Eugene Stefanovich

Elementary Particle Theory

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Elementary Particle Theory

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Postulates, statements, theorems, assertions

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Conventional notation

Quantum logic

$\mathcal{L}$	set of all propositions (logic) (page 10)
$\mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}, \mathcal{Z}$	logical propositions (page 9)
$\mathcal{A} < \mathcal{B}$	$\mathcal{B}$ follows from $\mathcal{A}$ and $\mathcal{B} \neq \mathcal{A}$ (page 11)
$\mathcal{A} \leq \mathcal{B}$	$\mathcal{B}$ follows from $\mathcal{A}$ (page 11)
$\mathcal{I}$	maximal proposition (page 11)
$\emptyset$	minimal proposition (page 11)
$\mathcal{A}^{\perp}$	orthocomplement to $\mathcal{A}$ (NOT $\mathcal{A}$) (page 11)
$\mathcal{A} \vee \mathcal{B}$	join ($\mathcal{A}$ OR $\mathcal{B}$) (page 10)
$\mathcal{A} \wedge \mathcal{B}$	meet ($\mathcal{A}$ AND $\mathcal{B}$) (page 10)
$\mathcal{A} \leftrightarrow \mathcal{B}$	$\mathcal{A}$ compatible with $\mathcal{B}$ (page 21)
$\mathfrak{S}$	set of all states (page 10)
ϕ	state (page 9)
$(\phi \vert \mathcal{X})$	probability measure (page 16)

Hilbert space

$\mathscr{H}$	Hilbert space (page 209)
$\mathscr{H}^*$	dual space (page 210)
$\mathscr{H}_1 \otimes \mathscr{H}_2$	tensor product (page 212)
$\vert x\rangle$	ket vector in $\mathscr{H}$ (page 210)
$\langle x\vert$	bra vector in $\mathscr{H}^*$ (page 210)
$\mathscr{A} \cap \mathscr{B}$	intersection of subspaces $\mathscr{A}$ and $\mathscr{B}$ (page 23)
$\mathscr{A} \subseteq \mathscr{B}$	$\mathscr{A}$ is a subspace in $\mathscr{B}$ (page 189)
$\mathscr{A} \uplus \mathscr{B}$	linear span of $\mathscr{A}$ and $\mathscr{B}$ (page 23)
$\mathscr{A}'$	orthogonal complement of $\mathscr{A}$ (page 225)
$\mathscr{A} \perp \mathscr{B}$	orthogonal subspaces (page 225)
$\mathscr{A} \oplus \mathscr{B}$	direct sum of subspaces (page 225)
$[A, B]$	commutator of operators $AB - BA$ (page 217)
$\{A, B\}$	anticommutator $AB + BA$ (page 217)
$\mathrm{Tr}(F)$	trace of operator F (page 217)
$\hat{F}$	operator acting on wave functions (page 93)
$P_{\mathscr{A}}$	projection on the subspace $\mathscr{A}$ (page 225)
P_f	spectral projection for eigenvalue f (page 26)
ϕ	phase of a wave packet (page 151)

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3D space

$\mathbb{R}$	real numbers (page 24)
$\mathbb{C}$	complex numbers (page 24)
$i, j, k, \dots$	3-vector indices ($1, 2, 3 = x, y, z$) (page 192)
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	basis vectors in $\mathbb{R}^3$ (page 191)
$\boldsymbol{a} \cdot \boldsymbol{b}$	scalar product of two 3-vectors (page 191)
$[\boldsymbol{a} \times \boldsymbol{b}]$	vector product of two 3-vectors (page 197)
δ_{ij}	Kronecker delta (page 193)
ϵ_{ijk}	Levi-Civita tensor (page 195)

Minkowski space–time

$\mathcal{M}$	Minkowski space–time (page 237)
$\tilde{A}$	4-vector (page 237)
$\tilde{A} \cdot \tilde{B}$	pseudoscalar product (page 237)
$\mu, \nu, \sigma, \dots$	4-vector indices (0, 1, 2, 3) (page 237)
$\eta_{\mu\nu}$	metric tensor (page 237)

