COMPUTATIONAL MODELS IN ARCHITECTURE

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COMPUTATIONAL MODELS IN ARCHITECTURE — TOWARDS COMMUNICATION IN CAAD. SPECTRAL CHARACTERISATION AND MODELLING WITH CONJUGATE SYMBOLIC DOMAINS

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ABSTRACT

This work deals with computational models in architecture, with the ambition of accomplishing three objectives:

1 To position the established computational models in architecture within the broader context of mathematical and computational modelling.
2 To challenge computational models in architecture with contemporary modelling approaches, in which computation is regarded from the perspective of communication between different domains of a problem.
3 To show how within the paradigm of communication, it is possible to computationally address architectural questions that cannot be adequately addressed within the current computational paradigm.

The first part of the work begins in the 19th century, delves into the body of thinking from which computation emerged and traces two general attitudes towards mathematical modelling, which will each eventually lead to different interpretations of computation. The first one, described as the *logicist tradition*, saw the potential of formal, mechanised reasoning in the possibility of constructing the absolute foundation of mathematics, its means of explanation and proof. The second one, the *algebraist tradition*, regarded formalisation within a larger scope of model-theoretic procedures, characterised by creatively applying abstraction towards a certain goal. The second attitude proved to be a fertile ground for the redefinition of both mathematics and science, thus paving a way for contemporary physics and information technology. On the basis of the two traditions, this dissertation identified a discrepancy between the computational models in architecture, following the first tradition, and those commonly used in information technology, following the second.

The Internet revolution, initiated by the development of search engines and social media, is recognised as indicative of the changing role of computers, from "computing machinery" towards the generic infrastructure for communication. In this respect, three contemporary models of communication, proponents of the algebraic tradition, are presented in detail in the second part of the work. As a result, the *self-organizing model* is introduced as the concrete implementation of the ideas appropriated from communication models.

In the last part of the work, the self-organizing model is applied to the problem of similarity between spaces, on the basis of their architectural representation. By applying partition and generalisation procedures of the self-organizing model to a large number of floor plan images,

a finite collection of elementary geometric expressions was extracted, and a symbol attached to each instance. This collection of symbols is regarded as the alphabet, by means of which any plan created by the same conventions can be described as the writing of that alphabet. Finally, each floor plan is represented as a chain of probabilities, based upon its individual alphabetic expression of a written language, and its values used to compute similarities between plans.

PREFACE

Some might find this doctoral thesis unconventionally written. Instead of circumscribing its scope and concentrating its efforts on accomplishing a single objective within that scope, it engages with an unusually extensive body of knowledge with the aim of providing additional angles to its principal research domain: computational models in architecture. This body of knowledge involves early analytic philosophy, computability and probability theory, formal logic, quantum physics, abstract algebra, computer-aided design, computer graphics, glossematics, machine learning and architecture. However, the reason for such a comprehensive approach and perhaps radical gesture is not to claim any expertise nor mastery over the aforementioned fields of knowledge. To the contrary, it is a matter of methodology, aiming to operate in a more architectural manner, without losing the necessary rigour and consistency required of an academic work. An architect's effort towards creating a masterful work, whether it is a building or a theory, always involves the integration of a wide variety of aspects laying outside of his/her own area of expertise. I see this apparent difficulty as a potential to enrich my work, and as a source of inspiration towards finding new, unexplored research perspectives. One more reason in favour of such approach can be justified by the very theories cited within this work, especially the concept of communication. To communicate with someone or something involves a responsive spectrum of frequencies on both sides, and tuning oneself to become sensitive to the potential resonances. The wider and richer this spectrum is, the more meaningful communication becomes. In this sense, the aim of this work is to make the spectrum of the research as resonant as possible, hoping to establish a more satisfying communication with the field of computational models in architecture, as well as with the reader.

