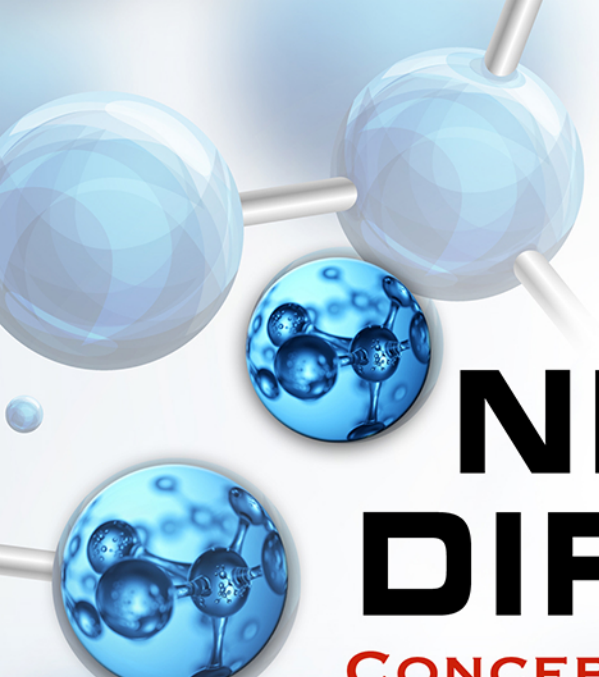
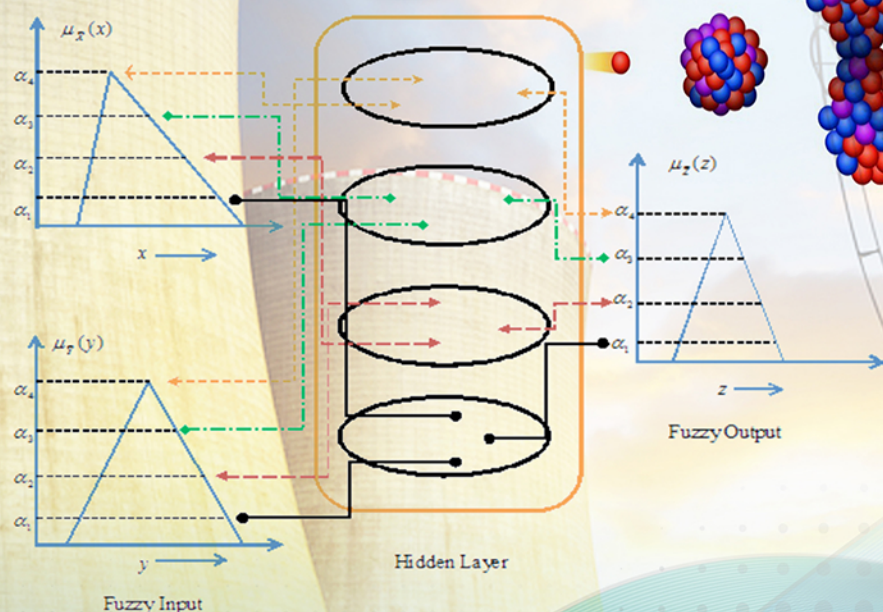




NEUTRON DIFFUSION

CONCEPTS AND UNCERTAINTY
ANALYSIS FOR ENGINEERS
AND SCIENTISTS



S. Chakraverty
Sukanta Nayak



CRC Press
Taylor & Francis Group

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Concepts and Uncertainty Analysis for
Engineers and Scientists



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Preface

The design of the reactor core is based on the description of the production, transport and absorption of neutrons. As neutrons move within a medium, viz. gas, liquid or solid, they collide with the nuclei of the atoms in the medium. During such collisions, neutrons may be absorbed by the nuclei, which are either elastic or inelastic. Absorption of neutrons may result in a loss or an increase in the number of neutrons by fission. Fission neutrons usually have different energies and move in different directions than incident neutrons. As a result, there may be scattering of neutrons, which changes the position, energy and direction of the motion of the neutrons. Furthermore, the scattering of neutron collision inside a reactor depends upon the geometry of the reactor, diffusion coefficient and absorption coefficient. It may also be noted that the parameters responsible for the diffusion (i.e. the scattering of neutron) may not always be crisp, rather they may be uncertain.