Poincaré group

$\{\boldsymbol{\eta}\}$	element of a Lie group (page 203)
$\{\boldsymbol{v}(\boldsymbol{\theta}); \boldsymbol{\varphi}; \boldsymbol{r}; t\}$	inertial transformation (page 38)
$\mathcal{H}$	generator of time translations (page 41)
$\boldsymbol{\mathcal{J}}$	generator of rotations (page 41)
$\boldsymbol{\mathcal{P}}$	generator of space translations (page 41)
$\boldsymbol{\mathcal{K}}$	generator of boosts (page 41)
$R_{\boldsymbol{\varphi}}$	matrix of rotation (page 192)
$\boldsymbol{\phi}, \boldsymbol{\varphi}$	rotation vectors (page 197)
$\boldsymbol{\theta}, \boldsymbol{\vartheta}$	boost rapidity vectors (page 50)
$\boldsymbol{\theta} \circ \boldsymbol{\varphi}$	composition of transformations from the Lorentz subgroup (page 69)
C_{bc}^{a}	structure constants (page 204)

Group representations

$\mathbb{K}$	isomorphism of subspaces in $\mathscr{H}$ (pages 56, 125)
$\Bbbk$	unitary or antiunitary generator of $\mathbb{K}$ (pages 56, 125)
$[U_g]$	ray of unitary transformations (page 57)
U_g	unitary representation of a group (page 231)
$\mathfrak{U}_g$	unitary projective representation of a group (page 61)

$\mathfrak{T}$	generator of a projective representation (page 62)
$\mathfrak{Q}$	central charge (page 63)
$[A, B]_L$	Lie bracket (page 205)
$\mathfrak{h}_n$	Heisenberg algebra (page 232)
$\boldsymbol{\kappa}$	standard momentum (page 102)
$\mathscr{D}^s$	irreducible representation of the rotation group (page 235)
$\varphi_W(\boldsymbol{p}, \boldsymbol{\theta})$	Wigner angle (page 101)
$U(\Lambda; \tilde{a})$	unitary representation of the Poincaré group (page 69)
$\tilde{\boldsymbol{\theta}}$	4×4 pseudo-orthogonal matrices of boosts (page 239)

Observables

H	Hamiltonian (energy) (page 72)
$\boldsymbol{K}$	boost operator (page 72)
$\boldsymbol{J}$	total angular momentum (page 72)
$\boldsymbol{P}$	total linear momentum (page 72)
$H_0, \boldsymbol{P}_0, \boldsymbol{J}_0, \boldsymbol{K}_0$	generators of the noninteracting representation (page 128)
V	potential energy (page 132)
$\boldsymbol{Z}$	potential boost (page 132)
M	operator of mass (page 74)
N	interaction in the mass operator (page 133)
$\boldsymbol{R}$	center-of-energy position (page 77)
$\boldsymbol{S}$	spin (page 77)
$\boldsymbol{V}$	velocity (page 73)
$\tilde{W}$	$\equiv (W_0, \boldsymbol{W})$ Pauli–Lubanski operator (page 75)
$\boldsymbol{r}_i$	particle position (page 125)
$\lvert\boldsymbol{p}\rangle^{\tau}$	photon state with momentum $\boldsymbol{p}$ and helicity τ (page 118)
$\omega_{\boldsymbol{p}}$	$\equiv \sqrt{m^2c^4 + p^2c^2}$ one-particle energy (page 96)
$\mathrm{pow}(A)$	power of operator A (page 85)
S	S-operator (page 162)
Φ	scattering phase operator (page 166)
$\underline{Y(t)}$	$\equiv -\frac{i}{\hbar}\int_{-\infty}^{t} Y(t')dt'$ (page 165)
$\underbrace{Y(t)}$	$\equiv -\frac{i}{\hbar}\int_{-\infty}^{+\infty} Y(t')dt'$ (page 165)
$\langle F\rangle$	expectation value of observable F (page 29)
$<x>$	physical dimension of observable x (page 49)
$\mathcal{S}$	phase space (page 9)
$[A, B]_P$	Poisson bracket (page 155)
m_e	electron mass (page 92)
m_p	proton mass (page 92)

Preface

Theoretical physics, as a science, began with Newton. His ideas were based on particles – corpuscles,[1] and the first realistic model of interactions was the Newtonian theory of gravity, in which the planets and the Sun were attracted to each other by instantaneous forces at a distance.

Newton himself was very unhappy about this model. He wrote [61]

> That Gravity should be innate, inherent and essential to Matter, so that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it.

