

Axel Rühl

The normal dispersion regime
in passively mode-locked
fiber oscillators



Cuvillier Verlag Göttingen
Internationaler wissenschaftlicher Fachverlag

The normal dispersion regime in passively mode-locked fiber oscillators

**Von der Fakultät für Mathematik und Physik
der Gottfried Wilhelm Leibniz Universität Hannover
zur Erlangung des Grades**

**Doktor der Naturwissenschaften
- Dr. rer. nat. -**

genehmigte Dissertation

**von
Dipl.-Phys. Axel Rühl
geboren am 6.8.1975 in Berlin**

2008

Bibliografische Information der Deutschen Nationalbibliothek

Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über <http://dnb.ddb.de> abrufbar.

1. Aufl. - Göttingen : Cuvillier, 2008

Zugl.: Hannover, Univ., Diss., 2008

978-3-86727-831-7

Referent: Prof. Dr. Uwe Morgner, Leibniz Universität Hannover

Korreferent: Prof. Dr. Boris N. Chichkov, Leibniz Universität Hannover

Tag der Promotion: 5.12.2008

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Nonnenstieg 8, 37075 Göttingen

Telefon: 0551-54724-0

Telefax: 0551-54724-21

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1. Auflage, 2008

Gedruckt auf säurefreiem Papier

978-3-86727-831-7

Abstract

The subject of this thesis is the development and study of passively mode-locked fiber oscillators operating in the normal dispersion regime. Inspired by the concept of similariton amplifiers, pulse evolution towards a similariton opened new perspectives for fiber oscillators. In contrast to solitary laser oscillators, pulse shaping towards similaritons offers the possibility of significant energy scaling because of strongly chirped intracavity dynamics while the pulses can be efficiently compressed outside the cavity. These properties make fiber oscillators in the normal dispersion regime versatile femtosecond light sources whose performance can be tailored for various applications.

Ytterbium- and erbium-doped fiber oscillators mode-locked by nonlinear polarization evolution were set up and utilized for the study of this pulse regime. The oscillators exhibit promising performances and the pulse energy could be significantly scaled compared to previous concepts. Supported by numerical simulations, various aspects of the intracavity pulse formation and its targeted influence were investigated. Based on the analysis of the intracavity pulse dynamics, the role of dissipative effects in the formation of a stable steady-state was pointed out and its influence on the frequency as well as on the temporal domain was investigated. Filtering in the frequency domain cannot only be provided by spectral filters and a limited gain bandwidth, but also by intrapulse Raman-scattering. By using additional mode-locking mechanisms constituting nonlinear temporal filters, the output characteristic could be significantly tuned and the increased self-starting capability gives access to various cavity designs.

The impact of dispersion also allows the adaption and optimization of laser parameters. Changes of the output characteristic for varying group-delay dispersion were addressed and its limits were discussed. Furthermore, the self acting nonlinear compensation of third-order dispersion was experimentally demonstrated for the first time and verified by numerical simulations. Beside experimental and numerical works, this thesis also contains a general discussion of particular features of this operation regime. By comparison with analytic solutions of wave-equations and averaged models for laser oscillators, the presented results were classified. The discussion about the validity of theoretical models provides not only a basis for the description of oscillators operating in the normal dispersion regime but might also be useful for the transfer of other amplifier concepts to (fiber) oscillators.

Zusammenfassung

Thema der vorliegenden Arbeit ist die Entwicklung und Untersuchung passiv modengekoppelter Faseroszillatoren im normalen Dispersionsregime. Inspiriert durch das Konzept der Similariton Verstärker, eröffnete die Pulsentwicklung hin zum Similariton neue Perspektiven. Im Gegensatz zu solitären Lasern, bietet diese Pulsformung auf Grund der stark gechirpten Dynamik im Resonator die Möglichkeit einer deutlichen Energieskallierung während die Pulse außerhalb des Resonators effizient komprimiert werden können. Diese Eigenschaften machen Faserlaser im normalen Dispersionsregime zu vielseitigen fs-Quellen, deren Charakteristik für diverse Anwendungen angepasst werden kann.

