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HEAT AND MASS TRANSFER



A. F. MILLS



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THE RICHARD D. IRWIN SERIES IN HEAT TRANSFER

Heat Transfer

by Anthony F. Mills, 1992

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CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an informa business

Reprinted 2009 by CRC Press

CRC Press
6000 Broken Sound Parkway, NW
Suite 300, Boca Raton, FL 33487
270 Madison Avenue
New York, NY 10016
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK

Computer Graphics by Geograf. by permission of Geocomp
Corporation, 66 Commonwealth Avenue, Concord, MA 01742

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Sponsoring editor: Elizabeth Jones
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Art coordinator: Heather Burbridge
Artist: David Wood
Cover designer: Joshua Paul Carr
Compositor: Publication Services, Inc.
Typeface: 10/12 Times Roman
Printer: R. R. Donnelley & Sons Company

Library of Congress Cataloging-in-Publication Data

Mills, Anthony F.
Heat and mass transfer / Anthony F. Mills.
p. cm. — (Irwin heat transfer series)
Includes bibliographical references and index.
ISBN 0-256-11443-9
1. Heat—Transmission. 2. Mass transfer. I. Title. II. Series.
TJ260.M518 1995
621.402'2—dc20 94-146

2 3 4 5 6 7 8 9 0 DO 1 0 9 8 7 6 5

To Brigid
For your patience and understanding.



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PREFACE

Heat and Mass Transfer has been written for undergraduate and first-year graduate students in mechanical engineering programs. Apart from the usual lower-division mathematics and science courses, the preparation required of the student is introductory courses in fluid mechanics and thermodynamics, and preferably the usual junior-level engineering mathematics course. The ordering of the material and the pace at which it is presented have been carefully chosen so that the beginning student can proceed from the most elementary concepts to those that are more difficult. As a result, the book should prove to be quite versatile. It can be used as the text for an introductory course during the junior or senior year, although the coverage is sufficiently comprehensive for use in a more advanced course and as a reference work in laboratory and design courses. Throughout, the emphasis is on engineering calculations, and each topic is developed to a point that will provide the student with the tools needed to practice the art of design. In addition, the student is introduced to design methodology for heat and mass transfer equipment.

Heat and Mass Transfer is planned to complement *Heat Transfer*, which was published in 1992. The results of a market survey indicate that, whereas a majority of schools prefer a heat transfer text containing no coverage of mass transfer, a large minority of schools consider it essential to include some mass transfer in a heat transfer course sequence. *Heat and Mass Transfer* was developed by adding three chapters on mass transfer to *Heat Transfer*. As a result, *Heat and Mass Transfer* contains 11 chapters and three appendixes:

Chapter 1: Elementary Heat Transfer

Chapter 2: Steady One-Dimensional Heat Conduction

Chapter 3: Multidimensional and Unsteady Conduction

Chapter 4: Convection Fundamentals and Correlations

Chapter 5: Convection Analysis

Chapter 6: Thermal Radiation

Chapter 7: Condensation, Evaporation, and Boiling

Chapter 8: Heat Exchangers

Chapter 9: Elementary Mass Transfer

Chapter 10: High Mass Transfer Rate Theory

Chapter 11: Mass Exchangers

Appendix A: Property Data

Appendix B: Units, Conversion Factors, and Mathematics

Appendix C: Charts

The mathematics and physics of heat transfer and mass transfer are very similar. Yet most instructors agree that students find mass transfer to be a much more difficult subject than heat transfer. Students have less experience and poorer intuition regarding species concentrations and mass fluxes than they do regarding temperature and heat flux. Also, mass transfer analysis is applied to a large variety of physical and chemical systems, and it does take time to become familiar with the essential physics and chemistry. For these reasons, I believe that students should be introduced to mass transfer only after they have developed a proficiency in heat transfer. When the two subjects are taught simultaneously, the student is exposed to too many new concepts in too short a time interval, and may become needlessly confused. For example, once the student is fully familiar with the Prandtl and Nusselt numbers (having used them in homework exercises; for example) introduction of the Schmidt and Sherwood numbers is easily accomplished. Also, when the student is familiar with the pertinent mathematics, more attention can be paid to the physics and chemistry of the problem. For example, the student who is familiar with ε - N_{tu} relations for heat exchangers is better prepared to learn about the more complex mass exchangers and simultaneous heat and mass exchangers.

Chapter 1 is a brief but self-contained introduction to heat transfer. All problem solving can be performed without use of the property data in Appendix A. It serves to give the student an overview of the subject and to provide material needed in subsequent chapters. For example, the analysis of single-stream heat exchangers in Section 1.6 introduces the student to a most important engineering application of heat transfer, but also provides a result essential for heat convection in Chapter 4; most texts refer ahead to a subsequent heat exchanger chapter for this result, which is most unsatisfactory. But perhaps most significantly, the student can be given meaningful and interesting engineering problems at the earliest opportunity and thereby develop an enthusiastic interest in the subject. Chapters 2 and 3 present a relatively conventional treatment of heat conduction, except that an introduction to moving-boundary problems is included. The treatment of finite-difference numerical methods for conduction has been kept relatively brief and focused.

Most instructors agree that heat convection is a particularly difficult topic to teach in a first heat transfer course, and many hold strong opinions as to the most

appropriate approach to follow. In keeping with the overall philosophy of the book, my objective in Chapter 4 is to develop the student's ability to calculate convective heat transfer coefficients. In a brief introduction, the physics of convection is explained and the heat transfer coefficient is defined. Dimensional analysis using the Buckingham pi theorem is used to quickly introduce the required dimensional groups and to allow a discussion of the important role played by laboratory experiments. The rather large number of correlation formulas that follow might be overwhelming if it were not for the associated computer program CONV. The instructor can discuss selected configurations in class as time allows; the student is able, with the aid of CONV, to reliably calculate heat transfer coefficients, and skin friction coefficients or pressure drop, for a much wider range of configurations. Analysis of convection is deferred to Chapter 5. High-speed flows are treated first in Section 5.2 since an understanding of the recovery temperature concept enhances the student's problem-solving capabilities. Each of the topics in Sections 5.3 through 5.8 is essentially self-contained, and the instructor can select as few or as many as required. My own opinion, however, is that convection analysis belongs more properly in a graduate-level course; it is more difficult, and of less practical use, than the analysis of conduction, radiation, heat exchangers, and so on for the beginning student.

The treatment of thermal radiation in Chapter 6 has some special features. Radiation properties are initially defined on a total basis, and the shape factor is introduced as a simple geometrical concept without discussion of the concept of radiation intensity. This approach gives the student the base required to solve most simple engineering radiation exchange problems. Only subsequently need the student tackle the more difficult directional and spectral aspects of radiation. I believe that this approach is justified because an understanding of the directional and spectral aspects of radiation does not lead to a significant increase in engineering problem-solving capability at this level. For gas radiation, the ubiquitous Hottel charts have been replaced by the more accurate models developed by Edwards; the accompanying computer program makes their use particularly simple.

The treatment of condensation and evaporation heat transfer in Chapter 7 is reasonably original, while the treatment of pool boiling is quite conventional. Forced-convection boiling and condensation is taken far enough for the student to be able to calculate both pressure drop and heat transfer. Heatpipes are dealt with in some detail, enabling the student to calculate the wicking limit and to analyze the performance of simple gas-controlled heatpipes. The thermal analysis of heat exchangers in Chapter 8 is also conventional. The student is then taken further into the design process than is customary. Additional topics include the calculation of exchanger pressure drop, thermal-hydraulic design, heat transfer surface selection for compact heat exchangers, and economic analysis leading to the calculation of the benefit-cost differential associated with heat recovery operations. A simple design program, HEX2, serves to introduce the student to computer-aided design of heat exchangers.

Chapter 9 considers diffusion in a stationary medium, and low mass transfer rate convection. As was the case for heat convection in Chapter 4, mass convection is introduced using dimensional analysis and the Buckingham pi theorem. The analogy between low mass transfer rate convection and heat transfer to an impermeable surface is thoroughly exploited. Simultaneous heat and mass transfer is considered,

with an emphasis on problems involving evaporation of water, such as the wet- and dry-bulb psychrometer. Diffusion and chemical reaction in porous catalysts are analyzed in order to provide results needed for the study of automobile catalytic converters in Chapter 11. The chapter closes with methods for calculating transport properties, with special emphasis on gas mixtures. Chapter 10 is considerably more advanced than Chapter 9. Velocities and fluxes in a mixture or solution are rigorously defined and the general species conservation equation derived. Important problems such as diffusion with one component stationary and combustion of volatile hydrocarbon fuel droplets are carefully analyzed. The Couette-flow model is introduced to obtain blowing factors for high mass transfer rate convection. These factors are then improved to account for both flow geometry and variable properties to give a unique engineering problem-solving facility. Mass, momentum, and heat transfer in a constant laminar boundary layer on a flat plate are rigorously analyzed for a binary mixture, and complements the analysis of momentum and heat transfer in a pure fluid given in Chapter 5. The chapter closes with a generalized formulation of steady convective heat and mass transfer, based on the well-known contributions of D. B. Spalding.