Nikola Marinčić, Spring 2019

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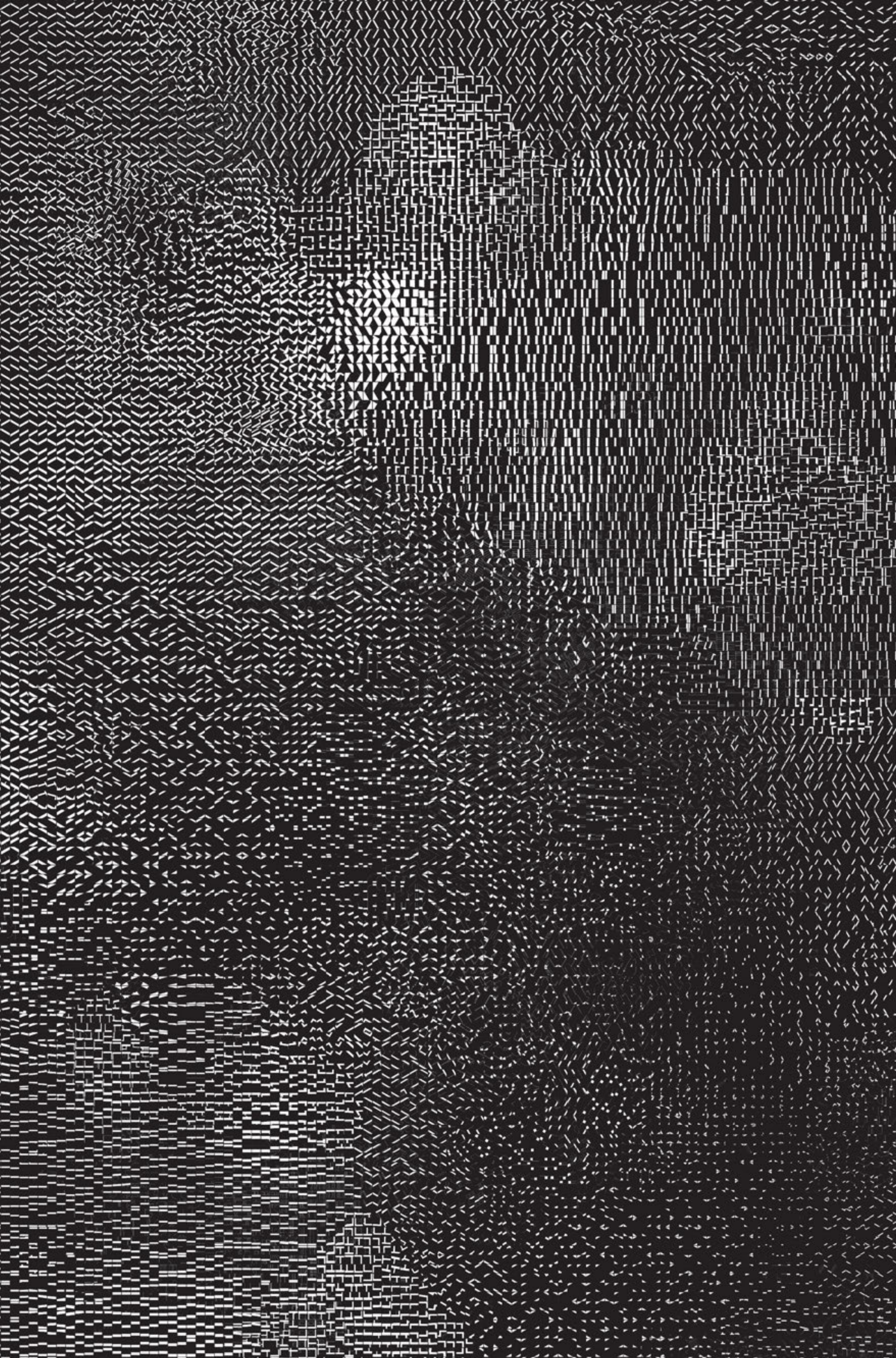
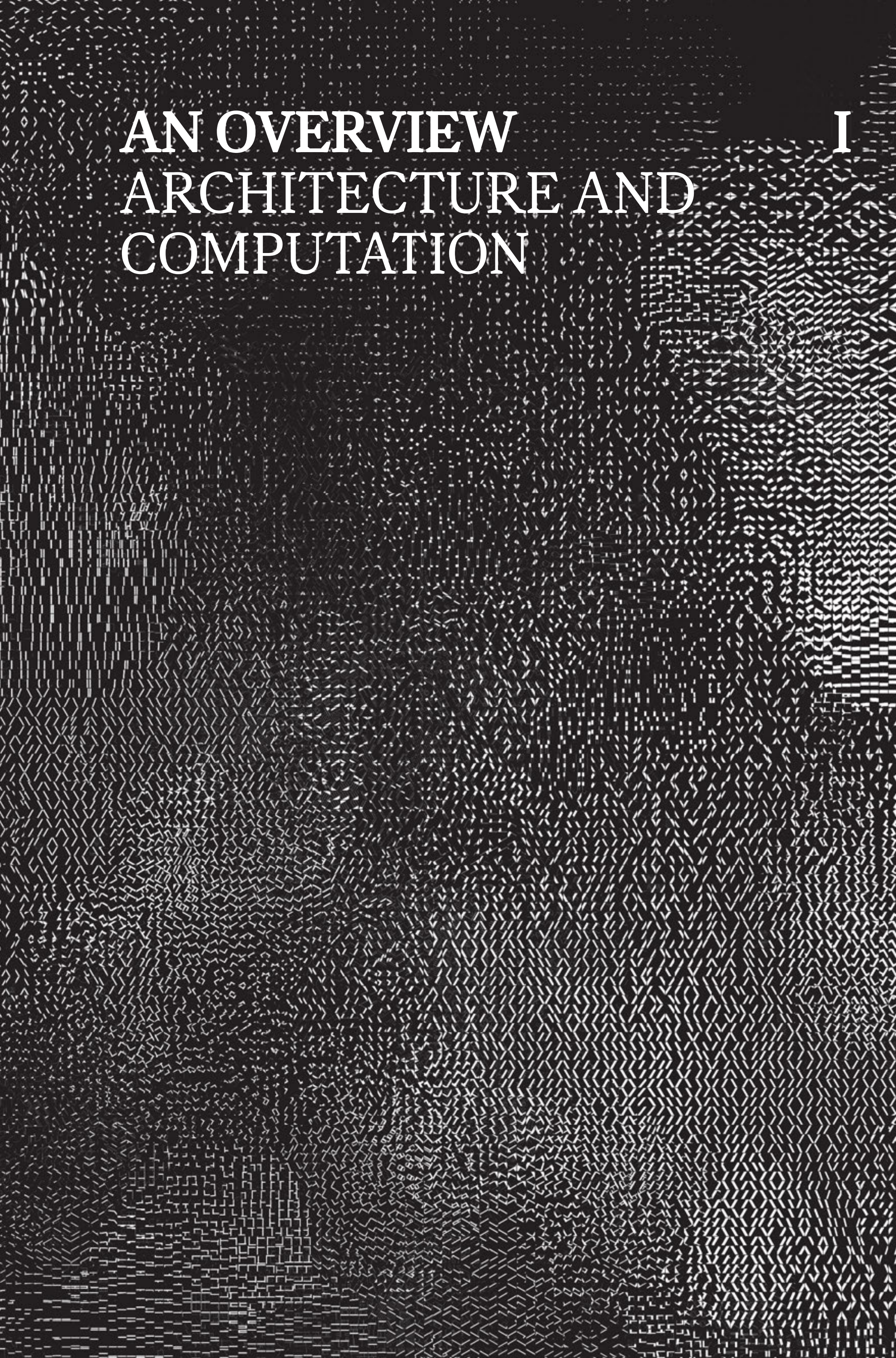
Nikola Marinčić, Spring 2019



AN OVERVIEW I

ARCHITECTURE AND COMPUTATION



> Architecture and information technology… two species similar in kind, neither of them being in the least disciplinal: both affect everything, both are arts of gathering things. The one, 2,500 years old and dignified, and the other, just fifty years of age and impatient.
>
> L. Hovestadt, *"Cultivating the Generic"* (2014: 9)

What today seems to be a passionate love affair between architecture and information technology, is in fact quite a delicate relationship, full of misunderstandings. The first "universal machines," built in the 1940s, emerged as a side effect of the resolution of the 19th century attempt to ground all formalized, mathematical knowledge in logic.[1] Soon, computation was seen essentially as the mechanised treatment of logic.[2] Nevertheless, computers quickly got the attention of almost every field of human endeavour, including architecture.[3] With a certain amount of scepticism, acquired in the long tradition built upon mastership, architecture did not embrace its new potential "partner" very easily. Early researchers saw a lot of promise in computation, but for the large majority of practitioners, it seemed to be in poor taste[4] to simply embrace "logic" as a means to mechanise their articulations with the promise of greater efficiency and formal clarity. However, with the expansion of personal computers and intuitive computer-aided design software, the resistance became futile. An architecture was born out of generic drafting and modelling solutions, which employed computation to mimic the established modes of design.[5] While information technology started exploring new ideas, architectural research remained on the path of the "logicist" tradition.

Today we live in a different world. Computers are omnipresent in our existence, and are no longer about logic. As the old identities slowly dissolve, new ideas are emerging on what computers are all about. These new ideas come from a higher level of abstraction and offer new unexpected vistas.[6] In this chapter, I will give an account of both old and new, with a hope that architecture might just find a very good partner in information technology, and hopefully reinvent itself in the digital.

