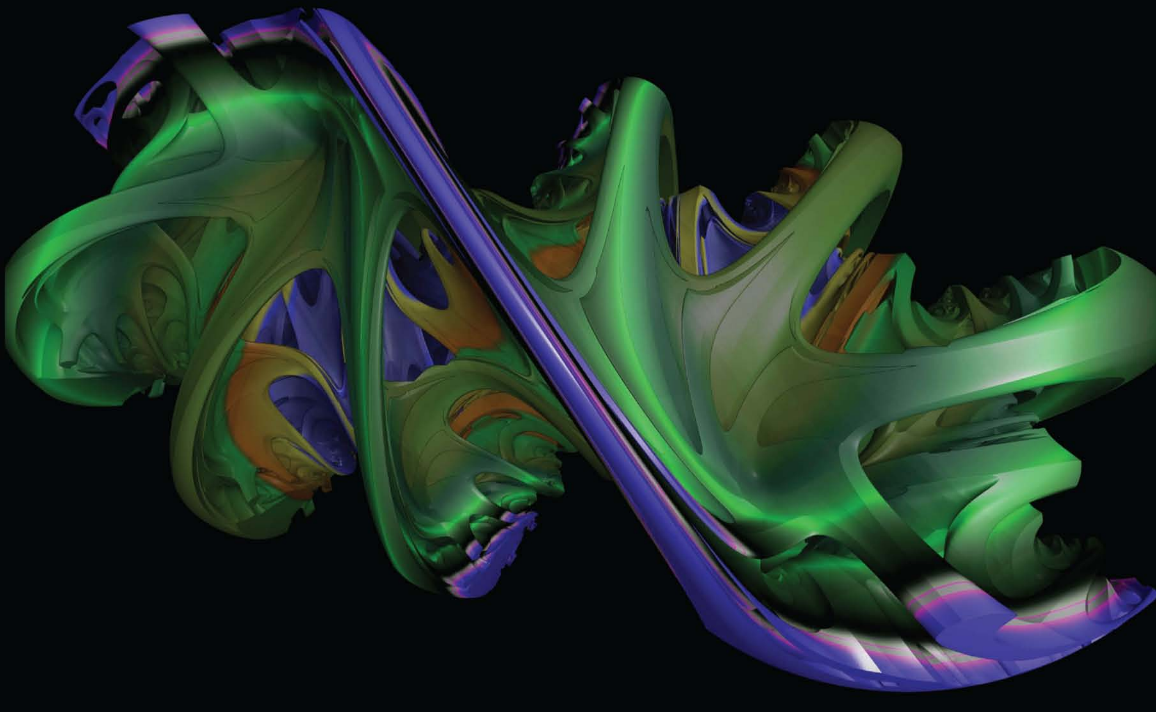
A Concise Introduction to Hypercomplex Fractals



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Preface

Since I started working with fractals, my family, friends, colleagues, and students were puzzled hearing about hypercomplex fractals. This is mainly due to the specific nomenclature used both in the fractal theory as well as in the hypercomplex number theory, with all of their attractors, repellers, octonions, zero divisors, and nilpotents. These words naturally scare them! And at the same time, they love colorful images generated by my computer simulations. These images, however, are just nice pictures to them. I am convinced that many non scientists, who had contact with complex and hypercomplex fractals have the same feelings. Therefore, I decided to write a book which would present complex and hypercomplex fractals in a concise and comprehensible manner omitting mathematical formalism as much as possible. This idea has germinated in my mind for a few years, and finally I can place the fruit of a few years' work into your hands.

This book concisely presents the complete story on complex and hypercomplex fractals, starting from the very first steps in complex dynamics and resulting complex fractal sets, through the generalizations of Julia and Mandelbrot sets on a complex plane and the Holy Grail of fractal geometry – a 3-D Mandelbrot set, and ending with hypercomplex, multicomplex, and multihypercomplex fractal sets that are still under consideration of scientists. I tried to write this book in a simple way in order to make it understandable to most people whose math knowledge covers the fundamentals of complex numbers only. Moreover, the book is full of illustrations of generated fractals and stories about great mathematicians, number spaces, and related fractals. In most cases, only information required for proper understanding of a nature of a given vector space or a construction of a given fractal set is provided; nevertheless, a more advanced reader may treat this book as a fundamental compendium on hypercomplex fractals, with references to purely scientific issues like dynamics and stability of hypercomplex systems.