Chapter 11 deals with mass exchangers, such as catalytic converters and gas scrubbers; particle removal equipment, such as filters and electrostatic precipitators; and simultaneous heat and mass exchangers, such as humidifiers and cooling towers. All the analyses are based on low mass transfer rate theory, and the high mass transfer rate theory of Chapter 10 is not a prerequisite to the study of Chapter 11. Humidifiers and cooling towers are usually considered within the scope of mechanical engineering, but mass exchangers have been traditionally the province of chemical engineers. Particle removal equipment has not been emphasized in either discipline. However, environmental concerns are now having profound effects on the design of mechanical engineering systems. Tuning and operating of the modern automobile engine is now dictated by requirements of the catalytic converter used to reduce exhaust emissions. A significant portion of the investment in a modern coal-fired central power plant is for equipment used to clean the stack gas, including a scrubber to remove sulfur oxides, and cyclones, baghouses, or electrostatic precipitators to remove particulates. Incinerators for destroying toxic wastes have even more stringent requirements on stack gas cleanup. A major concern of nuclear engineers has become the design of safety systems to prevent escape of radioactive gases and particulates into the atmosphere in the event of an accident involving damage to the reactor core. Major aerospace engineering projects, such as the space station, require complex life-support systems that involve a variety of mass exchangers. My goal in Chapter 11 is to prepare mechanical, aerospace, and nuclear engineers for effective participation in the design of the systems described above, complementing rather than competing with chemical engineers on the design team.

The extent to which engineering design should be introduced in a heat transfer course is a controversial subject. In the recent past, the practice at most universities in the United States has been to teach heat transfer as an engineering science course, and textbooks have generally reflected this philosophy. However, due partly to pressure from ABET and the engineering profession, there is now a move toward including more design in introductory heat transfer courses. Some educators are of

the opinion that this should be done by including “open-ended” exercises for the student. In surveying examples of this approach, I have found that, whereas many of these exercises are indeed excellent and challenging problems for the student, they are seldom good vehicles for teaching the elements of design methodology. The fact that a problem does not have a unique answer does not, in itself, ensure that it will provide a satisfactory design experience. It is my opinion that the student can be best introduced to design methodology through an increased emphasis on equipment such as heat and mass exchangers. It is in the context of such equipment that students can be introduced to design topics such as synthesis, parametric studies, trade-offs, optimization, economics, and material or health constraints. The thermal-hydraulic design of a heat exchanger or cooling tower should surely be regarded as an essential topic in a heat transfer course. Thus, in *Heat and Mass Transfer* I present a more extensive coverage of heat and mass exchangers than is found in comparable textbooks. If this material is taught, I am confident that ABET guidelines will be met; but, more importantly, I believe that engineering undergraduates are better served by exposure to this material, even if it means studying somewhat less heat transfer science.

Based on my experience at UCLA, I suggest two possible strategies for teaching the mass transfer component of this text. In a one-semester introductory heat and mass transfer course, I would teach most of Chapter 9 and selected single-stream exchangers from Chapter 11; for example, the automobile catalytic converter and the adiabatic humidifier. Alternatively, in a two-quarter or two-semester sequence, I would teach Chapter 9, portions of Chapter 10, and most of Chapter 11.

When writing a heat transfer textbook, the author must take into account that the student’s preparation in mathematics varies from school to school. It is imperative, nonetheless, that engineering courses follow up with the use of topics taught in mathematics courses, or else the student will surely never become proficient. The more advanced mathematics used in *Heat and Mass Transfer* is as follows.

1. First- and second-order linear ordinary differential equations
2. Sets of n linear algebraic equations
3. The use of separation of variables to solve partial differential equations, Fourier series, and special functions
4. Elementary numerical methods including integration, Newton-Raphson iteration, matrix inversion or Gauss-Siedel iteration, and integration of nonlinear ordinary differential equations
5. Finite-difference solution methods for partial differential equations

When using classical mathematics in the text, I have given complete details so as to reinforce what the student learned in previous mathematics courses. Of course, an instructor can choose to omit the details if appropriate. However, when using the numerical methods listed under item 4 above (which occurs almost exclusively in more advanced examples and exercises), I have assumed that the student is either able to write an appropriate computer program or has access to standard subroutines on a programmable calculator or computer. The number of programming languages

in use, the wide variety of software available, and rapid changes in the field make such an approach essential. Computing resources vary from school to school. I was particularly impressed by the situation at the University of Auckland, where I taught from 1983 to 1985. In their sophomore year, engineering students had lectures on elementary numerical methods accompanied by a series of laboratory sessions, where they implemented these methods and, at the same time, became familiar with the school's computing resources. Thus, when teaching at the junior or senior level, the instructor could assume that the students were fully ready to use such methods as needed.

A unique feature of *Heat and Mass Transfer* is that it has a fully integrated package of computer software. With the aid of generous grants from the IBM Corporation, the School of Engineering and Applied Science at UCLA has been able to equip a number of computer classrooms and laboratories. Also, the Chancellor's Committee on Instructional Improvement made funds available for software development to enable these computers to be used effectively for undergraduate instruction. These funds assisted development of the software package, and this support is gratefully acknowledged. The software is intended to serve primarily as a tool for the student, both at college and after graduation as a practicing engineer. Most of the programs are designed to reduce the effort required to obtain reliable numerical results, and thereby increase the efficiency and effectiveness of the engineer. I have found the impact of the software on the educational process to be encouraging. It is now possible to assign more meaningful and interesting problems, because the students need not get bogged down in lengthy calculations. Parametric studies, which are the essence of engineering design, are relatively easily performed. Of course, computer programs are not a substitute for a proper understanding. My practice has been to require the student to perform various hand calculations, using the computer to give immediate feedback. For example, the student does not have to wait a week or two until homework is returned to find that a calculated convective heat transfer coefficient was incorrect because a property table was misread.

The *Heat Transfer* software package has been well received, and the additional programs that accompany *Heat and Mass Transfer* should prove even more useful. The calculations required to solve practical mass transfer problems are often very long and tedious, owing to the complexity of the analysis and the need to evaluate many thermodynamic and transport properties of mixtures. The software provided radically reduces the effort required to obtain reliable numerical results. For example, with the interactive programs provided, the student can easily perform parametric studies of the performance of scrubbers and cooling towers, and thereby develop a much deeper understanding of these systems than has been possible hitherto. Furthermore, at UCLA I have seen how programs such as SCRUB and CTOWER have certainly increased the students' interest in such equipment: previously the lengthy calculations required all but killed their enthusiasm for the subject. In providing this software to the student, it is not my intent to discourage students from writing their own computer programs. But, during a typical semester or quarter course, students do not have time to write more than two or three computer programs of any substance. Hopefully, the *Heat and Mass Transfer* software will demonstrate the value

of computer programs for routine engineering calculations, and encourage the student and engineer to write such programs when circumstances suggest that they will be cost-effective.

Some of the material in *Heat and Mass Transfer*, mostly in the form of examples and exercises, has been taken from an earlier text which was co-authored by my ex-colleagues at UCLA, D. K. Edwards and V. E. Denny (*Transfer Processes*, 1st ed., Holt, Rinehart & Winston, 1973; 2nd ed., Hemisphere-McGraw-Hill, 1979). I have also used material on radiation heat transfer from a more recent text by D. K. Edwards (*Radiation Heat Transfer Notes*, Hemisphere, 1981). D. B. Spalding introduced me to the subject of mass transfer as a student at the Imperial College of Science and Technology, London: his influence is surely in evidence in this text. I gratefully acknowledge the contributions of these gentlemen, both to this book and to my professional career. The computer software for *Heat and Mass Transfer* was expertly written by Baek Youn and Hae-Jin Choi, with able assistance from Benjamin Tan. I also wish to acknowledge the contributions made by many others. The late D. N. Bennion provided a chemical engineering perspective to some of the material on mass exchangers. R. Greif of the University of California, Berkeley, and J. H. Lienhard V of the Massachusetts Institute of Technology provided detailed critiques of the mass transfer chapters. Reviewers commissioned by Richard D. Irwin, Inc., suggested numerous improvements, most of which I was able to incorporate in the final manuscript. My students have been most helpful—in particular, S. W. Hiebert, R. Tsai, B. Cowan, E. Myhre, B. H. Chang, D. C. Weatherly, A. Gopinath, J. I. Rodriguez, B. P. Dooher, M. A. Friedman, and C. Yuen. My special thanks to the secretarial staff at UCLA and the University of Auckland—in particular, Phyllis Gilbert, Joy Wallace, and Julie Austin, for their enthusiastic and expert typing of the manuscript.

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ACKNOWLEDGMENTS

Richard D. Irwin, Inc., would like to thank the following reviewers for their contribution to the development of *Heat and Mass Transfer*.

Martin Crawford, University of Alabama—Birmingham

Prakash R. Damshala, University of Tennessee—Chattanooga

Tom Diller, Virginia Polytechnic Institute and State University

Glenn Gebert, Utah State University

Clarke E. Hermance, University of Vermont

John H. Lienhard V, Massachusetts Institute of Technology

Jennifer Linderman, University of Michigan—Ann Arbor

Robert J. Ribando, University of Virginia

Jamal Seyed-Yagoobi, Texas A&M University—College Station

The author worked hard to improve the book based on the comments of each reviewer. Please feel free to contact us with suggestions for improvement of future editions. We hope you enjoy using the book.

