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# Geometrical and Statistical Methods of Analysis of Star Configurations

## Dating Ptolemy's Almagest

A. T. Fomenko,
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# Preface

This book is devoted to a problem that lies at the crossroad of several sciences: statistics, geometry, celestial mechanics and computational astronomy, the problem of dating ancient star catalogs from an analysis of their contents, on the basis of modern knowledge of how the visible picture of the sky evolves with time. A vivid example is the problem of dating the star catalog of the famous Ptolemy's *Almagest*. The problem has a long and involved history; see a review of publications on the subject in the book of R. Newton[1].

The *Almagest* is traditionally attributed to Claudius Ptolemy (about the 2nd century AD). Yet, some investigations (mainly, the ones carried out in the 18th–19th centuries) revealed some contradictions between the astronomic data contained in the catalog and the astronomic reality of the 2nd century AD. This led to a hypothesis that Ptolemy had in fact used for the *Almagest* a star catalog compiled by Hipparchus (whose lifetime is traditionally attributed to the 2nd century BC), presumably having added some observations of his own. The reader can find a discussion of this hypothesis (and some others) in classical works[2,3]. A more recent book of R. Newton[1] presents a thorough statistical and astronomical analysis of the *Almagest* as a whole, and in particular of the star catalog it contains. R. Newton contends that his analysis gives an irrefutable proof of most observational data contained in the catalog being counterfeit. In any case R. Newton insists on the necessity of an overall revision of our views of the position and the role of the *Almagest* in the history of science. In fact, a similar conclusion and the inference that an essential redating of the *Almagest* is necessary had been suggested long before

R. Newton by N. A. Morozov in his fundamental book *History in the Light of Natural Sciences*, published in 1928–1932 under the title *Khristos* (*Christ*) (see Ref. 4). It should be noted that the astronomical and mathematical arguments of N. A. Morozov are diverse from the ones of R. Newton, but they lead to a similar conclusion about the necessity of a revision of the traditional views of the *Almagest*. A lot of additional criticism on the subject can be found in the cycle of works of A. T. Fomenko[5–13], devoted to the development of new empirico-statistical methods for detecting dependent narrative texts and for dating the events they describe (in particular, astronomic events).

We stress, however, that the investigations we expose in this book are completely independent of the methods and arguments used in the aforementioned works and that we do not use the hypotheses suggested therein.

In this book we suggest a new method for dating ancient star catalogs. The method uses, in particular, the investigation of proper motions of stars. Since these motions are now measured with a very high accuracy (on the basis of astronomic observations of the last two centuries), it is possible to compute the positions of stars in the past. Comparing these with the ones indicated in a star catalog, we can try to determine the time when the observations were made, and consequently the approximate time of compilation of the catalog. However, a practical implementation of this seemingly simple idea encounters major difficulties, both of technical and fundamental nature. Coping with these difficulties requires the new statistico-geometrical method we present in this book. The foundations of the method have been exposed in Refs. 14 and 15. Our approach involves both statistical and geometrical ideas; the latter are necessary because of the geometrical nature of the object we deal with, the evolution of a point set (the set of stars) in the celestial sphere.

We have tested the method on some reliably dated medieval star catalogs, and also on some artificially created catalogs. In the latter case the catalogs were compiled with the help of a computer; of course, the compiler knew the "date of compilation", but the researcher did not. The date was sealed in an envelope to be unsealed only after getting a date from the method. The procedure proved the efficiency of the method: the "date of compilation" was always within the interval it produced.

Then we applied the method to the star catalog of the *Almagest*. The results thus obtained contradict the traditionally accepted date and imply the necessity of its considerable "rejuvenating".

The main body of this book does not involve any historical questions or questions concerning the origins of the data. Thus, we concentrate on the contents of the star catalog itself, and do not even raise any questions concerning the rest of the *Almagest* (the star catalog constitutes the seventh and the eighth books of the *Almagest*).

However, for the reader's convenience, we have supplemented the book with the *Addendum* containing an exposition of some problems and conjectures on dating the *Almagest* as a whole. We should stress once more that the main body of the book is entirely independent of the *Addendum*. The

*Addendum* is intended for a reader wishing to proceed with the study of the questions we raise in the main body of the book toward understanding the origins of the data. A reader interested in mathematical and astronomical aspects alone may confine himself to the main body of the book.

The structure of the book is the following.

The *Introduction* provides a brief review of the contents of the *Almagest*, and in particular of its star catalog. We also give a brief review of other star catalogs and explain our interest in the problem of dating catalogs.

*Chapter 1* provides some necessary information from astronomy, astrometry and history of observational equipment and methods for measuring coordinates of stars.

In *Chapter 2*, we carry out a preliminary analysis of the star catalog of the *Almagest*. We discuss here various problems that arise in connection with the catalog (for example, the ambiguity in identification of stars), the accuracy of altitudes and longitudes in the catalog, and some peculiarities of the catalog (such as the Peters' sine curve).

In *Chapter 3*, we analyze some attempts to date the star catalog of the *Almagest* based on the most obvious ideas. We show that no straightforward elementary methods lead to a reliable date, and reveal the difficulties behind these failures.

In *Chapter 4*, we start the description of our new method for dating star catalogs. Here we discuss the "Who is who?" problem, the problem of identification of the stars described in the catalog with the ones known in modern astronomy.

*Chapter 5* presents mathematical backgrounds for the statistical analysis of the catalog. Here we classify various errors that occur in the catalog, and suggest methods for their detection and for compensation for the systematic component.

In *Chapter 6*, we carry out a global statistical processing of the catalog and of its basic parts. We apply several statistical characteristics to various pieces of the celestial sphere, which enables us to distinguish the "well-measured" and "poorly measured" pieces. The ensuing decomposition of the sky into the "homogeneous areas" (with contrasting accuracy of measurement) implies a new view of the structure of the *Almagest*.

In *Chapter 7*, we apply two different dating procedures, statistical and geometrical, to the catalog of the *Almagest*; the two estimates turn out to agree.

In *Chapter 8*, we suggest an explanation for the "Peters' sine curve", based on the previous results; we also discuss here the value of the angle between the equatorial plane and the ecliptic given in the *Almagest*.

In *Chapter 9*, we apply our method to the catalogs of Tycho Brahe, Ulugh Beg, Hevelius and Al Sûfi (As-Sûfi).

*Chapter 10* is devoted to determination of the date using other parts of the *Almagest*. The ensuing results demonstrate perfect agreement with our date for the star catalog. Finally, we obtain the period of time that captures the observations fixed in the *Almagest* (500–1350 AD), and reconstruct the "Ptolemaic chronology", that is, Ptolemy's concepts of global chronology (nowadays concealed by the erroneous tradition of recalculating Ptolemy's dates into the years AD). It turns out that similar concepts can be found in several sources of the 13th–14th centuries. Thus, the *Almagest* keeps to a chronological tradition, nowadays forgotten, but actual in the 13th–14th centuries, which differs much from the chronology we are used to today.