The preparation of this book would not be possible without the reviewers: Professor Wojciech Chojnacki from the University of Adelaide, Australia, who is an outstanding specialist in computer science, computer graphics, and mathematics related to these disciplines, and Dr. Krzysztof Gdawiec from the University of Silesia, Poland, who is the eminent fractal researcher working on developing new types of fractal sets. They both contributed many corrections and additions to make this book even better, and I am very grateful to them for their valuable comments and discussion.

I would like to thank my dear fiancée, Angelika, who supported me during my writing of this book, read its draft version, and provided a great feedback. I am very grateful to her for this help.

Finally, I would like to thank CRC editorial staff, especially Rick Adams, Jessica Vega, and Robin Lloyd-Starkes, for their great and professional support during the entirety of the publishing process.

Andrzej Katunin
January, 2017

CHAPTER 1

Introduction to Fractals on a Complex Plane

Fractal. This mysterious word penetrates modern pop-culture and society. But what does it mean? What is the definition of a fractal?

Actually, considering a great variety of types, shapes, and properties of objects that are called fractals, it is really hard to formulate a universal definition. In 1975, the word "fractal" was introduced by Benoit B. Mandelbrot, the father of a fractal geometry, and popularized in his famous book, *The Fractal Geometry of Nature* [79]. It comes from the Latin *fractus*, which means "broken" or "fractured." This name explains its nature, which is usually characterized by a very complex shape and, in most cases, fractional dimension. Beyond this property, fractals have few other differences from other geometrical objects. The next property is the self-similarity of fractals, which means that they looks exactly the same no matter how big the magnification of the fractals are. Roughly speaking, fractals are constructed from smaller copies of themselves. And the last thing that characterizes a fractal (which results from already presented properties) – it cannot be represented by a closed form expression, but by a recurrent dependency.

However, fractals and hierarchical structure of objects were known a long time before the Mandelbrot. The best examples are the Indian temples built in the Middle Ages[1] (see example in Figure 1.1). Many proofs of self-similarity of Hindu temples were given by numerous

[1] Striking examples of Indian temples that use self-similarity in their constructions are the Kandariya Mahadeva Temple built in 1030 in Khajuraho, the towers of Meenakshi Amman Temple built in 1623–1655 in Madurai, the Shveta Varahaswamy Temple built in 1673–1704 in Mysore, and many others.

Figure 1.1: A view of Chennakesava Temple, Somanathapura, India, built in 1268 (photo courtesy of Arlan Zwegers).

researchers [31, 106, 117]. Additional proofs were provided by Ron Eglash, who works in the area of ethnomathematics, and are outlined in his book [36]. He wrote that Africans have been widely using fractals in their culture (e.g., in architecture and textile design) for centuries. In the 20th century, fractals conquered arts, they appeared in works of artists who used the decalcomania technique; several paintings of the 20th-century surrealists also consist of a fractal hierarchy (e.g. *The Face of War* painted in 1940 by Salvador Dalí).

The first objects that we call now fractals appeared at the turn of the 19th and 20th centuries, and started from the simplest fractal — the Cantor set, named after Georg Cantor, a German mathematician. This fractal was constructed by Cantor in 1883, and inspired by earlier studies of Karl Weierstrass, a German mathematician who introduced everywhere continuous but nowhere differentiable function (which is known now as the Weierstrass function), the prototype of a fractal. Two decades later, the next fractal function appeared — the Koch curve, and then Koch snowflake, which was proposed by Swedish mathematician Helge von Koch in 1904.

In the meantime, two fractal curves of a special type appeared,