Continuous Quantum Measurements and Path Integrals

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To my parents

B M and V A Mensky

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Preface

"Quantum measurement" is a professional jargon word meaning "measurement with quantum effects taken into account". This topic has been of great interest, for specialists as well as for the general public, since the beginning of quantum mechanics up the present time. In fact, the interest to this area has grown during the last decade.

From the very beginning the interest in quantum measurements was provoked by their unusual and paradoxical features, such as the impossibility of measuring the position and momentum of an elementary particle simultaneously and with arbitrarily high accuracy. Nowadays this interest continues for both theoretical and practical reasons.

From the practical point of view, even macroscopic measurements (such as the measurement of position of elements of gravitational-wave antenna) have become very precise and require quantum effects to be taken into account. Moreover, in certain conditions these effects are the main restriction on measurement sensitivity.

The theoretical origin of constant and growing interest in quantum measurements is connected with the general interest in the foundations of quantum mechanics, of which the quantum theory of measurements is a central point. Real insight into the principles of quantum mechanics is impossible without an understanding of quantum measurements.

It must be said, though, that from a practical point of view in most cases neither a "deep understanding" of quantum mechanics nor even explicit consideration of quantum measurements is necessary for the quantum mechanical description of real systems and processes. But this is true only for narrow task of the most immediate applications. Anyone who is interested in the first principles of quantum mechanics, or even in more distant applications, inevitably runs into the question of quantum measurements.

The first principles of quantum mechanics are especially important at the present time since quantum theory is undergoing fast development and expansion of its sphere of application. Particularly acute are questions of principle in such new areas of quantum mechanics as quantum gravity and quantum cosmology, the scientific directions developing before our eyes. However, the development of new technology encounters complicated quantum-mechanical problems and also requires deep penetration into the principles of quantum theory.

The subject of the present book is continuous quantum measurements, i.e. measurements prolonged in time. It turned out that the most appropriate tool for describing such a measurement is the Feynman path integral. In the Feynman approach one describes the evolution of a quantum system as though it follows a path, passing through one point after another. This is just like a classical system. However, if for a classical system each individual path forms a complete description of its evolution, for a quantum system only the summation (integration) over all paths according to certain rules describes the evolution.

If a continuous measurement is performed during the evolution of the system, it produces certain information about the path the system takes. It is evident that summation in this case should be made not over all paths but over only those paths compatible with this information. This is the main idea of the path-integral approach to continuous quantum measurements. The technical development of this idea gives formulae for concrete physical effects.

An evident extension of this idea is from continuous measurements of quantum-mechanical systems to continual measurements of quantum fields. The quantum field dynamics can be described by an integral over all field configurations (this integral is often called the path integral too). The dynamics of the same field under a continual (stretched in time and space) measurement should evidently be described by an integral over those field configurations that are compatible with the measurement result (output).

What conclusions can be drawn from the theory of continuous (and continual) measurements? The main one is a specific uncertainty principle for processes (as distinct from the well known uncertainty principle for states). This principle can be formulated in terms of the action $S[q]$ of the physical system and therefore may be called the action uncertainty principle. It states that the history $[q]$ of a system can be traced, with the help of continuous measurement, with an error not less than the error corresponding to the uncertainty of the action δS equal to the quantum of action, $\hbar$.

In practice, this means there is an optimal accuracy for each continuous measurement. If the measurement is rougher than the optimal one, it is inefficient because of the measurement error, as classical measurement theory predicts. If, on the contrary, the measurement is finer than the optimal one, it is inefficient because of large quantum fluctuations (which may be called quantum measurement noise). The optimal regime of measurement is at the boundary between the classical and quantum regimes, and its error places an absolute limit on the precision of a given type of measurement. The only way to overcome this limit is to choose the measurement from the class of so-called quantum-nondemolition (QND) measurements, which have no quantum regime and no limit at all.

Many concrete applications of this general statement will be derived, from the sensitivity of the measurement of an oscillator's frequency components to the emergence of time in a quantum Universe as a consequence of its self-measurement. Both practical and theoretical aspects of the theory will be considered.

The book is organized in such a way that its main ideas can be understood without thorough study. Thus some sections or even whole chapters may be skipped. When this is possible without detriment to understanding, the corresponding remarks are made. The minimum possible path through the book is following:

Chapter 1 → Chapter 4 → ... → Chapter 11.

Chapters 5–10 can be selected by the reader according to taste.

