Martin Strehler

Signalized Flows

Optimizing Traffic Signals and Guideposts and Related Network Flow Problems

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Optimizing Traffic Signals and Guideposts and Related Network Flow Problems

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Abstract

Guideposts and traffic signals are important devices for controlling inner-city traffic and their optimized operation is essential for efficient traffic flow without congestion. In this thesis, we develop a mathematical model for guideposts and traffic signals in the context of network flow theory. Guideposts lead to *confluent flows* where each node in the network may have at most one outgoing flow-carrying arc. The complexity of finding maximum confluent flows is studied and several polynomial time algorithms for special graph classes are developed. For traffic signal optimization, a *cyclically time-expanded model* is suggested which provides the possibility of the simultaneous optimization of offsets and traffic assignment. Thus, the influence of offsets on travel times can be accounted directly. The potential of the presented approach is demonstrated by simulation of real-world instances.

Zusammenfassung

Vorwegweiser und Lichtsignalanlagen sind wichtige Elemente zur Steuerung innerstädtischen Verkehrs und ihre optimale Nutzung ist von entscheidender Bedeutung für einen staufreien Verkehrsfluss. In dieser Arbeit werden Vorwegweiser und Lichtsignalanlagen mittels der Netzwerkflusstheorie mathematisch modelliert. Vorwegweiser führen dabei zu *konfluenten Flüssen*, bei denen Fluss einen Knoten des Netzwerks nur gebündelt auf einer einzigen Kante verlassen darf. Diese konfluenten Flüsse werden hinsichtlich ihrer Komplexität untersucht und es werden Polynomialzeitalgorithmen für das Finden maximaler Flüsse auf ausgewählten Graphenklassen vorgestellt. Für die Versatzzeitoptimierung von Lichtsignalanlagen wird ein *zyklisch zeitexpandiertes Modell* entwickelt, das die gleichzeitige Optimierung der Verkehrsumlegung ermöglicht. So kann der Einfluss geänderter Versatzzeiten auf die Fahrzeiten direkt berücksichtigt werden. Die Leistungsfähigkeit dieses Ansatzes wird mit Hilfe von Simulationen realistischer Szenarien nachgewiesen.

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Cottbus, November 2011 — Martin Strehler

Contents

Introduction

Urban traffic congestion is increasing day by day. Examples can be found around the world. Growing cities and increasing population doubled the traffic volume in the last two decades in Europe and North America and an even higher rise has to be expected for the urban regions in Asia or South America in the next years. In Germany, the population travels about 1000 billion kilometers every year, and 85 percent of this distance is covered by individual motor car traffic [25, 56]. The city of São Paulo, Brazil, is famous for its record-breaking traffic jams. The 20 million inhabitants own about six million vehicles. On an evening in June 2009, the traffic congestion the city reached a new record of 293 kilometers in total [47]. In August 2010, there was a 60-mile, nine-day traffic jam near Beijing, China, that even made headlines in Europe.

Traffic congestion causes delays which add up to huge costs for society and business. The urban mobility report 2009 [136] states a total loss of 4.2 billion hours and 87.2 billion dollars for the 439 urban areas in the United States in only one year. Wasted fuel of 2.8 billion gallons, noise, and pollution accumulate. A huge problem has to be solved.

But what can mathematics do to support the quest for stress-free, environment-friendly, and safe traveling? In this thesis we will have a close look at two familiar systems for traffic control–*guideposts* and *traffic signals*. If you do not like traffic jams you are invited to read on and find out how an optimization of guideposts and traffic signals can be used to direct and improve inner-city traffic flow.

Guideposts. Guideposts have been used for a long time. They provide guidance, especially in unfamiliar regions. With increasing mobility, we cannot imagine traveling without guideposts and we can find them everywhere. Even modern GPS-based satellite navigation systems can be seen as small, virtual guideposts inside our cars. Everything seems clear–just follow the guideposts!

But we missed an important question. Where should we install these guideposts? And what consequences arise from our choice?

Assume we want to find a certain point of interest in an unfamiliar network. Fortunately, this network is equipped with guidepost pointing towards our destination wherever a routing decision has to be made. Further, we may assume that these guideposts point in an unique direction, i.e. they exclude all but one road at each intersection. Nothing would be more confusing than two guidepost naming the same destination but pointing in two different directions. Most likely, other traffic participants with the same destination will follow these guideposts, too. If we meet one of them, she will make the same routing decisions just like us. Thus, we will travel on the same route until we reach our destination. Consequently, a group of road users starting at the same origin will share the same path, even if there would exist several alternatives. The capacity of this path limits the amount of traffic participants that can reach the common destination. With road users starting all over the network, a bad choice of guideposts may lead to congestion, although the traffic flow in the network is far away from the network's capacity at free route choice.