The book is concluded by the *Addendum*, containing a brief review of problems connected with dating the *Almagest* as a whole. We treat this material as supplementary, and do not use it in the main body of the book, although it is probably of some epistemological interest.

The book is supplemented with tables containing some astronomic data we use in the text.

The book contains a lot of material represented in tabular and graphical form. We call reader's attention to figures and graphs, which contain much important information, necessary for a fuller understanding of the book. We number the figures and tables consecutively within every chapter and chapter number precedes the number of the figure. Thus "see Figure 2.1" means "see Figure 1 in Chapter 2".

We use the techniques of mathematical statistics, modern geometry, celestial mechanics and astrometry. Therefore some chapters require an acquaintance with basic mathematical notions. Yet, we tried to make the mathematics we used as simple as possible, and we hope that this book will be accessible to a reader familiar with the elements of mathematics at the level of a second-year student in mathematics. The book is intended not only for specialists in natural sciences, but also for the historians interested in modern mathematical and statistical methods. See also the book: A. T. Fomenko, *Empirico-Statistical Methods for Analysis of Narrative and Numerical Sources with Applications to the Problems of Ancient and Medieval History and Chronology*, vols. 1,2. Kluwer Acad. Publ. (in print).

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# Contents

# Introduction

## 1. Brief description of the *Almagest*

The *Almagest* is a famous work of an Alexandrite astronomer, mathematician and philosopher Claudius Ptolemy, whose lifetime is traditionally attributed to the 2nd century AD. We give some information about Ptolemy below; it should be noted, however, that "history treated somewhat strangely the person and the works of Ptolemy. Historians of his time never mention his life and activities . . . . No facts of his life, neither the dates of his birth and death are known" (Ref. 16, p. 96).

It is traditionally considered that the *Almagest* was created in the reign of Roman emperor Antoninus Pius (131–161 AD).

The *Almagest* contains 13 books, about 1000 pages in total volume (in modern editions).

The first book contains basic concepts and constructions, of which the following should be mentioned: 1) The firmament is spherical, and rotates like a sphere; 2) The earth is a sphere, disposed at the center of the universe; 3) The earth can be considered as a point in comparison with the distances to the sphere of fixed stars; 4) The earth does not alter its position in space (does not move). As Ptolemy notes, these principles are based on the conclusions of Aristotle's philosophy. Further, the first and the second books contain an exposition of elements of spherical astronomy (theorems on spherical triangles, a method for calculating arcs (angles) from the lengths of their spans, etc.).

The third book presents a theory of visible solar motion and a discussion of the dates of equinoxes, the length of the year, etc. The fourth book treats the length of the synodic month and the theory of lunar motion. The fifth book is devoted to the construction of some astronomic instruments and to a further development of the theory of the moon. The sixth book exposes a theory of solar and lunar eclipses.

The famous star catalog (comprising more than 1000 stars) is contained in the seventh and eighth books of the *Almagest*. The books contain the catalog and a discussion of properties of fixed stars, of motion of the celestial sphere, etc.

The last five books of the *Almagest* are devoted to the theory of motion of planets (Ptolemy considers five planets, Saturn, Jupiter, Mars, Venus and Mercury).

## 2. A brief review of the history of the *Almagest*

It is commonly accepted that the *Almagest* was created in the reign of Antoninus Pius (131–161 AD) and that the last observation included therein had been made on February 2, 141 AD. The Greek title of the *Almagest*, *μαθηματικη συνταξις*, implies that the *Almagest* exposes the state-of-the-art of contemporary Greek astronomy. Nowadays it is not known whether any other astronomical treatises comparable to the *Almagest* existed at the time. Usually, the tremendous success of the *Almagest* (with astronomers, as well as with other scientists) is attributed[17] to the loss of most of the astronomic treatises of the time. The *Almagest* had become the basic textbook in astronomy (as is considered nowadays) for more than a thousand years. It influenced greatly the late medieval astronomy, both in Islamic and Christian regions, up to the 16th century. The influence of this book might be only compared to the influence of Euclid's *Elements* on the medieval science.

As noted, for example, by Toomer (Ref. 17, p. 2), it is extremely difficult to trace the history of the *Almagest* and its influences from the 2nd century AD to the Middle Ages. Commentaries of Pappus and Theon of Alexandria are the usual source for judgment on the role of the *Almagest* as a standard textbook in astronomy for "advanced students" in the schools at Alexandria in late antiquity. Further, a "period of darkness" comes. We will only note here the following description of this period: "After the exciting blossoming forth of antique culture, on the European continent a long period of stagnation, sometimes even of regress, began, usually referred to as Middle Ages . . . Over more than 1000 years not a single essential discovery in astronomy was made . . . " (Ref. 17, p. 73).

Furthermore, it is believed that in the 8th and the 9th centuries, in connection with growing interest in Greek science in the Islamic world, the *Almagest* was "raised from the darkness" and was translated, first into Syrian and later, several times, into Arabic. By the middle of the 12th century at least five versions of the translations existed. It is presumed that while in the East (in particular, in Byzantium), the work of Ptolemy, originally written in Greek, was being copied and, to some extent, studied, "all knowledge of it was lost to western Europe by the early middle ages. Although translations from the Greek text into Latin were made in medieval times, the principal channel for the recovery of the *Almagest* in the west was the translation from the Arabic by Gerard of Cremona, made at Toledo and completed in 1175. Manuscripts of the Greek text (of the *Almagest* — *Authors*) began to reach the west in the fifteenth century, but it was Gerard's text which underlay (often at several removes) books on astronomy as late as the Peurbach-Regiomontanus epitome of the *Almagest*... It was also the version in which the *Almagest* was first printed (Venice, 1515). The sixteenth century saw the wide dissemination of the Greek text (printed at Basel by Hervagius, 1538), and also the obsolence of Ptolemy's astronomical system, brought about not so much by the work of Copernicus (which in form and concepts is still dominated by the *Almagest*) as by that of Brahe and Kepler" (Ref. 17, pp. 2–3).