In general, these uncertainties occur due to the vague, imprecise and incomplete information about the variables and parameters, as a result of errors in measurement, observation, experiment or applying different operating conditions or due to maintenance-induced errors, which are uncertain in nature. So, to overcome these uncertainties, one may use the fuzzy/interval or stochastic environmental parameters and variables in place of crisp (fixed) parameters. With these uncertainties, the governing differential equations turn fuzzy/interval or stochastic. Practically, it is sometimes difficult to obtain the solution of fuzzy equations due to the complexity in the fuzzy arithmetic; for example, addition and multiplication are not the inverse operations of subtraction and division, respectively. On the other hand, we may model the problem as stochastic only when sufficient data are available. Moreover, the combination (hybridization) of fuzzy and stochastic is also a new vista. These were found to be important and challenging areas of study in recent years. As such, one may need to understand the nuclear diffusion principles/theories corresponding to reliable and efficient techniques for the solution of such uncertain problems.

Accordingly, the objective of this book is to provide first the basic concepts of reactor physics as well as neutron diffusion theory. The main aim of the book, however, is about handling uncertainty in neutron diffusion problems. Hence, uncertainties (i.e. fuzzy, interval, stochastic) and their applications in nuclear diffusion problems have been included here in a systematic manner, along with the recent developments. This book may be an essential reference for students, scholars, practitioners, researchers and academicians in the assorted fields of engineering and science, particularly nuclear engineering.

[Chapter 1](#) describes the preliminaries of basic reactor principles. Here, a few important and related terminologies for the nuclear reactor are briefly explained. In [Chapter 2](#), neutron diffusion theory and the formulation of the neutron diffusion equation have been presented, which give first-hand scientific insights to study various nuclear design problems. The transportation of the scattered neutrons is modelled and formulated mathematically. In [Chapter 3](#), the fundamentals of uncertainties are discussed. In this chapter, uncertainties are addressed with respect to three categories, viz. interval, fuzzy and probabilistic. The operations of these uncertainties are demonstrated through various examples. Furthermore, [Chapter 4](#) elaborates the uncertain modelling (considering interval/fuzzy parameters as uncertain) of neutron diffusion. The factors involved in the reactor system, which are responsible for uncertainness, are modelled in terms of interval/fuzzy.

One-group models with respect to crisp parameters are explained in [Chapter 5](#). This chapter includes both the analytical and numerical approaches to investigate one-group models, whereas in [Chapter 6](#), uncertain (considering interval/fuzzy parameters as uncertain) one-group models are discussed, and example problems are investigated. In this chapter, the uncertain neutron diffusion equation for a bare square homogeneous reactor is discussed, which has been modelled by the fuzzy finite element method. The multigroup neutron diffusion equation has been generalized in [Chapter 7](#), and again the finite element method has been used to solve the same. In [Chapter 8](#), the uncertain multigroup neutron diffusion model is investigated. Accordingly, a benchmark problem is solved under an uncertain environment, and the sensitivity of the uncertain parameters is also analyzed.

[Chapter 9](#) includes the theory of point kinetic diffusion. Here, different terminologies related to point kinetic diffusion of the one-group bare reactor are discussed, and the point kinetic diffusion equation for crisp parameters is formulated. Next, the stochastic point kinetic diffusion equation is modelled in [Chapter 10](#). The basic concept of the birth–death stochastic process and stochastic point kinetic diffusion are discussed here. Finally, in [Chapter 11](#), hybridized uncertainty (i.e. both probabilistic and fuzzy) is considered, and uncertain point kinetic diffusion is modelled. Furthermore, the hybridized uncertainty for this model is demonstrated through an example problem.

This book aims to provide a systematic understanding of nuclear diffusion theory along with uncertainty, viz. fuzzy/interval/stochastic and hybrid methods. The book will certainly prove to be a benchmark for graduate and postgraduate students, teachers, engineers and researchers in this field. It provides comprehensive results and an up-to-date and self-contained review of the topic along with application-oriented treatment of the use of newly developed methods of different fuzzy, stochastic and hybrid computations in various domains of nuclear engineering and sciences. It is worth mentioning that the presented methods may very well be used in/extended to various other engineering disciplines, viz. electronics, marine, chemical and mining, and sciences such as physics, chemistry and biotechnology, where one wants to model physical problems with respect to non-probabilistic (interval/fuzzy) and hybrid uncertainties for understanding the real scenario.