Elizabeth Jones
Sponsoring Editor

Kelley Butcher
Senior Developmental Editor



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NOTES TO THE INSTRUCTOR AND STUDENT

These notes have been prepared to assist the instructor and student and should be read before the text is used. Topics covered include conventions for artwork and mathematics, the format for example problems, organization of the exercises, comments on the thermophysical property data in Appendix A, and a guide for use of the accompanying computer software.

ARTWORK

Conventions used in the figures are as follows.

	Conduction or convection heat flow
	Radiation heat flow
	Fluid flow
	Species flow
	Temperature or concentration profile

MATHEMATICAL SYMBOLS

Symbols that may need clarification are as follows.

$\approx$	Nearly equal
$\sim$	Of the same order of magnitude
$ _x$	All quantities in the term to the left of the bar are evaluated at x

EXAMPLES

Use of a standard format for engineering problem solving is a good practice. The format used for the examples in *Heat and Mass Transfer*, which is but one possible approach, is as follows.

Problem statement

Solution

Given:

Required:

Assumptions: 1.
2. etc.

Sketch (when appropriate)

Analysis (diagrams when appropriate)

Properties evaluation

Calculations

Results (tables or graphs when appropriate)

Comments

- 1.
2. etc.

It is always assumed that the problem statement precedes the solution (as in the text) or that it is readily available (as in the *Solutions Manual*). Thus, the *Given* and *Required* statements are concise and focus on the essential features of the problem. Under *Assumptions*, the main assumptions required to solve the problem are listed; when appropriate, they are discussed further in the body of the solution. A sketch of the physical system is included when the geometry requires clarification; also, expected temperature and concentration profiles are given when appropriate. (Schematics that simply repeat the information in the problem statements are used sparingly. I know that many instructors always require a schematic. My view is that students need to develop an appreciation of when a figure or graph is necessary, because artwork is usually an expensive component of engineering reports. For example, I see little use for a schematic that shows a 10 m length of straight 2 cm–O.D. tube.) The analysis may consist simply of listing some formulas from the text, or it may require setting up a differential equation and its solution. Strictly speaking, a property should not be evaluated until its need is identified by the analysis. However, in routine calculations, such as evaluation of convective heat transfer coefficients, it is

often convenient to list all the property values taken from an Appendix A table in one place. The calculations then follow with results listed, tabulated, or graphed as appropriate. Under *Comments*, the significance of the results can be discussed, the validity of assumptions further evaluated, or the broader implications of the problem noted.

In presenting calculations for the examples in *Heat and Mass Transfer*, I have rounded off results at each stage of the calculation. If additional figures are retained for the complete calculations, discrepancies in the last figure will be observed. Since many of the example calculations are quite lengthy, I believe my policy will facilitate checking a particular calculation step of concern. As is common practice, I have generally given results to more significant figures than is justified, so that these results can be conveniently used in further calculations. It is safe to say that no engineering heat transfer calculation will be accurate to within 1%, and that most experienced engineers will be pleased with results accurate to within 10% or 20%. Thus, preoccupation with a third or fourth significant figure is misplaced (unless required to prevent error magnification in operations such as subtraction).

EXERCISES

The diskette logo next to an exercise statement indicates that it can be solved using the *Heat and Mass Transfer* software, and that the sample solution provided to the instructor has been prepared accordingly. There are many additional exercises that can be solved using the software but that do not have the logo designation. These exercises are intended to give the student practice in hand calculations, and thus the sample solutions were also prepared manually.

The exercises have been ordered to correspond with the order in which the material is presented in the text, rather than in some increasing degree of difficulty. Since the range of difficulty of the exercises is considerable, the instructor is urged to give students guidance in selecting exercises for self-study. Answers to all exercises are listed in the *Solutions Manual* provided to instructors. Odd- and even-numbered exercises are listed separately; the instructor may choose to give either list to students to assist self-study.

PROPERTY DATA

A considerable quantity of property data has been assembled in Appendix A. Key sources are given as references or are listed in the bibliography. Since *Heat and Mass Transfer* is a textbook, my primary objective in preparing Appendix A was to provide the student with a wide range of data in an easily used form. Whenever possible, I have used the most accurate data that I could obtain, but accuracy was not always the primary concern. For example, the need to have consistent data over a wide range of temperature often dictated the choice of source. All the tables are in SI units, with temperature in kelvins. The computer program UNITS can be used for conversions to other systems of units. Appendix A should serve most

needs of the student, as well as of the practicing engineer, for doing routine calculations. If a heat transfer research project requires accurate and reliable thermophysical property data, the prudent researcher should carefully check relevant primary data sources.

SOFTWARE

The *Heat and Mass Transfer* software has menus that describe the content of each program. The programs are also described at appropriate locations in the text. The input format and program use are demonstrated in example problems in the text. Use of the text index is recommended for locating the program descriptions and examples. There is a one-to-one correspondence between the text and the software. In principle, all numbers generated by the software can be calculated manually from formulas, graphs, and data given in the text. Small discrepancies may be seen when interpolation in graphs or property tables is required, since some of the data are stored in the software as polynomial curve fits.

The software facilitates self-study by the student. Practice hand calculations can be immediately checked using the software. When programs such as CONV, PHASE, BOIL, and SCRUB are used, properties evaluation and intermediate calculation steps can also be checked when the final results do not agree.

Since there is a large thermophysical property database stored in the software package, the programs can also be conveniently used to evaluate these properties for other purposes. For example, in CONV both the wall and fluid temperatures can be set equal to the desired temperature to obtain property values required for convection calculations. We can even go one step further when evaluating a convective heat transfer coefficient from a new correlation not contained in CONV: if a corresponding item is chosen, the values of relevant dimensionless groups can also be obtained from CONV, further simplifying the calculations.

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HEAT AND MASS TRANSFER





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ELEMENTARY HEAT TRANSFER

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1.1 INTRODUCTION

The process of heat transfer is familiar to us all. On a cold day we put on more clothing to reduce heat transfer from our warm body to cold surroundings. To make a cup of coffee we may plug in a kettle, inside which heat is transferred from an electrical resistance element to the water, heating the water until it boils. The engineering discipline of **heat transfer** is concerned with methods of calculating **rates** of heat transfer. These methods are used by engineers to design components and systems in which heat transfer occurs. Heat transfer considerations are important in almost all areas of technology. Traditionally, however, the discipline that has been most concerned with heat transfer is mechanical engineering because of the importance of heat transfer in energy conversion systems, from coal-fired power plants to solar water heaters.

Many *thermal design* problems require reducing heat transfer rates by providing suitable *insulation*. The insulation of buildings in extreme climates is a familiar example, but there are many others. The space shuttle has thermal tiles to insulate the vehicle from high-temperature air behind the bow shock wave during reentry into the atmosphere. Cryostats, which maintain the cryogenic temperatures required for the use of superconductors, must be effectively insulated to reduce the cooling load on the refrigeration system. Often, the only way to ensure protection from severe heating is to provide a fluid flow as a heat “sink.” Nozzles of liquid-fueled rocket motors are cooled by pumping the cold fuel through passages in the nozzle wall before injection into the combustion chamber. A critical component in a fusion reactor is the “first wall” of the containment vessel, which must withstand intense heating from the hot plasma. Such walls may be cooled by a flow of helium gas or liquid lithium.

A common thermal design problem is the transfer of heat from one fluid to another. Devices for this purpose are called *heat exchangers*. A familiar example is the automobile radiator, in which heat is transferred from the hot engine coolant to cold air blowing through the radiator core. Heat exchangers of many different types are required for power production and by the process industries. A power plant, whether the fuel be fossil or nuclear, has a *boiler* in which water is evaporated to produce steam to drive the turbines, and a *condenser* in which the steam is condensed to provide a low back pressure on the turbines and for water recovery. The condenser patented by James Watt in 1769 more than doubled the efficiency of steam engines then being used and set the Industrial Revolution in motion. The common vapor cycle refrigeration or air-conditioning system has an *evaporator* where heat is absorbed at low temperature and a *condenser* where heat is rejected at a higher temperature. On a domestic refrigerator, the condenser is usually in the form of a tube coil with cooling *fins* to assist transfer of heat to the surroundings. An oil refinery has a great variety of heat transfer equipment, including rectification columns and thermal crackers. Many heat exchangers are used to transfer heat from one process stream to another, to reduce the total energy consumption by the refinery.

Often the design problem is one of *thermal control*, that is, maintaining the operating temperature of temperature-sensitive components within a specified range. Cooling of all kinds of electronic gear is an example of thermal control. The development

of faster computers is now severely constrained by the difficulty of controlling the temperature of very small components, which dissipate large amounts of heat. Thermal control of temperature-sensitive components in a communications satellite orbiting the earth is a particularly difficult problem. Transistors and diodes must not overheat, batteries must not freeze, telescope optics must not lose alignment due to thermal expansion, and photographs must be processed at the proper temperature to ensure high resolution. Thermal control of space stations of the future will present even greater problems, since reliable life-support systems also will be necessary.