## 3. Basic medieval star catalogs

*The catalog of the Almagest is the only extant antique star catalog*; it is traditionally dated about the 2nd century AD. It is considered, however, that Ptolemy used the star catalog compiled by his predecessor Hipparchus about the 2nd century BC. The *Almagest* catalog (as well as other catalogs of later origin) comprises about 1000 stars, whose positions are described in terms of their longitudes and latitudes (see below for details). After Ptolemy, the "period of darkness and regress" in the history of astronomy (and in the history of all natural sciences) begins, and we know of no other star catalogs up to the 10th century. Finally, only as late as in the 10th century (according to the traditional chronology) was the first medieval catalog created, the one composed by Arabic astronomer As-Sûfi (Abd Al Rahman Al Sûfī, 903–966) in Baghdad. This catalog has come down to us. The next at our disposal is the Ulugh Beg star catalog (1394–1449, Samarkand). The three catalogs are not very precise: they indicate the coordinates of stars to an accuracy within 10 minutes of arc. The next extant is the famous catalog of Tycho Brahe (1546–1601), the precision of which is an order of magnitude better than that of the three preceding catalogs. Brahe's catalog is the acme of skill reached with the help of medieval methods and instruments for astronomical observations. We stop our enumeration here and do not list the catalogs created after Tycho Brahe (there were many, and they are of no interest to us here).

## 4. Why dating star catalogs is interesting

Every star catalog comprising about 1000 stars is a result of a lot of observations made by an astronomer (even more likely, by a group of professional observers), which required much effort, thoroughness and professionalism, and also an utmost use of available measuring instruments, which were made at the highest contemporary level. Moreover, a catalog required a proper astronomic theory, a world view. Thus, every ancient catalog is a focus of the astronomic mind of the age. So, analyzing a catalog we can learn much about the available accuracy of measurement, the astronomic ideas of the time, etc.

But to understand properly the results of the analysis, we need to know the time when the catalog was compiled. Any variation in the date alters automatically our estimates and conclusions about the catalog. Meanwhile, to determine the date of compilation of a catalog is far from easy. This is very well seen in the case of the *Almagest*. First (in the 18th century), the traditional version attributing the catalog to Ptolemy, about the 2nd century AD, was indisputably accepted. In the 19th century, a more thorough analysis of longitudes of stars indicated in the *Almagest* showed (we describe the details below) that they are more likely to belong to the 2nd century BC, that is, to the time of Hipparchus.

The catalog, contained in the seventh and the eighth books of the *Almagest*, comprises 1028 stars (three of which are duplicates). It contains not a single star that could be observed by Ptolemy from Alexandria, but not by Hipparchus from Rhodes. Moreover, Ptolemy claimed that he had determined, from comparison of his observations with the ones of Hipparchus and others, the magnitude of precession 38′ (which is erroneous), treated by Hipparchus as the least possible value, and by Ptolemy, as the final estimate. The positions of stars as indicated in Ptolemy's catalog are nearer to their real positions in the time of Hipparchus, with the purported 38′ yearly correction, than to their real positions in the time of Ptolemy. So, it looks very likely that the catalog is not a result of Ptolemy's own observations, but the catalog of Hipparchus, corrected for precession, with a few alterations from observations of Ptolemy or other astronomers (see Ref. 2, pp. 68–69).

Thus, in this case the date of compilation of the catalog acquires a paramount importance. For several centuries astronomers and historians of astronomy analyzed the catalog (and the *Almagest* in the whole) trying to "sort" the data contained therein to separate the observations of Hipparchus from the ones of Ptolemy. A lot of literature is devoted to this dating problem. We do not dwell on a review of this literature here; an interested reader will find a guide thereto in Ref. 1.

In this book we consider the question: Is it possible to create a method for dating star catalogs "intrinsically", that is, using only the numeric information contained in the coordinates of stars indicated in the catalog? Our answer is YES. We have worked out such a method, tested it on several reliably dated catalogs and applied it, in particular, to the *Almagest*. The reader will learn our results from this book.

# Part One
# Preliminary Analysis

# Chapter 1
# Some Concepts of Astronomy and History of Astronomy

## 1. Ecliptic, equator and precession

Let us consider the orbital motion of the earth around the sun. It is conventional to treat this motion as the motion of the so-called *barycenter*, the mass center of the system earth-moon. The barycenter is about six thousand kilometers from the center of the earth (hence under the earth's surface). This distance is unessential for our further treatment, so we will make no distinction between the motion of the earth and the motion of the barycenter. Gravitational pull from other planets brings about steady rotation of the orbital plane of the barycenter. The principal sinusoidal component of the rotation has a very large period, and in small intervals of time may be treated as linear. The real motion is the sum of this component with minor oscillations, which we will neglect. The rotating plane that contains the orbit is called the *ecliptic plane*. The circumference where the ecliptic plane meets the *sphere of fixed stars* is called *the ecliptic*. We assume that the center of the sphere of fixed stars $O$ lies in the ecliptic plane (Figure 1.1). Since the ecliptic moves, it is called the *moving ecliptic*. The position of the ecliptic at a given moment of time is called the *instantaneous ecliptic*. For example, we can speak of the *instantaneous ecliptic of January 1, 1900*. It should be clear that we can use any fixed instantaneous ecliptic as a frame of reference for other ecliptics.

Celestial mechanics usually treats the earth as a *rigid body*. A rotation of a rigid body is usually described in terms of its *moment ellipsoid*, determined by its axes, called the *axes of inertia*. A particular rotation of a rigid body is

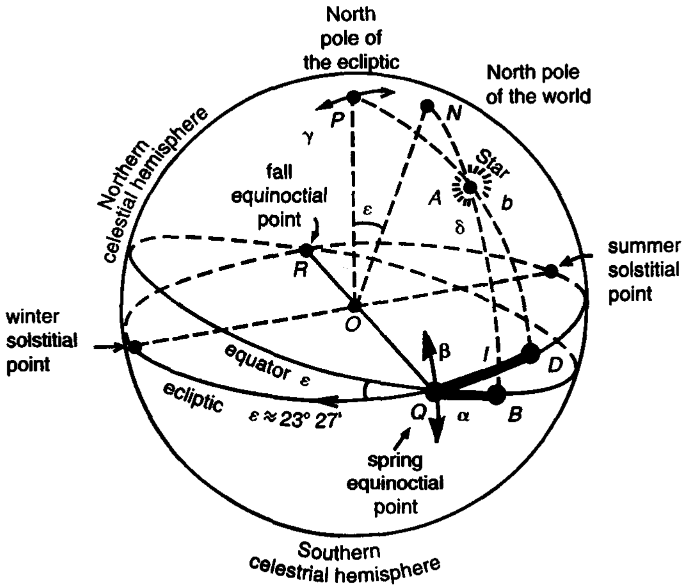


**Figure 1.1.** The sphere of fixed stars, with the ecliptic and the equatorial coordinate systems.

characterized by the *vector of angular velocity* $\omega$, sometimes called the *instantaneous rotation axis* of the body (in our case, of the earth). Since the axes of inertia $A$, $B$, $C$ ($A > B > C$) are orthogonal, we can use them as the axes of a rectangular coordinate system. Now we can consider the projections $x$, $y$, $z$ of the vector $\omega$ on the axes $A$, $B$ and $C$ as the coordinates of $\omega$. The rotation of a rigid body can now be described by the *Euler-Poisson equations*:

$$
\begin{aligned}
&A\dot{x} + (C - B)yz = M_A \\
(1)\qquad &B\dot{y} + (A - C)xz = M_B \\
&C\dot{z} + (B - A)xy = M_C
\end{aligned}
$$

where $M_A$, $M_B$, $M_C$ are the projections on the axes of a vector $M$, called the *moment of outer forces about the barycenter*. The moment $M$ is mainly due to the gravitational pull of the sun and the moon on the ellipsoid that is the earth. Usually, the earth is assumed to be an ellipsoid of revolution (that is, the greater semiaxes $A$ and $B$ are assumed equal). The position of $M$ with reference to the axes $A$, $B$ and $C$ varies with time very fast and in a very complicated way; however, modern theories of lunar and solar motion enable us to compute it to a high accuracy for any moment of time. Consequently, we can solve the Euler-Poisson equations, thus determining the evolution of $\omega$. Usually, the *Tables of the Motion of the Earth on its Axis and around the Sun* by the well-known astronomer S. Newcomb[19] are used to take into account all irregularities of the motion. A study of the Euler-Poisson equations from the point of view of the existence of *exact* solution constitutes an important field of modern theoretical mechanics, physics and geometry; see a short review hereof, for example, in Ref. 20.

The vector of instantaneous angular velocity of the earth $\omega$ determines the (instantaneous) *axis of rotation*. The points where the axis of rotation pierces the surface of the earth are called the *instantaneous poles of the earth*, and the points where the axis meets the celestial sphere (the sphere of fixed stars) are called the (North and South) *poles of the world* (Figure 1.1). The intersection of the plane through the center of the earth perpendicular to the axis of rotation with the surface of the earth is called the (instantaneous) *equator*, and its intersection with the celestial sphere is called the (*true*) *equator of the celestial sphere*.

Let us now consider a coordinate system that does not rotate together with the earth, for example, the one associated with the ecliptic. Conventionally, the following axes are used as coordinates in this system: the normal to the ecliptic plane, the axis where the ecliptic plane intersects the equatorial plane (the *equinoctial axis*) and the axis of inertia $C$. The projections of $\omega$ on the three axes are denoted by $\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$. Thus, we have expanded the velocity of rotation of the earth into three components. What is their geometrical sense? $\dot{\psi}$ is called the *velocity of precession of the earth*. It characterizes the motion of the axis of precession $C$ (the third axis of inertia) along a circular cone about the normal $OP$ (see Figure 1.2); thus, the vector $\omega = ON$ moves along the same cone. Note that the axes $\omega$ and $OC$ are very close to each other, so in calculations that do not require high accuracy we may assume that the vector $\omega$ is parallel to $OC$. Because of the precession, the equinoctial axis rotates in the ecliptic plane.

The component $\dot{\theta}$ characterizes variation of the angle $\theta$ the axis $OC$ makes with the ecliptic plane. As for $\dot{\varphi}$, it determines the velocity of the earth's rotation about the axis $OC$; in theoretical mechanics this magnitude is called *the velocity of proper rotation*. This velocity is much greater than $\dot{\psi}$ and $\dot{\theta}$. From the point of view of theoretical mechanics, this reflects the principle according to which a rotation of a rigid body is stable when its axis is close to the axis of the greatest moment of inertia, that is, to the shortest axis of the ellipsoid of inertia.

Thus, $\omega = \dot{\psi} + \dot{\theta} + \dot{\varphi}$ where + stands for summation of vectors. Each of $\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$ is the sum of a constant (or almost constant) component and many minor periodic summands, called *nutations*. Neglecting nutations, we come to the following picture of rotation of the earth.

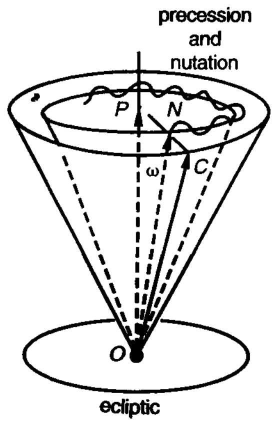


**Figure 1.2.** Trajectory of motion of the earth's precession.

1) The (almost) constant component of $\dot{\psi}$ is called the *longitudinal precession*; it moves the axis $OC$ uniformly along a circular cone (see Figure 1.2) at the rate of approximately 50″ per year; the equinoctial axis rotates in the ecliptic plane clockwise if looked at from the North pole of the ecliptic. The vector of precession is directed toward the South pole of the ecliptic.

2) The constant component of $\dot{\theta}$ is now approximately equal to 0.5″ per year.

3) The constant component of $\dot{\varphi}$ is the *mean proper rotation of the earth* about the axis $OC$, anticlockwise if looked at from the North pole of the earth; the period of rotation is 24 hours.

Note that the axis $OP$ (the normal to the ecliptic plane), the vector $\omega$ (the instantaneous angular velocity of the earth) and the axis $OC$ lie in the same plane. The precession turns this plane about the axis $OP$.

Nutational addends in $\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$ distort the above picture of rotation. Therefore the vector $\omega$ moves not along an ideal circular cone, but along a "wavy" surface near the cone (Figure 1.2). In Figure 1.2, the trajectory of the endpoint of $\omega$ is depicted by a wavy line. Two circumferences in the celestial sphere, the ecliptic and the equator, meet at the angle $\varepsilon \approx 23°27'$ at points $Q$ and $R$ (Figure 1.1). These are the points where the sun passes the equator in its yearly motion along the ecliptic. The point $Q$, where the sun enters the Northern hemisphere, is called the *spring equinoctial point* (when the sun is at this point, day and night have equal length all over the surface of the earth). The point $R$ is the *fall equinoctial point* (Figure 1.1). As the moving ecliptic turns, *the spring equinoctial point moves steadily along the equator (shifting simultaneously along the ecliptic)*. The rate of this motion of the equinoctial point along the ecliptic is exactly the longitudinal precession. The shift of the equinoctial points thus produces a shift of dates of equinoxes (Figure 1.1).

## 2. Equatorial and ecliptic coordinates

Recording observations of heavenly bodies requires a convenient coordinate system. Several coordinate systems are used to that end. The *equatorial coordinate system* is defined as follows. Figure 1.1 shows the North pole $N$ and the celestial equator, containing the arc $QB$. We may assume with sufficiently high accuracy that the plane of the celestial equator contains also the earth's equator; furthermore, we assume that the center of the earth coincides with the center $O$ of the celestial sphere; $Q$ is the spring equinoctial point. Let $A$ be a fixed star and $NB$ the meridian through the North pole and $A$; here $B$ is the point where the meridian meets the equatorial plane. The arc $QB = \alpha$ is the *equatorial longitude* of the star $A$, also called *the direct ascent* of the star. The ascent is counted in the direction opposite to the one of the motion of the spring equinoctial point $Q$. Consequently, due to precession, *the direct ascents of stars slowly increase with time*. The arc $\delta$ of the meridian $AB$ in Figure 1.1 is called the *equatorial latitude*, or the *declination* of the star $A$. If we neglect oscillations of the ecliptic, *the declinations of stars in the Northern hemisphere slowly decrease with time* (because of the shift of the spring equinoctial point), *and the declinations of stars of the Southern hemisphere slowly increase*. The diurnal rotation of the earth does not affect declinations, and the direct ascents vary uniformly at the velocity of the earth's rotation.