1 Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem." Turing's seminal paper on computability comes as an answer to Hilbert's decision problem (Entscheidungsproblem) but it starts with providing a mechanically constructed arithmetic of "computable" numbers.

2 "Kurt Gödel has reduced mathematical logic to computation theory by showing that the fundamental notions of logic … are essentially recursive. Recursive functions are those functions which can be computed on Turing machines, and so mathematical logic may be treated from the point of view of automata." Burks, editor's introduction to *Theory of Self-Reproducing Automata*, 25.

3 Mitchell, foreword to *Architecture's New Media*, xi.

4 "Computational methods to support the synthesis of design solutions have fascinated architectural researchers and horrified the practitioners." Kalay, *Architecture's New Media*, 237.

5 Kalay, 181.

6 See: Hovestadt, "Elements of Digital Architecture," 28–116.

NECESSITY OR CONTINGENCY?

The story of symbolic computation emerged out of a peculiar state of affairs that started in the 19th century and got its epilogue in the first half of the 20th century. Before that time, mathematics appeared to be intimately linked to our physical reality, a phenomenon that Erich Reck described with the metaphor of an umbilical cord.[7] Geometry was safely grounded in Euclid's axiomatic method, dating from around 300 BC. This method consisted of three parts: *Axioms*, or *postulates* are statements which are accepted without proof, and act as foundations of a theory; *theorems* are statements that are derived from the axioms and act as a superstructure (of knowledge) built upon the foundations; *logic* is a formal apparatus used to deduce theorems from axioms. Logic was established in antiquity and can be traced back to Plato and Aristotle. Aristotelian logic introduced three laws of reasoning in the natural language: *laws of identity*, *contradiction* and the *excluded middle.*[8] The interesting thing about logic was that it preserved the truthfulness of the statements it derived from axioms. It was believed that if the axioms were true, *everything* that was logically deducible from the axioms necessarily needed to be true as well. In fact, geometry and logic were so stable that their link to physical reality was not questioned for thousands of years.[9]

It was not recognised for a long time that the truthfulness of axioms of logic and geometry was in fact accepted on the basis of intuition, which could only confirm that such statements are in fact self-evident.[10] An example of such evident truths were Euclid's four postulates of planar geometry, which seemed to conceptualise our experience of space:

1 Let it have been postulated to draw a straight-line from any point to any point.
2 And to produce a finite straight-line continuously in a straight-line.
3 And to draw a circle with any centre and radius.
4 And that all right-angles are equal to one another.[11]

Euclid's fifth postulate about parallel lines in two-dimensional geometry seems to be of a different kind than the previous four:

7 "With this first conception, geometry is firmly attached to physical reality—the umbilical cord between them is still in place." Reck, "Frege, Natural Numbers, and Arithmetic's Umbilical Cord," 431.

8 *Encyclopædia Britannica*, s.v. "Laws of thought," accessed September 1, 2017, https://www.britannica.com/topic/laws-of-thought.

9 "As late as 1787, the German philosopher Immanuel Kant was able to say that since Aristotle formal logic 'has not been able to advance a single step, and is to all appearances a closed and completed body of doctrine.'" Nagel and Newman, *Gödel's Proof*, 30.

10 Burge, "Frege on Knowing the Third Realm," 1.

11 Euclid, *Elements of Geometry*, 7.

5 And that if a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself whose sum is) less than two right-angles, then the two (other) straight-lines, being produced to infinity, meet on that side (of the original straight-line) that the (sum of the internal angles) is less than two right-angles (and do not meet on the other side).[12]

This postulate is logically equivalent to the assumption that only one parallel can be drawn through a point outside a given line.[13] The fifth postulate introduced a great deal of problems to mathematicians, as it is neither self-evident, nor can be proved within Euclid's axiomatic system.[14] Nevertheless, it somehow appears to be a correct statement. Such incoherencies were something that science and mathematics of the 19th century were determined to eradicate.

Unlike other branches of mathematics, geometry was considered to be the most stable due to its axiomatic method. It seemed natural to ask whether such a secure axiomatic system could also be established elsewhere. Soon, many branches of mathematics were supplied with "what appeared to be adequate sets of axioms."[15] It was of the utmost importance to establish an adequate axiomatic system of arithmetic, as it would securely ground other branches of mathematics on top of it.[16] In an attempt to use algebra to ground infinitesimal calculus, Cantor, Cauchy, Weierstrass, Dedekind, and others, showed how different notions in analysis could be defined in arithmetical terms.[17] The promise of axiomatisation was great: For each area of inquiry, having such a set of axioms would yield endless amounts of true propositions.

In the mid 19th century, the work of Lobachevsky, Bolyai, Gauss and Riemann[18] began to challenge Euclid's axiomatic system. In 1829, Lobachevsky developed a "geometry" by appropriating the first four axioms of Euclid, asserting that in his geometry the famous fifth

12 Euclid, 7.
13 Nagel and Newman, *Gödel's Proof*, 6.
14 "The chief reason for this alleged lack of self-evidence seems to have been the fact that the parallel axiom makes an assertion about infinitely remote regions of space. Euclid defines parallel lines as straight lines in a plane that, "being produced indefinitely in both directions," do not meet. Accordingly, to say that two lines are parallel is to make the claim that the two lines will not meet even 'at infinity'." Nagel and Newman, 6.
15 Nagel and Newman, 3.
16 Nagel and Newman, 3.
17 For example: "instead of accepting the imaginary number '–1' as a somewhat mysterious "entity," it came to be defined as an ordered pair of integers (0, 1) upon which certain operations of "addition" and "multiplication" are performed. Similarly, the irrational number $\sqrt{2}$ was defined as a certain class of rational numbers—namely, the class of rationals whose square is less than 2." Nagel and Newman, *Gödel's Proof*, 32. See also: Gauthier, *Towards an Arithmetical Logic*, 1.
18 "However, the geometric starting point of Riemann was not the non-Euclidean geometry, of which Riemann apparently had not even taken note, but rather the theory of surfaces developed by Carl Friedrich Gauss." Jost, historical introduction to *On the Hypotheses Which Lie at the Bases of Geometry*, 26.