From the foregoing examples, it is clear that heat transfer involves a great variety of physical phenomena and engineering systems. The phenomena must first be understood and quantified before a methodology for the thermal design of an engineering system can be developed. Chapter 1 is an overview of the subject and introduces key topics at an elementary level. In Section 1.2, the distinction between the subjects of heat transfer and thermodynamics is explained. The first law of thermodynamics is reviewed, and closed- and open-system forms required for heat transfer analysis are developed. Section 1.3 introduces the three important modes of heat transfer: **heat conduction**, **thermal radiation**, and **heat convection**. Some formulas are developed that allow elementary heat transfer calculations to be made. In practical engineering problems, these modes of heat transfer usually occur simultaneously. Thus, in Section 1.4, the analysis of heat transfer by combined modes is introduced. The remainder of Chapter 1 deals with changes that occur in engineering systems as a result of heat transfer processes. In Section 1.5, the first law is applied to a very simple model closed system to determine the temperature response of the system with time. In Section 1.6, the first law for an open system is used to determine the change in temperature of a fluid flowing through a simple heat exchanger. Appropriate relations are developed that allow the design engineer to evaluate the exchanger performance. Finally, in Section 1.7, the International System of units (SI) is reviewed, and the units policy that is followed in the text is discussed.

1.2 HEAT TRANSFER AND ITS RELATION TO THERMODYNAMICS

When a hot object is placed in cold surroundings, it cools: the object loses internal energy, while the surroundings gain internal energy. We commonly describe this interaction as a *transfer of heat* from the object to the surrounding region. Since the caloric theory of heat has been long discredited, we do not imagine a “heat substance” flowing from the object to the surroundings. Rather, we understand that internal energy has been transferred by complex interactions on an atomic or subatomic scale. Nevertheless, it remains common practice to describe these interactions as transfer, transport, or flow, of heat. The engineering discipline of heat transfer is concerned with calculation of the rate at which heat flows within a medium, across an interface, or from one surface to another, as well as with the calculation of associated temperatures.

It is important to understand the essential difference between the engineering discipline of heat transfer and what is commonly called thermodynamics. Classical thermodynamics deals with systems in equilibrium. Its methodology may be used

to calculate the energy required to change a system from one equilibrium state to another, but it cannot be used to calculate the rate at which the change may occur. For example, if a 1 kg ingot of iron is quenched from 1000°C to 100°C in an oil bath, thermodynamics tells us that the loss in internal energy of the ingot is mass (1 kg) $\times$ specific heat (~ 450 J/kg K) $\times$ temperature change (900 K), or approximately 405 kJ. But thermodynamics cannot tell us how long we will have to wait for the temperature to drop to 100°C. The time depends on the temperature of the oil bath, physical properties of the oil, motion of the oil, and other factors. An appropriate heat transfer analysis will consider all of these.

Analysis of heat transfer processes does require using some thermodynamics concepts. In particular, the **first law of thermodynamics** is used, generally in particularly simple forms since work effects can often be ignored. The first law is a statement of the *principle of conservation of energy*, which is a basic law of physics. This principle can be formulated in many ways by excluding forms of energy that are irrelevant to the problem under consideration, or by simply redefining what is meant by energy. In heat transfer, it is common practice to refer to the first law as the *energy conservation principle* or simply as an *energy* or *heat balance* when no work is done. However, as in thermodynamics, it is essential that the correct form of the first law be used. The student must be able to define an appropriate system, recognize whether the system is *open* or *closed*, and decide whether a steady state can be assumed. Some simple forms of the energy conservation principle, which find frequent use in this text, follow.

A closed system containing a fixed mass of a solid is shown in Fig. 1.1. The system has a volume V [m³], and the solid has a density ρ [kg/m³]. There is heat transfer into the system at a rate of $\dot{Q}$ [J/s or W], and heat may be generated within the solid, for example, by nuclear fission or by an electrical current, at a rate $\dot{Q}_v$ [W]. Solids may be taken to be incompressible, so no work is done by or on the system. The principle of conservation of energy requires that over a time interval Δt [s],

$$\begin{aligned} \text{Change in internal energy} &= \text{Heat transferred} + \text{Heat generated} \\ \text{within the system} &\quad \text{into the system} \quad \text{within the system} \\ \Delta U &= \dot{Q}\Delta t + \dot{Q}_v\Delta t \end{aligned} \quad (1.1)$$

Dividing by Δt and letting Δt go to zero gives

$$\frac{dU}{dt} = \dot{Q} + \dot{Q}_v$$

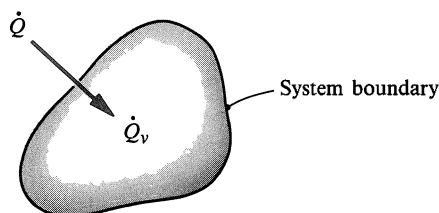


Figure 1.1 Application of the energy conservation principle to a closed system.

The system contains a fixed mass (ρV); thus, we can write $dU = \rho V du$, where u is the specific internal energy [J/kg]. Also, for an incompressible solid, $du = c_v dT$, where c_v is the constant-volume specific heat [J/kg K], and T [K] is temperature. Since the solid has been taken to be incompressible, the constant-volume and constant-pressure specific heats are equal, so we simply write $du = c dT$ to obtain

$$\rho V c \frac{dT}{dt} = \dot{Q} + \dot{Q}_v \quad (1.2)$$

Equation (1.2) is a special form of the first law of thermodynamics that will be used often in this text. It is written on a *rate* basis; that is, it gives the rate of change of temperature with time. For some purposes, however, it will prove convenient to return to Eq. (1.1) as a statement of the first law.

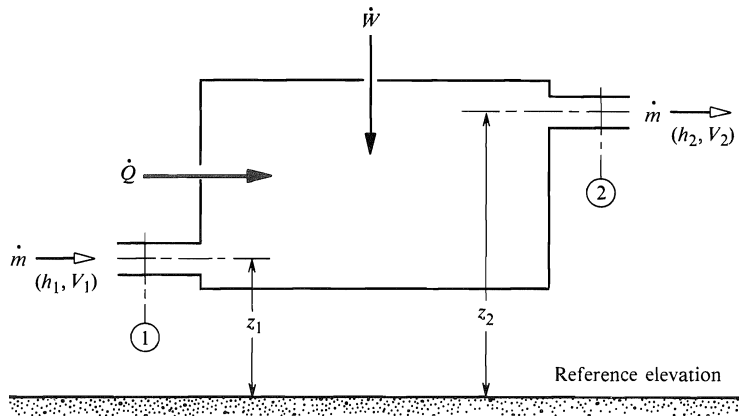


Figure 1.2 Application of the energy conservation principle to a steady-flow open system.

Figure 1.2 shows an *open* system, for which a particularly useful form of the first law is the **steady-flow energy equation**. It is used widely in the thermodynamic analysis of equipment such as turbines and compressors. The equation is

$$\dot{m} \Delta \left(h + \frac{V^2}{2} + gz \right) = \dot{Q} + \dot{W} \quad (1.3)$$

where $\dot{m}$ [kg/s] is the mass flow rate, h [J/kg] is the specific enthalpy, V [m/s] is velocity, g [m/s²] is the gravitational acceleration, z is elevation [m], $\dot{Q}$ [W] is the rate of heat transfer, as before, and $\dot{W}$ [W] is the rate at which external (shaft) work is done on the system.¹ Notice that the sign convention here is that external work done *on* the system is positive; the opposite sign convention is also widely used. The symbol ΔX means $X_{\text{out}} - X_{\text{in}}$, or the change in X . Equation (1.3) applies to a pure

¹ Equation (1.3) has been written as if h , V , and z are uniform in the streams crossing the control volume boundary. Often such an assumption can be made; if not, an integration across each stream is required to give appropriate average values.

substance when conditions within the system, such as temperature and velocity, are unchanging over some appropriate time interval. Heat generation within the system has not been included. In many types of heat transfer equipment, no external work is done, and changes in kinetic and potential energy are negligible; Eq. (1.3) then reduces to

$$\dot{m}\Delta h = \dot{Q} \quad (1.4)$$

The specific enthalpy h is related to the specific internal energy u as

$$h = u + Pv \quad (1.5)$$

where P [N/m² or Pa] is pressure, and v is specific volume [m³/kg]. Two limit forms of Δh are useful. If the fluid enters the system at state 1 and leaves at state 2:

1. For ideal gases with $Pv = RT$,

$$\Delta h = \int_{T_1}^{T_2} c_p dT \quad (1.6a)$$

where R [J/kg K] is the gas constant and c_p [J/kg K] is the constant-pressure specific heat.

2. For incompressible liquids with $\rho = 1/v = \text{constant}$

$$\Delta h = \int_{T_1}^{T_2} c_v dT + \frac{P_2 - P_1}{\rho} \quad (1.6b)$$

where $c_v = c_p$. The second term in Eq. (1.6b) is usually negligible.

Equation (1.4) is the usual starting point for the heat transfer analysis of steady-state open systems.