The list of references provided in this book is not complete. It contains only those papers that have been used in the author's work. A number of additional references, mostly to books and review papers, have been included when discussing related topics. However, their choice depends on the specific point of view of the author in the subject discussed. Many important books and papers have not been included because their inclusion would require consideration of the same subjects from other points of view, and this was difficult in the framework of the present book. An attempt was made to compensate partly for this incompleteness by including short remarks on the literature. The aim is to give an idea about areas of quantum measurement theory not considered in this book and to provide a preliminary directions to the literature in these areas.

I am indebted to V B Braginsky, who awoke my interest in the theory of precise measurements and specifically in practical aspects of the quantum theory of measurement. V N Rudenko was the first to point out the importance of evaluating quantum effects in continuous measurements. This was a starting point for the investigation, which led later to the path-integral theory of continuous quantum measurements. My deep gratitude is to K S Thorne, who considered my first paper on the subject (see Mensky 1979a) to be interesting and recommended it for *Physical Review.* I am obliged to G A Golubtsova, who was my collaborator and a coauthor of an important paper on quantum-nondemolition measurements. I also had useful discussions of some questions with C M Caves, H Borzeszkowski, R Onofrio, C Presilla, J Halliwell and many other colleagues.

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Introduction to Continuous Quantum Measurements

The main topic of this book is the path-integral theory of continuous quantum measurements. In this introductory chapter we shall expose the principal ideas of this theory on a qualitative level with a minimum of mathematical apparatus.

All physical systems are in fact quantum, but in certain circumstances some of them may approximately be described as classical. This depends on the error with which the action of the system is known (section 1.1). If the system should be considered as a quantum one, then a specific quantum description is necessary and specific quantum features in the behaviour of the system arise. The main distinction in the description of a quantum system is the concept of a probability amplitude (section 1.2), and the principal feature of the quantum system is an uncertainty principle.

A detailed analysis of the concept of an amplitude in the situation when the system undergoes some measurement allows one to obtain a theory of quantum measurement even if the measurement is continuous (prolonged in time). In the latter case, the different paths the system moves along should be considered as alternatives for the motion and characterized by amplitudes (section 1.3).

The uncertainty principle in its well known form $\Delta q\,\Delta p \gtrsim \hbar$ is appropriate to instantaneous measurements. For continuous measurements a modified uncertainty principle can be formulated in terms of the action (section 1.4). According to this principle (in its simplest but weak form) a continuous measurement produces information such that the uncertainty in the action is not less than the quantum of action $\delta S \gtrsim \hbar$.

The reader may skip Chapters 2 and 3, and go directly from this chapter to Chapter 4 without any detriment to understanding of the main points of the theory. Chapter 2 is necessary only for those who have special interest in the link between von Neumann's theory of instantaneous quantum measurements and the path-integral theory of continuous quantum measurements (though the latter can and will be developed quite independently).

Chapter 3 will be useful for a deeper study of the mathematical formalism of path integrals than the level used in Chapter 4.

1.1 QUANTUM AND CLASSICAL SYSTEMS

Quantum mechanics appeared as a theory of microscopic bodies when it had been proved that the motion of microscopic systems cannot be described in the framework of classical physics. However, quantum effects may be important even for macroscopic bodies. The main criterion is in fact inaccuracy in the value of the action S typical for description of the motion in the framework of the given approximation.

The action S is a functional characterizing the dynamics of a system:

$$S[q] = \int_{t'}^{t''} L(q, \dot{q}, t)\, dt.$$

Here L is the Lagrangian of the system, which in the simple case of a one-dimensional mechanical system takes the form

$$L = \frac{1}{2} m\dot{q}^2 - V(t, q),$$

and

$$[q] = \{q(t) \,|\, t' \leq t \leq t''\}$$

is a path (a trajectory) of the system. It is important that the action functional $S[q]$ may be evaluated not only for the actual path the classical system takes but also for an arbitrary path in the configuration space of the system. In fact, nonclassical paths play a key role in quantum mechanics and specifically in the theory of continuous measurements.

To judge whether the system is quantum or not it is necessary to compare its action with the *Planck constant*, or the *quantum of action*, $\hbar = 1.055 \times 10^{-27}$ erg s.

Let us make this more precise. Any system is in fact a quantum one. However, in an approximate description the quantum features of a certain system may turn out to be negligible. Then this system in this approximation may be considered to be a classical one.