Für die Untersuchung wurden mittels nichtlinearer Polarisationsentwicklung modengekoppelte Ytterbium- und Erbium-dotierte Faseroszillatoren aufgebaut. Die Pulsenergie konnte im Gegensatz zu vorherigen Konzepten deutlich gesteigert werden. Auf numerische Simulationen gestützt, wurden verschiedene Aspekte der Pulsentstehung und deren gezielte Beeinflussung untersucht. Basierend auf der Analyse der Pulsdynamik im Resonator, wurde die Rolle dissipativer Effekte bei der Bildung stabiler Gleichgewichtszustände aufgezeigt und deren Einfluß in der Zeit- und Frequenzdomäne untersucht. Filtern in der Frequenzdomäne kann dabei nicht nur durch spektrale Filter oder durch eine begrenzte Verstärkungsbandbreite, sondern auch durch intrapuls Ramanstreuung erfolgen. Durch die Verwendung zusätzlicher Modenkopplungsmechanismen als zeitliche Filter, konnten die Oszillatoren durchgestimmt werden, während die erhöhte Selbststartfähigkeit diverse Resonatoraufbauten ermöglichte.

Der Einfluß der Dispersion erlaubt ferner die Adaption und Optimierung einzelner Laserparameter. Die Änderungen der Ausgangscharakteristik bezüglich der Dispersion zweiter Ordnung wurden untersucht und ihre Grenzen diskutiert. Darüber hinaus wurde die selbsttätige nichtlineare Kompensierung der Dispersion dritter Ordnung erstmalig experimentell gezeigt und durch numerische Simulationen bestätigt. Neben den experimentellen und numerischen Untersuchungen enthält die vorliegende Arbeit auch eine allgemeine Diskussion typischer Eigenschaften dieses Betriebsregimes. Durch den Vergleich mit analytischen Lösungen von Propagationsgleichungen und gemittelten Lasernmodellen wurden die hier dargestellten Ergebnisse klassifiziert. Die Diskussion über die Gültigkeit der theoretischen Modelle kann auch für die Übertragung anderer Verstärkerkonzepte auf (Faser)-Oszillatoren von Nutzen sein.

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1 Introduction

Ultrashort laser pulses with durations in the fs-range become more and more important in many branches of science. The unique properties of fs-pulses such as broad optical spectra, high peak powers and a high phase coherence facilitate various applications. They are not only used in many branches of physics but also in other disciplines like medicine, chemistry and biology.

All fs-laser systems are based on mode-locked oscillators where the ultrashort pulses are generated. The dominant approach to generate fs-pulses in a laser oscillator is the usage of Ti:sapphire crystal allowing a broadband amplification. This material is capable to produce the shortest pulses from an oscillators with pulse energies in the μJ -range. On the other hand, this concept has several drawbacks which hinder the spread of applications especially to the commercial sector. The necessity of gas lasers or frequency-doubled solid-state lasers for optical pumping results in low overall efficiencies and high costs. The setups of Ti:sapphire based oscillators are quite complicated, require periodic reallignment and are therefore often unreliable for applications outside research laboratories.

Rare earth-doped fibers are a promising alternative to bulk gain materials. Due to the amorphous structure of the fused silica host, the laser levels are significantly broadened, making rare earth-doped fibers ideal for the generation and amplification of ultrashort pulses. The waveguide nature of optical fibers results in a well-controlled spatial mode quality which is insensitive to external perturbations. The large surface to volume ratio reduces thermal effects and the availability of fiber based components from the telecommunication industry allows for compact and low-cost setups. The usage of laser diodes for optical pumping further reduces the costs for fiber based oscillators.

Passively mode-locked oscillators based on optical fibers have been studied for about three decades and are nowadays commercially available. The pulse formation is based on an interplay between nonlinear phase contributions and the impact of anomalous dispersion which allows a solitonic balance between these two effects. Mainly operating

around $1\text{ }\mu\text{m}$ and $1.5\text{ }\mu\text{m}$, passively mode-locked fiber lasers routinely generated pulses below 200-fs with pulse energies of a few nJ. The low pulse energy constitutes the major drawback of fiber oscillators compared to their solid-state counterparts. Most applications require higher pulse energy and thus additional amplifiers increasing costs and the complexity of the system.

