years the COLI seemed to be not eligible for a target index in official statistics due to the problems involved in the notion of operationalizing the concept of utility in an index number context. This changed, however, when W. Erwin Diewert (1976, 1978) succeeded in showing that certain symmetrically weighted price index formulas (using the quantities in the base period as well as in the current reference period as weights in a symmetric fashion) like the indices of Fisher (1922), Törnqvist (1937) or Walsh (1901, 1921) are *superlative* indices in that they approximate the COLI under fairly general conditions (and also have favourable axiomatic properties). Two papers in this special issue of the *Jahrbücher für Nationalökonomie und Statistik*[5] explicitly refer to the COLI concept of a CPI.

In addition to vigorously advocating the COLI paradigm, the Boskin Report also emphasized some important practical aspects of index methodology. Two examples are the *hedonic approach to quality adjustment* and the choice of the "best" unweighted *elementary index*, which is an index for the lowest level of aggregation as compared to upper level aggregation where expenditure weights are available. Both hedonics and index compilation in two stages are comparatively new topics of index theory, and they are both addressed in contributions to this special issue.

Elementary indices such as the formula of Dutot (1738), Carli (1764) or Jevons (1865) are unweighted indices while in the second stage of aggregation, when traditional formulas like Laspeyres (1871) and Paasche (1874) indices are calculated, quantities are introduced as weights for the prices. Another possibility, examined in the paper of Diewert and von der Lippe in this special issue, is to use *unit values* rather than the unweighted elementary price indices used in the first stage of a two stage index compilation.

This special issue is another example of a productive and prolific international co-operation with authors coming from various countries and being influenced by different schools and traditions of thinking in economics and statistics. We hope that the benefits of a joint multinational effort will become visible in this issue and we also appreciate that this journal, with its long tradition in index number theory, was open to devoting an issue to such often neglected topics like index number theory and price statistics.

In the following sections, we will provide brief introductions to the papers collected for this special issue of the *Jahrbücher für Nationalökonomie und Statistik* (the *Journal of Economics and Statistics*).

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[5] See the papers by Balk and by Greenlees and Williams in this issue.

2 Drobisch, unit value price indexes and elementary indexes

This journal takes pride in the fact that many famous German index number theorists have published their seminal papers in this very journal. Not only Laspeyres and Paasche did so but also the less well known Moritz Wilhelm Drobisch. It is therefore no coincidence that this special issue of the journal starts with the paper of Ludwig **von Auer**, *Drobisch's Legacy to Price Statistics*, which is a sketch of life and work of a sadly neglected German scholar who among other things also contributed to index theory. According to von Auer, Drobisch (1871) proposed among other index formulas his *unit value index* (UVI), defined as the ratio of two all-items unit values, as his favourite price index. This index may be used as an index at lower levels of aggregation, where only closely related items are aggregated, but it is not a useful price index at higher levels of aggregation, where quantity units cannot be summed in a meaningful manner. However for Drobisch, his price index was intended to be an upper aggregation level index and he thought that his index stood on the same footing as the formula of Laspeyres (1871) or Paasche (1874). Unfortunately, the term *unit value index* is used for both situations; both when aggregating over similar items as well as when aggregating over very different products. This leads to some terminological problems, which had to be solved in the paper of Diewert and von der Lippe. These authors preferred to call the unit value index for the second, more problematic case of an all-item index, the *Drobisch price index*.

In a rather formal manner, the indices (throughout this issue we use "indices" and "indexes" as synonyms) of Laspeyres and Paasche, P_L and P_P respectively, could be regarded as special cases of Drobisch's index (they emerge if either the base period quantity vector q_0 or the current period quantity vector q_t are used in both the numerator and denominator of the Drobisch formula). However, Drobisch rejected the formulas P_L and P_P which later were to become famous. Moreover Drobisch did not effectively realise the axiomatic shortcomings of his formula. According to von Auer there must have been a vigorous academic dispute between Drobisch and Laspeyres and of course, Drobisch came out on the losing end of this dispute. Loosing also seems to have made Drobisch feel to have been robbed of the merits of being the inventor of both the Laspeyres and the Paasche formula. In our view (with which von Auer might perhaps disagree), Drobisch's complaint for lack of academic recognition as compared to Laspeyres, who acquired a lot of posthumous fame, is understandable though possibly not justified since he examined the formulas P_L and P_P and he explicitly rejected them in favour of his quite objectionable and rightly much less acceptable formula.