The *second law of thermodynamics* tells us that if two objects at temperatures T_1 and T_2 are connected, and if $T_1 > T_2$, then heat will flow spontaneously and irreversibly from object 1 to object 2. Also, there is an entropy increase associated with this heat flow. As T_2 approaches T_1 , the process approaches a reversible process, but simultaneously the rate of heat transfer approaches zero, so the process is of little practical interest. All heat transfer processes encountered in engineering are irreversible and generate entropy. With the increasing realization that energy supplies should be conserved, efficient use of available energy is becoming an important consideration in thermal design. Thus, the engineer should be aware of the irreversible processes occurring in the system under development and understand that the optimal design may be one that minimizes entropy generation due to heat transfer and fluid flow. Most often, however, energy conservation is simply a consideration in the overall economic evaluation of the design. Usually there is an important trade-off between energy costs associated with the operation of the system and the capital costs required to construct the equipment.

1.3 MODES OF HEAT TRANSFER

In thermodynamics, *heat* is defined as energy transfer due to temperature gradients or differences. Consistent with this viewpoint, thermodynamics recognizes only two modes of heat transfer: *conduction* and *radiation*. For example, heat transfer across a steel pipe wall is by conduction, whereas heat transfer from the sun to the earth or to a spacecraft is by thermal radiation. These modes of heat transfer occur on a molecular or subatomic scale. In air at normal pressure, conduction is by molecules that travel a very short distance ($\sim 0.65 \mu\text{m}$) before colliding with another molecule and exchanging energy. On the other hand, radiation is by photons, which travel almost unimpeded through the air from one surface to another. Thus, an important distinction between conduction and radiation is that the energy carriers for conduction have a short *mean free path*, whereas for radiation the carriers have a long mean free path. However, in air at the very low pressures characteristic of high-vacuum equipment, the mean free path of molecules can be much longer than the equipment dimensions, so the molecules travel unimpeded from one surface to another. Then heat transfer by molecules is governed by laws analogous to those for radiation.

A fluid, by virtue of its mass and velocity, can transport momentum. In addition, by virtue of its temperature, it can transport energy. Strictly speaking, *convection* is the transport of energy by motion of a medium (a moving solid can also convect energy in this sense). In the steady-flow energy equation, Eq. (1.3), convection of internal energy is contained in the term $\dot{m}\Delta h$, which is on the left-hand side of the equation, and heat transfer by conduction and radiation is on the right-hand side, as $\dot{Q}$. However, it is common engineering practice to use the term *convection* more broadly and describe heat transfer from a surface to a moving fluid also as convection, or *convective heat transfer*, even though conduction and radiation play a dominant role close to the surface, where the fluid is stationary. In this sense, convection is usually regarded as a distinct mode of heat transfer. Examples of convective heat transfer include heat transfer from the radiator of an automobile or to the skin of a hypersonic vehicle. Convection is often associated with a change of phase, for example, when water boils in a kettle or when steam condenses in a power plant condenser. Owing to the complexity of such processes, boiling and condensation are often regarded as distinct heat transfer processes.

The hot water home heating system shown in Fig. 1.3 illustrates the modes of heat transfer. Hot water from the furnace in the basement flows along pipes to radiators located in individual rooms. Transport of energy by the hot water from the basement is true convection as defined above; we do not call this a heat transfer process. Inside the radiators, there is convective heat transfer from the hot water to the radiator shell, conduction across the radiator shell, and both convective and radiative heat transfer from the hot outer surface of the radiator shell into the room. The convection is *natural* convection: the heated air adjacent to the radiator surface rises due to its buoyancy, and cooler air flows in to take its place. The radiators are heat exchangers. Although commonly used, the term *radiator* is misleading since

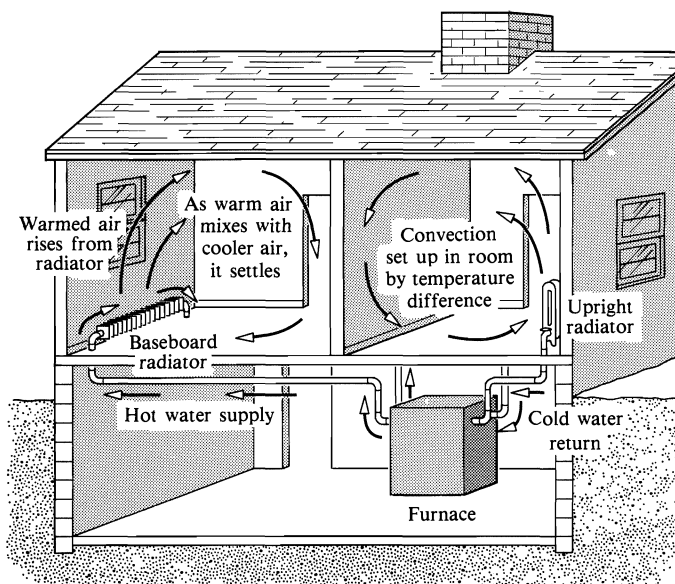


Figure 1.3 A hot-water home heating system illustrating the modes of heat transfer.

heat transfer from the shell surface can be predominantly by convection rather than by radiation (see Exercise 1–19). Heaters that transfer heat predominantly by radiation are, for example, electrical resistance wire units.

Each of the three important subject areas of heat transfer will now be introduced: conduction, in Section 1.3.1; radiation, in Section 1.3.2; and convection, in Section 1.3.3.

1.3.1 Heat Conduction

On a microscopic level, the physical mechanisms of conduction are complex, encompassing such varied phenomena as molecular collisions in gases, lattice vibrations in crystals, and flow of free electrons in metals. However, if at all possible, the engineer avoids considering processes at the microscopic level, preferring to use *phenomenological laws*, at a macroscopic level. The phenomenological law governing heat conduction was proposed by the French mathematical physicist J. B. Fourier in 1822. This law will be introduced here by considering the simple problem of one-dimensional heat flow across a plane wall—for example, a layer of insulation.² Figure 1.4 shows a plane wall of surface area A and thickness L , with its face at $x = 0$ maintained at temperature T_1 and the face at $x = L$ maintained at T_2 . The heat flow $\dot{Q}$ through the wall is in the direction of decreasing temperature: if

² In thermodynamics, the term *insulated* is often used to refer to a *perfectly* insulated (zero-heat-flow or adiabatic) surface. In practice, insulation is used to *reduce* heat flow and seldom can be regarded as perfect.

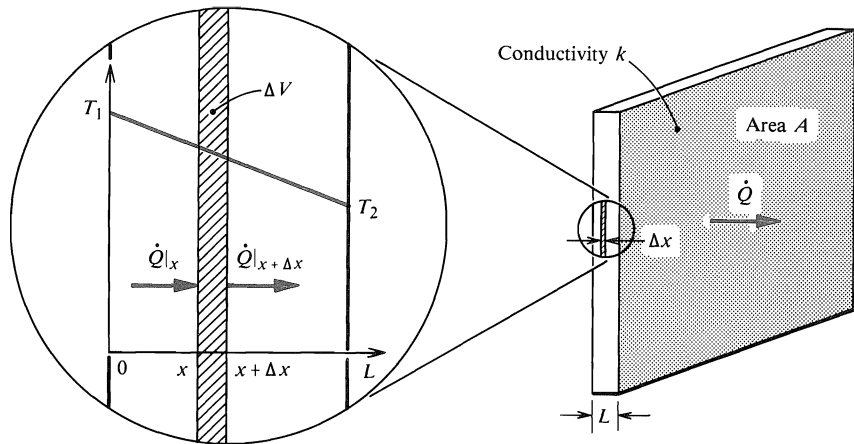


Figure 1.4 Steady one-dimensional conduction across a plane wall, showing the application of the energy conservation principle to an elemental volume Δx thick.

$T_1 > T_2$, $\dot{Q}$ is in the positive x direction.³ The phenomenological law governing this heat flow is **Fourier's law of heat conduction**, which states that in a homogeneous substance, the local heat flux is proportional to the negative of the local temperature gradient:

$$\frac{\dot{Q}}{A} = q \quad \text{and} \quad q \propto -\frac{dT}{dx} \quad (1.7)$$

where q is the heat flux, or heat flow per unit area perpendicular to the flow direction [W/m^2], T is the local temperature [K or $^{\circ}\text{C}$], and x is the coordinate in the flow direction [m]. When dT/dx is negative, the minus sign in Eq. (1.7) gives a positive q in the positive x direction. Introducing a constant of proportionality k ,

$$q = -k \frac{dT}{dx} \quad (1.8)$$

where k is the **thermal conductivity** of the substance and, by inspection of the equation, must have units [$\text{W}/\text{m K}$]. Notice that temperature can be given in kelvins or degrees Celsius in Eq. (1.8): the temperature gradient does not depend on which of these units is used since one kelvin equals one degree Celsius ($1 \text{ K} = 1^{\circ}\text{C}$). Thus, the units of thermal conductivity could also be written [$\text{W}/\text{m}^{\circ}\text{C}$], but this is not the recommended practice when using the SI system of units. The magnitude of the thermal conductivity k for a given substance very much depends on its microscopic structure and also tends to vary somewhat with temperature; Table 1.1 gives some selected values of k .

³ Notice that this $\dot{Q}$ is the heat flow in the x direction, whereas in the first law, Eqs. (1.1)–(1.4), $\dot{Q}$ is the heat transfer into the whole system. In linking thermodynamics to heat transfer, some ambiguity in notation arises when common practice in both subjects is followed.