In general, there are two mechanisms limiting energy scaling of mode-locked (fiber) oscillators. The first is an insufficient suppression of cw-operation at high power. Once net gain is provided, a cw-background starts to grow and destabilizes mode-locked operation. This problem arises from saturable absorber mechanisms, whose transmittance or reflectivity does not increase monotonically with increasing power and energy, respectively. The second limitation is the stability of the generated pulse itself. Nonlinear phase contributions can only be tolerated up to a certain level, constituting the main drawback of previous approaches in (fiber) oscillators. This can be regarded as the more general physical limitation as it is independent of the cavity design and the saturable absorber mechanism.

The tight confinement of the electric field in an optical fiber leads to much higher nonlinear phase contributions than in bulk gain media. Although the Kerr-nonlinearity of fused silica is comparably small, the long interaction length and the tight confinement of the electric field in the core result in much stronger nonlinear effects. The limitations by excessive nonlinearities are therefore more stringent in fiber based oscillators constituting precursors for the development of new operation regimes. Research on high-energy pulse formation in mode-locked fiber oscillators is thus necessary to be competitive to solid-state oscillators.

Pulses tolerating for higher nonlinear phase contributions can be formed in the presence of normal dispersion which has been demonstrated in several propagation experiments. The generation of similaritons representing an analytical class of ultrashort pulses led to a new generation of ultrafast fiber amplifiers. For the generation in a system with feedback, the pulse shaping process needs to be consistent with the boundary conditions of cavities. Due to the fact that the pulse evolution at normal dispersion is monotonic, the boundary conditions cannot be fulfilled offhand. The stability of the dynamics in such nonlinear dissipative systems therefore requires additional mechanisms. Beside the study of stable pulse formation at normal dispersion and its targeted influence, the demonstration of fiber oscillators with improved performance are the main subjects of this thesis.

Organization of the thesis

The thesis is organized as follows. Chapter 2 provides a brief introduction to short-pulse propagation in normal dispersive fibers. After the description of the governing wave-equations and the general solution of optical solitons, the phenomenon of optical wave breaking is subject of Sec. 2.2. This optical shock constitutes the main instability mechanism for ultrashort pulses propagating at normal dispersion. Sec. 2.3 deals with similaritons and parabolic pulses, respectively, which were shown to resist optical wave breaking. The attempt to generate this class of analytic solutions in laser oscillators constitutes the main motivation for the operation in the normal dispersion regime.

Chapter 3 describes some fundamentals of passively mode-locked fiber oscillators. After introducing the basic mode-locking techniques suited for fiber lasers, a general overview on the operation regimes of fiber oscillators and its limitations is given in Sec. 3.2. The theoretical approach of master equations for the description of mode-locked lasers is given in Sec. 3.3 with a focus on the normal dispersion regime (NDR).

Chapter 4 presents a passively mode-locked ytterbium³⁺-doped fiber oscillator operating in the normal dispersion regime. After introducing the experimental setup and the numerical model in Sec. 4.1 and 4.2, the output characteristics are presented in Sec. 4.3. The subject of the last section in this chapter is the impact of dispersion on the output characteristics. Pulse shaping in the fiber section is basically an interplay between the dispersion and the dominating Kerr-nonlinearity. For high power operation, the non-linear phase contributions are usually maximized, so the parameters of the dispersion slope remain free and can be adapted for stable steady-states. Beside the consequences of the group-delay dispersion, the impact of third-order dispersion is evaluated. A self-acting compensation of the third-order dispersion in an oscillator could be demonstrated for the first time.

Chapter 5 contains detailed analysis of the operation regime and attempts to classify the experimental results. The pulse dynamics inside the resonator are analyzed on the basis of numerical results and the main mechanisms for pulse generation are deduced in Sec. 5.1. It will be shown that the dynamic is an interplay between the evolution in the fiber section and dissipative effects in the time and frequency domain. Sec. 5.2 discusses the importance of those for the formation of a stable steady-state and the possibilities of influencing the dynamic with additional filtering. (Experiments concerning filtering in the time domain via an additional mode-locking mechanism is subject of chapter 6.) For

classifying the operation regime, the experimental results are compared with theoretical predictions in Sec. 5.3. Beside the analytical solutions of similaritons found for propagation in normal dispersive fiber sections, the solution of the master equation approach for the normal dispersion regime is also utilized. The classification of the results and the parallels to other resonator designs are carried out on this basis. The last section of this chapter discusses the necessity of decoupling amplification from pulse shaping. This design guide line was introduced in the first publications on fiber oscillators operating in the normal dispersion regime and limits the range of use to core-pumped ytterbium³⁺-doped fiber oscillators. It will be shown that a steady-state based on pulse shaping in gain fibers is possible and opens the way to advanced fiber designs and the transfer to other wavelengths.