Such disputes may have been responsible for the fact that Drobisch's interest in index theory was not lasting, and that he never properly responded to the valid criticism of his formula and therefore failed to become as prominent as Laspeyres for example (who by the way also was unable to fully realise in his lifetime the relevance of the formula which later was to bear his name).

It is just because Drobisch fell into oblivion that von Auer's interesting narrative of life and work of this almost unknown German mathematician and philosopher is a valuable contribution.

An extensive analysis of the just defined *Drobisch price index* P_D can be found in W. Erwin **Diewert** and Peter **von der Lippe** in their paper *Notes on Unit Value Index Bias*. The problem with the index P_D is that it is affected by the changing structure of the quantity weights in the numerator and denominator of the Drobisch index. The weights S^t in the period t unit value $P^t \equiv p^t \cdot (q^t/\Sigma_{n=1} q_n^t) = p^t \cdot S^t$ therefore reflect a changing structural component in the numerator and denominator of the Drobisch price index, $P_D \equiv P^t/P^0 = p^t \cdot S^t/p^0 \cdot S^0$. Thus the Drobisch index can change even if prices do not change; i. e., we can have $P_D \neq 1$ even if $p^0 = p^t$.[6] Another unfavourable axiomatic property of the Drobisch index when aggregation takes place over very heterogeneous items is that it is not invariant to changes in the units of measurement.[7] However, the focus of this paper is not on the axiomatic properties of the Drobisch price index P_D, but on determining the *bias* of P_D when aggregating over "reasonably" homogeneous (but not completely homogeneous) products as compared to standard indexes, like P_L, P_P and P_F. It turns out that the bias in P_D relative to these indexes can be defined in terms of covariances between prices in the base period 0 and various measures of the change in the (structure of) quantities between base and current periods.

Another topic discussed in this paper is an index which is called *hybrid Paasche index* (P_{HP}) (and its Laspeyres counterpart) by Diewert and von der Lippe and hitherto only rarely examined in detail in the literature on unit value indexes.[8] It is clear that the Drobisch index requires that quantities of items can meaningfully be added up to a total quantity (for which a common unit of measurement across all items is a necessary but not sufficient condition) and that this condition is not fulfilled for an aggregate as wide as an all items CPI for example. It is, however, reasonable to make use of ratios of unit values as building blocs on the lower aggregation level of an index compiled in two stages. The P_{HP} index uses unit values in the first stage and the Paasche formula in the second stage of aggregation (correspondingly, the *hybrid Laspeyres index* P_{HL} uses the Laspeyres formula in the second stage). Again a bias formula is derived, now for the bias of P_{HP} (rather than P_D) relative to P_P and again it turns out that the bias is determined by a covariance between base period prices and changes in the quantities. One of conditions for a vanishing covariance suggests the following rule for choosing how to construct the subaggregates: in order to minimize bias (relative to the Paasche price index), use unit value aggregation over products that sell for the same price in the base period.

On the basis of the results concerning P_D and P_{HP} the paper also considered the question: does disaggregation of a unit value into more homogeneous subgroups reduce the unit value bias? The answer seems to be: probably yes.

Jens **Mehrhoff** in his paper *Aggregate Indices and Their Corresponding Elementary Indices* also deals with the problem of price indices compiled in two stages. This paper explicitly takes into account, that in practice, price indices are in general compiled in a two stage procedure. The problem Mehrhoff addressed is to find for the three second stage or weighted indices of Laspeyres (P^L), Paasche (P^P) and Fisher (P^F) the "best" corresponding elementary indices (first stage indices). Here the first stage uses well known unweighted price indices; that is, the formulas of Carli and Jevons are considered (or more precisely: all indices that can be derived as special cases of the so called *power means* or *generalised means* or *means of order r*).[9] The practical implication of this might be that there can be situations which do not permit calculation of P^L, P^P and P^F, perhaps

6 See the paper by Diewert and von der Lippe for definitions of the notation.
7 See Balk (1995, 1998) and (2008) on the axiomatic approach to index number theory.
8 Párniczky (1974) initiated this line of research and Balk (2008) significantly added to it.
9 See Hardy et al. (1934) for the properties of means of order r.

due to missing information about quantities, and it then may be reasonable to approximate the desired index by an unweighted price index which uses prices only.

