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# UPGRADED METALLURGICAL SILICON FOR

**APPLICATION IN THE PHOTOVOLTAIC INDUSTRY** 





## UPGRADED METALLURGICAL SILICON FOR APPLICATION IN THE PHOTOVOLTAIC INDUSTRY

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Von der Fakultät V, Verkehrs- und Maschinensysteme Der Technischen Universität Berlin Zur Verleihung des akademischen Grades - Doktor-Ingenieur (Dr.-Ing.) -

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#### Preface

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Berlin, 9. November 2011

Samo Semenič

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#### List of Abbreviations

- $\lambda$  Wavelength
- $E_{\lambda}$  Photon energy
- *H* Planck's constant
- c Speed of light
- $E_G$  Semiconductor band gap
- $\tau_{Aug}$  Auger-limit lifetime
- C<sub>Aug</sub> Auger coefficient
- $\tau_{SRH}$  Shockley-Read Hall lifetime
- $\tau_{\rm Rad}$  Radiative lifetime
- $N_{\rm A}$  Dopant concentration
- $X_{\rm S}$  Solid solubility of impurities
- $k_0$  Segregation (distribution) coefficient
- *C*<sub>S</sub> Solid solubility limit at the melting point
- k Boltzmann constant
- *L* Solubility
- $\Delta S$  Entropy change
- $\Delta H$  Enthalpy change
- T Temperature
- $J_n$  Current density of electrons
- q Charge
- *u<sub>n</sub>* Mobility
- *E* Electric field
- *k*<sub>n</sub> Segregation coefficient of n-type impurities
- *k*<sup>p</sup> Segregation coefficient of p-type impurities
- $n^{r}$  Carrier concentration
- p Resistivity
- *N*<sub>a</sub> Acceptors density
- *N*<sub>d</sub> Donors density
- *L*<sub>e</sub> Electron minority carrier diffusion length
- *u*<sub>e</sub> Electron mobility
- $\tau_{e}$  Electron minority carrier lifetime
- *L*<sub>h</sub> Hole diffusion length
- *u*<sub>h</sub> Hole mobility
- $\tau_{\rm h}$  Hole lifetime
- *N*<sub>P</sub> Density of phosphorus
- $N_{\rm B}$  Density of boron
- $\rho_{Si}$  Mass density of silicon
- $m_{\rm P}$  Mass of phosphorus atom
- *X*<sub>P</sub> Phosphorus concentration in ppmw
- *X*<sub>B</sub> Boron concentration in ppmw
- $\rho$  Static resistivity
- *E* Magnitude of the electric field
- J Magnitude of the current density
- *R* Electrical resistance of the material
- l Length
- A Cross-section
- $\sigma$  Conductivity

- $\Delta G$  Gibbs energy
- *k*<sub>0</sub> Segregation coefficient
- $X_{\rm S}$  Concentration of impurities in the solid stage
- $X_{\rm L}$  Concentration of impurities in the liquid stage
- $T_{\rm L}$  Liquidus temperature
- $T_{\rm S}$  Solidus temperature
- $X_0$  Initial concentration of the impurity
- $f_S$  Volume fraction of solid phase
- $f_{\rm B}$  Coefficient of activity
- *p* Partial pressures
- $p^*$  Vapor pressure
- a Activity
- N Molar concentration
- *p*<sub>A</sub> Actual vapor pressure of component A
- $p_{\rm B}$  Actual vapor pressure of component B
- $a_{\rm A}$  Activity of component A
- *a*<sub>B</sub> Activity of component B
- $p_{\rm A}^*$  Saturation vapor pressure of component A
- $p_{\rm B}^*$  Saturation vapor pressure of component B
- *N*<sub>A</sub> Concentration of component A
- $N_{\rm B}$  Concentration of component B
- $\gamma_A$  Activity coefficients of component A
- $\gamma_{\rm B}$  Activity coefficients of component B
- $\beta_A$  Ratio of the concentrations of components in the two phases