Ethirajan Rathakrishnan

Instrumentation, Measurements, and Experiments in Fluids

Second Edition

CRC Press
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Ethirajan Rathakrishnan

Department of Aerospace Engineering
Indian Institute of Technology Kanpur, India

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

First issued in hardback 2019

ISBN-13: 978-1-4987-8485-6 (hbk)

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Dedication

This book is dedicated to my parents,

Mr. Thammanur Shunmugam Ethirajan

and

Mrs. Aandaal Ethirajan

Ethirajan Rathakrishnan

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Preface

The first edition of this book, developed to serve as the text for a course on experiments in fluids at the introductory level for undergraduate courses and advanced level courses at the graduate level, was well received all over the world because of its completeness and easy-to-follow style.

Over the years the feedback received from the faculty and students made the author realize the need for adding exercise problems at the end of different chapters. Also, while using the chapter on wind tunnel in the lectures, students expressed that adding a section on internal balance would be useful. Some faculty conveyed that adding material on PIV would make the chapter on flow visualization more effective. Users of optical visualization for high-speed flows expressed the need for highlighting the possibility of obtaining quantitative information from the schlieren and shadowgraph pictures.

Considering the feedback from faculty and students, the following material is added in this edition.

- A section on internal balance in Chapter 3 on wind tunnels.
- A subsection highlighting the use of schlieren and shadowgraph pictures for qualitative and quantitative analysis of high-speed flows in Chapter 4.
- A detailed section describing the theory and application of particle image velocimetry in Chapter 4.
- A section on water flow channels giving the calibration and use of water flow channels for visualizing vortex formation in Chapter 4.
- The design, fabrication, calibration, and use of Hele-Shaw apparatus has been given in detail, highlighting the capability of this devise to simulate potential flow field, in Example 6.1.
- Some other important aspects, such as the limiting value pressure probe blockage for neglecting the blockage correction, is addressed through a detailed example in Chapter 7 on pressure measurements.
- Some new examples in the chapters on wind tunnel, pressure measurement, temperature measurement, and mass flow measurement.
- A considerable number of exercise problems at the end of many chapters.

I am indebted to Professor Junjiro Iwamoto, Department of Mechanical Engineering, Tokyo Denki University, Japan, for allowing me to take the schlieren and shadowgraph pictures used in Section 4.3.8 in his jet facility.

I thank Dr. Yasumasa Watanabe, Assistant Professor, Department of Aeronautical and Astronautical Engineering Department, University of Tokyo, Japan, for fabricating, calibrating, and testing the Hele-Shaw device during my stay with the Department of Advanced Energy, Graduate School of Frontier Sciences, University of Tokyo, Japan, as Visiting Professor in 2011.

My sincere thanks to Professor Shouiio Iio, Department of Mechanical Engineering, Shinshu University, Nagano, Japan, for providing his PIV facility and

working with me in the visualization of the jet, discussed in Section 4.4, during my stay in his lab in the summer of 2014, while on JSPS Fellowship.

Finally, I would like to thank the faculty and students all over the world for adopting this book for their courses.

For instructors, a companion Solutions Manual that contains typed solutions to all the end-of-chapter problems and lecture slides for the complete book are available from the publisher.

Ethirajan Rathakrishnan

About the Book

The first edition was well received. Many faculty wanted exercise problems along with answers, PIV systems of flow visualization, water flow channels for flow visualization, and good pictures with schlieren and shadowgraph, with possible quantitative information extracted from these images. Also, there was a request for adding some more examples with specific details on probe blockage in the case of pressure probes and the theory of internal balances for wind tunnels.

Most of the chapters are provided with exercise problems along with answers. This will enable the users to prepare for the exams in a more effective manner. There is no single book covering the full spectrum of a complete experimental flows course, which should include: wind tunnels, flow visualization, hot-wire anemometers, analogue methods, pressure, temperature, volume/mass and force measurements, data acquisition, and uncertainty. Many books need to be referred to in order to acquire this material. This book covers all the requirements for a full course on experimental fluid mechanics.

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About the Author

Ethirajan Rathakrishnan is professor of Aerospace Engineering at the Indian Institute of Technology Kanpur, India. He is well known internationally for his research in the area of high-speed jets. The limit for the passive control of jets, called the *Rathakrishnan Limit*, is his contribution to the field of jet research, and the concept of *breathing blunt nose (BBN)*, which simultaneously reduces the positive pressure at the nose and increases the low pressure at the base, is his contribution to drag reduction at hypersonic speeds. Positioning the twin-vortex Reynolds number at around 5000, by changing the geometry from cylinder, for which the maximum limit for the Reynolds number for positioning the twin-vortex was found to be around 160, by von Karman, to flat plate, is his addition to vortex flow theory. He has published a large number of research articles in many reputed international journals. He is a Fellow of many professional societies including the Royal Aeronautical Society. Rathakrishnan serves as the Editor-in-Chief of the *International Review of Aerospace Engineering* (IREASE) and *International Review of Mechanical Engineering* (IREME) journals. He has authored 11 other books: *Gas Dynamics*, 5th ed. (PHI Learning, New Delhi, 2013); *Fundamentals of Engineering Thermodynamics*, 2nd ed. (PHI Learning, New Delhi, 2005); *Fluid Mechanics: An Introduction*, 3rd ed. (PHI Learning, New Delhi, 2012); *Gas Tables*, 3rd ed. (Universities Press, Hyderabad, India, 2012); *Theory of Compressible Flows* (Maruzen Co., Ltd. Tokyo, Japan, 2008); *Gas Dynamics Work Book*, 2nd ed. (Praise Worthy Prize, Napoli, Italy, 2013); *Applied Gas Dynamics* (John Wiley, New Jersey, USA, 2010); *Elements of Heat Transfer* (CRC Press, Taylor & Francis Group, Boca Raton, Florida, USA, 2012); *Theoretical Aerodynamics* (John Wiley, New Jersey, USA, 2013); *High Enthalpy Gas Dynamics* (John Wiley & Sons Inc., 2015); and *Dynamique Des Gaz* (Praise Worthy Prize, Napoli, Italy, 2015).

In addition to the technical books above, Professor Ethirajan Rathakrishnan has authored the following literary books in Tamil: *Krishna Kaviam* (book on the life of Lord Krishna, from classical Tamil poetry) (Shantha Publishers, Royapettah, Chennai, India, 2014); *Naan Kanda Japan* (The Japan I Saw, in Tamil) (Shantha Publishers, Royapettah, Chennai, India, 2014); *Japanin Munnilai Ragasiam* (The Secrecy of Japanese Success, in Tamil) (Shantha Publishers, Royapettah, Chennai, India, 2015), and *Vallalaar Kapiam* (on St. Ramalinga Swamy, from classical Tamil poetry) (Vanathi Pathippagam, T. Nagar, Chennai, India, 2016).

Chapter 1

Needs and Objectives of Experimental Study

1.1 Introduction

As we know, the theory of potential flow is based on simplifying assumptions such as the fluid is barotropic, inviscid, and so on. Therefore, the potential flow theory cannot account for the profile drag acting on an object present in the flow field and for the boundary layer effects. Also, we know that the mathematical theory of boundary layer motion is highly complex. A considerable amount of theoretical work has already been done on inviscid compressible fluid flow at subsonic and supersonic speeds. But the theory of boundary layer for compressible flow has to develop a lot. Therefore, the theory available for fluid flow analysis is incomplete and needs to be supplemented by experiments. From the design point of view, experiments have two principal objectives:

1. They make it possible to determine the influence of various features of design, and modifications to them, in a safe, quick, direct, and relatively less expensive manner.

2. They provide information of a fundamental nature, usually in conjunction with theoretical work. By this means, the theory is confirmed or extended, thereby laying the foundation for future design improvements of a fundamental character.

The aim of this book is to discuss the fundamental aspects of the *experimentation in fluids*. In other words, our aim here is to acquaint ourselves with the need for experimental study and to gain insight concerning various applications of the available techniques for experimental study of fluid flows.

1.2 Some Fluid Mechanics Measurements

A large variety of measurement techniques are used in the field of experimental fluid mechanics. To have an idea about the different types of measurements, let us see the examples given below:

1.2.1 Wind Tunnel Studies

Wind tunnels are used for numerous investigations ranging from fundamental research to industrial aerodynamics. Many wind tunnel studies aim at the determination of forces on scaled models of aircraft, aircraft components, automobiles, buildings, and so on. Forces such as lift and drag acting on the models being tested are known to obey the following law of similitude

$$\boxed{F = \frac{1}{2}\rho V^2 S C_N} \tag{1.1}$$

where S is the surface area or cross-sectional area of the model, depending on the application. The force coefficient C_N is known to be a function of several nondimensional parameters. The prime ones, among such dimensionless parameters used, in aerodynamics are

$$\text{Reynolds number} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V l}{\mu}$$

$$\text{Mach number} = \frac{\text{Inertia force}}{\text{Elastic force}} = \frac{V}{a}$$

where ρ and V are the density and velocity of flow, respectively, μ is the dynamic viscosity coefficient, l is the characteristic length, and a is the speed of sound.