Indeed, over time, the idea of Newtonian corpuscles began to lose its appeal. The first blow was caused by the wave theory of light by Young and Fresnel. The second blow was Maxwell's theory of electromagnetic phenomena. The culmination of these misfortunes was Einstein's theory of relativity. By 1905, a harmonious system of views had developed, which denied the Newtonian action-at-a-distance. The theory of relativity forbade the superluminal transmission of any signals and interactions. The Maxwell–Liénard–Wiechert theory explained that the carrier of the retarded interaction between charges is the electromagnetic field propagating at the speed of light. Energy and momentum flowing between charges are temporarily stored in the field, so that conservation laws are not violated even in the case of such a retarded transmission of forces.

For a short period of time this field picture was shaken by the arrival of quantum mechanics. In particular, to explain the photoelectric effect, Einstein revived the Newtonian corpuscles of light – photons [4]. It turned out that these corpuscles (their wave functions) can also interfere, and to explain the structure of the atom it was sufficient to solve the Schrödinger equation for particles interacting via the instantaneous Coulomb potential.

However, early quantum theory was soon criticized for its alleged incompatibility with the principle of relativity and replaced with quantum field theory (QFT). The fantastic agreement of this theory with experiments, it would seem, has forever discouraged the return to the corpuscular past. It is enough to go over titles of some articles in respected journals,[2] to understand that in today's physics particles are in deep disgrace.

1 Even light was understood by Newton as a stream of a huge number of microscopic particles.

2 "No place for particles in relativistic quantum theories?" [35], "There are no particles, there are only fields" [37], "Why there cannot be a relativistic quantum mechanics of (localizable) particles" [53].

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> In its mature form, the idea of quantum field theory is that quantum fields are the basic ingredients of the universe and particles are just bundles of energy and momentum of the fields – S. Weinberg [96].

However, the "particles vs. fields" argument is still far from a happy resolution. Modern field theories face two difficult problems.

The first problem is ultraviolet divergences. All realistic quantum field theories suffer from divergent loop integrals occurring in calculations of scattering amplitudes. These divergent theories are "renormalized" by adding infinite counterterms to their Hamiltonians. In fact, the renormalization sweeps the problem of ultraviolet divergences "under the carpet," because it results in a poorly defined formally infinite energy operator, which is not suitable for describing the time-dependent dynamics of states. On closer examination, it turns out that the problem of divergences is related to the self-interaction of particles in QFT. In this theory, the electron interacts with itself, which is often depicted by diagrams in which an electron absorbs its own emitted virtual photons.

In the third volume of our book, we shall see that the problems of self-interaction and renormalization can be solved by introducing the so-called dressed interaction theory. This will bring us back to Newton's corpuscles, interacting with each other through instantaneous potentials. But how can one reconcile this action-at-a-distance with the theory of relativity, which prohibits superluminal propagation of interactions?

To answer this question, we turn to the second important problem of theoretical physics. It is sometimes formulated as the problem of quantum gravity, although, in fact, quantum mechanics is poorly compatible even with Einstein's special relativity theory. In special relativity, positions and time are treated on an equal basis as coordinates in the four-dimensional Minkowski space–time. However, in quantum mechanics these two quantities play quite different roles. The spatial coordinate (like any other physical observable) is described by an Hermitian operator, whereas time is simply a numerical parameter that cannot be converted into an operator without contradictions.

Our main goal is to understand the essence of contradictions between quantum mechanics and the special theory of relativity. For this, we will have to return to the very foundations of theoretical physics. We begin with indisputable postulates of *quantum mechanics* and the *principle of relativity*. Strict adherence to these postulates will lead us to the idea of unitary representations of the Poincaré group in a Hilbert space of states as the basis of the entire mathematical apparatus of our theory. Although applications of this approach to interacting systems are well known since the fundamental work of Dirac [23], it was not recognized that Dirac's interacting generators of boosts[3] imply that Lorentz transformations cannot be exact and universal,

3 The generators of boosts are interaction-dependent in the instant form of Dirac dynamics. In Volume 3, we will argue that only this form should be used to describe nature.

as required by special relativity. Boost transformations of observables must depend on the particular physical system and forces acting therein. This important observation will enable us to lift the prohibition on superluminal propagation of interactions and formulate a theory of particles acting on each other by means of instantaneous potentials. At the same time, we will be able to avoid conflicts with the unshakable principles of relativity and causality.

In the third volume, we will analyze in detail the recent experiment [21] conducted by the team of professor Pizzella at the Frascati Research Center. With this experiment they discovered the superluminal propagation of Coulomb forces, which, in our opinion, is the most convincing validation of the theory presented in this book. In some sense, the ultimate goal of the entire book is to demonstrate that Pizzella's unusual results are naturally expected in a rigorous approach to quantum relativistic physics.