postulate was not a true statement. This will become known as Bolyai–Lobachevskian geometry. Compared to Euclidean geometry, which was considered to mirror physical reality[19], the hyperbolic geometry of Lobachevsky and Bolyai was radically different. This geometry was able to describe a world that could not be observed empirically, while at the same time remaining a perfectly valid mathematical construction. Finally, in 1868, Beltrami demonstrated the independence of the fifth postulate from the other axioms. Implications of these events shook the very idea of mathematical foundations. If the validity of mathematical statements could not be guaranteed by the *truthfulness* of the axioms, as they need not be self-evident or mirror reality anymore, what remains of the ideas of grounding and validation? Moreover, if mathematics is not about the truths of our world, then what it is about? Gradually, it became clear that the position of necessity within mathematics was to be shifted from the truthfulness of its axioms to the validity of the inferences it employed.[20] Mathematics became abstract and stripped of meaning, as illustrated by the famous quote from Russell:

> ... mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.[21]

What replaced the method of validating a system of premises on the basis on its truthfulness, was a new idea of internal coherence, known as *consistency*. If an axiomatic system was to be consistent, it needed to guarantee that no mutually contradictory theorems can be deduced from the postulates.[22] With that requirement, an important question begged to be asked: Are even the axioms of Euclid's system consistent? There was no single approach to the idea of creating a consistent system, and the interest in this question by two equally rigorous but ideologically quite distinct schools of thought warrants attention. The approach of the first group of mathematicians, including George Boole, Richard Dedekind and David Hilbert, among others, can be characterised as an algebraic approach to the idea of consistency. On the other side, Gottlob Frege, Bertrand Russell and their school of thought established an approach based on formal logic.

19 "Against Leibniz and Wolff, Kant thus emphasises and elaborates the axiomatic nature of geometry, i.e., that geometry has real axioms and that the propositions of geometry cannot be obtained analytically from definitions." Jost, 28.

20 "We repeat that the sole question confronting the pure mathematician (as distinct from the scientist who employs mathematics in investigating a special subject matter) is not whether the postulates he assumes or the conclusions he deduces from them are true, but whether the alleged conclusions are in fact the necessary logical consequences of the initial assumptions." Nagel and Newman, *Gödel's Proof*, 8.

21 Russell, *Mysticism and Logic*, 58.

22 Nagel and Newman, *Gödel's Proof*, 10.

ALGEBRAIST TRADITION The 'algebraist' approach heavily relied on abstraction as the operative means to create coherent but contingent frameworks that did not offer a unifying consensual definition of the basis. The objectivity which they sought to establish within algebra was not something they considered as already given, but rather something that needed to be produced.

George Boole was the first to revolutionise the study of logic after Aristotle. In his 1847 book *The mathematical analysis of Logic*, he established the study of logic on a purely algebraic basis. His algebra of logic provided a precise notation "for handling more general and more varied types of deduction than were covered by traditional logical principles."[23] In 1854, he published his second monograph on algebraic logic, known as *An Investigation of the Laws of Thought*. The most important invention in his work was the equational treatment of logical statements, which allowed him to assess the validity of logical problems, and to extend their scope. In his book, he demonstrated how to transform any logical problem into an operative algebraic equation. By solving the algebraic equation, the logical problem was able to be resolved.[24]

One of the most misunderstood algebraists of the time was the mathematician Richard Dedekind.[25] His approach to the problem of mathematical foundations was to arithmetise mathematics, but without appealing to numbers and the operations on them as naturally given. For Dedekind, natural numbers were a free creation of the human mind and abstraction was a tool to think with. In his essay "On Continuity and Irrational Numbers" (1872), he attempted to rigorously define the notion of a continuous magnitude, which at the time rested upon geometrical intuitions.[26] His method, known today as the *Dedekind cut*, constructed irrational and real numbers by freeing them from any content.[27] Dedekind considered the application of ordinal numbers as central, which allowed him to identify numbers structurally. He defined the cut as a separation which possesses one property, namely that it separates the domain

23 Nagel and Newman, 31.
24 Boole, *An Investigation of the Laws of Thought*, 24–38.
25 "… great philosophers, such as Cantor and Dedekind, are treated as philosophical naïfs, however creative, whose work provides, at best, fodder for philosophical chewing. Not only have we inherited from Frege a poor regard for his contemporaries, but, taking the critical parts of his *Grundlagen* as a model, we in the Anglo-American tradition of analytic philosophy have inherited a poor vision of what philosophy is." Tait, "Frege Versus Cantor and Dedekind: on the Concept of Number," 215.
26 "The statement is so frequently made that the differential calculus deals with continuous magnitude, and yet an explanation of this continuity is nowhere given; even the most rigorous expositions of the differential calculus do not base their proofs upon continuity but, with more or less consciousness of the fact, they either appeal to geometric notions or those suggested by geometry, or depend upon theorems which are never established in a purely arithmetic manner." Dedekind, *Essays on the Theory of Numbers*, 2.
27 Tait, "Frege Versus Cantor and Dedekind: on the Concept of Number," 222.

of rational numbers into two classes A_1 and A_2, where every number a_1 belonging to A_1, is smaller than every number a_2 from A_2.[28]

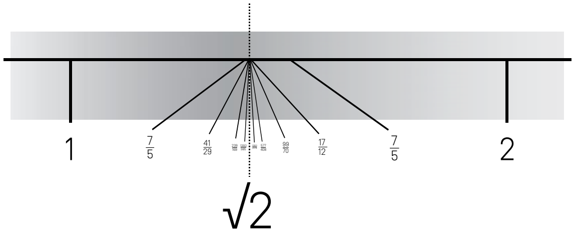


FIG. 1
Dedekind cut
(Hyacinth, 2015)

His construction of natural numbers goes "beyond logic because it appeals to entities which, although created by the intellect, are nevertheless objectively available to it."[29] In the chapter "Architectonics of Communication: How Different Natures Communicate," we will investigate the mathematical framework of category theory in the light of Dedekind's legacy of free creation of numbers, and his attention to structural properties.

David Hilbert's early work was greatly inspired by the advances in the axiomatic treatment of geometry. In *The Foundations of Geometry* (1899)[30], Hilbert devised a set of twenty axioms as a foundation of Euclidean geometry.[31] Unlike Euclid's system, Hilbert's axioms are not about the physical space, but "rather, they are taken to form the definition or characterisation of a certain abstract structure."[32] In other words, Hilbert's axioms are not self-evident truths, but contingent truths, which employ algebra to construct their consistency.[33] Since the algebraic characterisation cannot be "accommodated within any one ideal and elemental order,"[34] Hilbert provided the contingent basis as the source of consistency.[35] Some of the approaches to establish consistency required an infinite number of elements; others simply shifted