To correlate data, velocity is measured with pitot–static tube or hot–wire anemometer or laser Doppler anemometer, and the temperature and pressure are obtained with appropriate instrumentation. The forces and the moments on a model are usually determined with specially designed balance or surface pressure measurements. The density is usually calculated from the measured pressure and temperature.

1.2.2 Analogue Methods

By analogue methods, problems may be solved by setting up another physical system, such as an electric field, for which the governing equations are of the same form as those for the problem to be solved, with corresponding boundary conditions. The solution of the original problem may be obtained experimentally from measurements on the analogous system. Some of the well-known analogue methods for solving fluid flow problems are the *Hele–Shaw analogy*, the *electrolytic tank*, and the *surface waves in a ripple tank*.

1.2.3 Flow Visualization

Apart from the conventional methods of experimentally investigating flow patterns by means of pressure and velocity surveys, fluid flows lend themselves to numerous visualization techniques. Some of the popularly employed flow visualization methods for fluid flow analysis are flow visualization with *smoke*, *tuft*, *chemical coating*, *interferometer*, *schlieren*, and *shadowgraph*.

1.3 Measurement Systems

Basically, the main components of a measuring system may be classified into the following three categories.

1. The sensing element.
2. The signal converter.
3. The display.

1.3.1 Sensing Element

A sensing element is also called a *transducer*. For instance, the bulb of a mercury–in–glass thermometer, the diaphragm in a pressure transducer are sensing elements. The transducer is in some way "in contact" with the quantity to be measured and produces some signal which is related to the quantity being measured. A typical sensing process is illustrated schematically in Figure 1.1.

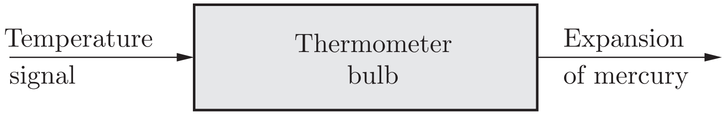

Figure 1.1: Sensing element response

1.3.2 Signal Converter

A signal converter is a device to convert the output from the sensing element to a desired form and feed the same to the display unit. A typical example of a signal converter is the amplifier which receives a small signal from the sensing element and makes it large enough to activate the display.

1.3.3 Display

The display is yet another vital part of a measuring system. It is here that the information from the sensing element, which is converted into a desired form by the signal converter, is read by the experimenter. A typical example of a display system is the combination of a dial and a pointer, as in the case of a dial-type pressure gauge.

1.3.3.1 Performance Terms

There are some commonly used performance terms associated with measurement systems. They are

* Accuracy	* Range
* Error	* Resolution
* Repeatability	* Sensitivity
* Reliability	* Dead space
* Reproducibility	* Threshold
* Lag	* Hysteresis

and so on. Now let us consider these performance terms one by one.

Accuracy

The accuracy of a measuring devise may be defined as the extent to which the reading given by it is close to the exact value. For example, if the accuracy of a mercury manometer is $\pm$ 1 mm, it means that when a mercury manometer is used to measure pressure of a flow it can only be stated that the pressure of the flow is lying within $\pm$ 1 mm of the manometer reading. Thus, a reading of 500 mm of mercury means that the exact value of the measured pressure is somewhere between 499 and 501 mm of mercury. Accuracy is generally expressed as a percentage of full–scale reading of the instrument.

Error

The error is the difference between the measured value and the true value of the quantity being measured.

Repeatability

The repeatability of an instrument is its ability to display the same reading as long as its sensor element is fed the same signal.

Reliability

The reliability of a measuring system is the probability that it will operate with an agreeable accuracy under the conditions specified for its operation.

Reproducibility

The reproducibility of a measuring device is its ability to display the same reading when it is used to measure the same quantity over a period of time or when that quantity is measured on a number of instants. Reproducibility of a device is also termed as *stability of the device.*

Range

The range of an instrument is the limits between the minimum and the maximum readings measurable by it. For example, the range of a thermometer which is capable of measuring temperatures between −10°C and 110°C is −10°C and 110°C.

Resolution

The resolution of an instrument is the smallest change in the quantity being measured that will produce an observable change in the reading of the instrument.

Sensitivity

The sensitivity of an instrument is its response to any change in the quantity being measured.

Dead space

The dead space of a measuring device is the range of values of the quantity being measured for which it gives no reading.

Threshold

The threshold is the minimum level of the quantity that is being measured, which has to be reached before the instrument responds and gives a detectable reading. In other words, it is just a dead space which occurs when an instrument is used for reading from the minimum limit of its range.

Lag

The lag of an instrument is the time interval between the time of input and the time of display of the reading.

Hysteresis

The hysteresis is that characteristic which makes an instrument give different readings for the same value of measured quantity depending on whether the value has been reached by a continuously increasing change or a continuously decreasing change.

1.4 Some of the Important Quantities Associated with Fluid Flow Measurements

Pressure, temperature, and volume flow/mass flow rate are the prime quantities associated with any fluid flow experimentation. In fact, once these quantities are measured independently, many quantities of practical importance, like acceleration of a fluid stream, density of flow, energy associated with a flow, force acting on an object placed in a flow field, and so on can easily be determined. The units commonly employed for the above quantities are given in Tables 1.1 and 1.2.

Table 1.1 Units

Quantity	Unit
Pressure	1 Pa = 1 N/m^2
	1 bar = 10^5 Pa
	1 torr = 1 mm of mercury
	1 atm = 101.325 kPa
	= 1.01325 bar
	= 760 mm of Hg at 0°C
Specific volume	1 m^3/kg = 1000 l/kg
	= 1000 cm^3/g
Temperature	T (K) = T (°C) + 273.15
	ΔT (K) = ΔT (°C)
Volume	1 m^3 = 1000 = 10^6 cm^3(cc)

Table 1.2 Conversion factors

Dimension	Unit
Acceleration	1 m/s^2 = 100 cm/s^2
Area	1 m^2 = 10^4 cm^2 = 10^6 mm^2 = 10^{-6} km^2
Density	1 g/cm^3 = 1 kg/l = 1000 kg/m^3
Energy, Heat,	1 kJ = 1000 J = 1000 N.m = 1 kPa.m^2
Work	1 kJ/kg = 1000 m^2/s^2
	1 kWh = 3600 kJ
Force	1 N = 1 kg.m/s^2
Length	1 m = 100 cm = 1000 mm
	1 km = 1000 m
Mass	1 kg = 1000 g
	1 metric ton = 1000 kg
Power	1 W = 1 J/s
	1 kW = 1000 W = 1.341 hp

1.5 Summary

From the discussions on the need and objective of experimental study, it is evident that the experimental studies are important from both fundamental and applied research points of view. The experimental investigations can broadly be classified into

- Direct measurements
- Analogue methods
- Flow visualization

A measurement system basically consists of

- The sensing element
- The signal converter
- The display

In an experimental investigation, an experimenter must have a clear idea about the experimental technique and the measurement system used. In addition to these, the researcher must have a thorough understanding of the problem to be studied, the principles of flow physics associated with the problem being studied, and the principles of operation of the instruments being used. A thorough knowledge of the fundamentals of fluid mechanics is inevitable for any successful experimentalist. Therefore, let us have a quick look at the basic principles underlying the fluid flow processes in Chapter 2, before actually getting into the experimental techniques.

Chapter 2

Fundamentals of Fluid Mechanics

2.1 Introduction

Gases and liquids are generally termed as fluids. Though the physical properties of gases and liquids are different, they are grouped under the same heading since both can be made to flow unlike a solid. Under dynamic conditions, the nature of governing equations is the same for both gases and liquids. Hence, it is possible to treat them under the same heading, namely, fluid dynamics or fluid mechanics. However, certain substances known as viscoelastic materials behave like liquids as well as solids, depending on the rate of application of the force. Pitch and silicone putty are typical examples of viscoelastic material. If the force is applied suddenly the viscoelastic material will behave like a solid, but with gradually applied pressure the material will flow like a liquid. The properties of such materials are not considered in this book. Similarly, non-Newtonian fluids, low-density flows, and two-phase flows such as gas liquid mixtures are also not considered in this book. The experimental techniques described in this book are for well-behaved simple fluids such as air.

2.2 Properties of Fluids

A fluid may be defined *as a substance which will continue to change shape as long as there is a shear stress present, however small it may be.* That is, the basic feature of a fluid is that it can flow, and this is the essence of any definition of it. Examine the effect of shear stress on a solid element and a fluid element, shown in Figure 2.1.