S. Chakraverty
Sukanta Nayak

Basic Reactor Principles

The principle of the nuclear reactor is based on the transport of neutrons and their interaction with matter within a reactor. Various terminologies involved in the design of nuclear reactors, based on the fission process, have been included in this chapter.

1.1 Atomic Structure

An atom is a small part of an element which consists of a positively charged nucleus surrounded by negatively charged *electrons*. Atomic nuclei are made up of two fundamental particles, viz. protons and neutrons. The *proton* carries a unit positive charge (+e) and a mass about 1836 times the electronic mass (m_e). In fact, the proton is identical with the nucleus of a hydrogen atom. On the other hand, the *neutron* is electrically neutral and slightly heavier than the proton. The neutrons and protons are together called the *nucleons*.

For each element, protons represent the atomic number of the atomic nucleus and are denoted by the symbol Z . It is also the same as the ordinal number of the element in the familiar periodic table of the elements. The total number of neutrons (N) and protons (Z) inside the nucleus is called the *mass number* (A), that is $A = N + Z$. In general, a nucleus of an atom X of atomic number Z and mass number A is symbolically written as A_ZX . For example, the helium atom is denoted by ${}^4_2\text{He}$, with the atomic number 2 and mass number 4. These are also known as α -particles. The protons and neutrons are symbolically denoted as p and n , respectively. The subscript to the left (i.e. atomic number) of the atomic symbol is often omitted and it looks like AX .

Nuclei with the same atomic number (Z) but a different mass number (A) are called *isotopes*. A particular element with a given Z may have isotopes of different mass numbers. Their nuclei possess the same number of protons, whereas a different number of neutrons. For example, carbon has the isotopes ${}^{12}\text{C}$, ${}^{13}\text{C}$ and ${}^{14}\text{C}$ with mass numbers 12, 13 and 14, respectively. On the other hand, nuclei with the same mass number, but different atomic numbers, are known as *isobars*, while those with the same number of neutrons are known as *isotones*. Chlorine and potassium are isotones that have 20 neutrons in their nuclei.

The mass of each atom is expressed in terms of the atomic mass unit (u), which is exactly $1/12$ of the mass of the ${}^{12}\text{C}$ atom. Uranium is the most important element for the release of nuclear energy. In nature, it exists in three different forms with mass numbers 234, 235 and 238, respectively. Apart from this, another important element is thorium, with the atomic number 90 and mass number 232 (Hetrick 1971).

1.2 Binding Energy

In nuclear physics, the nuclear masses are determined by using the mass spectrometers (Ghoshal 2010). The actual mass is always less than the sum of the masses of the constituent nucleons. The difference is called the *mass defect*. The mass defect is related to the energy binding the particles in the nucleus. In other words, if we want to break up a nucleus of Z protons and N neutrons completely so that they are all separated from each other, a certain minimum amount of energy is to be supplied to the nucleus. This energy is known as the *binding energy*.

Let us consider the mass of protons, neutrons and electrons as m_p , m_n and m_e , respectively. The mass defect is defined as (Stacey 2007)

$$\text{Mass defect} = [Z(m_p + m_e) + (A - Z)m_n] - M,$$

where

$Z(m_p + m_e) + (A - Z)m_n$ is the masses of the constituents of the atom

M is the observed mass of the atom

Using the Einstein equation, the binding energy can be calculated as follows:

$$E = \Delta mc^2 \quad (1.1)$$

where

Δm is the mass defect

c is the velocity of light ($\approx 3 \times 10^8$ m/s)

In other words, binding energy = $\Delta m \times 931.5$ MeV, where $1 \text{ u} = 931.5$ MeV.

1.2.1 Calculation of 1 u

It is well known that 1 mole of ^{12}C has the mass of 12 g or 12×10^{-3} kg. Since 1 mole contains 6.02205×10^{23} atoms (Avogadro number), the mass of each ^{12}C atom is

$$\frac{12 \times 10^{-3}}{6.02205 \times 10^{23}} \text{ kg} = 12 \times 1.66056 \times 10^{-27} \text{ kg}.$$

Hence, the unit of atomic mass is

$$1 \text{ u} = \frac{1}{12} \times \frac{12 \times 10^{-3}}{6.02205 \times 10^{23}} \text{ kg} = 1.66056 \times 10^{-27} \text{ kg}. \quad (1.2)$$

The energy equivalent of this amount of mass is

$$\begin{aligned} 1 \text{ u} &= 1.66056 \times 10^{-27} \times c^2 \\ &= 1.66056 \times 10^{-27} \times 8.98755 \times 10^{16} \\ &= 931.502 \text{ MeV} \end{aligned} \quad (1.3)$$

If Δm is in gram, then E will be in MeV per gram atom.