Another frequently used system (especially in ancient catalogs) is the *ecliptic coordinate system*. Consider the celestial meridian through the pole of the ecliptic $P$ and the star $A$ (Figure 1.1). The meridian meets the ecliptic plane at a point $D$. The arc $QD$ in Figure 1.1 is the *ecliptic longitude* $l$, and the arc $AD$ is the *ecliptic latitude* $b$ of $A$. Because of precession, the arc $QD$ increases with time (at the rate of about $1^\circ$ per century), so *the ecliptic longitudes uniformly increase with time*. If we neglect oscillations of the ecliptic, we can assume to a first approximation that the ecliptic latitudes $b$ do not vary with time. This circumstance made the ecliptic coordinates popular among medieval astronomers. The advantage of the ecliptic coordinates over equatorial is for uniform (and easily computable) variation of $l$ and the constancy of $b$. As for the variations of equatorial coordinates generated by precession, they are described by more complicated formulas (taking into account the turn through the angle between the equator and the ecliptic). This is the reason why medieval astronomers chose to compile their catalogs in ecliptic coordinates, despite the fact that equatorial coordinates are easier to measure from observations. The disappointing discovery of oscillations of the ecliptic brought about the use of equatorial coordinates in modern catalogs.

## 3. Methods of measuring equatorial and ecliptic coordinates

Here we dwell for a while on a brief description of concrete measurements of equatorial and ecliptic coordinates. We will describe a simple geometrical idea that underlies such measuring instruments as *quadrant, sextant, meridian circle*, etc.

Suppose the vantage point $H$ is at the latitude $\varphi$ on the surface of the earth (Figures 1.3 and 1.4). It is not difficult to determine the straight line $HN'$ towards the North pole of the world (the line parallel to $ON$). Find the meridian through $H$ and erect a vertical wall along the meridian (Figures 1.3, 1.4). If we draw on the wall the ray $HN'$, we can also find the equatorial line $HK'$ parallel to $OK$, lying at a right angle off $HN'$. Sectoring the right angle into degrees of arc, we get an astronomic goniometer. The idea of this instrument underlies modern meridian instruments. The instrument can be used to measure declinations of stars, i.e., their equatorial latitudes, and to fix the moments when the stars pass the meridian. Since we can determine the equatorial plane (at a given latitude of the vantage point) with a sufficiently high accuracy from a series of consequent independent observations, this instrument enables us to measure declinations with a fairly high accuracy. Meanwhile, as can be seen from the above description of elementary concepts of celestial mechanics, *measuring longitudes requires fixing moments of stars' passing the meridian, for which we need either a sufficiently precise clock, or an additional instrument for fast measurement of longitudinal distance between the star and the meridian.* In any case, measurement of longitudes is a much more complicated operation, so it looks likely that medieval astronomers measured direct ascents with much lower accuracy than declinations.

To determine ecliptic coordinates, the observer $H$ must first determine the position of the ecliptic in the sky. This nontrivial procedure requires a fair knowledge of the geometry of basic elements of motion of the earth and the sun. Some ancient methods for determination of inclination of the ecliptic to the equator and for finding the position of the equinoctial axis are described in Ref. 1. It is important to note that an immediate measurement of ecliptic coordinates of stars is impossible unless we have a clockwork able to compensate for the rotation of the earth and to keep fixed the direction towards the equinoctial point. The obvious difficulty of this problem made the astronomers as they calculated ecliptic coordinates either use the formulas for the turns of the celestial sphere, or celestial globes carrying frames both of equatorial and ecliptic coordinates, thus making it possible to recalculate immediately. Of course, this procedure inevitably added errors originating in determination of the position of the ecliptic in relation to the equator and to the equinoctial axis.

The above brief discussion of methods of measuring ecliptic coordinates leads to the conclusion that the following algorithm was used:

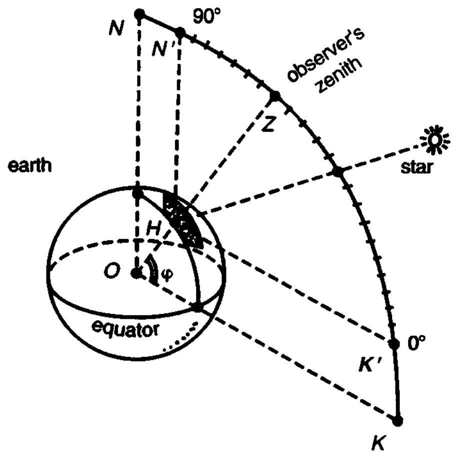


**Figure 1.3.** The procedure of measurement of equatorial latitude of a star with the help of a meridian circle (1).

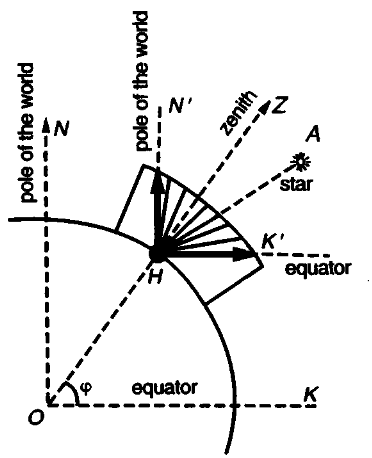


**Figure 1.4.** The procedure of measurement of equatorial latitude of a star with the help of a meridian circle (2).

1) Find equatorial coordinates (latitudes were determined with a higher accuracy than longitudes).
2) Calculate the position of the ecliptic and the equinoctial axis in relation to the equator.

3) Recalculate equatorial coordinates into ecliptic with the help of trigonometric formulas, or an instrument, or a double-framed celestial globe.

Furthermore, since all observational instruments were earthbound, the above algorithm is the only realistic way of finding ecliptic coordinates the medieval astronomers could use. The fact that the observational instrument is attached to the surface of the earth and hence shares the earth's rotation means that the instrument is bound to the equatorial coordinate system.

Below, we will get a confirmation for the assumption that the above algorithm (or a similar procedure) was used for the star catalog of the *Almagest* from our statistical analysis.