28 Dedekind, *Essays on the Theory of Numbers*, 12–13.
29 Potter, *Reason's Nearest Kin*, 282.
30 The original title is *Grundlagen der Geometrie*.
31 Hilbert, *The Foundations of Geometry*, 2–16.
32 Reck, "Frege, Natural Numbers, and Arithmetic's Umbilical Cord," 431.
33 "The geometric statement that two distinct points uniquely determine a straight line is then transformed into the algebraic truth that two distinct pairs of numbers uniquely determine a linear relation; the geometric theorem that a straight line intersects a circle in at most two points, into the algebraic theorem that a pair of simultaneous equations in two unknowns (one of which is linear and the other quadratic of a certain type) determine at most two pairs of real numbers." Nagel and Newman, *Gödel's Proof*, 15.
34 Bühlmann, "Continuing the Dedekind Legacy Today," 6.
35 Riemann's "...idea is that if the metric properties of the space do not necessarily follow from its structure, then the space can carry several possible metrics, and the mathematician then can specify any such hypothetical relations and examine the resulting structures and distinguish them with regard to their characteristics. Hilbert will then raise this as the axiomatic method to a systematic program." Jost, presentation of the text *On the Hypotheses Which Lie at the Bases of Geometry* 46.

the problem of consistency of one system, by placing the responsibility on another system used as its base. Hilbert found these approaches unsatisfactory. In the next twenty years, he became obsessed with the idea of proving the *absolute consistency* of an axiomatic system, which led to the development known as *Hilbert's program.*[36]

LOGICIST TRADITION The 'logicist' approach to consistency emerged into a dominant paradigm whose followers appropriated computation as a child of their own tradition. Its proponents wished to encapsulate an ultimate objectivity within a system of foundations, upon which the whole of mathematics could rest. The implementation of this idea required grounding all of mathematics in logic. The objective was to construct an ideal, foolproof reasoning apparatus on the logical basis, which could, ideally, (in) validate any logical statement.

The most prominent member of the logicist party was Gottlob Frege. If Boole's idea was to ground logic within mathematics by means of algebra, Frege's idea was quite the opposite. He wished to ground the whole of mathematics in arithmetic by means of the powerful deductive logic. Frege claimed that all the axioms of arithmetic could be "deduced from a small number of basic propositions certifiable as purely logical truths."[37] In *Begriffsschrift* (1879), Frege invented quantification theory, which was a first step towards a precise notion of purely logical deduction.[38] The "conceptual notation" he defined allowed him to represent mathematical statements involving, for example, an infinite number of prime numbers.[39] In 1884, in *The Foundations of Arithmetic*[40], Frege introduced his own number theory, made to emulate formal logic.[41] He wished to show that arithmetic could be reduced to logical fundamentals, without any basis in intuition. Moreover, he regarded arithmetic as a completely objective "realm." His central claim in *The Foundations* was that:

> In arithmetic, we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to our reason and, as its nearest kin, utterly transparent to it.[42]

Today, we can more easily recognise the alarming implications of such a statement. By regarding mathematics as a transparent, objective

36 Nagel and Newman, *Gödel's Proof*, 25.
37 Nagel and Newman, 32.
38 Tait, "Frege Versus Cantor and Dedekind: on the Concept of Number," 213.
39 Tait, 217. However, he did so by utilising cardinal numbers, which was the misunderstanding of the notion of infinity introduced by Cantor.
40 Originally published *as Die Grundlagen der Arithmetik*.
41 Gauthier reformulates Frege's question into: "How far can we go into arithmetic with deductive logic alone?" Gauthier, *Towards an Arithmetical Logic*, 1.
42 Frege, *The Foundations of Arithmetic*, §105:115.

reality,[43] which was simply to be accessed by reason, Frege diminished the role of human creativity and invention. His statement completely rejects the possibility of the creative abstraction that Dedekind was fighting for.[44] Despite all this, Frege is considered to be the founder and the "hero" of abstraction until this day.[45]

Very soon, a complete new set of problems had emerged out of Frege's program. In 1901, Bertrand Russell showed that Frege's logical axioms were inconsistent.[46] He discovered that Frege's approach could lead to the construction of paradoxical sets, which was named *Russell's paradox.*[47] The source of Frege's inconsistencies lied in its self-referentiality. The logicists sought to remain within the same paradigm, while avoiding self-reference at all costs. In the period from 1910 to 1913, Russell and Whitehead wrote *Principia Mathematica,* a cornerstone of the logicist paradigm. It was a three-volume work of mathematical foundations that attempted to establish a set of axioms and rules powerful enough to prove all mathematical truths. It was meticulously designed to keep the inconsistencies out "in a most staunch and watertight manner."[48] *Principia Mathematica* also appeared to be the final solution for the problem of consistency, as it reduced the problem of consistency of arithmetic to the problem of the consistency of formal logic itself.[49]

This was the moment where Russell and Whitehead's work became closely intertwined with Hilbert's search for absolute consistency, which consisted in the complete formalisation of a deductive system by "draining" it from any meaning, as described by Nagel and Newman:

> The postulates and theorems of a completely formalised system are "strings" (or finitely long sequences) of meaningless marks, constructed according to rules for combining the elementary signs of the system into larger wholes. Moreover, when a system has been completely formalised, the derivation of theorems from postulates is nothing more than the transformation (pursuant to rule) of one set of such "strings" into another set of strings.[50]

43 Burge, "Frege on Knowing the Third Realm," 2. He called it "The third realm."
44 Bühlmann, "Continuing the Dedekind Legacy Today," 8.
45 "However, more important to me in this paper than the question of Frege's own importance in philosophy is the tendency in the literature on philosophy to contrast the superior clarity of thought and powers of conceptual analysis that Frege brought to bear on the foundations of arithmetic, especially in the Grundlagen, with the conceptual confusion of his predecessors and contemporaries on this topic." Tait, "Frege Versus Cantor and Dedekind: on the Concept of Number," 215.
46 Irvine and Deutsch, "Russell's Paradox."
47 Irvine and Deutsch, "Russell's Paradox." famous "set of all sets that are not members of themselves."
48 Hofstadter, preface to *Gödel, Escher, Bach*, 4.
49 Nagel and Newman, *Gödel's Proof*, 33.
50 Nagel and Newman, 20.

The defining trait of formal systems lies in their simplicity. They include a limited number of signs, a grammar which defines how to create well-formed strings, a set of strings taken as axioms, and a set of transformation rules.[51] This introduces two levels from which a formal system can be considered: The first, "lower" level consists of the "meaningless marks" that are produced mechanically; the second accommodates high-level reasoning about the processes of the lower level. Hilbert defined the higher level as a *meta-language*. His goal was to a find method that could prove the absolute consistency of a system. He believed that the solution lied on the "lower" level and was interested in demonstrating the "impossibility of deriving certain contradictory formulas"[52] within it. In other words, Hilbert's hope was that a purely formal language could be used to prove its own consistency.