In which sense does a first stage unweighted (elementary) index *correspond* to a second stage weighted index in Mehrhoff's study? According to Mehrhoff, *corresponding* is defined as *best numerically approximating the target index* P^L, P^P or P^F respectively. This type of methodology was also used by Shapiro and Wilcox (1997).

In Mehrhoff's view, the distinctive feature of his approach to assess elementary indices is an attempt to give an (intuitive or economic) interpretation to the notion of correspondence. His interpretation rests on assuming that the joint probability distribution of prices and quantities in both periods, 0 and t is a bivariate log-normal distribution (LND). This possibly doubtful LND assumption provides a framework in which the relationship between elementary (unweighted) and second stage (weighted) indices can be studied because the LND assumption determines both the distribution of the price relatives (the raw material of elementary indices) and the distribution of the expenditure shares (as weights for the second stage). His assumption may also help to make the choice of the unweighted index on the first stage data-driven. Note that the power mean can take on any value between the lowest and the highest price relative depending on the power r ($-\infty < r < +\infty$) of the mean of order r. Thus the task of finding the elementary index (equal to a mean of order r) that *corresponds* to P^L, P^P or P^F now boils down to determination of the correct power r of the generalised mean. Mehrhoff succeeded in showing that the expectations of P^L and P^P and the order r of the generalised mean can all be expressed in terms of LND-moments.

Next Mehrhoff attempts to give an interpretation to the moments. For this purpose Mehrhoff introduced a partial adjustment model (PAM) which allows him to give an elasticity interpretation to the moments of the LND. It should be borne in mind, however, that his elasticity (β in the paper) is not a price elasticity of demand or supply because there is no demand or supply function involved, but his parameter is associated with an equation describing an adjustment process. The economic meaning of the PAM model and the consistency of the assumptions underlying the LND and the PAM may be called in question. Nonetheless the ideas of Mehrhoff are innovative and worthy of notice and discussion.

3 Problems associated with the construction of a Consumer Price Index

In this section, we briefly describe the two papers in this special issue that deal with measurement problems associated with the construction of a Consumer Price Index (CPI).

The first paper is by Bert **Balk**, who holds positions at Statistics Netherlands and the Rotterdam School of Management at Erasmus University. He is one of the most prolific writers on index number theory of all time. The title of his paper is *Lowe and Cobb Douglas Consumer Price Indices and their Substitution Bias.*

Many statistical agencies use the Lowe (1823) index as the basis for their CPI. This index uses an implicit quantity vector pertaining to household expenditures in a base year b in order to price out this basket at the prices of the current month t and at the prices of a base month 0 and then the ratio of these expenditures is defined to be the monthly CPI for the country. Chapter 17 of the *Consumer Price Index Manual*[10] written by Balk and

[10] See the ILO (2004).

Diewert works out the likely bias of the Lowe index relative to a Konüs (1924) True Cost of Living Index. In the present paper, Balk undertakes a similar exercise where he attempts to determine the bias of a Cobb-Douglas price index[11] relative to a Cost of Living index. The Cobb-Douglas index is a weighted geometric mean of the monthly price relatives where the weights are the household expenditure shares in the base year b. This index is just as easy to construct as the Lowe index and hence it is a practical alternative that statistical agencies should consider using as their CPI concept. In the present paper, Balk reworks and improves the earlier analysis on the bias in the Lowe index and he develops some new formulae for the bias in the Cobb-Douglas index. Unfortunately, as Balk notes towards the end of his paper, it is difficult to draw a firm conclusion about the magnitude of the likely bias in the Cobb-Douglas or Geometric Young index relative to a Cost of Living index.