It is seen from this figure that the change in shape of the solid element is characterized by an angle $\Delta\alpha$ when subjected to a shear stress. Whereas, for the fluid element there is no such fixed $\Delta\alpha$ even for an infinitesimal shear stress. A continuous deformation persists as long as shearing stress is applied. The rate of deformation, however, is finite and is determined by the applied shear force and the fluid properties.

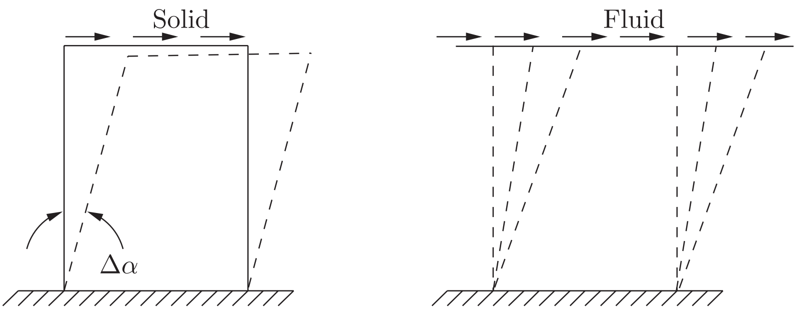

Figure 2.1: Solid and fluid elements under shear stress

2.2.1 Pressure

Pressure may be defined as the *force per unit area which acts normal to the surface of any object which is immersed in a fluid*. For a fluid at rest, at any point the pressure is the same in all directions. The pressure in a stationary fluid varies only in the vertical direction and is constant in any horizontal plane. That is, in stationary fluids the pressure increases linearly with depth. This linear pressure distribution is called *hydrostatic pressure distribution*. The hydrostatic pressure distribution is valid for moving fluids also provided there is no acceleration in the vertical direction. This distribution finds extensive application in manometry.

When a fluid is in motion, the actual pressure exerted by the fluid in the direction normal to the flow is known as the *static pressure*. If there is an infinitely thin pressure transducer which can be placed in a flow field without disturbing the flow, and it can be made to travel with the same speed as that of the flow then it will record the exact static pressure of the flow. From this stringent requirement of the probe for static pressure measurement, it can be inferred that exact measurement of static pressure is impossible. However, there are certain phenomena, like "*the static pressure at the edge of a boundary layer is impressed through the layer,*" which are used for the proper measurement of static pressure. *Total pressure* is that pressure which a fluid will experience if its motion is brought to rest. It is also called *impact pressure*. The total and static pressures are used for computing the flow velocity.

Since pressure is intensity of force, it has the dimensions

$$\frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} = \left[ML^{-1}T^{-2}\right]$$

and is expressed in the units of newton per square meter (N/m^2) or simply pascal (Pa). At standard sea level condition, the atmospheric pressure is 101325 Pa, which corresponds to 760 mm of mercury column height.

2.2.2 Temperature

In any form of matter the molecules are in motion relative to each other. In gases the molecular motion is a random movement of appreciable amplitude ranging from about 76×10^{-9} m under normal conditions to some tens of millimeters at very low pressures. The distance of free movement of a molecule of a gas is the distance it can travel before colliding with another molecule or the walls of the container. The mean value of this distance for all molecules in a gas is called the molecular *mean free path* length. By virtue of this motion the molecules possess kinetic energy, and this energy is sensed as *temperature* of the solid, liquid, or gas. In the case of a gas in motion it is called the *static temperature*. Temperature has units kelvin (K) or degrees celsius (degC), in SI units. For all calculations in this book, temperature will be expressed in kelvin, i.e., from absolute zero. At standard sea level conditions the atmospheric temperature is 288.15 K.

2.2.3 Density

The total number of molecules in a unit volume is a measure of the density ρ of a substance. It is expressed as mass per unit volume, say kg/m^3. Mass is defined as weight divided by acceleration due to gravity. At standard atmospheric temperature and pressure (288.15 K and 101325 Pa), the density of dry air is 1.225 kg/m^3.

Density of a material is a measure of the amount of material contained in a given volume. In a fluid, density may vary from point to point. Consider the fluid contained within a small spherical region of volume $\delta\mathbb{V}$ centered at some point in the fluid, and let the mass of fluid within this spherical region be δm. Then the density of the fluid at the point on which the sphere is centered can be defined by

$$\rho = \lim_{\delta\mathbb{V} \to 0} \frac{\delta m}{\delta\mathbb{V}} \tag{2.1}$$

There are practical difficulties in applying the above definition of density to real fluids composed of discrete molecules, since under the limiting condition the sphere may or may not contain any molecule. If it contains a molecule the value obtained for the density will be fictitiously high. If it does not contain a molecule the resultant value of density will be zero. This difficulty can be avoided over the range of temperatures and pressures normally encountered in practice, in the following two ways.

1. The molecular nature of a gas may be ignored and the gas is treated as continuum, i.e., does not consist of discrete particles.

2. The decrease in size of the imaginary sphere may be assumed to be carried to a limiting size. This limiting size of the sphere is such that, although it is small compared to the dimensions of any physical object present in a flow field, e.g., an aircraft, it is large compared to the fluid molecules and, therefore, contains a reasonably large number of molecules.

2.2.4 Viscosity

The property which characterizes the resistance that a fluid offers to applied shear force is termed *viscosity*. This resistance, unlike for solids, does not depend upon the deformation itself but on the *rate of deformation*. Viscosity is often regarded as the stickiness of a fluid and its tendency is to resist sliding between layers. There is very little resistance to the movement of the knife-blade edge-on through air, but to produce the same motion through a thick oil needs much more effort. This is because viscosity of oil is higher compared to that of air.

2.2.5 Absolute Coefficient of Viscosity

The absolute coefficient of viscosity is a direct measure of the viscosity of a fluid. Consider the two parallel plates placed at a distance h apart, as shown in Figure 2.2.

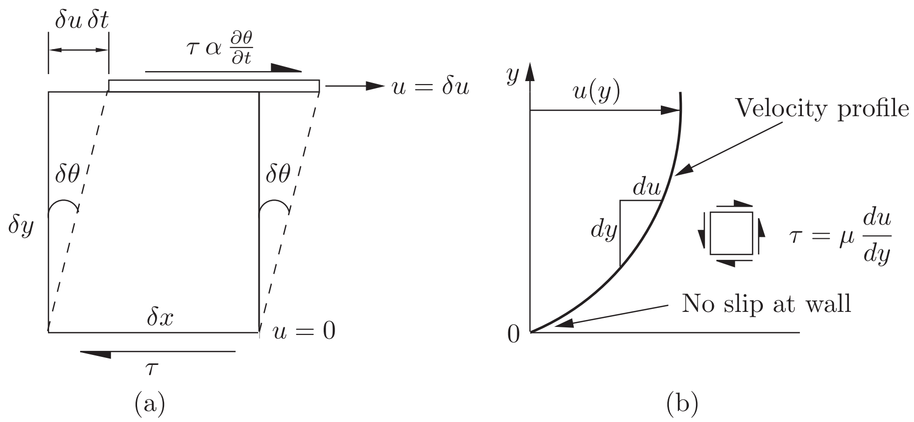

Figure 2.2: Parallel plates with fluid in between

The space between them is filled with a fluid. The bottom plate is fixed and the other is moved in its own plane at a speed u. The fluid in contact with the lower plate will be at rest, while that in contact with the upper plate will be moving with speed u, because of no-slip condition. In the absence of any other influence, the speed of the fluid between the plates will vary linearly, as shown in Figure 2.2. As a direct result of viscosity, a force F has to be applied to each plate to maintain the motion, since the fluid will tend to retard the motion of the moving plate and will tend to drag the fixed plate in the direction of the moving plate. If the area of each plate in contact with fluid is A, then the shear stress acting on each plate is F/A. The rate of slide of the upper plate over the lower is u/h.

These quantities are connected by Maxwell's equation, which serves to define the absolute coefficient of viscosity μ. The equation is

$$\frac{F}{A} = \mu \left(\frac{u}{h}\right) \tag{2.2}$$

Hence,

$$ML^{-1}T^{-2} \;= [\mu] \;\; LT^{-1}L^{-1} \;= [\mu] \;\; T^{-1}$$

i.e.,

$$[\mu] = \;\; ML^{-1}T^{-1}$$

and the unit of μ is therefore kg/(m s). At 0 degC the absolute coefficient of viscosity of dry air is 1.71×10^{-5} kg/(m s). The absolute coefficient viscosity μ is also called the *dynamic viscosity coefficient*.

Equation (2.2) with μ constant does not apply to all fluids. For a class of fluids, which includes blood, some oils, some paints, and so called thixotropic fluids, μ is not constant but is a function of du/dh. The derivative du/dh is a measure of the rate at which the fluid is shearing. Usually μ is expressed as (N.s)/m^2 or gm/(cm s). One gm/(cm s) is known as a *poise*.