## 4. Modern starry sky

1. If we want to date an ancient or medieval star catalog from the coordinates of stars it contains, we must be able to compute positions of stars at various moments of time in the past. The starting point is the now existing starry sky. We will only be interested in coordinates of stars, their proper velocities and their *star magnitudes*, that characterize visible brightness (the less the star magnitude, the brighter is the star). Star magnitudes are indicated in the most ancient catalogs. In particular, the *Almagest* indicates magnitudes for all stars it contains. The scale it uses matches in general with the one now in use, but modern catalogs indicate fractional values of the magnitudes. For example, Arcturus, which has magnitude 1 in the *Almagest*, has magnitude 0.24 in modern catalogs[21]; Sirius, also having magnitude 1 in the *Almagest*, has magnitude $-1.6$ (negative) in modern catalogs. Thus, Sirius is brighter than Arcturus, while Ptolemy considered them as equally bright. In the Middle Ages, the brightness (star magnitude) was judged by eye. The color of the star, the brightness of nearby stars and other factors influenced the result. So, star magnitudes were determined rather roughly. Nowadays star magnitudes are measured with the help of photometry. A comparison of the *Almagest*'s star magnitudes with modern precise values shows[22] that the difference usually does not exceed two units. We used the catalog[21], comprising about nine thousand stars up to the eighth star magnitude. Recall that only sixth to seventh magnitude stars are visible to the unaided eye, and the catalog of the *Almagest*, as Ptolemy claims, contains all stars of the visible part of the sky up to the sixth magnitude. In fact, though, there are many more stars of sixth and lesser magnitudes in the visible sky than in Ptolemy's catalog. This is one of the causes of ambiguities that arise in attempts to identify the stars in the *Almagest* with the stars in modern catalogs (computed back to the past).

The astronomer of the 17th century I. Bayer suggested to denote stars in a constellation by Greek letters: the brightest star is denoted by $\alpha$, the second in brightness by $\beta$, and so on. For example, $\alpha$ Leo is the brightest star in the constellation Leo. Later on, J. Flamsteed (1646–1720) assigned to stars the numbers (in the constellation): the westernmost star acquired number 1, the next to the east number 2, and so on. The Flamsteed number and the Bayer letter are usually written together in the denotation of a star, for example, 32 $\alpha$ Leo. Furthermore, a star can have a proper name. There are comparatively few "named" stars; the names were only given to the stars which had special significance in antique and medieval astronomy. For example, 32 $\alpha$ Leo has the proper name *Regul* (*Regulus*).

We used the following characteristics of stars from Ref. 21:

1) Direct ascent of the star in 1900, denoted by $\alpha_{1900}$ and measured in hours, minutes and seconds.

2) Declination of the star in 1900, denoted by $\delta_{1900}$ and measured in degrees, minutes and seconds of arc.

3) Velocities of the proper motion of the star in declination and in ascent, that is, the projections of the velocity of proper motion on the equatorial coordinate axes in 1900.

The velocities of proper motions of stars are rather small; as a rule, they do not exceed 1″ per year, and the fastest stars visible by unaided eye ($o^2$ Eri, $\mu$ Cas) move at the rate of about 4″ per year. In the interval of time we are interested in, about two to three thousand years long, the proper motion may be assumed uniform in each coordinate in a fixed coordinate system. For us, this coordinate system is the equatorial coordinate system of 1900. For the reader's convenience, we adduce in the *Appendix* two lists of characteristics of stars taken from Ref. 21. Table Ap. 1 is the list of fast stars. It contains all stars whose proper motion in at least one of the coordinates $\alpha_{1900}$, $\delta_{1900}$ is not less than 0.5″ per year. Table Ap. 2 is the list of named stars. The two tables have a common part: some named stars have a notable proper motion; such stars are especially useful for dating purposes (see below).

## 5. Computation of the starry sky to the past. Catalogs $K(t)$. Newcomb's theory

1. Having at our disposal the coordinates and the velocities of proper motion of stars in our time, we can calculate a precise catalog for an arbitrary epoch. We had to do this many times and for various epochs as we investigated the *Almagest* and other ancient catalogs. Compiling these "theoretical" catalogs, we first computed the positions of stars at the year $t$ in coordinates $\alpha_{1900}$ and $\delta_{1900}$, and then recalculated into ecliptic coordinates $l_t$ and $b_t$ for the year $t$. Below, we give the necessary formulas making it possible to take into account the precession and, in particular, to recalculate from $\alpha_s$, $\delta_s$ into

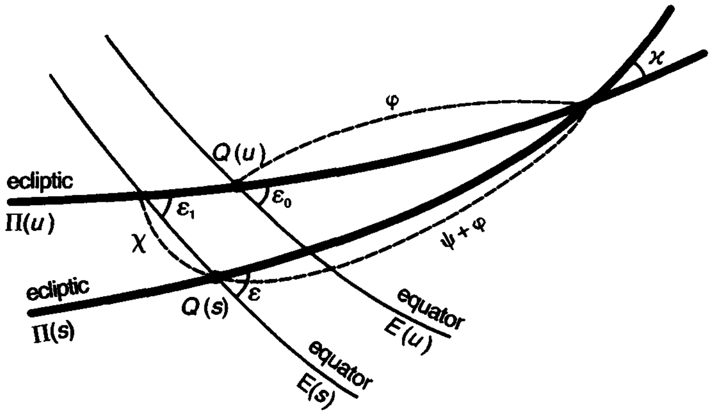


**Figure 1.5.** Relations between ecliptic and equatorial coordinates in various epochs.

$l_u$, $b_u$ for any two epochs $s$ and $u$. These formulas, as well as Figure 1.5 are taken from Ref. 23. They were obtained on the basis of a theory of Newcomb, modified by Kinoshita. The procedure of recalculating coordinates is described in Subsection 2 below. In the formulas, we assume that the epochs $u$ and $s$ are counted in Julian centuries from 2000 AD, and that $\theta = u - s$ (see Figure 1.5).

(1)
$$\begin{aligned}\varphi(s,u) &= 174^\circ 52' 27''\!.66 + 3289''\!.80023u + 0''\!.576264u^2 \\ &\quad - (870''\!.63478 + 0''\!.554988u)\theta + 0''\!.024578\theta^2\end{aligned}$$

(2)
$$\begin{aligned}\varkappa(s,u) &= (47''\!.0036 - 0''\!.06639u + 0''\!.000569u^2)\theta \\ &\quad + (-0''\!.03320 + 0''\!.000569u)\theta^2 + 0''\!.000050\theta^3\end{aligned}$$

(3)
$$\begin{aligned}\varepsilon(s,u) &= 23^\circ 26' 21''\!.47 - 46''\!.81559u \\ &\quad - 0''\!.000412u^2 + 0''\!.00183u^3 \\ &\quad + (-46''\!.8156 - 0''\!.00082u + 0''\!.005489u^2)\theta \\ &\quad + (-0''\!.00041 + 0''\!.005490u)\theta^2 + 0''\!.001830\theta^3\end{aligned}$$