The binding energy of a nucleus, when divided by the number of nucleons, gives the mean binding energy per nucleon.

The binding energy per nucleon is a measure of the stability of the nucleus. The greater the binding energy per nucleon, the more stable the nucleus.

It is noted that if the value of binding energy is negative then the product of the nucleus or nuclei will be less stable than the reactant nucleus or nuclei. If the binding energy is positive, then the product nucleus is more stable than the reactant nucleus.

1.3 Nuclear Reactions

The reactions in which the nucleus of an atom itself undergoes a spontaneous change or interacts with other nuclei of lighter particles resulting in new nuclei (and one or more lighter particles) are called nuclear reactions. Here, the neutron reactions are mainly categorized into scattering, capture and fission. The exchange of energy between a neutron and a nucleus are based on the scattering reactions. In scattering reactions, scattering is used to describe the movement of the neutron after interaction. Scattering may be either elastic or inelastic. In elastic scattering, the exchange of energy between the neutron and the nucleus is kinetic. But in inelastic scattering, the kinetic energy of the neutron is transferred to the nucleus as potential energy.

Neutrons lose much of their kinetic energy due to the scattering collision with various nuclei in the medium through which the neutrons move and become slow with energies of an electron volt or less. Finally, the kinetic energy may be reduced to such an extent that the average is the same as that of the atoms (or molecules) of the medium. Since the value of the kinetic energy depends on the temperature, it is called thermal energy. Hence, the neutrons whose energies have been reduced to this extent are called thermal neutrons.

In inelastic scattering, the excited compound nucleus can emit its excess energy as gamma (γ) radiation and this process is known as radiative capture or capture. For example, ^{238}U is the most abundant naturally occurring isotope that has the following radiative capture:



Here, the resulting nucleus ^{239}U is radioactive and decays with the emission of a negative beta particle.

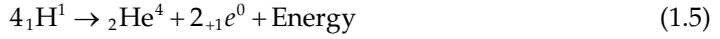
Finally, the third type of interaction between neutrons and nuclei is fission, or it is called nuclear fission. This is an essential process in nuclear reactors, which will be discussed in the subsequent sections.

1.4 Nuclear Fusion

The process of combining lighter elements into a stable heavier nucleus is known as *nuclear fusion*. However, such processes can take place at reasonable rates only at very high temperatures of the order of several million degrees, which exist only in the interior of stars.

Such processes are therefore called thermonuclear reactions. Once a fusion reaction is initiated, the energy released in the process is sufficient to maintain the temperature and to keep the process going on.

For example, the hydrogen bomb is based on the fusion of hydrogen nuclei into heavier ones by the thermonuclear reactions with the release of enormous energy.



1.5 Radioactivity

There are another class of nuclei that are unstable. They disintegrate (break up) spontaneously with the emission of electromagnetic radiation of very high energy. As such, the process of spontaneous transformation of a nucleus is known as radioactivity. Some of the unstable high atomic weight elements are radium, thorium and uranium. The emission or decay is associated with the radiation from the atomic nucleus, which is either an alpha particle (helium nucleus) or a beta particle (an electron). In many cases, gamma radiation comes out with the particle emission. The process of disintegration continues and after a number of stages of radiation, a stable nucleus is formed.

1.5.1 Rate of Radioactive Decay

The nuclei of a given radioactive species have a definite probability of decaying in unit time; this decay probability has a constant value characteristic of the particular nuclide. It remains the same, irrespective of the chemical or physical state of the element at all readily accessible temperatures and pressures. In a given specimen, the rate of decay at any instant is directly proportional to the number of radioactive atoms of the nuclide under considerations present at that instant. When N is the number of the particular radioactive atoms present at time t , then the decay rate can be represented as

$$\frac{dN}{dt} = -\lambda N \quad (1.6)$$

where λ is the decay constant of the radioactive nuclide.