In this book, we will focus on systems of charged particles and photons as well as on electromagnetic forces acting in such systems. Traditionally, these phenomena are described by *quantum electrodynamics* (QED). Our approach will lead us to another theory, which we call *relativistic quantum dynamics*, or RQD. This theory is exactly equivalent to the renormalized QED as long as one is interested in properties related to the *S*-matrix (scattering cross sections, lifetimes, energies of bound states, etc.). However, unlike QED, our approach can also describe the time evolution and boost transformations in interacting systems.

This book is divided into three volumes.[4] This is Volume 1, where we will try to avoid contradictory issues and will, basically, adhere to the generally accepted views on relativistic quantum theory. We will define our basic assumptions, notation, and terminology and also try to trace a logical path starting from the postulates of relativity and probability and leading to relativistic quantum theory of interacting systems. In this volume, we confine ourselves to interactions that do not change the number of particles in the system, which is an acceptable approximation for low-energy processes within the framework of elementary *quantum mechanics*.

Volume 1 consists of seven chapters.

In Chapter 1, *Quantum logic*, we derive the basic laws of quantum theory from simple axioms of measurements and probability (= quantum logic). We turn to the old, but not yet very popular idea that in order to understand quantum laws it is necessary to replace some of the postulates of classical logic. Despite the apparent radicalism of this approach, it leads to the well-known quantum formalism with wave functions and Hermitian operators in the Hilbert space. For us it will be important to emphasize that, being rooted in logic, the foundations of quantum mechanics are solid and unshak-

4 This work is based partially on our earlier publications [82, 83], which were rewritten, updated and improved in significant ways.

able. Therefore, we do not expect any modification of the laws of quantum mechanics[5] in the foreseeable future.

In Chapter 2, *Poincaré group*, we introduce the Poincaré group as a set of transformations connecting different (but equivalent) inertial reference frames. This chapter is central to understanding the principle of relativity. In our approach, the group properties of inertial transformations are at the core of the relativistic description of nature.

Chapter 3, *Quantum mechanics and relativity*, will combine the two theories presented above and establish unitary representations of the Poincaré group as the most general and complete mathematical description of any isolated physical system. This is the most adequate language for a relativistic quantum description of nature. One can even say that the rest of this book is simply an exercise in constructing and analyzing various unitary representations of the Poincaré group.

In Chapter 4, *Observables*, we examine the correspondence between known physical quantities (such as mass, energy, momentum, spin, position, etc.) and specific Hermitian operators in the Hilbert space of states. The most important point is the connection between physical observables and generators of the Poincaré group representation. From this connection we derive the commutation relations of observables and how these operators change with respect to inertial transformations of the observers.

Chapter 5, *Elementary particles*, is devoted to the Wigner theory of unitary irreducible representations of the Poincaré group. This theory fully describes the basic properties and dynamics of isolated stable elementary particles. For us, the special importance of this chapter is that Wigner's elementary particles are the most fundamental ingredients in our model of the world. As we explain in Volume 3, quantum fields are just formal technical constructions, and real physical systems are composed of elementary particles that interact directly with each other.

In Chapter 6, *Interaction*, we discuss relativistically invariant interactions in many-particle systems. Here we emphasize the most important conclusion of Dirac [23], that relativistically invariant interactions require modification not only of the Hamiltonian (as in the familiar non-relativistic theory) but also of other generators of the Poincaré group. We will base our theories on the Dirac instant form of dynamics, where interaction is present in both the Hamiltonian and the boost generators. In Volume 3 this will lead us to the conclusion that Lorentz transformations of special relativity are, strictly speaking, inapplicable to interacting systems.

Chapter 7, *Scattering*, is devoted to the quantum-mechanical description of particle collisions. Scattering is important first because it is the most informative experimental method for studying subatomic phenomena and second because the scattering

5 Such modifications are sometimes contemplated in attempts to develop a quantum theory of gravity. See, for example, [47] and references therein.

matrix is the main target of QFT (see Volume 2). In this book, we will build our theory (RQD) by modifying QFT, so for us the scattering matrix is of central importance. We will pay special attention to the notion of scattering equivalence, when two different Hamiltonians lead to the same S-operator. This property will play an important role in the derivation of the "dressed" Hamiltonian in Volume 3.

Some useful mathematical facts and technical calculations are collected in *Appendix*.