Table 1.1 Selected values of thermal conductivity at 300 K ($\sim 25^\circ\text{C}$).

Material	k W/m K
Copper	386
Aluminum	204
Brass (70% Cu, 30% Zn)	111
Mild steel	64
Stainless steel, 18–8	15
Mercury	8.4
Concrete	1.4
Pyrex glass	1.09
Water	0.611
Neoprene rubber	0.19
Engine oil, SAE 50	0.145
White pine, perpendicular to grain	0.10
Polyvinyl chloride (PVC)	0.092
Freon 12	0.071
Cork	0.043
Fiberglass (medium density)	0.038
Polystyrene	0.028
Air	0.027

Note: Appendix A contains more comprehensive data.

Figure 1.4 shows an elemental volume ΔV located between x and $x + \Delta x$; ΔV is a closed system, and the energy conservation principle in the form of Eq. (1.2) applies. If we consider a steady state, then temperatures are unchanging in time and $dT/dt = 0$; also, if there is no heat generated within the volume, $\dot{Q}_v = 0$. Then Eq. (1.2) states that the net heat flow into the system is zero. Since heat is flowing into ΔV across the face at x , and out of ΔV across the face at $x + \Delta x$,

$$\dot{Q}|_x = \dot{Q}|_{x+\Delta x}$$

or

$$\dot{Q} = \text{Constant}$$

But from Fourier's law, Eq. (1.8),

$$\dot{Q} = qA = -kA \frac{dT}{dx}$$

The variables are separable: rearranging and integrating across the wall,

$$\frac{\dot{Q}}{A} \int_0^L dx = - \int_{T_1}^{T_2} k dT$$

where $\dot{Q}$ and A have been taken outside the integral signs since both are constants. If the small variation of k with temperature is ignored for the present we obtain

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) = \frac{T_1 - T_2}{L/kA} \quad (1.9)$$

Comparison of Eq. (1.9) with Ohm's law, $I = E/R$, suggests that $\Delta T = T_1 - T_2$ can be viewed as a driving potential for flow of heat, analogous to voltage being the driving potential for current. Then $R \equiv L/kA$ can be viewed as a **thermal resistance** analogous to electrical resistance.

If we have a composite wall of two slabs of material, as shown in Fig. 1.5, the heat flow through each layer is the same:

$$\dot{Q} = \frac{T_1 - T_2}{L_A/k_A A} = \frac{T_2 - T_3}{L_B/k_B A}$$

Rearranging,

$$\dot{Q} \left(\frac{L_A}{k_A A} \right) = T_1 - T_2$$

$$\dot{Q} \left(\frac{L_B}{k_B A} \right) = T_2 - T_3$$

Adding eliminates the interface temperature T_2 :

$$\dot{Q} \left(\frac{L_A}{k_A A} + \frac{L_B}{k_B A} \right) = T_1 - T_3$$

or

$$\dot{Q} = \frac{T_1 - T_3}{L_A/k_A A + L_B/k_B A} = \frac{\Delta T}{R_A + R_B} \quad (1.10a)$$

Using the electrical resistance analogy, we would view the problem as two resistances in series forming a **thermal circuit**, and immediately write

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \quad (1.10b)$$

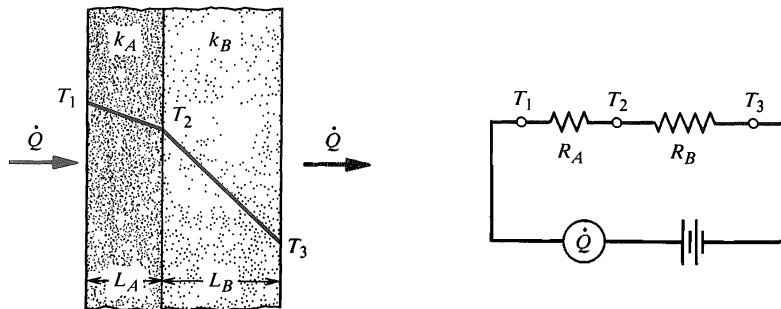


Figure 1.5 The temperature distribution for steady conduction across a composite plane wall and the corresponding thermal circuit.

EXAMPLE 1.1 Heat Transfer through Insulation

A refrigerated container is in the form of a cube with 2 m sides and has 5 mm-thick aluminum walls insulated with a 10 cm layer of cork. During steady operation, the temperatures on the inner and outer surfaces of the container are measured to be -5°C and 20°C , respectively. Determine the cooling load on the refrigerator.

Solution

Given: Aluminum container insulated with 10 cm-thick cork.

Required: Rate of heat gain.

Assumptions: 1. Steady state
2. One-dimensional heat conduction (ignore corner effects)

Equation (1.10) applies:

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \quad \text{where } R = \frac{L}{kA}$$

Let subscripts A and B denote the aluminum wall and cork insulation, respectively. Table 1.1 gives $k_A = 204 \text{ W/m K}$, $k_B = 0.043 \text{ W/m K}$. We suspect that the thermal resistance of the aluminum wall is negligible, but we will calculate it anyway. For one side of area $A = 4 \text{ m}^2$, the thermal resistances are

$$R_A = \frac{L_A}{k_A A} = \frac{(0.005 \text{ m})}{(204 \text{ W/m K})(4 \text{ m}^2)} = 6.13 \times 10^{-6} \text{ K/W}$$

$$R_B = \frac{L_B}{k_B A} = \frac{(0.10 \text{ m})}{(0.043 \text{ W/m K})(4 \text{ m}^2)} = 0.581 \text{ K/W}$$

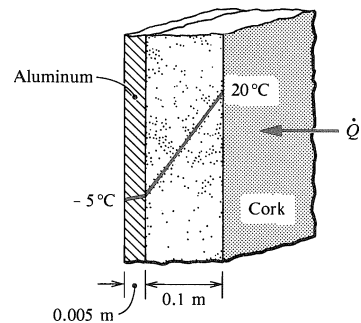
Since R_A is five orders of magnitude less than R_B , it can be ignored. The heat flow for a temperature difference of $T_1 - T_2 = 20 - (-5) = 25 \text{ K}$, is

$$\dot{Q} = \frac{\Delta T}{R_B} = \frac{25 \text{ K}}{0.581 \text{ K/W}} = 43.0 \text{ W}$$

For six sides, the total cooling load on the refrigerator is $6.0 \times 43.0 = 258 \text{ W}$.

Comments

1. In the future, when it is obvious that a resistance in a series network is negligible, it can be ignored from the outset (no effort should be expended to obtain data for its calculation).
2. The assumption of one-dimensional conduction is good because the 0.1 m insulation thickness is small compared to the 2 m-long sides of the cube.



3. Notice that the temperature difference $T_1 - T_2$ is expressed in kelvins, even though T_1 and T_2 were given in degrees Celsius.
4. We have assumed perfect thermal contact between the aluminum and cork; that is, there is no thermal resistance associated with the interface between the two materials (see Section 2.2.2).

1.3.2 Thermal Radiation

All matter and space contains electromagnetic radiation. A particle, or *quantum*, of electromagnetic energy is a photon, and heat transfer by radiation can be viewed either in terms of electromagnetic waves or in terms of photons. The flux of radiant energy incident on a surface is its **irradiation**, G [W/m^2]; the energy flux leaving a surface due to emission and reflection of electromagnetic radiation is its **radiosity**, J [W/m^2]. A **black surface** (or **blackbody**) is defined as a surface that absorbs all incident radiation, reflecting none. As a consequence, all of the radiation leaving a black surface is emitted by the surface and is given by the **Stefan-Boltzmann law** as

$$J = E_b = \sigma T^4 \quad (1.11)$$

where E_b is the **blackbody emissive power**, T is absolute temperature [K], and σ is the Stefan-Boltzmann constant ($\approx 5.67 \times 10^{-8} \text{ W}/\text{m}^2 \text{ K}^4$). Table 1.2 shows how $E_b = \sigma T^4$ increases rapidly with temperature.

Table 1.2 Blackbody emissive power σT^4 at various temperatures.

Surface Temperature K	Blackbody Emissive Power W/m^2
300 (room temperature)	459
1000 (cherry-red hot)	56,700
3000 (lamp filament)	4,590,000
5760 (sun temperature)	62,400,000

Figure 1.6 shows a convex black object of surface area A_1 in a black isothermal enclosure at temperature T_2 . At equilibrium, the object is also at temperature T_2 , and the radiation flux incident on the object must equal the radiation flux leaving:

$$G_1 A_1 = J_1 A_1 = \sigma T_2^4 A_1$$

Hence

$$G_1 = \sigma T_2^4 \quad (1.12)$$

and is uniform over the area. If the temperature of the object is now raised to T_1 , its radiosity becomes σT_1^4 while its irradiation remains σT_2^4 (because the enclosure reflects no radiation). Then the net radiant heat flux through the surface, q_1 , is the

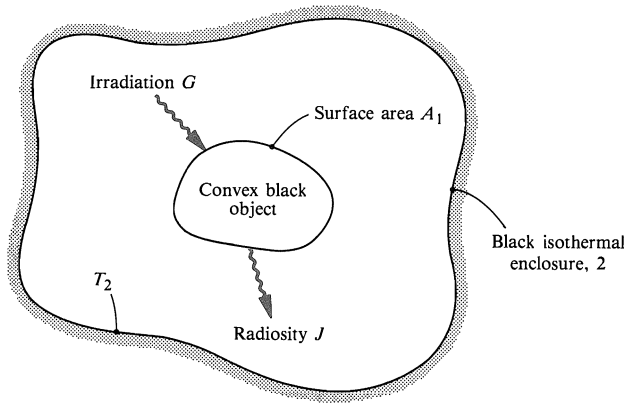


Figure 1.6 A convex black object (surface 1) in a black isothermal enclosure (surface 2).

radiosity minus the irradiation:

$$q_1 = J_1 - G_1 \quad (1.13)$$

or

$$q_1 = \sigma T_1^4 - \sigma T_2^4 \quad (1.14)$$

where the sign convention is such that a net flux away from the surface is positive. Equation (1.14) is also valid for two large black surfaces facing each other, as shown in Fig. 1.7.