Traffic signals. With increasing car traffic, traffic signals managing the right of way at intersections became more and more important. However, the *red* traffic light seems to be dominant. But sometimes, we arrive at a traffic signal and it switches to *green* just in time, so that we can go on without stopping. And once in a blue moon[1], we get even four or five green lights in a row.

Such a traffic signal coordination is a difficult task. Of course, coordinating one road in one direction is rather easy, but with traffic in the opposite direction or traffic in a whole street network it becomes considerably harder.

Even more, changing the coordination also means changing travel times. After a while road users will learn about the fastest routes in the network and they will switch to these routes. This new distribution of traffic in the network may completely disturb our fine-tuned coordination.

Obviously, guideposts and traffic signals are important tools for controlling traffic and traffic control is the backbone in the management of traffic flows in our cities. The optimal use of these signals is essential when we are going to resolve traffic congestion. However, it is sometimes not even clear what 'optimal' means in this context. Guideposts are often installed with respect to the shortest distance towards the destination. Their influence on congestion is poorly studied. In contrast, traffic signal coordination has been investigated for a long time and many approaches and models have been proposed. But these models also recommend various definitions of optimality. The two most common objectives are *minimizing the delay/waiting time of vehicles facing red lights* and *minimizing the number of stops*. Furthermore, the majority of the approaches reveal some deficits like unrealistic modeling of inner-city traffic flows or no guarantee for an optimal solution.

Contribution

In this thesis, we tackle traffic congestion with the help of network flow theory from two sides. First, we advance guideposts from a theoretical point of view and introduce *confluent flows*. A flow is called confluent if the flow uses at most one outgoing arc at each node. Unlike previous results we consider heterogeneous arc capacities. We will focus on $\mathcal{NP}$-hardness results for maximum confluent flows, an approximation algorithm for graphs with treewidth bounded by a constant k, and polynomial time-algorithms for special graph classes.

Second, we advance traffic signals. Since this discussion is actually a practical one – most results presented in this part are an outcome of the ADVEST project that emerged

[1] A *blue moon* refers to the third full moon in a season with four full moons. A season with four full moons is very rare, this happens only once every 2 or 3 years.

between BTU Cottbus, TU Berlin, TU Braunschweig, and PTV AG[2] – we start our contribution with a new model for the simultaneous optimization of traffic signal coordination and traffic assignment. This combined approach accounts the feedback between red lights and route choices. We answer the time dependance of traffic signals by a cyclically time-expanded network. This time expansion will also allow capturing several other characteristics of inner-city traffic like platoons of cars and exact arrival times of these platoons at the intersections. Still, viewing inner-city traffic as a periodic process limits the time horizon of the expansion and leads to a compact formulation of the problem as a mixed-integer program. Solving the MIP yields a guarantee or at least bounds for the optimal solution. We investigate our approach with the help of real-world data and state-of-the-art simulation tools.

Outline of the thesis

In Chapter 1 we will fix the notation and terminology and present basic definitions. We assume the reader to be familiar with the basic concepts in graph theory, complexity theory as well as linear programming. However, for later reference and as a short refreshing of knowledge we recall some of the most important facts. For additional information we refer to [2, 66, 100, 122, 147].

In Chapter 2 we will derive the concept for flows in networks with guideposts. For that, we will introduce *confluent flows* and conclude some basic properties, e.g. the underlying tree structure. We will study complexity result for both the *transshipment* and the *maximum flow* variant. We also present *arc-confluent flows* and discuss *cuts* for confluent flows.

Afterwards, we present *polynomial time algorithms* for restricted graph classes, e.g. trees, planar graphs with at most k terminals on the boundary, and graphs without $K_{2,3}$ as a minor in Chapter 3. The relation between confluent flows and trees will lead to a pseudo-polynomial time solution for maximum confluent flows on graphs with *treewidth* bounded by a constant k. We use this result to develop a *fully polynomial time approximation scheme* (FPTAS) for confluent flows on this kind of graphs.