The second paper in this section that deals with possible bias in the Lowe and Geometric Young indexes as approximations to a Cost of Living index is by two researchers in the U. S. Bureau of Labor Statistics (BLS), John **Greenlees** and Elliot **Williams.** The title of their paper is *Reconsideration of Weighting and Updating Procedures in the US CPI.*

In 2002, the BLS made two important changes to their CPI, which was previously based on a Lowe index: (i) they increased the frequency of expenditure weight updates in their headline CPI, the CPI-U, from approximately 10 years to 2 years and (ii) they introduced a new supplemental CPI using the superlative formula Törnqvist formula,[12] the C-CPI-U. This new index was introduced to provide a closer approximation to a Cost of Living Index (COLI). The availability of eight calendar years of final C-CPI-U data along with expenditure data from four consecutive two-year CPI-U base periods gave Greenlees and Williams the opportunity to analyze both the effects of more frequent basket updating and the change to a superlative formula. Thus Greenlees and Williams compute several alternative indexes over the period 1999–2007 including several variants of the Lowe type CPI-U that vary the updating period as well as the superlative Törnqvist and Fisher indexes and they also compute the Geometric Young index which was studied in Balk's paper. The authors note that it is not possible to compute a superlative index in real time and hence the issue of whether more frequent updating of the annual basket will move the resulting index closer to a superlative index is important, since the CPI-U is widely used in government tax and indexed bond programs, largely because unlike the superlative C-CPI-U it is not subject to revision. Thus Greenlees and Williams note that although the BLS and many economists believe that the superlative C-CPI-U is a closer approximation to a COLI, improvements to the CPI-U are of the utmost practical importance.

The first important result that Greenlees and Williams establish is that the Fisher and Törnqvist indexes are quite close together in every year but one, 2000, although the year-to-year changes in the Törnqvist are always slightly higher. The authors attribute the apparent divergence in the first year of their sample period to the greater sensitivity of the Fisher index to extreme data points, which is a very interesting observation.

Greenlees and Williams also computed chained Laspeyres and Paasche indexes and they find that over their sample period, the chained Laspeyres index rises about 2.9 percentage points more than the chained Paasche. On a per year basis, they found that the

[11] This index is also known as a geometric Young index.
[12] See Törnqvist (1936) and Törnqvist and Törnqvist (1936).

chained Laspeyres index rises about 0.18 percentage point per year more rapidly, and the chained Paasche 0.18 percentage point more slowly, than the corresponding chained Fisher.

In their Figure 2, Greenlees and Williams show that the effects of changing the annual basket more frequently does cause the corresponding Lowe type CPI-U to come closer to the chained Törnqvist index but that more frequent basket updates cannot fully close the gap. In their Figure 3, they compare a Lowe index with 5 year basket updating with the CPI-U (a Lowe index with 2 year basket updating) and with the C-CPI-U (a superlative index) and finally with a Geometric Lowe index. The results are perhaps as one might expect, the Lowe indexes lie well above the corresponding superlative index and the Geometric Young index lies somewhat below the superlative index. Thus for U. S. data over this period, it appears that the Geometric Young index is closer to a COLI than the corresponding Lowe indexes. This seems to be an important result that other statistical agencies might want to look at very closely.[13]

We conclude this section noting that not both editors agree on the significance of this paper. The paper supports the conclusion that as a rule, more frequent updating of weights seems to be a good thing in that the result of this more frequent updating causes the fixed base index to come closer to a superlative index. This result is not much to the liking of one of us (Peter von der Lippe) who is an ingrained opponent of chain indices and strict adherent to the "pure price comparison" school of thought, according to which a price index should preferably be reflective of price movements only.[14] Thus while Diewert sees a superlative index as a reasonable target index for a CPI, von der Lippe sees a fixed base basket type index as the more reasonable target index. While Diewert recognizes the attraction of making fixed base basket type comparisons (they are very natural and easy to explain), there are some practical considerations which lead him to generally prefer chained indexes (at least for annual data):

- If statistical agencies took the von der Lippe point of view, then instead of providing a single time series of fixed base price and volume movements, in order to suit the needs of all users, logically, they should provide *matrices* of comparisons, where every year is compared with every other year (since different users might have different ideas of what the right base year is and how many years the fixed base should be held fixed). This could be done but many users would not be comfortable with this matrix presentation.