If N_0 is the number of the particular radioactive atoms present at time zero and a time t , later, when N of these nuclei remain, Equation 1.6 is written as

$$\ln \frac{N}{N_0} = -\lambda t. \quad (1.7)$$

Equation 1.7 may be changed and encrypted as follows:

$$N = N_0 e^{-\lambda t}. \quad (1.8)$$

The ratio between the number of atoms disintegrating in unit time to the total number of atoms present at that time is called the decay constant of that nuclide.

In 1904, Rutherford introduced a constant known as the half-life period of the radioactive element for evaluating its radioactivity or for comparing its radioactivity with the activities of other radioactive elements. The half-life period of a radioactive element is

defined as the time required by a given amount of the element to decay to one half of its initial value.

If $N = (N_0/2)$, then Equation 1.7 will be

$$\ln \frac{1}{2} = -\lambda t_{1/2} \Rightarrow t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \quad (1.9)$$

It has been observed that the half-life is inversely proportional to the decay constant.

The half-life period is a measure of the radioactivity of the element. The shorter the half-life period of an element, the greater is the number of the disintegrating atoms and the greater is its radioactivity (Sharma et al. 2008). The half-life period of different radioactive elements vary widely ranging from a fraction of a second to millions of years.

Since the total decay period of any element is not fixed, the total decay period of the radioactive element may be meaningless. As such, the average life is the ratio of sum of lives of the nuclei and total number of nuclei. In other words, the average life of an element is the inverse of its decay constant which is given as follows:

$$t_{av} = \frac{1}{\lambda}. \quad (1.10)$$

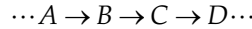
Substituting the value of λ in Equation 1.9, we get

$$t_{av} = \frac{t_{1/2}}{0.693} = 1.443 t_{1/2}. \quad (1.11)$$

The standard unit in radioactivity is curie (c), which is defined as that amount of any radioactive material, which gives 3.7×10^{10} disintegrations per second. In the SI system, the unit of radioactivity is Becquerel (Bq).

1.5.2 Radioactive Equilibrium

Let us consider a disintegration series in which A, B, C, D , etc. are some of the intermediate consecutive atoms (i.e. between the parent element and the final stable isotope)



A stage may come when the amounts of A, B, C, D , etc. become constant, which is so because their rates of disintegration become equal. Further, if N_A, N_B, N_C and N_D , etc. represent the number of atoms of A, B, C, D , etc. at equilibrium, then

$$-\frac{dN_A}{dt} = -\frac{dN_B}{dt} = -\frac{dN_C}{dt} = \cdots \quad (1.12)$$

But

$$-\frac{dN_A}{dt} = k_A N_A, \quad -\frac{dN_B}{dt} = k_B N_B \quad \text{and} \quad -\frac{dN_C}{dt} = k_C N_C.$$

Equation 1.12 becomes

$$k_A N_A = k_B N_B = k_C N_C = \cdots \quad (1.13)$$

$$\frac{N_A}{N_B} = \frac{k_B}{k_A} = \frac{(t_{1/2})_A}{(t_{1/2})_B} \quad (1.14)$$

Thus, the amounts present at equilibrium are inversely proportional to their disintegration constants or directly proportional to their half-lives.

1.5.3 Radioactive Disintegration Series

A new element is formed when a radioactive element disintegrates and emits α or β particle. The element which emits α or β particle is called the parent element and the new element formed is called the daughter element. If the daughter element is radioactive, it again disintegrates by emitting α or β particle forming a new element. Thus, the daughter element becomes a parent element and a new daughter element is produced. This process of disintegration goes on till the end product is a stable isotope.

It may be noticed that in these series, sometimes a branched disintegration takes place (the same radioactive element can disintegrate in two different ways forming two completely different radioactive isotopes). However, both the radioactive isotopes thus formed disintegrate further to form the same radioactive isotope.

1.5.4 Artificial Radioactivity

During the artificial transmutation of elements, it is observed that quite often the product obtained is also radioactive, though its half-life period is usually very small. This phenomenon in which the artificial disintegration of a stable nucleus leads to the formation of a radioactive isotope is called artificial radioactivity.

It may be mentioned that as the radioactive isotopes formed by the artificial transmutation have usually very short half-life periods, they are very rare in nature. This is because as soon as they are formed, they decay. In some cases, instead of positrons, electrons (β particle) are emitted by the artificial radioactive isotopes produced.