Due to the various approaches for traffic signal optimization we start with a short survey on this topic in Chapter 4. We will use this survey to make the reader familiar with concepts in traffic engineering and with terms related to traffic signals. We will also discuss the advantages and disadvantages of the considered approaches to motivate our new model. Herewith, the ground for the next two chapters should be prepared. Additional information can be found, e.g., in [69, 141].

In Chapter 5 we use the concept of dynamic flows and the periodicity of traffic signals to develop a cyclically time-expanded network. The model is completed by modeling intersections, traffic signals and traffic assignment. As a main result of this chapter we show how this model can be used to optimize traffic signal coordination and traffic assignment simultaneously. Aiming for a realistic modeling we also discuss the conse-

[2] PTV AG is a traffic planning company from Karlsruhe, Germany. It is well known for its traffic planning and simulation software VISUM and VISSIM.

quences of our approach to travel times and link performance in detail and derive further properties of the model.

Finally, Chapter 6 is designated for the simulation of inner-city traffic and the practical evaluation of the proposed model. We introduce the reader to two traffic simulation tools, namely VISSIM and MATSim. In detail, we consider the real-world inner-city networks of Cottbus, Braunschweig, Portland, and Denver. In particular, we emphasize the advantages of our simultaneous optimization of signal coordination and traffic assignment by comparing to a decomposed successive version of our approach.

About this thesis

A lot of results in this thesis were obtained during the ADVEST project, granted by the German ministry of education and research (BMBF). This also reflects in the thesis. First, some results were already published, see [53, 52, 98, 96, 97]. Second, the aims of the project lead to different kinds of results. On the one hand confluent flows were studied theoretically and are better understood now. But due to the combinatorial complexity practical applicability is – in the moment – poor. One the other hand, in a more experimental approach, a new model for simultaneous traffic signal coordination and traffic assignment was created, implemented, improved and tested with help of simulation tools. Hereby, a model of high practicability was developed, but it is difficult to prove the impact of the model also mathematically. Hence, the first part of the thesis will perhaps be more interesting for readers who focus on combinatorial optimization. The second part may more appeal to readers who are interested in the modeling of real world problems.

1 Basic Definitions and Notation

In this chapter we introduce and fix the basic notation for this thesis. Many fields of discrete mathematics are touched. First, we introduce the graph notation and we present some classical graph problems that we will refer to later. Due to the wide area of graph theory this description cannot be complete. For an introduction to graph theory we suggest, e.g., [152] or [49]. A good textbook on network flows is, for example, [2]. Good textbooks, covering network flows and other combinatorial optimization strategies, are [36, 100, 138]

Furthermore, we fix the notations for algorithms, complexity and approximation. Again, we can only give a short overview. For additional information, we refer to [66] and [9].

Linear Programming and *Integer Programming* are two basic approaches to solve network flow problems and combinatorial optimization problems. We will introduce both techniques in section 1.4 and suggest [147, 153] for further reading.

Please note that the following chapters also provide their own introductions and terms specific to these chapters are defined there.

1.1 Graphs

In this work we consider finite graphs $G = (V, E)$ where $V = V(G)$ is the *vertex set* and elements $v \in V$ are called *vertices* or *nodes*. $E = E(G)$ is the *edge set* of G. We consider both *undirected* and *directed* graphs (*digraphs*). In the case of undirected, loop free graphs the edge set is a subset of V^2, i.e. $E \subseteq \{\{u, v\} : u, v \in V,\ u \neq v\}$.

To denote directed edges, we also call them *arcs*. E is termed *arc set* A. A consists of ordered pairs of nodes, i.e. $A \subseteq V \times V = \{(u, v) : u, v \in V\}$. Therefore, each arc $a \in A,\ a = (u, v)$ is directed from its $\text{tail}(a) = u$ to its $\text{head}(a) = v$. For $v \in V$, we use $\delta^-(v) = \{a \in A :\ v = \text{head}(a)\}$ for the set of incoming arcs and $\delta^+(v) = \{a \in A :\ v = \text{tail}(a)\}$ for the set of outgoing arcs. A graph is called *bi-directed* if it contains for each arc $a = (u, v)$ also the arc in the opposite direction $a' = (v, u)$. A directed graph can be made undirected by simply deleting the directions of the arcs. To make an (undirected) graph a bi-directed one we add both directions for each edge/arc.