The subject of chapter 6 is a hybrid mode-locking scheme for fiber oscillators based on two passive saturable absorber mechanisms constituting nonlinear temporal filters. After introducing the experimental setup in Sec. 6.1, the results are presented in Sec. 6.2 and Sec. 6.3. Beside a drastic enhancement of the self-starting capability, this mode-locking scheme offers the possibility to tune the spectral output characteristic. This mode-locking scheme was also utilized for an oscillator with a photonic bandgap fiber for dispersion control which is presented in Sec. 6.4. Due to the birefringence properties of this fiber, the formation of undesired modes of operation became dominant. Its suppression and the formation of stable pulses is also discussed in this section.

Chapter 7 presents experimental results of an erbium³⁺-doped fiber oscillator operating in the normal dispersion regime constituting a first attempt to transfer this operation regime to other wavelengths. After an introduction to some special properties of the gain medium, the experimental setup is described in Sec. 7.2. The output characteristics of this oscillator are presented in Sec. 7.3, followed by a discussion of the impact of intrapulse Raman-scattering on the pulse formation. This nonlinear process was found to be significant during pulse formation. Accounting for its influence and the differences to ytterbium³⁺-doped fibers, the operation regime is discussed in Sec. 7.5. The subject of the last section in this chapter is the limitation of the pulse energy imposed by the mode-locking mechanism. Although only observed in this setup, the discussion contains general aspects concerning further energy scaling of passively mode-locked (fiber) oscillators.

The thesis closes with a conclusion in chapter 8 and an outlook on possible future studies in chapter 9.

2 Pulse propagation in normal dispersive fibers

This chapter deals with some theoretical fundamentals for the description of fs-pulses in optical fibers. The first section provides a brief introduction into soliton solutions with a focus on the NDR. In the following, the main limitation for pulse propagation in the NDR called optical wave breaking is described. The analysis of the criterion for optical wave-breaking led to a new class of ultrashort pulses called similaritons which are described in Sec. 2.3. This chapter mainly contains a qualitative description of pulse propagation focusing on its importance for mode-locked oscillators.

2.1 Wave-equations and solitons

Nonlinear Schrödinger equation

The propagation of an ultrashort pulse in a nonlinear medium is covered by the nonlinear Schrödinger equation (NLSE). This nonlinear partial differential equation describes the changes of the slowly varying envelope $A(z,t)$ in a reference frame moving with the pulse at the group velocity v_g . The so-called retarded frame T is related to the real time t by $T = t - z/v_g$. The slowly varying envelope approximation (SVEA) necessary to obtain the NLSE assumes that the spectral width of the pulse is significantly smaller than the central frequency. For an ultrashort pulse with a central wavelength in the infrared, the SVEA is valid for pulse duration as short as 100 fs. Additional simplifying assumptions are included in the NLSE: first, the nonlinear contributions to the polarization are treated as a small perturbation to the linear ones which is valid as the refractive index changes are typically in the order of 10^{-6} . Second, the polarization state of the electric field is assumed to remain unchanged during propagation. Owing to the fact that the NLSE is a conservative equation, only propagation in lossless and non-amplifying media can be covered. A detailed description of the derivation of the NLSE starting from Maxwell's equations, the approximations used and their validities can be found in

Agrawal's textbook on nonlinear fiber optics [1].

In its simplest form the NLSE accounts for the effects of group-velocity dispersion (GVD) and Kerr-nonlinearity:

$$\frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} = i \gamma |A|^2 A. \quad (2.1)$$