The main achievement of *Principia Mathematica*, was that it provided "a remarkably comprehensive system of notation, with the help of which all statements of pure mathematics (and of arithmetic in particular) can be codified in a standard manner."[53] The book's notation and deductive system presented themselves to Hilbert as a perfect medium for establishing an absolute proof of consistency. His work seemed to be on the right track until 1931, when Gödel's theorems proved that neither *Principia*, nor any other system of that kind, could ever achieve this goal.

LINGUISTIC TURN

The philosophy of the early 20th century experienced a crisis similar to the one of mathematics. The accounts that held philosophy as the fundamental discipline responsible for the questions of foundations and knowledge, started to lose appeal in the light of the clarity and the precision demonstrated by modern logic. An idea began to emerge that philosophical facts do not exist *per se*, but that they are above all language articulations. Accordingly, philosophy should have been dealing with clarification of thoughts on a logical basis by analysing the logical form of propositions.[54] As a consequence, the attention of philosophy turned to language as an operative medium for thought, and to grammar as an apparatus for coherent thinking. Within the relation between philosophy and language, another current emerged that was interested in the relation between grammar and logic. This interest introduced two schools of thought: The first one was established by the Austrian philosopher Ludwig Wittgenstein; the second by Rudolph Carnap and the Vienna Circle.[55]

51 Hofstadter, *Gödel, Escher, Bach*, 35.
52 Nagel and Newman, *Gödel's Proof*, 27.
53 Nagel and Newman, 33.
54 Wittgenstein, *Tractatus Logico-Philosophicus*, 45 (4.12).
55 Potter, *Reason's Nearest Kin*, 18.

Wittgenstein, an associate of Russell and a young admirer of Frege's work, is considered to be one of the progenitors of the linguistic turn. In 1921, he wrote *Tractatus Logico-Philosophicus*, which conveyed the idea that philosophical problems arise from an inconsistent nature of the language that is used to construct philosophical statements.[56] In the preface of the *Tractatus*, he summed up his argument as the following:

> What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.[57]

Tractatus was the first philosophical work putting the language at the centre of its inquiry, boldly stating that "the limits of my language mean the limits of my world."[58] This non-intuitive position was in strong contrast to most of the Western philosophical tradition. To free philosophy from incoherence, Wittgenstein required an ideal language for philosophical analysis, as "ordinary" language was full of ambiguities. Wittgenstein's conception of a language was not simply an instrument of logic. If this were the case, the argumentation would need to be set up so that it leads to an argument or a proof. It was a philosophical grammar, designed to draw a line separating valid philosophical language from nonsense.[59] By creating a philosophical system as an application of his rigorous grammar consisting of atomic facts, propositions and operators, Wittgenstein believed to have eliminated all philosophical problems. However, in all of his self-proclaimed success, he also realised "how little has been done when these problems have been solved."[60]

Decisive for the linguistic turn in the humanities were the works of yet another tradition, namely the structuralism of Ferdinand de Saussure and the ensuing movement of poststructuralism.[61] Saussure's general complaint was directed at the lack of systematicity in the study of language.[62] In his university lectures, collected and published only later by his students in *Course in General Linguistics* (1916),[63] Saussure referred to a number of approaches for studying language, finding them all inadequate.[64] For him, grammar was detached from language and too dependent upon (and limited by) logic, having the sole purpose of distinguishing between correct

56 Wittgenstein, preface to *Tractatus Logico-Philosophicus*, 23.
57 Wittgenstein, 23.
58 Wittgenstein, 74.
59 Wittgenstein, 23.
60 Wittgenstein, 24.
61 Wikipedia, s.v. "Linguistic turn," last modified March 24, 2017, 15:55, https://en.wikipedia.org/wiki/Linguistic_turn.
62 Saussure, *Course in General Linguistics*, 3–4.
63 Originally published as Saussure, Ferdinand de. *Cours de linguistique générale*. Publ. par Charles Bailly et Albert Sechehaye avec la collaboration de Albert Riedlinger. Lausanne: Pavot, 1916.
64 "At the same time scholars realised how erroneous and insufficient were the notions of philology and comparative philology. Still, in spite of the services that they rendered, the neogrammarians did not illuminate the whole question, and the fundamental problems of general linguistics still await solution." Saussure, 5.

and incorrect forms. He found that philology was not about language at all, but about the interpretation of texts "as a means to literary and historical insight."[65] He recognised some potential in comparative philology, and in its task of finding similarities and differences between languages.[66] Saussure's vision of linguistics was that it should be able:

- to describe and trace the history of all observable languages, which amounts to tracing the history of families of languages and reconstructing as far as possible the mother language of each family;
- to determine the forces that are permanently and universally at work in all languages and to deduce the general laws to which all specific historical phenomena can be reduced; and
- to delimit and define itself.[67]

For Saussure, language was a "system of signs that express ideas"[68] and was part of a larger whole, of a "science that studies the life of signs within society,"[69] which he termed *semiology.*[70] His concept of a *sign* challenged the traditional view, which considered words as mere labels attached to concepts. He defined sign as an entity that united a concept of a thing, the *signified*, and its sound image, the *signifier*. Since there cannot be a concept without it being named, the signified and the signifier necessarily exist as a pair. For Saussure, language was about symbolic manipulation, thus the "real things" did not play any role in the constitution of a sign. Another crucial view that he held was that signs possess differential, not natural, identity. In other words, a sign is being a sign only by the virtue of not being any other sign:

> … the concepts are purely differential and defined! Not by their positive content but negatively by their relations with the other terms of the system. Their most precise characteristic is in being what the others are not.[71]

MECHANISATION OF ARTICULATION

The advent of digital computers was rapid, overwhelming, and its development is still underway. There is no room here to mention every important contributor. For the purposes of this dissertation, the focus will be on the figures who have established the main computational paradigms and on those whose work has had the biggest influence on computer-aided architectural design.

65 Saussure, 1.
66 Saussure, 4–5.
67 Saussure, 6.
68 Saussure, 16.
69 Saussure, 16.
70 Saussure, 16.
71 Saussure, 117.

RECURSION AND COMPUTABILITY In the 1920's, Hilbert's program crystallised the main expectations of formal systems for the purpose of axiomatization, most notably ideas of:

- *Consistency*: No mutually contradictory theorems should be deducible from the axioms.
- *Completeness*: Axioms of a deductive system are "complete" if every true statement that can be expressed in the system is formally deducible from the axiom.[72]

At an international conference in 1928, Hilbert introduced a famous challenge that illustrated his hope of the potential of formal systems, what he referred to as the *Entscheidungsproblem*[73] (decision problem). He asked whether an algorithm could be made that takes two inputs: (i) a description of a formal language (for example *Principia Mathematica (PM)*) and (ii) a statement expressed in that language (for example, a theorem of PM), and outputs either true or false, depending whether the statement ii is provable within the formal language i. All that remained to settle the question of foundations once and for all was to solve the problem.