The cardinalities of the node and edge sets are denoted by $n = |V|$ and $m = |E|$. A graph with n vertices that contains all possible edges, is called a *complete graph* and denoted by K_n. Obviously, the complete graph has $m = \binom{n}{2} = \frac{n(n-1)}{2}$ edges. Sometimes, a graph is allowed to contain *multi-edges*, i.e., parallel edges. Hence, E, or A respectively, is defined as a multi-set in this case and G is called *multi-graph*. The *induced subgraph* on a vertex set $V' \subseteq V$ is denoted by $G[V']$. The induced arc set of $G[V']$ is denoted by $A[V']$.

A sequence $W = (a_1, \ldots, a_k)$, $a_i \in A$, of arcs is called a *walk* if it fits head to tail, i.e. $\text{head}(a_i) = \text{tail}(a_{i+1})\ \forall i \in \{1, \ldots, k-1\}$. For short, we will use $\text{tail}(W) := \text{tail}(a_1)$ and $\text{head}(W) := \text{head}(a_k)$. $V[W]$ is used for the set of vertices that occur in the arcs of W. To simplify matters, we use $a \in W$ to denote that the arc a is contained in the sequence of arcs in walk W. A *path* P is a walk which passes through every vertex at most once. A walk/path where the tail of the first arc and the head of the last arc coincide is called *cycle/circuit*. For $u, v \in V$, $\mathcal{P}_{u,v}$ denotes the set of all paths with tail u and head v. The *length* of a path (with respect to *unit edge lengths*) is the number of arcs in its sequence. Two paths P_1 and P_2 are (arc) *disjoint* if $A[V[P_1]] \cap A[V[P_2]] = \emptyset$. They are *node disjoint* if $V[P_1] \cap V[P_2] = \emptyset$. The composition of two paths $P_1 = (a_1, \ldots, a_k)$ and $P_2 = (b_1, \ldots, b_l)$ is defined as $P_1 \circ P_2 = (a_1, \ldots, a_k, b_1, \ldots, b_l)$.

Similarly, walks, paths, cycles, and circuits can be defined for undirected graphs.

A graph is *strongly connected* if for each pair of vertices (u, v) there exists a path P from u to v, i.e. $\text{tail}(P) = u$ and $\text{head}(P) = v$. A graph is *weakly connected* if its corresponding bi-directed graph is strongly connected. A node set $U \subseteq V$ is (strongly/weakly) connected if the induced graph $G[U]$ is (strongly/weakly) connected. The inclusion maximal (strongly/weakly) connected subgraphs of G are called *(strongly/weakly) connected components*.

A *cut* in G is an arc set C such that $G \backslash C = (V, A \backslash C)$ has at least one connected component more than G. The *value* of a cut is simply the number of arcs in the cut. An *s-t-cut* is defined as a partition of V into two subsets C_1 and C_2, such that $s \in C_1$ and $t \in C_2$. Let $C = \{a \in A : \text{tail}(a) \in C_1 \wedge \text{head}(a) \in C_2\}$ then there exists no directed path from s to t in $G \backslash C$. The value of the s-t-cut is $|C|$, i.e. the number of forward arcs from C_1 to C_2.

1.2 Flows and Networks

Flows have been a major planning tool for many applications ever since Ford and Fulkerson [58] studied them. Before considering flows with additional routing constraints in the following sections, standard flow is introduced here.

1.2.1 Definitions

Although there is consens what a *flow* in a *network* should be, we will use a slightly different and modular approach for defining flows[3].

A *flow function* is a non-negative function on the arc set $x : A \to \mathbb{R}_0^+$. In most practical applications the maximal flow on each arc is limited by a *capacity* bound, i.e., a maximal flow value that cannot be exceeded on this arc. The capacities are given by a function $u : A \to \mathbb{R}_0^+ \cup \{\infty\}$. A flow is *feasible* if $0 \leq x(e) \leq u(e)\ \forall e \in A$ holds. A graph together with capacities is called a *network* $G = (V, A, u)$. For some results in this thesis, we will limit the capacity function to integer values, i.e., $u : A \to \mathbb{N}_0$. This is no restriction for most applications, since rational values can simply be scaled to integer values and irrational numbers cannot exactly be represented in our computers anyway[4].Furthermore, a flow function is *integral* if it has only integral values.

[3]In some of the algorithms especially in Section 3, we will not be able to fulfill all requirements of a flow at once. A modular definition admits step-by-step procedures. For example, we will define and derive *preflows* with a slightly different flow conservation constraint.

[4]Irrational input may yield a unexpected behavior, the algorithm of Ford and Fulkerson is a prime example.