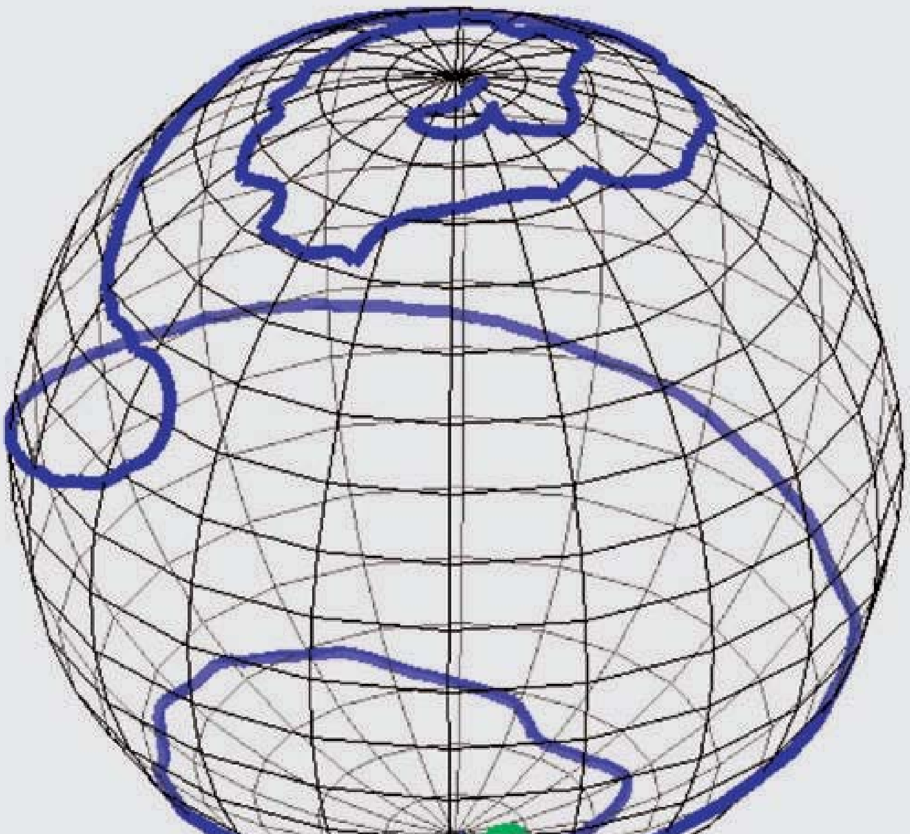
Nuala Patricia Timoney

Robust rotations & coherent quantum states with a single trapped ion



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Robust rotations & coherent quantum states with a single trapped ion

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Zusammenfassung

Kohärente Operationen werden für die Implementierung von einzelnen und Multiqubit Gattern mit gespeicherten Ionen benötigt. Es wird gezeigt, dass diese Operationen robust gegen Fluktuationen von experimentellen Parametern sind. Insbesondere werden einzelne Qubit Gatter, die mittels Optimaler Kontroll Theorie entwickelt wurden, zum ersten Mal an einzelnen Ionen gezeigt. Ihre Effizienz als Funktion von Fehlerparametern wird systematisch untersucht und mit Composite Pulsen verglichen.

Als quantenmechanisches Zweiniveausystem wird der $S_{1/2}$ F = 0 → $S_{1/2}$ F = 1 m_F = 0 Übergang eines einzelnen $^{171}Yb^+$ Ions ausgewählt, dass in einer Paul Falle gespeichert ist. Der Übergang wird mit einem Mikrowellenfeld bei 12.6 GHz getrieben. Die verwendeten Pulse wurden speziell zur Kompensation von Frequenz-, Zeit- und Leistungsfehlern des treibenden Feldes entwickelt. Bei Messungen mit Optimalen Kontroll Theorie Pulsen und Composite Pulsen ergibt sich eine höhere Fidelity als bei Messungen mit Rechteckpulsen. Es wurde eine gute Übereinstimmung zwischen den simulierten Ergebnissen und den gemessenen Resultaten erzielt.