Unfortunately for Hilbert, the complete opposite happened. In his 1931 paper "On formally undecidable propositions of Principia Mathematica and related systems," Austrian logician Kurt Gödel proved that Hilbert's requirements of consistency and completeness could not both be achieved in a formal system. Moreover, he exposed the fundamental limitations of all axiomatic systems, including those of arithmetic and logic.[74] The pinnacle of Gödel's paper are the two theorems known as *Gödel's incompleteness theorems*, which he proved in an ingenious way. The first theorem states that any system of a certain complexity—in which, for example, arithmetic can be developed—is essentially incomplete. In other words, true statements that cannot be derived from the axioms could be expressed in such a system. The second theorem shattered Hilbert's hope of achieving absolute consistency by showing that a formal system alone cannot be used to prove its own consistency. In his proof, Gödel applied the idea of recursive enumerability, and demonstrated how arithmetic, defined by recursive functions, could be made to emulate logic.

72 Nagel and Newman, *Gödel's Proof*, 73.
73 German for "decision problem."
74 "What is more, he (Gödel) proved that it is impossible to establish the internal logical consistency of a very large class of deductive systems— elementary arithmetic, for example—unless one adopts principles of reasoning so complex that their internal consistency is as open to doubt as that of the systems themselves." Nagel and Newman, *Gödel's Proof*, 3.

FIG. 2
A recursive definition from Gödel's famous paper. (Gödel, 1931)

Wir definieren nun nach dem Rekursionsschema (2) eine Funktion $\chi\,(x,\eta)$ folgendermaßen:

$$\chi\,(0,\eta) = 0$$

$$\chi\,(n+1,\eta) = (n+1)\cdot a + \chi\,(n,\eta)\cdot\alpha\,(a)$$

wobei $a = \alpha\,(\alpha\,(\rho\,(0,\eta)))\cdot\alpha\,(\chi\,(n,\eta))$

In 1936 and 1937, only five years after Gödel's paper, Alan Turing and Alonzo Church delivered another blow to Hilbert's hope that the *Entscheidungsproblem* could be solved. Both mathematicians proved that Hilbert's question cannot be positively answered.[75] In order to make Hilbert's notion of the algorithm operative, and its operations explicit, Turing conducted an experiment involving a hypothetical machine which operated with "empty" symbols mechanically. By mechanical means, the machine could automate the operations of finitistic formal systems. Like Gödel, Turing attempted to solve the *Entscheidungsproblem* within arithmetic, and in doing so introduced a novel constitution of arithmetic, utterly different from one based on deductive logic.

Turing introduced his paper with the notion of a *computable number*:

> According to my definition, a number is computable if its decimal can be written down by a machine.[76]
>
> The 'computable' numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means.[77]

The *computing machine* consisted of an infinite tape divided into a number of discrete elements called "squares." Each square could be empty but was also capable of bearing a symbol, for example 0 or 1. The machine could carry out only four actions: *read* the symbol from the square, *write* the symbol to the square, *erase* the symbol from the square, or *move* the tape one step left or right. Like Hilbert's formal system, such a machine was completely described by a finite number of conditions that he defined as "*m-configuration*."[78] Depending upon the symbol that was read from the square, the configuration assigned actions to be taken. For example, one such *m*-configuration c_1 would instruct the machine to write a symbol to the current square, move one step to the left and change its current configuration to c_2. Such simple procedures were to be repeated indefinitely. At any moment the machine was "directly aware" only of one symbol: the "scanned symbol" from the "scanned square." But the tape is what allowed the machine to

75 In other words, it is impossible to decide algorithmically whether statements in a finitistic formal system are true or false according to the description of the formal system.
76 Turing, 230.
77 Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," 230.
78 Turing, 231.

"effectively remember some of the symbols which it had seen (scanned) previously," thus serving as its memory.[79] It was shown later that such a simple mechanical machine could in fact emulate any possible formal system, but was prone to the same limitations discovered by Gödel.[80]

By explicating the machine's capabilities, Turing had effectively reduced the class of real numbers to the class of computable numbers, a class whose sole property was that it could accommodate for a finite mechanical calculation of all of its members. He showed that large classes of numbers were in fact computable, but not all numbers.[81] Those that were computable, were also necessarily *enumerable*[82], which was proved by Gödel. The same holds true for computable sequences. By instrumentalising them, however, Turing turned what appeared to be an inherent limitation into a new perspective:

> It is possible to invent a single machine which can be used to compute any computable sentence.[83]

With this statement, Turing turned the attention from the necessity implied in the limits of mechanically computing numbers, towards the contingency implied in the infinity of possible sequences. He implicitly pointed out that limitations are inherent to any formal system, but not to the creativity of an individual having such a system at his or her disposal. His statement transcended the computable machine into the *universal* or *any-machine.*

DIGITAL COMPUTERS AND ALGORITHMS According to Turing, the first mechanical computers, namely Babbage's difference and analytical engine were invented as early as the beginning of 19th century but failed to surpass the prototypical stage.[84] Inspired by Turing's early work, the first digital computers[85] came to existence in the 1940s. Hungarian-American polymath John von Neumann was one of the pioneers who streamlined the

79 Turing, 231. Or what we would call today—stored program.
80 "In short, it has become quite evident, both to the nominalists like Hilbert and to the intuitionists like Weyl, that the development of a mathematico-logical theory is subject to the same sort of restrictions as those that limit the performance of a computing machine." Wiener, *Cybernetics*, 13.
81 "Computable numbers include all numbers which could naturally be regarded as computable." - large classes of numbers are computable including PI, e, etc. "The computable numbers do not, however, include all definable numbers, and an example is given of a definable number which is not computable." Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," 230.
82 "...able to be counted by one-to-one correspondence with the set of all positive integers." Turing, 230.
83 Turing, 241.
84 Turing, "Computing Machinery and Intelligence," 439.
85 Burks, editor's introduction to *Theory of Self-Reproducing Automata*, 6–10. For example, ENIAC (1943–45) and EDVAC (1945).

design of digital computers[86] and wrote the first successful algorithms.[87] For him, computation was part of a larger umbrella of *automation*–theory that seeks "general principles of organisation, structure, language, information and control."[88] Such theory was meant to explain the processes inherent to natural systems by means of both analogue (natural automata) and digital computers[89] (artificial automata). However, Turing's construction in which arithmetic (thus logic) could be reduced to computation inspired von Neumann to think of them as one and the same thing. He introduced logic at the heart of the theory of automata, often referring to it as a "logical theory of automata."[90] Arthur Burks illustrated this point in his introduction to von Neumann's book *Theory of Self-Reproducing Automata*:

> To conclude, von Neumann thought that the mathematics of automata theory should start with mathematical logic and move toward analysis, probability theory, and thermodynamics. When it is developed, the theory of automata will enable us to understand automata of great complexity, in particular, the human nervous system.[91]

The early work of Claude Shannon has firmly established the technical foundation of digital computers in logic. In his famous 1937 master's thesis named "A Symbolic Analysis of Relay and Switching Circuits," he investigated the correspondence between Boolean algebra and electrical relays, which were the building blocks of electrical components of the time. He advanced the design of electrical switches by proposing that they be implemented as binary switches.[92] The logical basis of electrical switches became the cornerstone for the design of electronic digital computers[93], but its further development to transistors and computer chips was made possible only with development of quantum physics. In 1948, Shannon published his paper "A Mathematical Theory of Communication," which is considered to be the founding work of information theory. In this paper, Shannon defined *entropy* as the quantitative measure of information within the set-theoretical paradigm (which will be discussed and challenged in the part II).

86 ...by separating data from instructions, analogous to the Turing's tape, so that "by changing the program, the same device can perform different tasks." Kalay, *Architecture's New Media*, 28.

87 "He devised algorithms and wrote programs for computations ranging from the calculation of elementary functions to the integration of non-linear partial differential equations and the solutions of games." Burks, editor's introduction to *Theory of Self-Reproducing Automata*, 5.

88 Burks, 21.

89 Burks, 21.

90 Burks, 25.

91 Burks, 28.

92 Shannon, "A Symbolic Analysis of Relay and Switching Circuits," 3–4.

93 "In other words, the structure of the machine is that of a bank of relays, capable each of two conditions, say "on" and "off"; while at each stage the relays assume each a position dictated by the positions of some or all the relays of the bank at a previous stage of operation." Wiener, *Cybernetics*, 119.

German mathematician Norbert Wiener pushed the idea of automation further within an emerging thermodynamic understanding of the world, which, as he wrote, operated on the principles of conservation (and degradation) of energy[94], information and control.[95] Within such a paradigm, the distinct processes of both natural (animal) and artificial (machine) entities could be addressed from the perspective of energy and information exchange. Since the input and the output of each component of the system were necessarily interconnected, each event affected the state of the whole environment.

Wiener used the example of patients suffering from *ataxia*, whose muscles were completely healthy, but their brain was not able to establish control over their actions.[96] His hypothesis was that the brain was not simply an organ that gives orders to other organs, but also a monitoring device, that continuously, and in real time, adjusts its "outputs" according to the "inputs" it receives from the senses. Such a continuously adapting control process he called the *chain of feedback*[97], and named the entire field of "control and communication theory, whether in the machine or in the animal," cybernetics.[98] Accordingly, every system conceived upon the principle of feedback chains, is a *cybernetic system*.

For Wiener, the basis of a feedback chain lies in the anatomy of the brain. He considers *neurons* to be the elements of the human computation system "which are ideally suited to act as relays."[99] If the brain was using computation to control its own feedback chain, then digital computers had potential for controlling any system:

> It has long been clear to me that the modern ultra-rapid computing machine was in principle an ideal central nervous system to an apparatus for automatic control; and that its input and output need not be in the form of numbers or diagrams but might very well be, respectively, the readings of artificial sense organs, such as photoelectric cells or thermometers, and the performance of motors or solenoids.[100]

Wiener did not stop at defining the program for cybernetics, but rather developed it into a mathematical model that could be computationally implemented. He saw an enormous potential for the optimal governance of systems, going as far as proposing its use by psychopathologists for the control of physiological diseases.[101]

94 "The living organism is above all a heat engine, burning glucose or glycogen or starch, fats, and proteins into carbon dioxide, water, and urea. It is the metabolic balance which is the center of attention." Wiener, 41.

95 "the present time is the age of communication and control." Wiener, 39. "...the present age is as truly the age of servomechanisms as the nineteenth century was the age of the steam engine or the eighteenth century the age of the clock." Wiener, 43.

96 Wiener, 8.

97 Wiener, 96.

98 Wiener, 11.

99 Wiener, 120.

100 Wiener, 26.

101 In the chapter: "Cybernetics and Psychopathology" Wiener, 144–154.

Wiener's work has greatly influenced computer science, with his neuron model as a precursor to the neural network perspective to machine learning.[102] However, it is important to remember that cybernetics is primarily a control paradigm that models the world as a closed thermodynamic system. Its constitution, once it is set up, is, in principle, fixed. The social parallel to such a paradigm appears today as a tyrannical form of governance and care should be taken in proposing its application to systems more complex than a thermostat.

COMPUTATION AND LANGUAGE Around forty years after the *Principia Mathematica* and the peak of Hilbert's programme, researchers began to consider natural language from the perspective of formal systems. The most prominent figure in this respect was Noam Chomsky, whose most important work on the topic is *Syntactic Structures (*1957). Chomsky's interest in language was purely formal and pragmatic, focusing on two elementary notions:

- *Syntax*: "the study of the principles and processes by which sentences are constructed in particular languages."[103]
- *Grammar*: "device of some sort" for producing sentences of such a language.[104]

For Chomsky, grammar plays the role of a mechanical "judge," making efficient binary decisions whether a given sentence is correct or not, independent of any semantics. He gave a famous example:

> Sentences (1) and (2) are equally nonsensical, but any speaker of English will recognize that only the former is grammatical.

(1) Colorless green ideas sleep furiously.

(2) Furiously sleep ideas green colorless.[105]

Before presenting his generative model, Chomsky described two models that he considered incapable of dealing with the complexity of grammar: the Markov model, and the phase structure model. The fact that Chomsky dismissed the Markov model, which later became the *de facto* standard for machine translation (as well as search engine technology) well illustrates his misunderstanding of and disregard for mathematics.[106]

Chomsky presented his own generative *transformational model* as the most adequate and "natural" model for addressing linguistic structures. The model consists of three stages where simple transformations of strings are applied in succession. In the first stage, Chomsky applied

102 Wikipedia, s.v. "Cybernetics," last modified August 30, 2017, 17:55, https://en.wikipedia.org/wiki/Cybernetics.

103 Chomsky, *Syntactic Structures*, 11.

104 Chomsky, 11.

105 Chomsky, 15.

106 "I think that we are forced to conclude that grammar is autonomous and independent of meaning, and that probabilistic models give no particular insight into some of the basic problems of syntactic structure." Chomsky, 33.