The blackbody is an ideal surface. Real surfaces absorb less radiation than do black surfaces. The fraction of incident radiation absorbed is called the absorptance (or absorptivity), α . A widely used model of a real surface is the **gray surface**, which is defined as a surface for which α is a constant, irrespective of the nature of the incident radiation. The fraction of incident radiation reflected is the **reflectance** (or reflectivity), ρ . If the object is opaque, that is, not transparent to electromagnetic radiation, then

$$\rho = 1 - \alpha \quad (1.15)$$

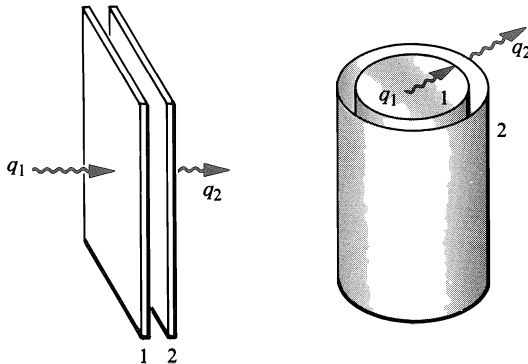


Figure 1.7 Examples of two large surfaces facing each other.

Table 1.3 Selected approximate values of emittance, ε (total hemispherical values at normal temperatures).

Surface	Emittance, ε
Aluminum alloy, unoxidized	0.035
Black anodized aluminum	0.80
Chromium plating	0.16
Stainless steel, type 312, lightly oxidized	0.30
Inconel X, oxidized	0.72
Black enamel paint	0.78
White acrylic paint	0.90
Asphalt	0.88
Concrete	0.90
Soil	0.94
Pyrex glass	0.80

Note: More comprehensive data are given in Appendix A. Emittance is very dependent on surface finish; thus, values obtained from various sources may differ significantly.

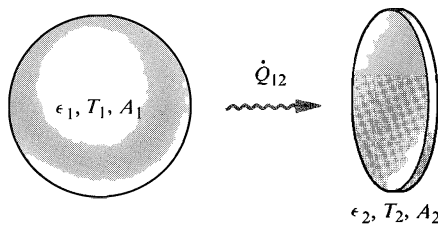
Real surfaces also emit less radiation than do black surfaces. The fraction of the blackbody emissive power σT^4 emitted is called the **emittance** (or emissivity), ε .⁴ A gray surface also has a constant value of ε , independent of its temperature, and, as will be shown in Chapter 6, the emittance and absorptance of a gray surface are equal:

$$\varepsilon = \alpha \quad (\text{gray surface}) \quad (1.16)$$

Table 1.3 shows some typical values of ε at normal temperatures. Bright metal surfaces tend to have low values, whereas oxidized or painted surfaces tend to have high values. Values of α and ρ can also be obtained from Table 1.3 by using Eqs. (1.15) and (1.16).

If heat is transferred by radiation between two gray surfaces of finite size, as shown in Fig. 1.8, the rate of heat flow will depend on temperatures T_1 and T_2 and emittances ε_1 and ε_2 , as well as the geometry. Clearly, some of the radiation leaving surface 1 will not be intercepted by surface 2, and vice versa. Determining the rate of heat flow is usually quite difficult. In general, we may write

$$\dot{Q}_{12} = A_1 \bar{\mathcal{F}}_{12} (\sigma T_1^4 - \sigma T_2^4) \quad (1.17)$$

**Figure 1.8** Radiation heat transfer between two finite gray surfaces.

⁴ Both the endings *-ance* and *-ivity* are commonly used for radiation properties. In this text, *-ance* will be used for surface radiation properties. In Chapter 6, *-ivity* will be used for gas radiation properties.

where $\dot{Q}_{12}$ is the net radiant energy interchange (heat transfer) from surface 1 to surface 2, and $\mathcal{F}_{12}$ is a **transfer factor**, which depends on emittances and geometry. For the special case of surface 1 surrounded by surface 2, where either area A_1 is small compared to area A_2 , or surface 2 is nearly black, $\mathcal{F}_{12} \simeq \varepsilon_1$ and Eq. (1.17) becomes

$$\dot{Q}_{12} = \varepsilon_1 A_1 (\sigma T_1^4 - \sigma T_2^4) \quad (1.18)$$

Equation (1.18) will be derived in Chapter 6. It is an important result and is often used for quick engineering estimates.

The T^4 dependence of radiant heat transfer complicates engineering calculations. When T_1 and T_2 are not too different, it is convenient to linearize Eq. (1.18) by factoring the term $(\sigma T_1^4 - \sigma T_2^4)$ to obtain

$$\begin{aligned} \dot{Q}_{12} &= \varepsilon_1 A_1 \sigma (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2) \\ &\simeq \varepsilon_1 A_1 \sigma (4T_m^3)(T_1 - T_2) \end{aligned}$$

for $T_1 \simeq T_2$, where T_m is the mean of T_1 and T_2 . This result can be written more concisely as

$$Q_{12} \simeq A_1 h_r (T_1 - T_2) \quad (1.19)$$

where $h_r = 4\varepsilon_1 \sigma T_m^3$ is called the **radiation heat transfer coefficient** [$\text{W/m}^2 \text{ K}$]. At 25°C ($= 298 \text{ K}$),

$$h_r = (4)\varepsilon_1 (5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4) (298 \text{ K})^3$$

or

$$h_r \simeq 6\varepsilon_1 \text{ W/m}^2 \text{ K}$$

This result can be easily remembered: The radiation heat transfer coefficient at room temperature is about six times the surface emittance. For $T_1 = 320 \text{ K}$ and $T_2 = 300 \text{ K}$, the error incurred in using the approximation of Eq. (1.19) is only 0.1%; for $T_1 = 400 \text{ K}$ and $T_2 = 300 \text{ K}$, the error is 2%.

EXAMPLE 1.2 Heat Loss from a Transistor

An electronic package for an experiment in outer space contains a transistor capsule, which is approximately spherical in shape with a 2 cm diameter. It is contained in an evacuated case with nearly black walls at 30°C . The only significant path for heat loss from the capsule is radiation to the case walls. If the transistor dissipates 300 mW, what will the capsule temperature be if it is (i) bright aluminum and (ii) black anodized aluminum?

Solution

Given: 2 cm-diameter transistor capsule dissipating 300 mW.

Required: Capsule temperature for (i) bright aluminum and (ii) black anodized aluminum.

Assumptions: Model as a small gray body in large, nearly black surroundings.

Equation (1.18) is applicable with

$$\dot{Q}_{12} = 300 \text{ mW}$$

$$T_2 = 30^\circ\text{C} = 303 \text{ K}$$

and T_1 is the unknown.

$$\dot{Q}_{12} = \varepsilon_1 A_1 (\sigma T_1^4 - \sigma T_2^4)$$

$$0.3 \text{ W} = (\varepsilon_1)(\pi)(0.02 \text{ m})^2 [\sigma T_1^4 - (5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(303 \text{ K})^4]$$

Solving,

$$\sigma T_1^4 = 478 + \frac{239}{\varepsilon_1}$$

(i) For bright aluminum ($\varepsilon = 0.035$ from Table 1.3),

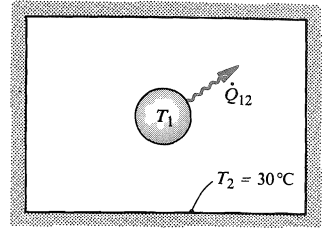
$$\sigma T_1^4 = 478 + 6828 = 7306 \text{ W/m}^2$$

$$T_1 = 599 \text{ K} (326^\circ\text{C})$$

(ii) For black anodized aluminum ($\varepsilon = 0.80$ from Table 1.3),

$$\sigma T_1^4 = 478 + 298 = 776 \text{ W/m}^2$$

$$T_1 = 342 \text{ K} (69^\circ\text{C})$$



Comments

1. The anodized aluminum gives a satisfactory operating temperature, but a bright aluminum capsule could not be used since 326°C is far in excess of allowable operating temperatures for semiconductor devices.
2. Note the use of kelvins for temperature in this radiation heat transfer calculation.