Die Experimente besitzen eine niedrige Obergrenze des Fidelity und manche haben eine Obergrenze von 76 %. Der wahrscheinlichste Grund für diese Obergrenze dürfte eine Fehlpräparation sein. Aus diesem Grund wurde das Präparationsverfahren geändert, um schneller und effizienter präparieren zu können. Während der Untersuchung von nicht erwünschtem Optischen Pumpen, wird die Rolle einer effektiven Vorselektion der Messdaten diskutiert.

Die Kohärenzzeit ist eine wichtige Eigenschaft eines Quantencomputers. Ein Schwachpunkt eines auf Ionenfallen basierenden Quantencomputers ist die eventuelle Abhängigkeit von magnetfeld sensitiven Niveaus. Störungen des Magnefeldes in der Falle verkürzt die Kohärenzzeit der magnetfeldempfindlichen Niveaus (5 ms) im Vergleich zu den magnetfeldunempfindlichen (500 ms).

Die Energieentartung eines “dressed states” soll bei einem “avoided crossing” gegenüber kleinen Verstimmungen der atomaren Resonanz sein. Ein “dressed state” System wird mit einem Mikrowellenfeld in den hyperfein aufgespalteten Niveaus des Grundzustands von $^{171}Yb^+$ erzeugt. Einzelne “dressed states” Zustände werden gezielt präpariert und Rabioszillationen zwischen den “dressed states” beobachtet. Die Kohärenzeiten von Systemen, die nach diesem Prinzip generiert wurden, werden gemessen.

Abstract

Coherent operations necessary for the implementation of single and multi-qubit quantum gates with trapped ions, that are robust against variations in experimental parameters and intrinsically indeterministic system parameters, are demonstrated. In particular, single qubit gates developed using optimal control theory are demonstrated for the first time with trapped ions. Their performance as a function of error parameters is systematically investigated and compared to composite pulses.

A two level quantum mechanical system is realized on the $S_{1/2}$ F = 0 $\rightarrow$ $S_{1/2}$ F = 1 m_F = 0 transition in $^{171}Yb^+$ confined in a Paul trap, driven by microwave radiation close to 12.6 GHz. Shaped pulses and composite pulses have been realized that are specifically designed to tackle off-resonance errors, timing errors or power variations of the driving field. Good agreement is seen between the simulated results and measured ones. Higher experimental fidelities are obtained with the aforementioned shaped and composite pulses over an extended parameter regime than with a simple pulse.

The experiments have baselines as low as 76 %. The suspected culprit of this result is the preparation procedure. To this end the preparation process was changed to be faster (0.5 ms) and more efficient (97 %). The essential role of effective preselection of the data is highlighted whilst investigating unwanted optical pumping effects.

Coherence time is an important property of a quantum computer. A weakness of an ion trap quantum computer is its potential dependence on magnetic field sensitive levels. Ambient magnetic field noise is the cause of the shorter coherence times of magnetic field sensitive levels (5 ms) compared to their field insensitive counterparts (500 ms) as observed in $^{171}Yb^+$.

At an avoided crossing the energy separation of a dressed state should be robust to small changes in detuning. Using a microwave field and the ground state hyperfine levels of $^{171}Yb^+$, such dressed states are prepared and Rabi oscillations are observed between them. The coherence times of systems built on this principle are measured.

Hope, like the gleaming taper's light,

Adorns and cheers our way;

And still, as darker grows the night,

Emits a brighter ray.

Oliver Goldsmith (1730 - 1774)

Contents

1 Introduction

1.1 Quantum computing

How is it possible to build a physical quantum computer? This question is an open one since Feynman and his contemporaries first pondered using a computer whose evolution is quantum mechanical. Quantum computing has in the meantime become a buzz term which grabs headlines. With modern computers developing at a rapid pace and processors becoming ever smaller, a quantum computer is seen as the ultimate goal. But just what is a quantum computer?