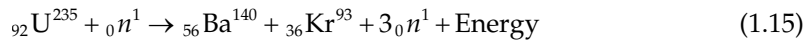
Natural Radioactivity	Artificial Radioactivity
It involves spontaneous disintegration of unstable nuclei with emission of α or a β particles or γ radiations giving rise to new nucleus.	Here, stable nuclei are bombarded with high-energy particles to produce radioactive elements.
It cannot be controlled.	It can be controlled by controlling the speed of the bombarding particles.
It is shown by the elements with high atomic number and mass number.	It can be induced even in the lighter elements.

1.6 Nuclear Fission

When a nucleus is bombarded with some subatomic particles (like α particles, neutrons, protons, etc.), then these particles are either captured by the nucleus or the nucleus disintegrates ejecting some other subatomic particles. So, the new element formed has a mass either slightly greater than or slightly smaller than that of the original element.

The splitting of a heavier atom into a number of fragments of much smaller mass by suitable bombardment with subatomic particles, with the liberation of a huge amount of energy, is called *nuclear fission*.

When uranium-235 was hit by slow neutrons, it was split up into a number of fragments, each of mass much smaller than that of uranium. The two fragments were barium and krypton (Glasstone and Sesonke 2004).



The neutron released from the fission of the first uranium atom can hit three other uranium atoms, each of which again releases three neutrons, each of which can further hit one uranium atom and so on. In this way, a chain reaction is set up and results into the liberation of a huge amount of energy. Another aspect which is extremely important for a chain reaction to continue is that the fissionable material must have a minimum size. If the size is smaller than the minimum size, the neutrons escape from the sample without hitting the nucleus, causing fission, and thus the chain reaction stops.

The minimum mass that the fissionable material must have so that one of the neutrons released in every fission hits another nucleus and causes fission (so that the chain reaction continues at a constant rate) is called the critical mass. If the mass is less than that of the critical mass, it is called sub-critical. In this case, many neutrons released in fission are able to hit the other nuclei and thus the numbers of fissions multiply in the chain reaction. The shape and the density of the packing of the material are also significant for the nuclear fission.

Nuclear Fission	Nuclear Fusion
It involves breaking up of a heavier nucleus into lighter nuclei.	It involves the union of two or more lighter nuclei to form a heavier nucleus.
It is a chain process.	It is not a chain process.
It is initiated by neutrons of suitable energy and does not need high temperature.	It is initiated by very high temperatures.
It can be controlled and the energy released can be harnessed for useful purposes.	It is difficult to control this process.
A larger number of radioactive isotopes are formed and there is nuclear waste.	There is no nuclear waste in this process.
It requires a minimum size fissionable material, and if the size of the material exceeds the critical size, the reaction becomes explosive.	There is no limit to the size of the fuel for the reaction to start. However, the fuel does not undergo fusion until heated to a very high temperature (a few million degrees).

1.7 Principles, Production, and Interaction of Neutrons with Matter

Nuclear reactions and the behaviour of subatomic particles are basic to core design. Here, the main purpose is to provide details at the review level. We treat such processes as radioactivity decay, neutron scattering, radiative capture, and fission. In general, these processes are associated with the emission of subatomic particles and radiations as well as their interaction with matter. We use the general term ‘radiation’ to include both material particles and true electromagnetic radiation.

1.7.1 Production of Neutrons

Alpha (α) particles are highly energetic positively charged particles, which are emitted by radioactive substances, and the amount of their positive electricity is equal to two units of electronic charge. The mass of the α particle is equal to that of a helium atom. They are emitted during the radioactive disintegration of certain heavy elements like uranium or radium, etc. When mono-energetic alpha particles emitted from a radioactive substance are allowed to pass through a very thin metal foil, they are found to be scattered in different directions with respect to the direction of the collimated beam of the incident particles. Though, by far, a great majority of the particles are scattered at small angles (greater than 90°).

Such large-angle scattering cannot be explained on the basis of the Thomson model of the atom. The electrostatic repulsive force between the positive charge of the scattering atom and α -particle depends inversely upon the square of the distance (q) of the α -particle from the centre of the charge of the latter and directly upon the portion of the positive charge of the scattering atom contained within the sphere of the radius q . The angle of scattering increases with the increasing effective charge of the atom repelling the α -particle and decreases with the increasing distance from the centre of the atom at which the α -particle passes by the atom. When the α -particle passes by at a relatively large distance from the centre of the atom ($\sim 10^{-10}$ m near the periphery of the atom), this distance becomes so large that the angle of scattering is quite small, even though the entire positive charge of the atom repels it.