For obtaining Eq. 2.1, the pulse amplitude A is assumed to be normalized, so that $|A|^2$ represents the optical power. Here, γ is a nonlinear parameter addressing Kerr-nonlinearities, namely its dominant contribution, the self-phase modulation (SPM)¹. SPM is the temporal analog of self-focussing and leads to an intensity dependent refractive index according to

$$n(\omega, I) = n_0(\omega) + n_2 |A|^2. \quad (2.2)$$

The nonlinear parameter γ is related to the nonlinear refractive index n_2 by

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}}, \quad (2.3)$$

where c is the speed of light, ω_0 the central frequency of the pulse and A_{eff} the effective core area. The parameter β_2 accounts for the GVD which is the lowest order of the Taylor series describing the frequency dependence of the refraction index around the carrier frequency ω_0 :

$$\beta(\omega) = \beta_0 + (\omega - \omega_0) \beta_1 + \frac{1}{2} (\omega - \omega_0)^2 \beta_2 + \frac{1}{6} (\omega - \omega_0)^3 \beta_3 + \dots, \quad (2.4)$$

where

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0}. \quad (2.5)$$

¹ Since SiO_2 is a symmetric molecule, contributions from the second-order susceptibility vanish in fused silica fibers, and third-order processes constitute the lowest-order nonlinearities.

In Eq. 2.4 the constant phase shift can be ignored and the second term corresponding to v_g is already included by the retarded time T . Beside the GVD, the third-order dispersion (TOD) might be of importance, too, depending on the situation under consideration [1].

Fundamental solitons

The NLSE can be integrated by the inverse scattering method leading to the well-known bright temporal solitons for propagation in the anomalous dispersion regime. Often expressed by dimensionless variables, the soliton solution has a hyperbolic secans shape:

$$A(z, T) = \sqrt{P_0} \cdot \text{sech} \left(\frac{T}{T_0} \right) \exp(ikz), \quad (2.6)$$

where k is the wave number given by $k = \frac{|\beta_2|}{\gamma T_0^2}$. For $\beta_2 < 0$, the quadratic phase contributions induced by GVD can be compensated by SPM as these two contributions are reciprocal. The pulse will adjust itself to fulfill this balance, thus solitons are inherently stable. This balance is reflected by the soliton area theorem given by

$$N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|}, \quad (2.7)$$

which relates the peak power P_0 to the pulse duration T_0 at a given GVD whereas N is the soliton number. The soliton number N assigns the relative importance of GVD and SPM effects. In this sense, N can also be used to describe non-solitonic pulses. For $N \ll 1$, GVD dominates whereas for $N \gg 1$, SPM dominates the pulse evolution. The relative importance can be treated by characteristic length scales over which the dispersive and nonlinear effects become important for the pulse evolution [1]. Eq. 2.7 becomes:

$$N^2 = \frac{L_D}{L_{NL}}, \quad (2.8)$$

where L_D is the dispersion length and L_{NL} is the nonlinear length. Both characteristic lengths depend on the parameters of the fiber and the pulse:

$$L_D = \frac{T_0^2}{|\beta_2|}, \quad (2.9)$$

$$L_{NL} = \frac{1}{\gamma P_0}. \quad (2.10)$$

For the discussion here, only the fundamental soliton with $N = 1$, where both effects are similar in strength, is of importance². Fundamental solitons propagate unchanged concerning envelope and phase and are chirp-free. The term soliton refers to the particle-like nature of these solutions.

The concept of solitons is widely used in optical systems especially in fiber optic communication as these pulses have some unique properties. Even when the initial conditions for a fundamental soliton is not fulfilled, a pulse will asymptotically evolve into a soliton. Energy can be exchanged with a non-solitonic background which allows to form a fundamental soliton for $0.5 < N < 1.5$. As a consequence, dissipative effects and perturbations can be ignored up to a certain level. A soliton can readjust itself to fulfill Eq. 2.7.

Dispersion-managed solitons

The energy coupling is determined by the phase velocities of the soliton and the non-solitonic background and hence by the GVD of the fiber. It can be suppressed by varying the GVD along the fiber section which avoids phase matching between the two components. For linear propagation, the full compensation of dispersion corresponds to a time reversal, so all GVD induced phase contributions vanish. The nonlinear contributions remain and are balanced by the residual group-delay dispersion (GDD) defined as the integral of the GVD over one periode of the dispersion map L:

$$\beta_2 L = \int_0^L \beta_2(z) dz. \quad (2.11)$$

Representing the GVD by $\beta_2(z)$, a dispersion map can be introduced to Eq. 2.1. Even if this equation is not integrable via the inverse scattering method, pulse-like and periodic

² High-order solitons, whose evolution is dominated by SPM are highly sensitive to perturbations (high-order dispersion and nonlinear as well as dissipative effects) and are difficult to stabilize in a resonator [2]. Properties of high-order solitons can be found in Ref. [1]

solutions have been found. They represent breathing pulses, oscillating concerning amplitude and width. Even their chirp varies along the dispersion map constituting the period. Although the shape is closer to a Gaussian, they can be regarded as “stroboscopic” solitons. The balancing between GVD and SPM is no longer local but averaged over one dispersion periode.