A quantum computer is based around the idea of mixing quantum mechanics and modern day computing. The modern day computer at its most fundamental level is a series of simple gates, put a 1 or a 0 in, get a (or many) 1 or a 0 out. If one has N gates, then N operations are performed. Two classical gates which are performed simultaneously do not have an influence on each other.

In contrast to this classical computer, the quantum computer doesn't merely have bits 0 and 1, rather it has quantum bits (qubits), which are superpositions of two orthogonal states. A qubit can be described by

$$\mid \Psi\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle, \tag{1.1}$$

where $\{\alpha, \beta\} \epsilon \mathbb{C}$ and $\mid \alpha \mid^2 + \mid \beta \mid^2 = 1$.

In addition, the qubits of a quantum computer can be intertwined or entangled, which means that the qubits are no longer purely individual entities. Operations on a qubit of the quantum register influence the evolution of the other entangled qubits. It is this possibility which vastly improves the computing power for the number of operations per switching unit. It is possible to have a unitary operation which effects all of the possible state permutations at once, i.e. the unitary operation performs 2^N operations on N qubits.

This increased power should mean faster operations if one is able to harness it. To this end algorithms have been developed which require a quantum computer to implement them. These algorithms can search a random database of N entries using $\sqrt{N}$ steps rather than the classical N/2 steps [Grover, 1996] or factorize large numbers in polynomial time frames, which is considerably faster than that required for known classical algorithms [Shor, 1997]. These commercially friendly applications are joined by the prospect of being able to solve dynamics of complex quantum mechanical problems [Kaye *et al.*, 2007]. What is required to execute such an algorithm?

In fact it is only necessary to show that individual qubit rotations and conditional logic are possible to execute a quantum algorithm. A rotation is an operation which changes the superposition of the orthogonal states by a prescribed amount. Conditional logic however is somewhat more complicated, but can be brought in analogy to the classical XOR gate [Barenco *et al.*, 1995].

This discussion, which has briefly described the main principles behind a quantum computer, has not answered the first question posed: How to physically implement it? It has been established that the physical system must have two basis states and qubits must interact with each other: What else is required? The DiVincenzo [DiVincenzo, 2000] criterium has five main points which describe the physical requirements for a system which is a quantum computer. There exist a further two points in this criteria, it is often referred to as the 5 (+2) conditions for a quantum computer. These extra two conditions refer to quantum communication techniques and are not discussed here. The five conditions on the the other hand are

1. The **system must be scalable** and the **qubits must be well characterized**. This means that a single qubit must be well understood in terms of its internal Hamiltonian, the couplings to other states, the interactions with other qubits and the coupling to external fields. The coupling with external fields is particularly important as this has a direct influence on the fidelity with which intended operations can be performed. It is also important that it is possible to entangle qubits and that the system can be scaled so that the advantage of the quantum computer can be obtained.

2. It must be possible to **prepare the qubits** to an intended state. Preparation means that an arbitrary qubit state can be obtained with high fidelity.

3. **Decoherence times** must be longer than gate operation times. The qubit is quite often subject to interactions with its environment. Such interactions can effect either the superposition of the orthogonal states of the qubit, or the phase between the occupation of the two states. The requirement of this point is saying that such processes can be dealt with, so long as they aren't likely to happen whilst performing the gates which make the calculation. A guide given in [DiVincenzo, 2000] is that the decoherence time should be $10^4 - 10^5$ times larger than the time needed for an individual gate. This estimation includes using the effect of quantum error correction techniques.

4. **Universal gates** must be possible. In the previously cited [Barenco *et al.*, 1995] it is shown that the set of one-bit quantum gates and the two bit

exclusive-or gate is sufficient to describe all unitary operations on arbitrary many bits. So for a unviersal gate to be possible, single qubit rotations and conditional logic operations must be possible.