The action of the system provides a quantitative criterion for this. If the errors, characteristic of the given approximation, lead to an indeterminacy ΔS in the action $S[q]$ large compared with the quantum of action, $\Delta S \gg \hbar$, then the system may be considered to be *classical*. If the action $S[q]$ is given with a rather small error, $\Delta S \lesssim \hbar$, then the system needs to be treated as a *quantum* one.

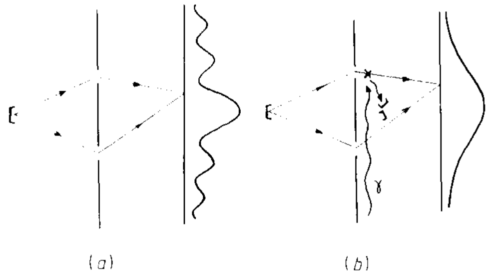

Figure 1.1: The two-slit experiment leads to an interference pattern if it is not known which slit the particle has passed through (a), but it gives no interference if an additional observation shows which slit was used (b).

1.2 AMPLITUDES AND ALTERNATIVES

From a certain point of view the main object in quantum mechanics is a probability amplitude because it expresses the principal difference between quantum and classical theory.[1] The ***probability amplitude*** of some event is a complex number A such that $p = |A|^2$ is the probability of this event.

Quantum mechanics differs in that not probabilities but probability amplitudes should be summed up for a quantum system. Suppose that some event can occur through one of two alternative channels and that the probability amplitudes for these channels are A_1 and A_2. Then the complete probability amplitude for the event under consideration is

$$A = A_1 + A_2 \tag{1.1}$$

and its probability is

$$p = |A|^2 = |A_1 + A_2|^2.$$

A typical example is a particle passing through one of two slits in an opaque screen (figure 1.1), with A_1 being the amplitude for the particle to pass through one of the slits and A_2 that for it to pass through the other. It is because of the law of amplitude summation (1.1) that passing through one of the slits cannot be considered independently from passing through the other slit. The consequence of this law is an interference pattern at the scintillation screen when a series of particles passes through two slits. This phenomenon is discussed in much detail in many textbooks on quantum mechanics (see for example Bohm 1952).

The formula (1.1) is valid, however, only if there is no means of knowing which of two possible alternatives has actually occurred. In this case the

[1] Dirac (1972) argued that the most important distinction of quantum theory is not in operators but in amplitudes.

alternatives are called, according to Feynman, *interfering*. They may also be called *quantum alternatives*.

If an additional observation (measurement) is performed giving information about which route has been followed, then the amplitude summation rule changes into the probability summation rule (see Feynman and Hibbs 1965):

$$p = p_1 + p_2 = |A_1|^2 + |A_2|^2.$$

As a consequence, no interference pattern will arise in the two-slit experiment if, for example, the flow of photons falls on the opaque screen so that scattering of the photons shows which of two slits the particle has passed through.[2] In such a situation Feynman called the alternatives *incompatible*. It also seems convenient to use the term *classical alternatives*.

The amplitude summation rule is also valid for many alternative channels,

$$A = A_1 + A_2 + \ldots + A_n \tag{1.2}$$

provided that there is no possibility of discovering which channel has been actually followed. If some observation (measurement) is performed giving information about the channel followed, then the amplitude summation rule must be corrected. The method of correction depends upon the information provided by the measurement.

The information may be complete so that the channel followed is known precisely. Then the probabilities of separate channels should be summed instead of their amplitudes:

$$p = p_1 + p_2 + \ldots + p_n, \qquad p_i = |A_i|^2.$$

For another type of observation information may be only partial. This means that the measurement is rougher. For example, let the measurement permit one to know whether the number of the channel followed belongs to one of the following pairs:

$$(1,2);\ (3,4);\ \ldots (n-1, n)$$

(we suppose that the total number of channels is even). Then the probabilities corresponding to separate pairs are to be summed but amplitudes should be summed inside the pairs:

$$p = p_1 + p_2 + \ldots + p_{n/2}, \qquad p_i = |A_{2i-1} + A_{2i}|^2. \tag{1.3}$$

In this case the alternatives inside each pair are interfering (quantum), while those in different pairs are incompatible (classical) alternatives for

[2] Of course, one can say that in this case quite a different physical system is being dealt with. But it is equally valid to talk about an additional observation of the same system. This is typical of the quantum theory of measurement: there is arbitrariness in what part of the real world is included in the system and what is included in the measuring device.