Newton's law of viscosity states that, "*the stresses which oppose the shearing of a fluid are proportional to the rate of shear strain,*" i.e., the shear stress τ is given by

$$\tau = \mu \frac{\partial u}{\partial y} \tag{2.3}$$

where μ is the absolute coefficient of viscosity and $\partial u/\partial y$ is the velocity gradient. The viscosity μ is a property of the fluid. Fluids which obey the above law of viscosity are called *Newtonian fluids*. Some fluids such as silicone oil, viscoelastic fluids, sugar syrup, tar, etc., do not obey the viscosity law given by Equation (2.3) and they are called *non-Newtonian fluids*.

We know that, in incompressible flow, it is possible to separate the calculation of velocity boundary layer from that of thermal boundary layer. But in compressible flow it is not possible, since the velocity and thermal boundary layers interact intimately and therefore, they must be considered simultaneously. This is because, for high-speed flows (compressible flows) heating due to friction as well as temperature changes due to compressibility must be taken into account. Further, it is essential to include the effects of viscosity variation with temperature. Usually large variations of temperature are encountered in high-speed flows.

The relation $\mu(T)$ must be found by experiment. The voluminous data available in literature leads to the conclusion that the fundamental relationship is a complex one and that no single correlation function can be found to apply to all gases. Alternatively, the dependence of viscosity on temperature can be calculated with the aid of the method of statistical mechanics, but as yet no completely satisfactory theory has evolved. Also, these calculations lead to complex expressions for the function $\mu(T)$. Therefore, only semiempirical relations appear to be the means to calculate the viscosity associated with compressible boundary layers. It is important to realize that, even though semiempirical relations are not extremely precise they are reasonably simple relations. For air, it is possible to use an interpolation formula based on D. M. Sutherland's theory of viscosity and express the viscosity coefficient as

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} \frac{T_0 + S}{T + S}$$

where μ_0 denotes the viscosity at the reference temperature, T_0, and S is a constant which assumes the value $S = 110$ K for air.

For air the Sutherland's relation can also be expressed [W.H. Heiser and D.T. Pratt, *Hypersonic air breathing propulsion*, 1994, AIAA Education Series] as

$$\boxed{\mu = 1.46 \times 10^{-6} \left(\frac{T^{3/2}}{T + 111}\right) \frac{\text{N s}}{\text{m}^2}}$$

where T is in kelvin. This equation is valid for the static pressure range of 0.01 to 100 atm, which is commonly encountered in atmospheric flight. The temperature range in which this equation is valid is up to 3000 K. The reasons that the absolute viscosity is a function only of temperature under these conditions are that the air behaves as a perfect gas, in the sense that intermolecular forces are negligible, and that viscosity itself is a momentum transport phenomenon caused by the random molecular motion associated with thermal energy or temperature.

Example 2.1

Determine the absolute viscosity of air at temperatures 0°C, 5°C and 10°C.

Solution

By Sutherland's relation we have

$$\mu = 1.46 \times 10^{-6} \left(\frac{T^{3/2}}{T + 111}\right) \frac{\text{N s}}{\text{m}^2}$$

Substituting the above temperatures in the Sutherland's relation, we get

$$\mu_0 = \boxed{1.71 \times 10^{-5} \frac{\text{N s}}{\text{m}^2}}$$

$$\mu_5 = \boxed{1.74 \times 10^{-5} \, \frac{\text{N s}}{\text{m}^2}}$$

$$\mu_{10} = \boxed{1.76 \times 10^{-5} \, \frac{\text{N s}}{\text{m}^2}}$$

The following program can calculate the viscosity of air at desired temperatures.

```
PROGRAM
------------------------
c    Estimation of viscosity
     real mu
     do it = 0,2000,10
     t=float(it)
     t = t + 273.15
     mu = 1.46E-6 *( t**(1.5)/(t + 111.0))
     print *, it, mu
     enddo
     stop
     end
```

2.2.6 Kinematic Viscosity Coefficient

The kinematic viscosity coefficient is a convenient form of expressing the viscosity of a fluid. It is formed by combining the density ρ and the absolute coefficient of viscosity μ according to the equation

$$\boxed{\nu = \frac{\mu}{\rho}} \tag{2.4}$$

The kinematic viscosity coefficient ν is expressed as m^2/s and 1 cm^2/s is known as *stoke*.

The kinematic viscosity coefficient is a measure of the relative magnitudes of viscosity and inertia of the fluid. Both dynamic viscosity coefficient μ and kinematic viscosity coefficient ν are functions of temperature. For liquids, μ decreases with increase of temperature, whereas for gases μ increases with increase of temperature. This is one of the fundamental differences between the behavior of gases and liquids. The viscosity is practically unaffected by the pressure.

2.2.7 Thermal Conductivity of Air

At high speeds, heat transfer from vehicles becomes significant. For example, re-entry vehicles encounter an extreme situation where ablative shields are necessary to ensure protection of the vehicle during its passage through the atmosphere. The heat transfer from a vehicle depends on the thermal conductivity K of air. Therefore, a method to evaluate K is also essential. For this case, a relation similar to Sutherland's law for viscosity is found to be useful, and it is

$$K = 1.99 \times 10^{-3} \left(\frac{T^{3/2}}{T + 112} \right) \frac{\text{J}}{\text{s m K}}$$

where T is temperature in kelvin. The pressure and temperature ranges in which this equation is applicable are 0.01 to 100 atm and 0 to 2000 K. For the same reason given for viscosity relation, the thermal conductivity also depends only on temperature.

2.2.8 Compressibility

The change in volume of a fluid associated with change in pressure is called compressibility. When a fluid is subjected to pressure it gets compressed and its volume changes. The bulk modulus of elasticity is a measure of how easily the fluid may be compressed, and is defined as the ratio of pressure change to volumetric strain associated with it. The bulk modulus of elasticity, k, is given by

$$k = \frac{\text{Pressure increment}}{\text{Volume strain}} = -\mathbb{V}\frac{dp}{d\mathbb{V}} \tag{2.5}$$

It may also be expressed as

$$k = \lim_{\Delta v \to 0} \frac{-\Delta p}{\Delta v / v} = \frac{dp}{(d\rho/\rho)} \tag{2.6}$$

where v is specific volume. Since $d\rho/\rho$ represents the relative change in density brought about by the pressure change dp, it is apparent that the bulk modulus of elasticity is the inverse of the compressibility of the substance at a given temperature. For instance, k for water and air are approximately 2 GN/m^2 and 100 kN/m^2, respectively. This implies that, air is about 20,000 times more compressible than water. It can be shown that, $k = a^2/\rho$, where a is the speed of sound. The compressibility plays a dominant role at high speeds. Mach number M (defined as the ratio of local flow velocity to local speed of sound) is a convenient nondimensional parameter used in the study of compressible flows. Based on M the flow is divided into the following regimes. When $M < 1$ the flow is called *subsonic*, when $M \approx 1$ the flow is termed *transonic flow*, M from 1.2 to 5 is called *supersonic regime*, and $M > 5$ is referred to as *hypersonic regime*. When flow Mach number is less than 0.3, the compressibility effects are negligibly small and hence the flow is called *incompressible*. For incompressible flows, density change associated with velocity is neglected and the density is treated as invariant.

2.3 Thermodynamic Properties

We know from thermodynamics that heat is energy in transition. Therefore, heat has the same dimensions as energy, and is measured in units of joule (J).

2.3.1 Specific Heat

The inherent thermal properties of a flowing gas become important when the energies are considered. The specific heat is one such quantity. The specific heat is defined as the amount of heat required to raise the temperature of a unit mass of a medium by one degree. The value of the specific heat depends on the type of process involved in raising the temperature of the unit mass. Usually constant volume process and constant pressure process are used for evaluating specific heat. The specific heats at constant volume and constant pressure processes, respectively, are designated by C_v and C_p. The definitions for these quantities are the following:

$$C_v \equiv \left(\frac{\partial u}{\partial T}\right)_v \tag{2.7}$$

where u is internal energy per unit mass of the fluid, which is a measure of the potential and more particularly kinetic energy of the molecules comprising the gas. The specific heat C_v is a measure of the energy-carrying capacity of the gas molecules. For dry air at normal temperature, $C_v = 717.5$ J/(kg K).

The specific heat at constant pressure is defined as

$$C_p \equiv \left(\frac{\partial h}{\partial T}\right)_p \tag{2.8}$$

where $h = u + p\,v$, the sum of internal energy and flow energy is known as the *enthalpy* or total heat constant per unit mass of fluid. C_p is a measure of the ability of the gas to do external work in addition to possessing internal energy. Therefore, C_p is always greater than C_v. For dry air at normal temperature, C_p = 1004.5 J/(kg K).