5. It must be possible to **measure the different qubits**.

1.1.1 Methods

The DiVincenzo criteria is a handy check list for knowing what is required for the physical manifestation of a quantum computer. In this section the current progress of the scientific community in pursuing this aim is summarized.

The earliest implementations of quantum computing systems were done in Nuclear Magnetic Resonance (NMR) experiments. Here algorithms were shown to work with up to seven qubits [Vandersypen & Chuang, 2005]. These implementations were only intended as proof of principle experiments, as the upper limit of NMR qubits is predicted at 30 [Jones, 2000].

A proposal which has made fast progress in recent years is based on nitrogen vacancy (NV) centres in diamond [Neumann *et al.*, 2008]. Two partite and three partite entanglement for two carbon atoms about the vacancy or for the two carbon atoms and the vacancy itself has been shown. Proposals exist for how to scale this system by entangling distant centers through emitted photons [Barrett & Kok, 2005].

A scheme which has been garnering many headlines in recent times is semiconductor qubits. Semiconductors as qubits is a young technology: it is only recently that two qubit algorithms have been demonstrated [DiCarlo *et al.*, 2009]. If technical problems such as coherence times (currently on the order of μs [Clarke & Wilhelm, 2008]) and gate fidelities You & Nori [2005] are overcome this may well prove to be a viable option for a quantum computer.

Orthogonally polarized photons are used in quantum communication as qubits. It is certainly not difficult to envisage how information can be transported if one is using photons, they travel quite easily! Experiments performed by [Lu *et al.*, 2007] show that it is possible to create six photon graph states where the fidelity with which these states are produced is $F = 0.593 \pm 0.025$.

In this thesis none of the above technologies are pursued to achieve a quantum computer. This thesis is concerned with individual ions in an electrodynamic trap. Although ion traps have existed for much of the previous century [Paul *et al.*, 1958], using them for quantum information is a relatively new development [Cirac & Zoller, 1995].

1.2 Ion trap quantum computers

The proposal to use ion traps as a quantum computer came from [Cirac & Zoller, 1995]. It was envisioned that the ions would be trapped in a linear trap. The

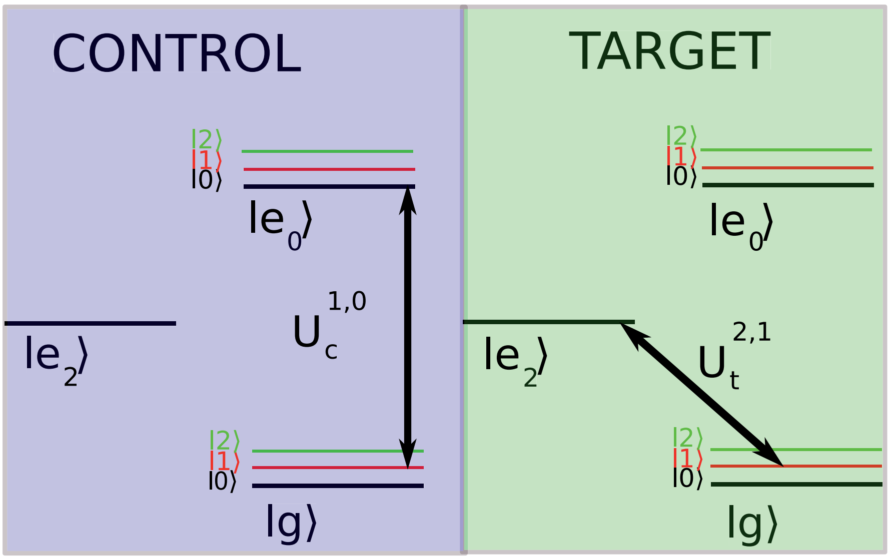

Figure 1.1: In this graphic the two operations which are used to create the Cirac Zoller CNOT scheme are shown. The left ion scheme represents the control qubit, the right the target qubit. The marked unitary operations correspond to those explained in table 1.1

ions would be individually addressable by different lasers being focussed on each individual ion. A universal rotation would be achieved by allowing a laser tuned to the the qubit transition to be switched on for a prescribed amount of time, where the phase and power of the light are controlled. Conditional logic is also possible: this is made possible through use of a bus qubit, which serves to the entangle ions. The bus qubit comes in the form of the collective quantized motion of the ions. The motion of an ion or ions in a trap can be quantized, much like a quantum mechanical harmonic oscillator. A string of N ions in a trap will have 3N common motional modes.