(4)
$$\begin{aligned}\varepsilon_0(s,u) &= 23^\circ 26' 21''\!.47 - 46''\!.81559u \\ &\quad - 0''\!.000412u^2 + 0''\!.00183u^3\end{aligned}$$

$$\begin{aligned}\varepsilon_1(s,u) = 23^\circ 26' 21''\!.47 - 46''\!.81559u \\ - 0''\!.000412u^2 + 0''\!.00183u^3 \\ + (0''\!.05130 - 0''\!.009203u)\theta^2 - 0''\!.0077734\theta^3\end{aligned} \tag{5}$$

$$\begin{aligned}\psi(s,u) = (5038''\!.7802 + 0''\!.49254u - 0''\!.00039u)\theta \\ + (-1''\!.05331 - 0''\!.001513u)\theta^2 - 0''\!.001530\theta\end{aligned} \tag{6}$$

$$\begin{aligned}\chi(s,u) = (10''\!.5567 - 1''\!.88692u - 0''\!.000144u)\theta \\ + (-2''\!.38191 - 0''\!.001554u)\theta^2 - 0''\!.0016661\theta^3\end{aligned} \tag{7}$$

$$\begin{aligned}\Psi(s,u) = (5029''\!.0946 + 2''\!.22280u + 0''\!.000264u^2)\theta \\ + (1''\!.13157 + 0''\!.000212u)\theta^2 + 0''\!.000102\theta^3\end{aligned} \tag{8}$$

We should note that the distinctions between the original Newcomb theory and its modification by Kinoshita[23], which we use here, are unimportant for us: for any moment of time $t$ in the interval we are interested in (600 BC–1900 AD), the difference between the ecliptic coordinates computed from the Newcomb theory and the ones from the modification is negligible in comparison with the errors of the *Almagest*. We used Ref. 23 because the formulas for precession are given there in a form convenient for computer calculations.

2. Let us now describe in details the algorithm of compilation of the catalog $K(t)$ reflecting, according to Newcomb's theory, the sky at the moment $t$. Henceforth we consider $t$ to be an arbitrary moment in the interval 600 AD–1900 BC, counted back in Julian centuries from 1900; thus, for example, $t = 1$ corresponds to 1800 AD, $t = 10$ to 900 AD, and $t = 18$ to 100 AD (the several days' difference that accumulates because of the difference between Julian and Gregorian calendars is absolutely immaterial for our purposes). The reason for this somewhat strange denotation is its matching the existing computer programs and our wish to avoid confusion that could initiate a change of notation. We will compare the catalogs $K(t)$ for various values of $t$ with the ancient catalog we study (say, with the *Almagest*); $t$ will serve as an a priori date for the catalog. Therefore $K(t)$ are to be compiled in ecliptic coordinates of the epoch $t$, because as we have already noted, ancient and medieval star catalogs used these coordinates.

So, suppose a star has equatorial coordinates $\alpha^0 = \alpha^0_{1900}$ and $\delta^0 = \delta^0_{1900}$ in a modern star catalog (say, in Ref. 21). These coordinates show the position of the star in 1900 in the spherical coordinate system the equator of which coincides with the earth's equator (hence lies in the plane of earth's rotation, which, as we have noted above, changes with time) in 1900. We need to determine the coordinates $l_t$ and $b_t$ (that is, coordinates in the spherical coordinate

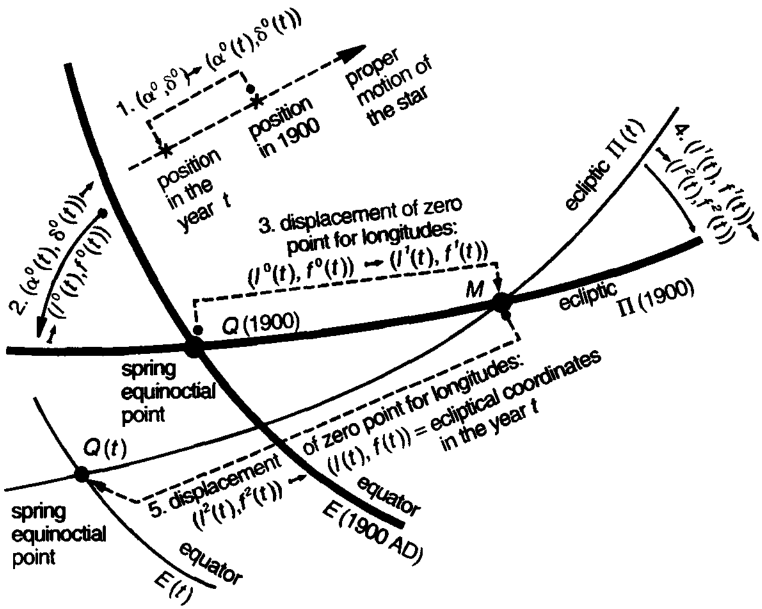


**Figure 1.6.** The recalculation of ecliptic coordinates of January 1, 1990 into ecliptic coordinates of an arbitrary epoch, taking into account the proper motion of stars.

system whose equator is the ecliptic in the year $t$). To that end it suffices to do the following (see Figure 1.6):

1) *Find the coordinates of the star $\alpha^0(t)$ and $\delta^0(t)$ for the year $t$ in the equatorial coordinates of 1900.* This can be done with the help of the proper motion velocities $v_\alpha$ and $v_\delta$ in the coordinates $\alpha$ and $\delta$ (see the fifth and the sixth columns of Tables Ap. 1 and Ap. 2). We have:

$$\alpha^0(t) = \alpha^0_{1900}(t) = \alpha^0 - v_\alpha t \tag{9}$$

$$\delta^0(t) = \delta^0_{1900}(t) = \delta^0 - v_\delta t \tag{10}$$

Indeed, as we have noted above, within the interval of time we are interested in, the proper motion of stars may be treated as uniform. The minuses in (9) and (10) come from our counting time to the past, while the signs of $v_\alpha$ and $v_\delta$ correspond to the natural time count.

2) *Pass from coordinates $\alpha_{1900}$, $\delta_{1900}$ to the coordinates $l_{1900}$, $b_{1900}$.* This gives us coordinates $l^0(t)$ and $b^0(t)$ of the star in the year $t$ in the spherical coordinate system bound to the ecliptic of 1900.

We have:

$$\sin b^0(t) = -\sin\alpha^0(t)\cos\delta^0(t)\sin\varepsilon^0 + \sin\delta^0(t)\cos\varepsilon^0 \tag{11}$$

$$\tan l^0(t) = \frac{\sin\alpha^0(t)\cos\delta^0(t)\cos\varepsilon^0 + \sin\delta^0(t)\sin\varepsilon^0}{\cos\alpha^0(t)\cos\delta^0(t)} \tag{12}$$

where

$$\varepsilon^0 = 23°27'8''.26 \tag{13}$$

These formulas enable us to restore uniquely the values of $b^0(t)$ and $l^0(t)$, because $-90° < \alpha < 90°$ and $|l^0(t) - \alpha^0(t)| < 90°$. The angle $\varepsilon^0$ is the inclination of the ecliptic to the equator in 1900 (see (4), where we put $u = -1$ to pass from year 2000 to 1900).