2.3.2 The Ratio of Specific Heats

The ratio of specific heats given by

$$\boxed{\gamma = \frac{C_p}{C_v}} \tag{2.9}$$

is an important parameter in the study of compressible flows. This is a measure of *the relative internal complexity of the molecules of the gas*. It has been determined from kinetic theory of gases that the ratio of specific heats can be related to the number of degrees of freedom, n, of the gas molecules by the relation

$$\gamma = \frac{n+2}{n}$$

At normal temperatures, there are six degrees of freedom, three translational and three rotational, for diatomic gas molecules. For nitrogen, which is a diatomic gas, $n = 5$ since one of the rotational degrees of freedom is small in comparison with the other two. Therefore,

$$\gamma = 7/5 = 1.4$$

A monatomic gas like helium has 3 translational degrees of freedom only, and therefore,

$$\gamma = 5/3 = 1.67$$

This value of 1.67 is the upper limit of the values which γ can take. In general γ varies from 1 to 1.67. i.e.,

$$\boxed{1 \leq \gamma \leq 1.67}$$

The specific heats of a compressible gas are related to the gas constant R. For a perfect gas this relation is

$$\boxed{R = C_p - C_v} \tag{2.10}$$

2.4 Surface Tension

Liquids behave as if their free surfaces were perfectly flexible membranes having a constant tension σ per unit width. This tension is called the *surface tension.* It is important to note that this is neither a force nor a stress but a *force per unit length.* The value of surface tension depends on

- The nature of the fluid.
- The nature of the substance with which it is in contact at the surface.
- The temperature and pressure.

Consider a plane material membrane, possessing the property of constant tension σ per unit width. Let the membrane be a straight edge of length l. The force required to hold the edge stationary is

$$p = \sigma\, l \tag{2.11}$$

Now, suppose that the edge is pulled so that it is displaced normal to itself by a distance x in the plane of the membrane. The work done, F, in stretching the membrane is given by

$$F = \sigma\, l\, x = \sigma\, A \tag{2.12}$$

where A is the area added to the membrane. We see that σ is the free energy of the membrane per unit area. The important point to be noted here is that, if the energy of a surface is proportional to its area, then it will behave exactly as if it were a membrane with a constant tension per unit width and this is totally independent of the mechanism by which the energy is stored. Thus, the existence of surface tension at the boundary between two substances is a manifestation of the fact that the stored energy contains a term proportional to the area of the surface. This energy is attributable to molecular attractions.

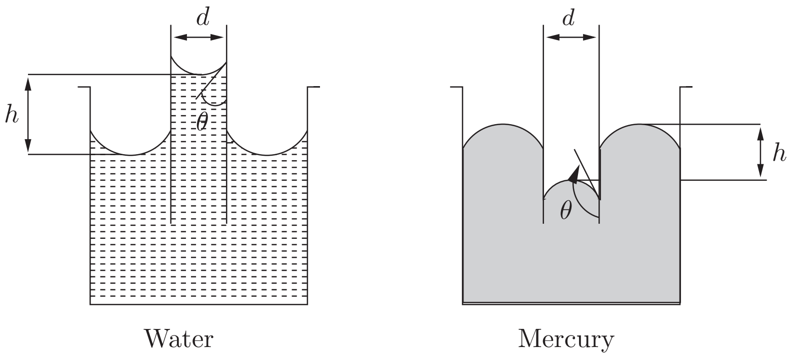

Figure 2.3: Capillary effect of water and mercury

An associated effect of surface tension is the capillary deflection of liquids in small tubes. Examine the level of water and mercury in capillaries, shown in Figure 2.3.

When a glass tube is inserted into a beaker of water, the water will rise in the tube and display a concave meniscus. The deviation of water level h in the tube from that in the beaker can be shown to be

$$h \propto \frac{\sigma}{d} \cos\theta \tag{2.13}$$

where θ is the angle between the tangent to the water surface and the glass surface. In other words, a liquid like water or alcohol, which wets the glass surface makes an acute angle with the solid, and the level of free surface inside the tube will be higher than that outside. This is termed as capillary action. However, when wetting does not occur, as in the case of mercury in glass, the angle of contact is obtuse and the level of free surface inside the tube is depressed, as shown in Figure 2.3.

Another important effect of surface tension is that, a long cylinder of liquid, at rest or in motion, with a free surface is unstable and breaks up into parts, which then assumes an approximately spherical shape. This is the mechanism of the breakup of liquid jets into drops.

2.5 Analysis of Fluid Flow

Basically two treatments are followed for fluid flow analysis. They are the *Lagrangian* and *Eulerian* descriptions. Lagrangian method describes the motion of each particle of the flow field in a separate and discrete manner. For example, the velocity of the n^{th} particle of an aggregate of particles moving in space can be specified by the scalar equations

$$\begin{aligned} (V_x)_n &= f_n(t) \\ (V_y)_n &= g_n(t) \\ (V_z)_n &= h_n(t) \end{aligned} \tag{2.14}$$

where $V_x,\ V_y,\ V_z$ are the velocity components in $x,\ y,\ z$ directions, respectively. They are independent of space coordinates and are functions of time only. Usually, the particles are denoted by the space point they occupy at some initial time t_0. Thus, $T(x_0, t)$ refers to the temperature at time t of a particle which was at location x_0 at time t_0.

This approach of identifying material points and following them along is also termed the *particle* or *material description*. This approach is usually preferred in the description of low-density flow fields (also called rarefied flows), moving solids, like in describing the motion of a projectile and so on. However, for a deformable system like a continuum fluid, there are infinite numbers of fluid elements whose motion has to be described, and the Lagrangian approach becomes unmanageable. Instead, we can employ spatial coordinates to help to identify particles in a flow. The velocity of all particles in a flow, therefore, can be expressed in the following manner.

$$\begin{aligned} V_x &= f(x, y, z, t) \\ V_y &= g(x, y, z, t) \\ V_z &= h(x, y, z, t) \end{aligned} \tag{2.15}$$

This is called the *Eulerian* or *field approach*. If properties and flow characteristics at each position in space remain invariant with time, the flow is called *steady flow*. A time dependent flow is referred to as *unsteady flow*. The steady flow velocity field would then be given as

$$\begin{aligned} V_x &= f(x, y, z) \\ V_y &= g(x, y, z) \\ V_z &= h(x, y, z) \end{aligned} \tag{2.16}$$

2.5.1 Relation between Local and Material Rates of Change

The rate of change of a property measured by probes at fixed locations is referred to as *local rate of change*, and the rate of change of properties experienced by a material particle is termed as the *material* or the *substantive rate of change*.

The local rate of change of a property η is denoted by $\partial\eta(x,t)/\partial t$, where it is understood that x is held constant. The material rate of change of property η shall be denoted by $D\eta/Dt$. If η is the velocity V, then DV/Dt is the rate of change of velocity for a fluid particle and thus, is the acceleration that the fluid particle experiences. On the other hand, $\partial V/\partial t$ is just a local rate of change of velocity recorded by a stationary probe. In other words, DV/Dt is the particle or material acceleration and $\partial V/\partial t$ is the local acceleration.

For a fluid flowing with an uniform velocity V_∞, it is possible to write the relation between the local and material rates of change of property η as

$$\frac{\partial \eta}{\partial t} = \frac{D\eta}{Dt} - V_\infty \frac{\partial \eta}{\partial x} \tag{2.17}$$

Thus, the local rate of change of η is due to the following two effects.

1. Due to the change of property of each particle with time.
2. Due to the combined effect of the spatial gradient of that property and the motion of the fluid.

When a spatial gradient exists, the fluid motion brings different particles with different values of η to the probe, thereby modifying the rate of change observed. This latter effect is termed a *convection effect*. Therefore, $V_\infty(\partial\eta/\partial x)$ is referred to as the convective rate of change of η. Even though Equation (2.17) has been obtained with uniform velocity V_∞, note that, in the limit $\delta t \to 0$ it is only the local velocity V which enters into the analysis and therefore, we have

$$\frac{\partial \eta}{\partial t} = \frac{D\eta}{Dt} - V \frac{\partial \eta}{\partial x} \tag{2.18}$$

Equation (2.18) can be generalized for a three-dimensional space as

$$\frac{\partial}{\partial t} = \frac{D}{Dt} - (V.\nabla) \tag{2.19}$$

where ∇ is the gradient operator $(= i\,\partial/\partial x + j\,\partial/\partial y + k\,\partial/\partial z)$ and $(V.\nabla)$ is a scalar product $(= V_x\ \partial/\partial x + V_y\ \partial/\partial y + V_z\ \partial/\partial z)$. Equation (2.19) is usually written as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V.\nabla \tag{2.20}$$

when η is the velocity of a fluid particle, DV/Dt gives acceleration of the fluid particle and the resultant equation is

$$\boxed{\frac{DV}{Dt} = \frac{\partial V}{\partial t} + (V.\nabla)V} \tag{2.21}$$

Equation (2.21) is known as *Euler's acceleration formula.*

2.5.2 Graphical Description of Fluid Motion

There are three important concepts for visualizing or describing flow fields. They are

1. Concept of *pathline.*
2. Concept of *streakline.*
3. Concept of *streamline.*

2.5.2.1 Pathline

Pathline may be defined as a line in the flow field describing the trajectory of a given fluid particle. From the Lagrangian viewpoint, namely, a closed system with a fixed identifiable quantity of mass, the independent variables are the initial position with which each particle is identified and the time. Hence, the locus of the same particle over a time period from t_0 to t_n is called the pathline.