In [Cirac & Zoller, 1995] an example of two ions in a trap is given. Both ions are in the motional ground state of the string. Here the basis of the conditional gate is that if one of the ions is excited to a higher motional state by means of a bus qubit, then both of them will be. Two qubits are shown in figure 1.1, one named the control qubit the other the target. The first pulse, a π-pulse, is tuned such that the the control qubit receives a motional quanta only if it is in the excited state. If the control qubit receives a motional quanta, the target qubit does too. The second operation, a 2π-pulse, is tuned in frequency such that it is only successful if the target qubit has a motional quanta. This 2π-pulse to an external level, adds a phase to the qubit which it would not otherwise have. Finally the sequence is closed by repeating the first π-pulse on the control qubit to remove the motional quanta from the system. The different input and output states are tabulated in table 1.1.

A suitable qubit transition must have a natural linewidth narrow enough so that the transitions involving different motional quanta are cleanly distinguishable. The description of the possible transitions on the various sidebands is given by the

Initial State	$U_c^{1,0}$	$U_t^{2,1}$	Final State $U_c^{1,0}$
$\vert g\rangle_c\ \vert g\rangle_t\ \vert 0\rangle$	$\vert g\rangle_c\ \vert g\rangle_t\ \vert 0\rangle$	$\vert g\rangle_c\ \vert g\rangle_t\ \vert 0\rangle$	$\vert g\rangle_c\ \vert g\rangle_t\ \vert 0\rangle$
$\vert g\rangle_c\ \vert e_0\rangle_t\ \vert 0\rangle$	$\vert g\rangle_c\ \vert e_0\rangle_t\ \vert 0\rangle$	$\vert g\rangle_c\ \vert e_0\rangle_t\ \vert 0\rangle$	$\vert g\rangle_c\ \vert e_0\rangle_t\ \vert 0\rangle$
$\vert e_0\rangle_c\ \vert g\rangle_t\ \vert 0\rangle$	-i$\vert g\rangle_c\ \vert g\rangle_t\ \vert 1\rangle$	i$\vert g\rangle_c\ \vert g\rangle_t\ \vert 1\rangle$	-$\vert e_0\rangle_c\ \vert g\rangle_t\ \vert 0\rangle$
$\vert e_0\rangle_c\ \vert e_0\rangle_t\ \vert 0\rangle$	-i$\vert g\rangle_c\ \vert e_0\rangle_t\ \vert 1\rangle$	-i$\vert g\rangle_c\ \vert e_0\rangle_t\ \vert 1\rangle$	$\vert e_0\rangle_c\ \vert e_0\rangle_t\ \vert 0\rangle$

Table 1.1: Cirac Zoller conditional logic gate. The unitary operations are illustrated in figure 1.1.

Hamiltonian term

$$H_{int} = \frac{1}{2}\,\hbar\,\Omega_R\,(\sigma_+ + \sigma_-)(e^{i[\eta(a_l+a_l^+)-\omega_M t]} + H.c.), \tag{1.2}$$

where Ω_R is the Rabi frequency of the driving field, ω_M is the rotating frequency of the driving field, t is time, $\sigma_\pm = \sigma_x \pm i\sigma_y$ the Pauli matrices, a_l and a_l^+ are the lowering and raising operators and η is the Lamb Dicke factor. This factor, which will later be explained as the most important factor in this Hamiltonian, is given by

$$\eta = \sqrt{\frac{\frac{(\hbar k)^2}{2m}}{\hbar\omega_1}}, \tag{1.3}$$

where $\vec{k}$ is the wave vector of the incoming light, m is the mass of the ion and ω_1 is the trap frequency. For a microwave transition the Lamb Dicke factor is 10^4 times smaller than that for an optical transition, due to the inverse proportionality of η to the wavelength λ.