the final event. In the case of yet rougher measurement all channels may be divided into triplets:

$$p = p_1 + p_2 + \ldots + p_{n/3}, \qquad p_i = |A_{3i-2} + A_{3i-1} + A_{3i}|^2. \tag{1.4}$$

Here p_i are probabilities for different results (outputs) of the measurement. For example, the value

$$p_i = |A_{2i-1} + A_{2i}|^2$$

in equation (1.3) is the probability for the ith pair to emerge as the output of the measurement. The value

$$A_{i\text{th pair}} = A_{2i-1} + A_{2i} \tag{1.5}$$

is nothing but the probability amplitude for the measurement to give the result expressed by the ith pair. More precisely, this is the amplitude for the event under consideration to occur and the measurement of pairs of channels to show the ith pair.

The situation described by formulae (1.3) and (1.4) can be modelled in a many-slit experiment. The photon flow should then be directed at the opaque screen in such a way that pairs or triplets of slits could be distinguished by the photon scattering rather than individual slits.[3]

1.3 PATHS AND CONTINUOUS MEASUREMENTS

This argument may be applied to *Feynman paths* considered as quantum alternatives. The amplitude $A(q'', q')$ for a particle to move from the point q' to the point q'' is called a *propagator*.[4] It has been expressed by Feynman (1948) in the form of sum (or rather integral) of the amplitudes $A[q]$ corresponding to all possible paths $[q]$ connecting the points q' and q'':

$$A(q'', q') = \int A[q]\, \mathrm{d}[q]. \tag{1.6}$$

Actually this formula is valid for a propagator of any quantum system if q is understood as a coordinate (or a set of coordinates) of the configuration space of this system. For most arguments it is sufficient to consider a one-dimensional system.

[3] It might be interesting to perform an experiment of this type for direct experimental investigation of quantum and classical alternatives and the modified amplitude summation rules (1.3) and (1.4).

[4] In the subsequent chapters we shall denote this amplitude by $U(q'', q')$ because it is closely connected with the evolution operator U.

The formula (1.6) is analogous to equation (1.2) but for paths in the role of quantum alternatives. And analogously to the above argument equation (1.6) for the propagator is valid only if there is no possibility of finding out which path is followed when a particle moves from q' to q''. This is usually the case. However, suppose that a *continuous measurement* is performed simultaneously with this transition. Let the output α of this measurement give some information about the path of the transition. Such information can be expressed by some set of paths I_α.[5] If the measurement gives the result (output) α then the transition follows one of the paths $[q]$ belonging to the set I_α. Then, in analogy with equation (1.5), the amplitude for the transition from q' to q'' can be expressed as an integral over paths belonging to I_α:

$$A_\alpha(q'', q') = \int_{I_\alpha} A[q]\, \mathrm{d}[q]. \tag{1.7}$$

The idea of using restricted path integrals in such a way was proposed in a short remark by Feynman (1948). Some attempts were made to elaborate this idea (see for example Bloch and Burba 1974) but, to our mind, not successfully. The present author, being unaware of this remark of Feynman and subsequent work, proposed this idea again and elaborated it (Mensky 1979a, b, 1983a). In this book other applications of this approach will be considered.[6]

A typical example of continuous measurement is monitoring of the coordinates of the system under consideration. (One may think, for example, of monitoring the position of an elementary particle). Then the measurement gives the value $a(t)$ of the coordinate $q(t)$ at each instant t (of some time interval) with the error Δa determined by the precision of measurement. Then the output of the measurement α can be identified with the path $[a]$ expressed by the curve $a(t)$.

Knowing the output $\alpha = [a]$ of the position monitoring, one knows in fact that the actual path of the system $[q]$ could differ from $[a]$ by no more than the value Δa. Therefore any path $[q]$ lying in the corridor I_α of width $2\Delta a$ around $[a]$ is possible, while no other path is impossible as an actual path of the system (taking the measurement output α into account). Information supplied by the measurement output α is expressed in this case by the corridor I_α of paths. Integration in the Feynman path integral should therefore be performed only over paths in the corridor I_α. Moreover, the corridor I_α may be identified with the output α of the measurement. In fact this corridor represents the output of position monitoring better

[5] In fact the set I_α represents the measurement output adequately. This is why we shall later identify the concepts and denote the measurement output and the corresponding set of paths by the same letter α.

[6] The author was probably influenced by Feynman's (1948) paper though unaware of it.