Solitons exist only at a certain power level which is given by Eq.2.7. Above that level, higher-order solitons, very sensitive to perturbations (especially high-order dispersion contributions), are excited and pulse break-up caused by soliton fission (split-up of higher-order solitons into multiple fundamental solitons) can occur [3].

Beside the formation of high-order solitons, modulation instability (MI) is another generic feature at anomalous dispersion which affects the stability of optical pulses especially at high power. MI refers to the exponential growth of a weak perturbation (amplitude or phase modulation) caused by the interplay of GVD and SPM. Cw-radiation breaks up into a train of pulses as gain is provided at sideband frequencies of ~ 0.2 THz [1]. MI occurs because its underlying four-wave mixing process is phase matched in the presence of anomalous dispersion causing an energy transfer to the MI-sideband frequencies.

MI constitutes the starting mechanism for soliton generation but also causes high-frequency ripples on the spectrum of a propagating pulse [4]. If noise, such as amplified spontaneous emission, copropagates, the pulse evolves randomly and is destabilized.

The destabilizing effects of soliton fission and MI are equivalent concerning the initial evolution of a pulse in the anomalous dispersion regime [5]. Both effects can be utilized for supercontinuum generation which relies on the break-up of a pump pulse and its subsequent spectral broadening. Their relative impact depends on the experimental situation and has been addressed in detail in Ref. [6]. These effects constitute the major instabilities for ultrashort pulses propagating at anomalous dispersion.

In contrast, at normal dispersion, phase matching of mixing processes can only be achieved by an extra degree of freedom. Therefore, MI does usually not occur during the propagation of pulses in normal dispersive fibers.

Gain-guided solitons

By expanding the NLSE, in order to account for dissipative effects like gain or loss, other classes of solutions can be found. In the context of mode-locked oscillators, the expansion

is of particular importance as lasers are highly dissipative systems. Mathematically, this can be simply realized by adding an additional linear term in Eq. 2.1. Depending on the experimental situation and required complexity, the gain/loss can be treated by various approaches. In its simplest form, the frequency dependence is treated by a parabolic gain approximation.

Only one of the exact solutions of this expanded NLSE found application in real optical systems and will therefore be described briefly. The stability of soliton solutions under the impact of a limited gain bandwidth was studied by Bélanger et al. in Ref. [7]. Assuming a homogeneously broadened gain, solutions with a hyperbolic secans shape similar to Eq. 2.6 can be obtained. The four parameters of the solution are determined by a set of equations restricting the parameter range for stable pulses. The fundamental soliton solution is recovered when an area theorem similar to Eq. 2.7 is fulfilled.

The so-called gain-guided solitons are weakly chirped pulses which are stable in the anomalous as well as in the normal dispersion regime. In the anomalous dispersion regime, the obtained solution underlies the same restrictions as mentioned above and will not be discussed further.

Solitons at normal GVD are at first astonishing as the phase behavior of GVD and SPM is cumulative resulting in temporal broadening during propagation. Gain-guided solitons are - compared to fundamental solitons - formed by a different mechanism. A limited gain spectrum (which is in practice always the case) provides a frequency-dependent amplification damping the high- and low-frequency wings. For a chirped pulse, spectral and temporal influences are equalized, so the pulse gets shortened. This effect can be balanced by GVD induced temporal broadening. In the frequency domain, gain narrowing of the pulse spectrum is balanced by the generation of new frequency components by SPM. This balance occurs locally which is reflected by the fact that gain-guided solitons are analytic solutions. Only these chirped solutions are stable in the presence of normal GVD.

In the context of this thesis one consequence derived in Ref. [7] is of particular interest. The solution is only valid if the pulse width is related to the gain bandwidth and the excess gain sustaining the pulse. Due to the competition between the solitary wave and cw-radiation, the stability relies on the last parameter because. This is basically the same restriction as for fundamental solitons when they are perturbed by gain and losses, respectively. From an experimental point of view, this indeed gives restrictions to the setup but allows to classify observed pulses.