2.5.2.2 Streakline

Streakline may be defined as the instantaneous locus of all the fluid elements that have passed the point of injection at some earlier time. Consider a continuous tracer injection at a fixed point Q in space. The connection of all elements passing through the point Q over a period of time is called the streakline.

2.5.2.3 Streamlines

Streamlines are imaginary lines, in a fluid flow, drawn in such a manner that the flow velocity is always tangential to it. Flows are usually depicted graphically with the aid of streamlines. These are *imaginary lines* in the flow field such that the velocity at all points on these lines are always tangential. Streamlines proceeding through the periphery of an infinitesimal area at some instant of time t will form a tube called *streamtube*, which is useful in the study of fluid flow.

From the Eulerian viewpoint, namely, an open system with constant control volume, all flow properties are functions of a fixed point in space and time, if the process is transient. The flow direction of various particles at time t_i forms streamline. The pathline, streamline, and streakline are different in general but coincide in a steady flow.

2.5.2.4 Timelines

In modern fluid flow analysis, yet another graphical representation, namely *timeline*, is used. When a pulse input is periodically imposed on a line of tracer source placed normal to a flow, a change in the flow profile can be observed. The tracer image is generally termed *timeline*. Timelines are often generated in the flow field to aid the understanding of flow behavior such as the velocity and velocity gradient.

From the above-mentioned graphical descriptions, it can be inferred that

- There can be no flow through the lateral surface of the streamtube.
- An infinite number of adjacent streamtubes arranged to form a finite cross-section is often called a bundle of streamtubes.
- Streamtube is a Eulerian (or field) concept.
- Pathline is a Lagrangian (or particle) concept.

- For steady flows, streamlines and streaklines are identical.

2.6 Basic and Subsidiary Laws for Continuous Media

In the range of engineering interest, four basic laws must be satisfied for any continuous medium. They are

- Conservation of matter (continuity equation).
- Newton's second law (momentum equation).
- Conservation of energy (first law of thermodynamics).
- Increase of entropy principle (second law of thermodynamics).

In addition to these primary laws, there are numerous subsidiary laws, sometimes called constitutive relations, that apply to specific types of media or flow processes (e.g., equation of state for perfect gas, Newton's viscosity law for certain viscous fluids, isentropic and adiabatic process relations are some of the commonly used subsidiary equations in flow physics).

2.6.1 Systems and Control Volumes

In employing the basic and subsidiary laws, any one of the following modes of application may be adopted.

- The activities of each and every given element of mass must be such that it satisfies the basic laws and the pertinent subsidiary laws.
- The activities of each and every elemental volume in space must be such that the basic laws and the pertinent subsidiary laws are satisfied.

In the first case, the laws are applied to an identified quantity of matter called the *control mass system*. A control mass system is an identified quantity of matter, which may change shape, position, and thermal condition, with time or space or both, but must always entail the same matter.

For the second case, a definite volume called *control volume* is designated in space, and the boundary of this volume is known as *control surface*. The amount and identity of the matter in the control volume may change with time, but the shape of the control volume is fixed, i.e., the control volume may change its position in time or space or both, but its shape is always preserved.

2.6.2 Integral and Differential Analysis

The analysis where large control volumes are used to obtain the aggregate forces or transfer rates is termed the *integral analysis*. When the analysis is applied to individual points in the flow field, the resulting equations are differential equations, and the method is termed the *differential analysis*.

2.6.3 State Equation

For air at normal temperature and pressure, the density ρ, pressure p, and temperature T are connected by the relation $p = \rho RT$, where R is a constant called gas constant. This is known as the state equation for a perfect gas. At high pressures and low temperatures, the above state equation breaks down. At normal pressures and temperatures the mean distance between molecules and the potential energy arising from their attraction can be neglected. The gas behaves like a perfect gas or ideal gas in such a situation. At this stage, it is essential to understand the difference between the ideal and perfect gases. An *ideal gas is frictionless and incompressible.* The perfect gas has viscosity and can therefore develop shear stresses, and it is compressible according to perfect gas state equation.

Real gases below critical pressure and above the critical temperature tend to obey the perfect-gas law. The perfect-gas law encompasses both Charles' law and Boyle's law. Charles' law states that, *for constant pressure, the volume of a given mass of gas varies directly as its absolute temperature.* Boyle's law (isothermal law) states that, *for constant temperature, the density varies directly as the absolute pressure.*

2.7 Kinematics of Fluid Flow

To simplify the discussions, let us assume the flow to be incompressible, i.e., the density is treated as invariant. The basic governing equations for an incompressible flow are the continuity and momentum equations. The continuity equation is based on the conservation of matter. For steady incompressible flow, the continuity equation in differential form is

$$\boxed{\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0} \tag{2.22}$$

Equation (2.22) may also be expressed as $\nabla . V = 0$, where $V\,(i\,V_x + j\,V_y + k\,V_z)$ is the flow velocity.

The momentum equation which is based on Newton's second law represents the balance between various forces acting on a fluid element, namely,

1. Force due to rate of change of momentum, generally referred to as inertia force.

2. Body forces such as buoyancy force, magnetic force, and electrostatic force.

3. Pressure force.

4. Viscous forces (causing shear stress).

For a fluid element under equilibrium, by Newton's second law, we have the momentum equation as

$$\boxed{\text{Inertia force} + \text{Body force} + \text{Pressure force} + \text{Viscous force} = 0}$$

For a gaseous medium, body forces are negligibly small compartialred to other forces and hence can be neglected. For steady incompressible flows, the momentum equation can be written as

$$\begin{aligned}
V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}\right) \\
V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_y}{\partial z^2}\right) \quad (2.23) \\
V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 V_z}{\partial x^2} + \frac{\partial V_z}{\partial y^2} + \frac{\partial V_z}{\partial z^2}\right)
\end{aligned}$$

Equations (2.23) are the x, y, z components of momentum equation, respectively. These equations are generally known as *Navier–Stokes* equations. They are nonlinear partial differential equations and there exists no known analytical method to solve them. This poses a major problem in fluid flow analysis. However, the problem is tackled by making some simplifications to the equation, depending on the type of flow to which it is to be applied. For certain flows, the equation can be reduced to an ordinary differential equation of a simple linear type. For some other type of flows, it can be reduced to a nonlinear ordinary differential equation. For the above types of Navier–Stokes equation governing special category of flows such as potential flow, fully developed flow in a pipe and channel, and boundary layer flows, it is possible to obtain analytical solutions.

It is essential to understand the physics of the flow before reducing the Navier–Stokes equations to any useful form, by making suitable approximations with respect to the flow. For example, let us examine the flow over an aircraft wing, shown in Figure 2.4.

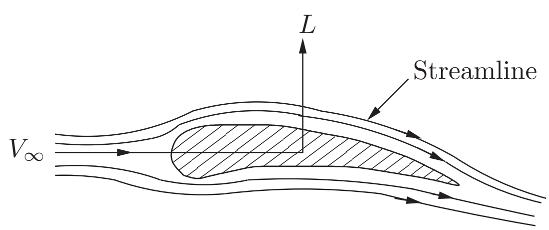

Figure 2.4: Flow past a wing

This kind of problem is commonly encountered in fluid mechanics. Air flow over the wing creates a pressure at the bottom which is larger than that at the top surface. Hence, there is a net resultant force component normal to the freestream flow direction called *lift*, L. The velocity varies along the wing chord as well as in the direction normal to its surface. The former variation is due to the shape of the aerofoil and the latter is due to the no-slip condition at the wall. In the direction normal to wing surface the velocity gradients are very large near the wall and the flow reaches asymptotically a constant velocity in a short distance. This thin region adjacent to the wall where the velocity increases from zero to freestream value is known as the *boundary layer*. Inside the boundary layer the viscous forces are predominant. Further, it so happens that the static pressure outside the boundary layer, acting in the direction normal to the surface, is transmitted through the boundary layer without appreciable change. In other words, the pressure gradient across the boundary layer is zero. Neglecting the interlayer friction between the streamlines, in the region outside the boundary layer, it is possible to treat the flow as inviscid. Inviscid flow is also called potential flow, and for this case the Navier–Stokes equation can be made linear. It is possible to obtain the pressures in the field outside the boundary layer and treat this pressure to be invariant across the boundary layer, i.e., the pressure in the freestream is impressed through the boundary layer. For low-viscous fluids such as air, we can assume with a high degree of accuracy that, the flow is frictionless over the entire flow field except for a thin region near solid surfaces. In the vicinity of solid surface, owing to high velocity gradients the frictional effects become significant. Such regions near solid boundaries where the viscous effects are predominant are termed *boundary layers*.