On the other hand, when the α -particle passes by at a relatively close distance from the centre of the atom, the effective charge repelling it is so small that the angle of scattering is again quite small.

In 1911, Rutherford proposed a new model of the atom. According to him, the entire positive charge of an atom and almost its entire mass are contained within a very small sphere near the centre. This positively charged core is known as the atomic nucleus. When an α -particle passes by very close to the centre of the atom, it feels a strong electrostatic repulsive force due to the entire positive charge of the atom and hence is scattered at a relatively large angle.

However, the Rutherford model of the atom has one serious drawback. As such, the electromagnetic theory of light predicts that the revolving electrons (due to their centripetal acceleration) should continually emit electromagnetic radiation so that they move spirally inwards and ultimately plunge into the nucleus. In 1913, Niels Bohr suggested a way out of the difficulty, which however involve entirely new concepts and that were at variance with some of the fundamental concepts of classical mechanics and Maxwell's electromagnetic theory of light.

This is known as Bohr's quantum theory. The quantum theory, in a more developed form at present, constitutes the theoretical basis of all subatomic phenomena.

In 1920, James Chadwick has determined the nuclear charge of several elements on the basis of Rutherford's theory of the α -particle scattering. To increase the number of scattered α -particles, Chadwick used a narrow ring-shaped scattering foil mounted on a suitable frame for the experiment. Since the atom as a whole is electrically neutral, the nuclear charge should be equal to the atomic number. Chadwick's determination of nuclear charge for the different elements confirmed this most conclusively.

1.7.2 Neutron Reactions and Radiation

The nature of the radiation emitted from a radioactive substance was investigated by Rutherford in 1904, by applying electric and magnetic fields to the radiations. It is

observed that on applying the field, the rays emitted from the radioactive substances are separated into three types, called α , β and γ rays. The α rays are deflected in a direction that shows that they carry a positive charge; the β rays are deflected in the opposite direction, showing that they carry a negative charge and γ rays are not deflected at all, showing that they carry no charge. Furthermore, β rays are deflected to a much greater extent than α rays, which indicates that particles in the β rays are much lighter than those in the α rays. The properties of each of these rays have been studied in detail and briefly described next (Barik et al. 2011).

1.7.2.1 Properties of α Ray

- The direction of deflection of the α rays in the electric and magnetic fields shows that they carry a positive charge. The charge and mass of the particles present in the α rays (called α -particles) have been investigated by suitable experiments. It is found that each α particle carries two units of positive charge and has mass nearly four times that of a hydrogen atom. Thus, the α -particles are like the helium nuclei.
- The velocity of α rays is found to be nearly 1/10th to 1/20th of that of light, depending upon the nature of source.
- Alpha (α) rays ionize the gas through which they pass. The ionization takes place due to the knockout of the electrons from the molecules of the gas when the high-speed heavy α -particles hit these molecules.
- Since α -particles are heavy, they cannot pass through thick sheets of metal. In other words, α rays have a low penetrating power. It has been observed that they can penetrate through air only to a distance of about 7 cm and then they are absorbed in the air. Similarly, they can be stopped by an aluminium foil less than 1/10th of a mm in thickness.
- Alpha (α) rays affect a photographic plate and produce luminescence when they strike a zinc sulphide screen.

1.7.2.2 Properties of β Ray

- The direction of deflection of β rays in the electric and magnetic fields shows that they carry negative charge. The charge and mass of the particles present in these rays have been determined, and it is found that these particles possess the same charge and mass as that of the electrons. Thus, β -particles are nothing but electrons.
- As in case of α rays, the velocity of the β rays depends upon the nature of the source. However, being much lighter than the α -particles, β -particles travel at a higher speed than the α -particles. The speed of the β -particles varies from 3% to 99% of that of light, that is in some cases it approaches the velocity of light.
- In spite of a higher speed than α -particles, β -particles have less momentum or kinetic energy than the α -particles because of their smaller mass. Hence, β -particles have lower ionizing power than α -particles. The ionizing power of β -particles is about 1/100th of that of α -particles.
- Because of their smaller mass and higher speed, β -particles are much more penetrating than the α -particles. They can penetrate through an aluminium foil several