3) *Pass from coordinates $l_{1900}$ and $b_{1900}$ to the coordinates $l^1$ and $b^1$*, which are also bound to the ecliptic of 1900, but whose zero point is at the intersection of the ecliptic of 1900 $\Pi(1900)$ and the ecliptic of the year $t$ $\Pi(t)$. The two coordinate systems are connected by the relations

$$\begin{aligned} l^1(t) &= l^0(t) - \phi \\ b^1(t) &= b^0(t) \\ \phi &= 173°57'38''.436 + 870''.0798t + 0''.024578t^2 \end{aligned} \tag{14}$$

Here $\phi$ is the arc of $\Pi(1900)$ between the spring equinoctial point of 1900 and the point of intersection of $\Pi(1900)$ and $\Pi(t)$; it can be found from (1) by putting $u = -1$ (then $\Pi(u)$ in Figure 1.5 will correspond to $\Pi(1900)$) and $\theta = -t$. Then $\Pi(s)$ in Figure 1.5 will depict the ecliptic of the epoch $t$. Indeed, $t$ is counted in centuries from 1900 to the past, and $\theta = s - u$ is counted in centuries from $u$ to the future; since we put $u = -1$, which corresponds to 1900 ($2000 - 100 = 1900$), we have to put $\theta = -t$ to make the epoch $s = u + \theta$ in (1) correspond to the epoch $t$.

4) *Pass from the coordinates $l^1$ and $b^1$ to the coordinates $l^2$ and $b^2$*, the spherical coordinates bound with the ecliptic $\Pi(t)$ and differing from the ecliptic coordinates $l_t$ and $b_t$ only for the choice of zero point of the longitudes. In the coordinates $l^2$ and $b^2$, the zero point is the intersection of $\Pi(1900)$ and $\Pi(t)$. The transfer formulas from $(l^1, b^1)$ to $(l_t, b_t)$ are similar to (14); we only have to replace $\varepsilon^0$ by the angle $\varepsilon_1$ between $\Pi(1900)$ and $\Pi(t)$; we have

$$\varepsilon = -47''.0706t - 0''.033769t^2 - 0''.000050t^3 \tag{15}$$

This expression can be obtained from (2) by putting $u = -1$ and $\theta = -t$.

5) Finally, we are to *pass from $l^2$ and $b^2$ to the ecliptic coordinates $l_t$ and $b_t$.* This can be done by the formulas

$$l_t = l^2 + \phi + \Psi$$
$$b_t = b^2 \tag{16}$$

where $\phi$ is as in (10) and $\Psi$ can be obtained from (8) by putting $u = -1$ and $\theta = -t$, that is,

$$\Psi = -5026''.872t + 1''.1314t^2 + 0''.0001t^3 \tag{17}$$

The sequence of steps 1)–5) is illustrated in Figure 1.6.

## 6. Astrometry. Some medieval astronomic instruments

In Section 3 we have exposed a general idea of an astronomic goniometer; an important feature of it is the possibility of a sufficiently accurate determination of the line of the celestial equator. The ray $HK'$ along which the observer's eye is directed does not leave the equator in the process of diurnal rotation. Of course, the setting of the ray $HK'$ depends on the geographical latitude of the vantage point. In principle, one can imagine the plane $HLM$ as attached to the quadrant (Figure 1.7). This plane is parallel to the equatorial plane, and intersects the celestial sphere along the celestial equator. This is in no way affected by the fact that $HLM$ actually does not pass through the center of the earth. Thus, at any point of the earth's surface it is possible to build a stationary instrument (oriented along the meridian) that allows a practically visual observation of the equator. *This makes a reliable measurement of equatorial latitudes of stars possible* (Figure 1.7), for example, at the moment when the star passes the vertical plane of the quadrant. As we already noted, for a professional medieval astronomer measuring equatorial latitudes was not a complicated procedure; it only required accuracy and a sufficient time for observations. In particular, we can expect that a thorough observer should not make a big systematic error in declinations of stars.

Let us now look at particular implementations of this idea in medieval astronomic instruments.

The first instrument, the so-called *meridian circle* is described by Ptolemy (Figure 1.8). The device is a flat metallic ring installed vertically on a firm support in the plane of the meridian. The ring is graduated, for example, into 360 degrees. A smaller ring, rotating freely inside the first ring in the same plane was installed (Figure 1.8). Two small metallic plates with arrows pointing to the divisions on the outer ring were attached at two diametrically opposite points of the inner ring (points $P$ in Figure 1.8). The device is

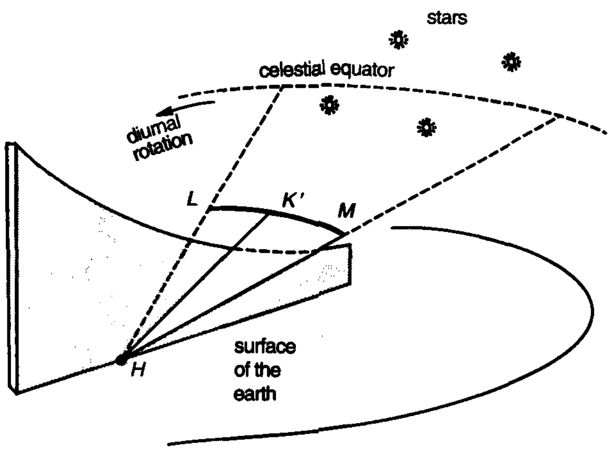


**Figure 1.7.** Visual determination of the equator's position in the celestial sphere.

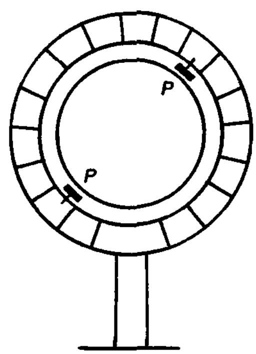


**Figure 1.8.** Meridian circle.

installed in the meridian plane with the help of a plumb; the direction of the meridian was determined from the shade of a vertical pole at noon. Then the zero division of the outer ring was matched with the zenith. The device could be used for measurement of the altitude of the sun (at the latitude of the vantage point); to that end, the inner ring is turned at noon so that the shade of one of the plates $P$ covered the other. Then the arrow on the upper plate points to the altitude of the sun in degrees on the outer ring. Note that we can read the result *after* fixing the plates; so we can read the altitude after the moment of noon. Furthermore, the meridian circle can be used to determine the angle $\varepsilon$ between the ecliptic and the equator.