In general, for streamlined bodies these boundary layers are extremely thin. There may be laminar and turbulent flow within the boundary layer, and its thickness and profile may change along the direction of the flow. Different zones of boundary layer over a flat plate are shown in Figure 2.5. The *laminar sublayer* is that zone near the boundary where the turbulence is suppressed to such a degree that only the laminar effects predominate. The various regions shown in Figure 2.5 are not sharp demarcations of different zones. There is actually a smooth variation from a region where certain effect predominates to another region where some other effect is predominant.

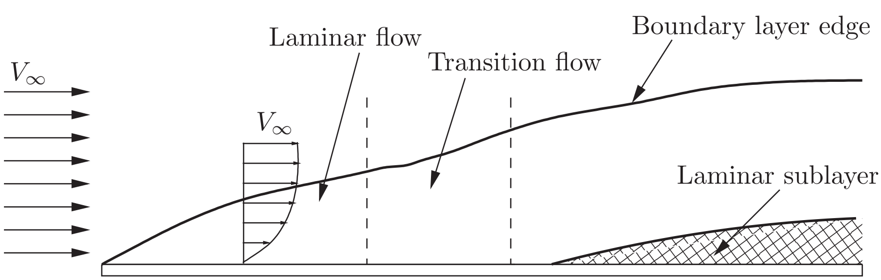

Figure 2.5: Flow over a flat plate

Although the boundary layer is thin, it plays a vital role in fluid dynamics. The drag on ships and missiles, the efficiency of compressors and turbines for jet engines, the effectiveness of ram and turbojets, and the efficiencies of numerous other engineering devices are all influenced by the boundary layer to a significant extent. The performance of a device depends on the behavior of boundary layer and its effect on the main flow. The following are some of the important parameters associated with boundary layers.

2.7.1 Boundary Layer Thickness

Boundary layer thickness, δ, may be defined as the distance from the wall in the direction normal to the wall surface, where the fluid velocity is within 1 percent of the local main stream velocity. It may also be defined as the distance, δ normal to the surface, in which the flow velocity rises from zero to some specified value (e.g., 99%) of its local main stream flow. The boundary layer thickness may be shown schematically as in Figure 2.6.

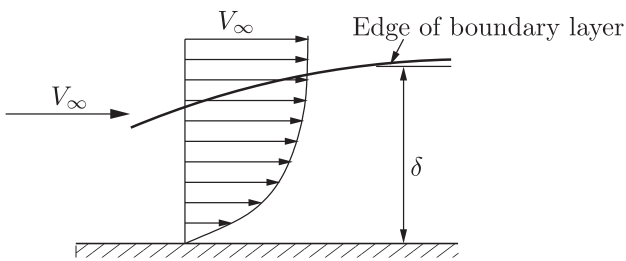

Figure 2.6: Illustration of boundary layer thickness

2.7.2 Displacement Thickness

Displacement thickness, δ^*, may be defined as the distance by which the boundary would have to be displaced if the entire flow were imagined to be frictionless and the same mass flow maintained at any section.

Consider unit width in the flow across an infinite flat plate at zero angle of attack, and let x-component of velocity to be V_x and y-component of velocity be V_y. The volume flow rate Δq through this boundary layer segment of unit width is given by

$$\Delta q = \int_0^\infty (V_m - V_x)\, dy$$

where V_m is the main stream frictionless velocity component and V_x is the actual local velocity component. To maintain the same volume flow rate, q, for the frictionless case as in the actual case, the boundary must be shifted out by a distance δ^* so as to cut off the amount Δq of flow.

Thus,

$$V_m\delta^* = \Delta q = \int_0^\infty (V_m - V_x) dy$$

$$\delta^* = \int_0^\infty \left(1 - \frac{V_x}{V_m}\right) dy \qquad (2.24)$$

The displacement thickness is illustrated in Figure 2.7.

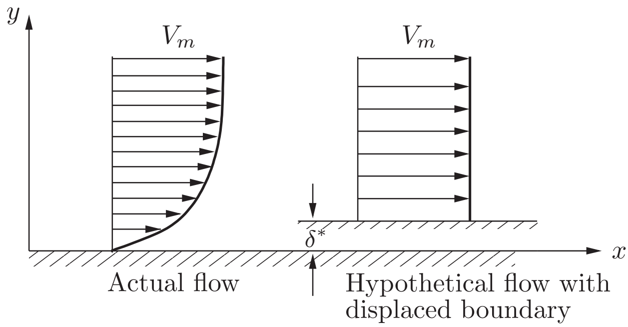

Figure 2.7: Displacement thickness

The main idea of this postulation is to permit the use of a displaced body in place of the actual body such that, the frictionless mass flow around the displaced body is the same as the actual mass flow around the real body. The displacement thickness concept is made use of in the design of wind tunnels, air intakes for jet engines, etc.

There are other (thickness) measures pertaining to the thickness of boundary layer, such as *momentum thickness,* θ, and *energy thickness,* δ_e. They are defined mathematically as follows.

$$\theta = \int_0^\infty \left(1 - \frac{V_x}{V_m}\right) \frac{\rho V_x}{\rho_m V_m} dy \qquad (2.25)$$

$$\delta_e = \int_0^\infty \left(1 - \frac{V_x^2}{V_m^2}\right) \frac{\rho V_x}{\rho_m V_m} dy \qquad (2.26)$$

where V_m and ρ_m are the velocity and density at the edge of the boundary layer and V_x and ρ are the velocity and density at any y location normal to the body surface. In addition to boundary layer thickness, displacement thickness, momentum thickness, and energy thickness, we can define the transition point and separation point also with the help of boundary layer.

2.7.3 Transition Point

Transition point may be defined as the end of the region at which the flow in the boundary layer on the surface ceases to be laminar and becomes turbulent.

2.7.4 Separation Point

Separation point is the position at which the boundary layer leaves the surface of a solid body. If the separation takes place while the boundary layer is still laminar, the phenomenon is termed *laminar separation*. If it takes place for a turbulent boundary layer it is called *turbulent separation*.

The boundary layer theory makes use of Navier–Stokes equation (Equation (2.23)) with the viscous terms in it but in a simplified form. On the basis of many assumptions, such as boundary layer thickness being small compared to the body length and similarity between velocity profiles in a laminar flow, the Navier–Stokes equation can be reduced to a nonlinear ordinary differential equation, for which special solutions exist. Some such problems for which Navier–Stokes equations can be reduced to boundary layer equations and closed form solutions can be obtained are flow past a flat plate or Blassius problem, Hagen–Poiseuille flow through pipes, Couette flow, and flow between rotating cylinders.

2.7.5 Rotational and Irrotational Motion

When a fluid element is subjected to a shearing force, a velocity gradient is produced perpendicular to the direction of shear, i.e., a relative motion occurs between two layers. To achieve this relative motion the fluid elements have to undergo rotation. A typical example of this type of motion is the motion between two roller chains rubbing each other, but moving at different velocities. It is convenient to use an abstract quantity called *circulation*, Γ, defined as the line integral of velocity vector between any two points (to define rotation of the fluid element). By definition, the circulation is given as

$$\Gamma = \oint_c V.\,dl \tag{2.27}$$

where dl is an elemental length. Circulation per unit area is known as *vorticity* ζ,

$$\zeta = \Gamma/A \tag{2.28}$$

In vector form ζ becomes

$$\zeta = \nabla \times V = \mathrm{curl}\, V \tag{2.29}$$

For a two-dimensional flow in xy plane, ζ becomes

$$\zeta_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \tag{2.30}$$

where ζ_z is the vorticity about the z-direction, which is normal to the flow field.

Likewise, the other components of vorticity about x and y axes are

$$\zeta_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}$$

$$\zeta_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}$$

If $\zeta = 0$, the flow is known as *irrotational flow.* Inviscid flows are basically irrotational flows.

2.8 Streamlines

These are imaginary lines in the flow field such that the velocity at all points on these lines is always tangential to them. Flows are usually depicted graphically with the aid of streamlines. Streamlines proceeding through the periphery of an infinitesimal area at some time t form a tube called a *stream tube*, which is useful for the study of fluid flow phenomena. From the definition of streamlines, it can be inferred that

- Flow cannot cross a streamline, and the mass flow between two streamlines is confined.
- Based on the streamline concept, a function ψ called *stream function* can be defined. The velocity components of a flow field can be obtained by differentiating the stream function.