So building a quantum computer should be easy right? A five step check list, powerful sounding algorithms and talented scientists it must all be a doddle....

Well not quite, but there have been nonetheless some interesting developments towards building a quantum computer in recent years. Not least, the results from Innsbruck where 8 qubits were shown to be entangled [Häffner *et al.*, 2005]. Entanglement with a higher number of ions has not been attempted due to the large amount of measuring time required and the increasing classical computational time and power required to reconstruct the density matrix. Theoreticians are currently working on more elegant ways that experimentalists can use to prove that qubits are entangled.

Experimentalists in recent years have focussed on how ion traps could be built on much larger scales. To this end it is intended to incorporate chip technology

[Labaziewicz *et al.*, 2008], which would involve several ion traps where ions can be ferried between different trap regions [Kielpinski *et al.*, 2002] whilst maintaining entanglement [Jost *et al.*, 2009]. The future with ion trap quantum computing looks bright indeed!

1.3 Thesis outline

This thesis does not build directly on the great achievements just listed but investigates more humble beginnings. All the experiments shown here are motivated by the hope that the results can be used in potential ion trap schemes or traps involving many ions. The experiments are all performed with a single ion. In particular technical problems foreseen in implementing a variation to the Cirac Zoller scheme based on an "ion spin molecule" which uses microwave radiation, are investigated.

The theory of ion traps, the ions and isotope used, how laser cooling works and finally the alternate "ion spin molecule" scheme are all presented in chapter 2. A problem which will arise in this scheme is an uncertainty in the resonant frequency of individual ions. Pulses which are robust to such uncertainaties are introduced. Decoherence and the main method to overcome it are discussed. Sensitivity to decoherence from ambient magnetic field noise is forseen as a problem in the "ion spin molecule" scheme. An alternative scheme based on the concept of dressed states is introduced which should be robust to such noise. In fact such a scheme could also be used with the "ion spin molecule" to increase interaction strengths between entangled ions. Dressed states themselves are described before examining different possible atomic configurations of implementing such a scheme. Preparation of the states and physical requirements to implement such a system are also considered.

Chapter 3 describes the experimental setup used for the experiments presented in this thesis. This includes the trap (section 3.1), the optics (section 3.2) including the laser systems and the λ-meter, the non optical electromagnetic fields (section 3.3) and the static fields (section 3.3.3). The detection is described (section 3.4) and a brief overview of the experimental control system is given in section 3.5. The data evaluation method used and a possible alternative is described in section 3.6

Chapter 4 describes how the robust pulses introduced in chapter 2 were tested. Two types of pulses are shown: shaped pulses which are based on optimal control theory and simpler composite pulses. These are compared to both expected theoretical values and simple rectangular pulses.

The measurements in chapter 5 are made to investigate the reason that the fidelities in the measurements in chapter 4 were lower than expected. Methods to determine this include an additional laser for state preparation (section 5.1), an additional repumper laser (section 5.2) and changing the photomultiplier (section 5.3). The negative effect of ambient magnetic field noise on the magnetic field sensitive levels is shown in section 5.4 and the improvement after triggering the experiment to the A/C line is also shown. The coherence time of the magnetic field insensitive transition is measured in section 5.5.

Finally chapter 6 describes some investigative measurements for the implementation of a tweaked "ion spin molecule" scheme based on dressed states. It is envisioned that this tweaked scheme could either be used to combat the decoherence caused by ambient magnetic field noise, or to amplify the existing scheme. Preparation methods of the dressed states are measured (section 6.1), the suitability of a two level system (section 6.2) and a three level system (section 6.3) are also measured.