- Bilateral comparisons between distant periods become more problematic for two reasons: (i) individual products cannot be matched to disappearing obsolete products and the introduction of new products and (ii) Paasche and Laspeyres spreads tend to become larger using direct comparisons as opposed to chained comparisons when comparing distant periods.[15] Thus using the chain principle (for annual data) will tend to diminish the importance of the choice of the index number formula.

[13] A Geometric Young index can be constructed in real time just as easily as a Lowe index.

[14] See von der Lippe (2001, 2007) for a more complete exposition of his position on chaining.

[15] Paasche and Laspeyres spreads can be narrowed between any two points by picking a path of bilateral comparisons based on the similarity of relative prices between pairs of observations. For examples of this methodology, see Hill (2004, 2009) and see Diewert (2009) for materials on how to measure the similarity of relative prices.

However, Diewert does concede that the use of the chain principle does not necessarily work when dealing with subannual data where price and quantity changes can be very large.[16] Thus this issue is not completely resolved. For a balanced viewpoint see Balk (2010b).

4 User costs, waiting services and the treatment of depreciation

W. Erwin Diewert in his paper, "*User Costs versus Waiting Services and Depreciation in a Model of Production*", addresses a problem that occurs when measuring the Multifactor or Total Factor Productivity growth of a production unit. *Productivity* is defined as the output of a production unit over a period of time divided by the input used and *Productivity Growth* is defined as the rate of growth of outputs divided by the rate of growth of inputs. This definition of Productivity Growth means that it could be measured by index number techniques; i. e., it could be set equal to an output quantity index divided by an input quantity index. However, such a gross output productivity index is not entirely suitable for making cross sectional comparisons for similar production units where the focus is on the amount of *value added* (gross outputs minus intermediate inputs) per unit of *primary input* (primarily labour and capital services). Thus Jorgenson and Griliches (1967) developed their Divisia index methodology to measure the value added productivity growth of an economy and they used the ratio of Törnqvist indexes of value added and primary inputs as approximations to their productivity concept. Diewert and Morrison (1986), using duality theory, showed how a slight variant of the Jorgenson and Griliches methodology could be justified using (flexible) translog functional forms and exact index numbers. The resulting methodology measures the growth of value added of the production unit per unit of primary input, where primary input is equal to labour and capital services. However, a component of capital services is *depreciation* or *deterioration*, which is not really a primary input. Primary input services should include the services of labour, land, waiting services and natural resource and other environmental services. Put otherwise, the user cost of capital includes a term reflecting the decline in capital services due to the aging of the asset; i. e., due to depreciation of the asset. A better measure of the effectiveness of a production unit in producing useful goods and services per unit of primary input would take depreciation out of the user cost of capital and treat it as an offset to gross investment. The resulting amended value added concept could be defined as a *net value added concept* as opposed to the usual gross value added concept.[17] If depreciation can be considered to be a decision variable by the production unit, then it turns out that the analysis of Diewert and Morrison (1986) can be adapted to this net value added framework and a rigorous productivity growth framework based on production theory can be readily be implemented using index number theory.[18] This brings us to the contribution of the Diewert paper in this special issue: he argues that depreciation is indeed a decision variable and can be extracted from the usual user cost of capital and treated as a separate offset to gross investment.

[16] See Ivancic et al. (2010) and Haan and van der Grient (2010) for some methods that combine fixed base indexes with chain indexes that can deal with large fluctuations in period to period price and quantity data.

[17] Balk (2010a) discusses these concepts in more detail; see also the earlier discussion by Schreyer (2001). If the production unit is the entire economy, net value added is equivalent to net domestic product and value added is equivalent to gross domestic product.

[18] See Diewert and Lawrence (2006).