In Volume 2 [84], we will formulate the foundations of the most successful quantum field theory – QED, explain the causes of ultraviolet divergences and demonstrate the renormalization of the S-matrix by introducing counterterms into the Hamiltonian. There is no new physics introduced in Volumes 1 and 2. They present textbook quantum mechanics and QFT, perhaps sometimes viewed from unusual angles, but still rather orthodox. The main goal of the first two volumes is to prepare the ground for the formulation of our unconventional approach, based on the notion of physical particles and "dynamical" relativity, in Volume 3 [85] of this book.

We use the Heaviside–Lorentz system of units,[6] in which the potential energy of the electron–proton interaction has the form $V = -e^2/(4\pi r)$, and the proton charge is $e = 2\sqrt{\pi}\times 4.803\times 10^{-10}$ statcoulomb. The speed of light is $c = 2.998\times 10^{10}$ cm/s, and the Planck constant is $\hbar = 1.055 \times 10^{-27}$ erg·s = $6.582 \cdot 10^{-16}$ eV·s, so that the *fine structure constant* is equal to $\alpha \equiv e^2/(4\pi\hbar c) \approx 1/137$.

I would like to express my sincere gratitude to Peter Enders, Rainer Grobe, Theo Ruijgrok, Alexander Shebeko and Boris Zapol, who read the draft of this book and gave me many priceless comments and much advice, which I tried to take into account in the final manuscript. I also thank Harvey R. Brown, William Klink, Vladimir Korda, Chris Oakley, Federico Piazza, Guido Pizzella, Wayne Polyzou, Mikhail Shirokov and Charles Su for enlightening discussions at various stages of this work. I enjoyed online communications with Juan Bernard Chaverondier, Wolfgang Engelhardt, Juan R. González-Álvarez, Bill Hobba, Igor Khavkine, Mike Mowbray, Arnold Neumaier and Dan Solomon. All these contacts and exchanges of ideas have formed my understanding of relativistic quantum physics and, ultimately, led to the writing of this book. However, this does not at all mean that the mentioned researchers share or approve my views. For all the misconceptions and errors contained in this book, the author bears full responsibility.

6 See Appendix in [39].

Introduction

As a result, it was almost three o'clock in the morning before the final result of my computations lay before me. The energy principle held for all the terms, and I could no longer doubt the mathematical consistency and coherence of the kind of quantum mechanics to which my calculations pointed. At first, I was deeply alarmed. I had the feeling that, through the surface of atomic phenomena, I was looking at a strangely beautiful interior, and felt almost giddy at the thought that I now had to probe this wealth of mathematical structures nature had so generously spread out before me.
Werner Heisenberg

In this Introduction, we will try to formulate more precisely what is the goal of theoretical physics, what are the fundamental concepts of this science and the relationship between them. Some of our statements may look self-evident or even trivial. However, it seems important to us to spell out these definitions and clarify our positions here and now, in order to avoid misunderstandings in further parts of the book.

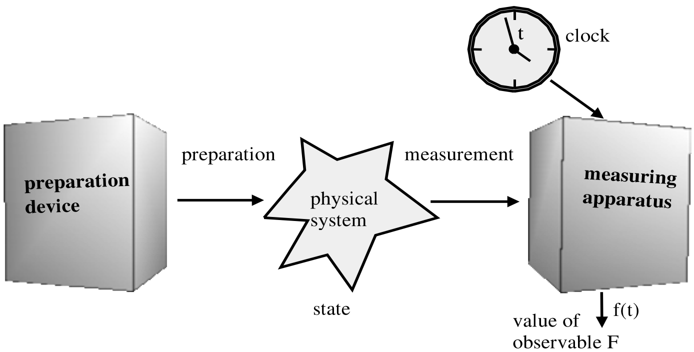

Figure 1: Schematic representation of the preparation/measurement act.

We get all information about the physical world through results of *measurements*, and the fundamental goal of theoretical physics is to describe and predict these results. Any act of measurement requires the presence of at least three objects (see Figure 1): the *preparation device*, the *physical system* and the *measuring apparatus*. The preparation device arranges the physical system in a specific *state*. This state has certain attributes or properties. If the state's attribute can be associated with a numerical value, it will be called a physical quantity or *observable* F. Observables are measured by bringing the system into contact with the measuring apparatus. The result of the measurement is a numerical value of the observable, i. e., a real number f. We assume that each measurement of the observable F always produces *some* result f, i. e., the measuring apparatus never misfires.

This is just a short list of important concepts. Let us now dwell on each of them in more detail.

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