2

Theory

2.1 Ion traps

The main principle which underlies an ion trap is that of like charges. A positive charge repels a positive charge. The ions used in the measurements in this thesis are all positively charged ions. A first iteration of an ion trap might be a metal ring surrounding the charged particle where the ring is held at a positive potential. In two dimensions the center of the ring will be where the charged particle is, equally repelled from all parts of the ring. Assuming a perfect ring, the charged particle will be held in the center. In three dimensions, the ion can escape along the axis perpendicular to the ring. Adding further electrodes at various potentials will not keep the particle in the ring when constant potentials are used. The potentials formed contain only saddle points, but no minima. This heuristic description of Earnshaw's theorem can be shown mathematically using the Laplace equation [Braun, 2007].

To make an electrodynamic trap in the previous example, the potential applied to the ring should oscillate at some radio frequency, rather than being static as above. This was first suggested by Wolfgang Paul in [Paul *et al.*, 1958]. A static time point of the potential is shown in left picture of figure 2.1. This potential has no minima in which a particle can be trapped. A time dependent potential evolves to the potential shown in the right hand figure of 2.1, where the potential has evolved after a time interval of π/Ω, where Ω is the angular frequency of the driving field. This cycle is repeated at the rate of the radio frequency field and as a result the charged particle stays in the trap. A useful physical picture to help visualize this, is if the potential drawn in the left hand picture of figure 2.1 is a solid surface and a ball is placed in the center. If the surface stays still, the ball will roll down the sides. If the surface spins, the exit path for the ball keeps on changing. If the surface spins fast enough, the ball won't leave.

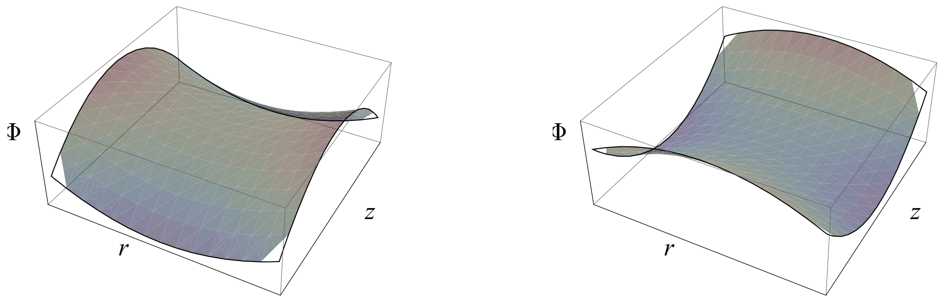

Figure 2.1: Sample potentials of a Paul trap potential as described by equation 2.1. The left and right figures represent the potential at different times which are separated in time by odd integer multiples of π/Ω, where Ω is the angular frequency of the driving field.

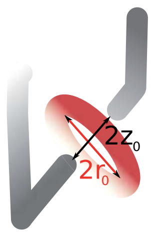

Figure 2.2: Illustration of the trap used in this thesis. The inner ring diameter ($2r_0$) is 2 mm and the distance between the z-cap electrodes ($2z_0$) is 1.2 mm.

There exists a mathematical description which describes the dynamics of a charged particle in an electrodynamic trap. The example of a ring trap, with endcap electrodes, which was used in the previous example is exactly the trap used in the experimental work presented in this thesis.

2.1.1 Mathieu Equation

The field resulting from the ring trap in the previous section is a quadrupole field. Adding the restraint that ${r_0}^2 = 2{z_0}^2$, where r_0 and z_0 are defined according to figure 2.2 the potential is given by [Ghosh, 1995]

$$\Phi = \Phi_0(r^2 - z^2). \tag{2.1}$$

In fact, this potential assumes that the end cap electrodes are hyperboloids of revolution about the z axis and the ring itself has a hyperbolic cross-section. This is not an essential requirement, the ring trap used in this thesis is only an approximation of the hyperbolic surface and functions effectively none the less.

The equations of motion from this potential are given by [Ghosh, 1995] as
where e is the charge of the particle in the trap, m its mass, U the magnitude