In terms of stream function, ψ, the velocity components of a two-dimensional incompressible flow are given as

$$V_x = \frac{\partial \psi}{\partial y}, \qquad V_y = -\frac{\partial \psi}{\partial x} \tag{2.31}$$

If the flow is compressible the velocity components become

$$\boxed{V_x = \frac{1}{\rho}\frac{\partial \psi}{\partial y}, \qquad V_y = -\frac{1}{\rho}\frac{\partial \psi}{\partial x}} \tag{2.32}$$

It is important to note that the stream function is defined only for two-dimensional flows, and the definition does not exist for three-dimensional flows. Even though some books define ψ for axisymmetric flows, they again prove to be equivalent to two-dimensional flow. We must realize that the definition of ψ does not exist for three-dimensional flows. This is because such a definition demands a *single tangent* at any point on a streamline, which is possible only in two-dimensional flows.

2.8.1 Relationship between Stream Function and Velocity Potential

For irrotational flows (the fluid elements in the field are free of angular motion), there exists a function ϕ called *velocity potential* or *potential function*. For two-dimensional flows, ϕ must be a function of x, y, and t. The velocity components are given by

$$V_x = \frac{\partial \phi}{\partial x}, \qquad V_y = \frac{\partial \phi}{\partial y} \tag{2.33}$$

From Equations (2.31) and (2.33), we can write

$$\boxed{\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}, \qquad \frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}} \tag{2.34}$$

These relations between stream function and potential function given by Equation (2.34) are the famous *Cauchy–Riemann equations* of complex-variable theory. It can be shown that the lines of constant ϕ or potential lines form a family of curves which intersect the streamlines in such a manner as to have the tangents of the respective curves always at right angles at the point of intersection. Hence, the two sets of curves given by ψ = constant and ϕ = constant form an orthogonal grid system or flow net.

Unlike stream function, potential function exists for three-dimensional flows also. This is because there is no condition such as the local flow velocity must be tangential to the potential lines imposed in the definition of ϕ. The only requirement for the existence of ϕ is that the flow must be potential.

2.9 Potential Flow

Potential flow is based on the concept that the flow field can be represented by a potential function ϕ such that,

$$\boxed{\nabla^2 \phi = 0} \tag{2.35}$$

This linear partial differential equation is popularly known as the *Laplace equation*. Derivatives of ϕ give velocities, as given in Equation (2.33), for a two-dimensional flow. Unlike the stream function ψ, the potential function can exist only if the flow is *irrotational*, that is, when viscous effects are absent. All inviscid flows must satisfy the irrotationality condition, namely,

$$\boxed{\nabla \times V = 0} \tag{2.36}$$

For two-dimensional potential flows, by Equation (2.30), we have the vorticity ζ as

$$\zeta_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = 0$$

Equation (2.36) can be rewritten, using Equation (2.33), as

$$\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial x \partial y} = 0$$

This shows that the flow is irrotational. For two-dimensional flows, the continuity equation given by Equation (2.22) becomes

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

Using Equation (2.33) this equation can be expressed as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

i.e.,

$$\boxed{\nabla^2 \phi = 0} \tag{2.37}$$

For flows with finite vorticity the potential function ϕ does not exist, and the linear equation $\nabla^2 \phi = 0$ cannot be obtained.

For potential flows, the Navier–Stokes equations (2.23) reduce to the form

$$\begin{aligned} V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{aligned} \tag{2.38}$$

Equations (2.38) are known as *Euler equations*.

2.9.1 Two-Dimensional Source and Sink

A type of flow in which the fluid emanates from origin and spreads radially outwards to infinity, as shown in Figure 2.8, is called a *source*. The volume flow rate q crossing a circular surface of radius r and unit depth is given by

$$q = 2\pi r V_r \tag{2.39}$$

where V_r is the radial component of velocity. For a source, the radial lines are the streamlines. Therefore, the potential lines must be concentric circles, represented by

$$\phi = A \ln(r)$$

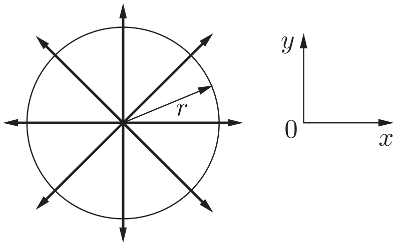

Figure 2.8: A two-dimensional source

where A is a constant. The radial velocity component $V_r = \partial\phi/\partial r = A/r$. Substituting this into Equation (2.39), we get

$$\frac{2\pi r A}{r} = q$$

or

$$A = \frac{q}{2\pi}$$

Thus, the velocity potential for a two-dimensional source of strength q becomes

$$\boxed{\phi = \frac{q}{2\pi}\ln(r)} \tag{2.40}$$

In a similar manner as above, the stream function for a source of strength q can be obtained as

$$\boxed{\psi = \frac{q}{2\pi}\theta} \tag{2.41}$$

where θ stands for the location of the streamline in θ-direction. Similarly, for a sink, which is *a type of flow in which the fluid at infinity flows radially towards the origin*, we can show that the potential and stream functions are given by

$$\boxed{\phi = -\frac{q}{2\pi}\ln(r)}$$

and

$$\boxed{\psi = -\frac{q}{2\pi}\theta}$$

where q is the strength of the sink. Note that the volume flow rate is termed as the strength of source and sink. Also, for both source and sink the origin is a singular point.

2.9.2 Simple Vortex

A *simple* or *free vortex* is a flow in which the fluid elements simply move along concentric circles, without spinning about their own axes. The fluid elements have only translatory motion in a free vortex. In addition to moving along concentric paths, if the fluid elements spin about their own axes, the flow is termed *forced vortex*.

A *simple* or *free vortex* can be established by selecting the stream function, ψ, of the source to be the potential function ϕ of the vortex. Thus, for a simple vortex

$$\boxed{\phi = \frac{q}{2\pi}\theta} \tag{2.42}$$

It can be easily shown from Equation (2.42) that the stream function for a simple vortex is

$$\boxed{\psi = -\frac{q}{2\pi}\ln(r)} \tag{2.43}$$

It follows from Equations (2.42) and (2.43) that, the velocity components of the simple vortex, shown in Figure 2.9, are

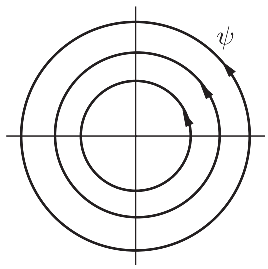

Figure 2.9: Simple vortex flow

$$V_\theta = \frac{q}{2\pi r}, \qquad V_r = 0 \tag{2.44}$$

Here again the origin is a singular point, where the tangential velocity approaches infinity, as seen from Equation (2.44). The flow in a simple or free vortex resembles part of the common whirlpool found while paddling a boat or while emptying water from a bathtub. An approximate profile of a whirlpool is as shown in Figure 2.10.

For the whirlpool shown in Figure 2.10, the circulation along any path about the origin is given by,

$$\begin{aligned}\Gamma &= \oint V.dl \\ &= \int_0^{2\pi} V_\theta \; r \; d\theta\end{aligned}$$

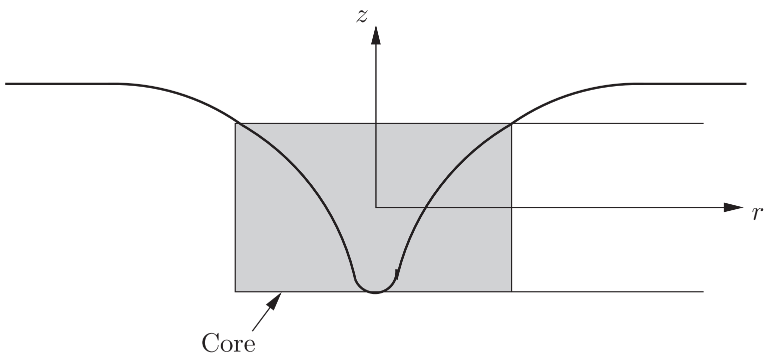

Figure 2.10: A whirlpool flow field

By Equation (2.44), $V_\theta = \dfrac{q}{2\pi r}$, therefore, the circulation becomes

$$\Gamma = \int_0^{2\pi} \frac{q}{2\pi r} r d\theta = q$$

Since there are no other singularities for the whirlpool shown in Figure 2.10, this must be the circulation for all paths about the origin. Consequently, q in the case of vortex is the measure of circulation about the origin and is also referred to as the *strength of the vortex*.

2.9.3 Source-Sink Pair

This is a combination of a source and sink of equal strength, situated (located) at a distance apart. The stream function due to this combination is obtained simply by adding the stream functions of source and sink. When the distance between the source and sink is made negligibly small, in the limiting case, the combination results in a *doublet*.

2.10 Viscous Flows

In the previous sections of this chapter, we have seen many interesting concepts of fluid flows. With this background, let us observe some of the important aspects of fluid flow from a practical or application point of view.

We are familiar with the fact that viscosity produces shear force which tends to retard the fluid motion. It works against inertia force. The ratio of these two forces governs (dictates) many properties of the flow, and the ratio expressed in the form of a nondimensional parameter known as the famous *Reynolds number*, Re_L.

$$\boxed{Re_L = \frac{\rho V L}{\mu}} \tag{2.45}$$