1.3.3 Heat Convection

As already explained, *convection* or *convective heat transfer* is the term used to describe heat transfer from a surface to a moving fluid, as shown in Fig. 1.9. The surface may be the inside of a pipe, the skin of a hypersonic aircraft, or a water-air interface in a cooling tower. The flow may be *forced*, as in the case of a liquid pumped

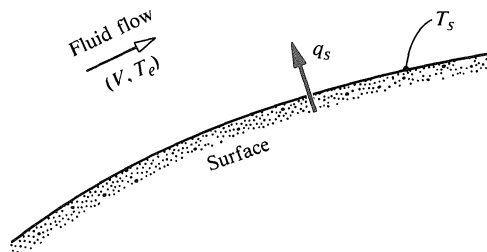


Figure 1.9 Schematic of convective heat transfer to a fluid at temperature T_e , flowing at velocity V past a surface at temperature T_s .

through the pipe or air on the flight vehicle propelled through the atmosphere. On the other hand, the flow could be *natural* (or *free*), driven by buoyancy forces arising from a density difference, as in the case of a natural-draft cooling tower. Either type of flow can be *internal*, such as the pipe flow, or *external*, such as flow over the vehicle. Also, both forced and natural flows can be either *laminar* or *turbulent*, with laminar flows being predominant at lower velocities, for smaller sizes, and for more viscous fluids. Flow in a pipe becomes turbulent when the dimensionless group called the **Reynolds number**, $Re_D = VD/\nu$, exceeds about 2300, where V is the velocity [m/s], D is the pipe diameter [m], and ν is the kinematic viscosity of the fluid [m²/s]. Heat transfer rates tend to be much higher in turbulent flows than in laminar flows, owing to the vigorous mixing of the fluid. Figure 1.10 shows some commonly encountered flows.

The rate of heat transfer by convection is usually a complicated function of surface geometry and temperature, the fluid temperature and velocity, and fluid thermophysical properties. In an external forced flow, the rate of heat transfer is approximately proportional to the difference between the surface temperature T_s and the temperature of the free stream fluid T_e . The constant of proportionality is called the **convective heat transfer coefficient** h_c :

$$q_s = h_c \Delta T \quad (1.20)$$

where $\Delta T = T_s - T_e$, q_s is the heat flux from the surface into the fluid [W/m²], and h_c has units [W/m² K]. Equation (1.20) is often called *Newton's law of cooling* but is a definition of h_c rather than a true physical law. For natural convection, the situation is more complicated. If the flow is laminar, q_s varies as $\Delta T^{5/4}$; if the flow is turbulent, it varies as $\Delta T^{4/3}$. However, we still find it convenient to define a heat transfer coefficient by Eq. (1.20); then h_c varies as $\Delta T^{1/4}$ for laminar flows and as $\Delta T^{1/3}$ for turbulent ones.

An important practical problem is convective heat transfer to a fluid flowing in a tube, as may be found in heat exchangers for heating or cooling liquids, in condensers, and in various kinds of boilers. In using Eq. (1.20) for internal flows, $\Delta T = T_s - T_b$, where T_b is a properly averaged fluid temperature called the **bulk temperature** or mixed mean temperature and is defined in Chapter 4. Here it is sufficient to note that enthalpy in the steady-flow energy equation, Eq. (1.4), is also the bulk value, and T_b is the corresponding temperature. If the pipe has a uniform wall temperature T_s along its length, and the flow is laminar ($Re_D \leq 2300$), then sufficiently far from the pipe entrance, the heat transfer coefficient is given by the exact relation

$$h_c = 3.66 \frac{k}{D} \quad (1.21)$$

where k is the fluid thermal conductivity and D is the pipe diameter. Notice that the heat transfer coefficient is directly proportional to thermal conductivity, inversely proportional to pipe diameter, and—perhaps surprisingly—*independent of flow velocity*. On the other hand, for fully turbulent flow ($Re_D \geq 10,000$), h_c is given

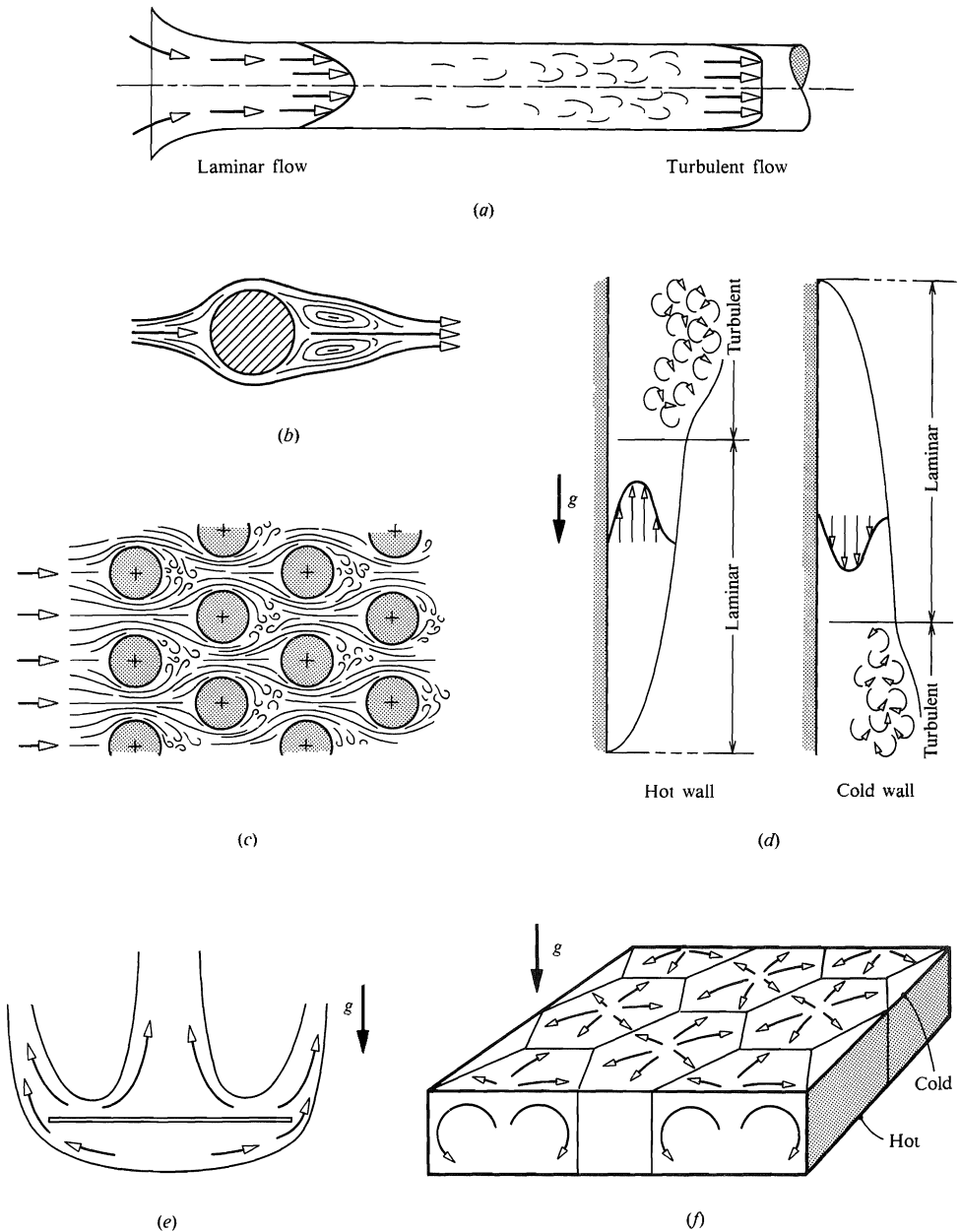


Figure 1.10 Some commonly encountered flows. (a) Forced flow in a pipe, $Re_D \approx 50,000$. The flow is initially laminar because of the "bell-mouth" entrance but becomes turbulent downstream. (b) Laminar forced flow over a cylinder, $Re_D \approx 25$. (c) Forced flow through a tube bank as found in a shell-and-tube heat exchanger. (d) Laminar and turbulent convection boundary layers on vertical walls. (e) Laminar natural convection about a heated horizontal plate. (f) Cellular natural convection in a horizontal enclosed fluid layer.

approximately by the following, rather complicated correlation of experimental data:

$$h_c = 0.023 \frac{V^{0.8} k^{0.6} (\rho c_p)^{0.4}}{D^{0.2} \nu^{0.4}} \quad (1.22)$$

In contrast to laminar flow, h_c is now strongly dependent on velocity, V , but only weakly dependent on diameter. In addition to thermal conductivity, other fluid properties involved are the kinematic viscosity, ν ; density, ρ ; and specific heat, c_p . In Chapter 4 we will see how Eq. (1.22) can be rearranged in a more compact form by introducing appropriate dimensionless groups. Equations (1.21) and (1.22) are only valid at some distance from the pipe entrance and indicate that the heat transfer coefficient is then independent of position along the pipe. Near the pipe entrance, heat transfer coefficients tend to be higher, due to the generation of large-scale vortices by upstream bends or sharp corners and the effect of suddenly heating the fluid.

Figure 1.11 shows a natural convection flow on a heated vertical surface, as well as a schematic of the associated variation of h_c along the surface. Transition from a laminar to a turbulent boundary layer is shown. In gases, the location of the transition is determined by a critical value of a dimensionless group called the **Grashof number**. The Grashof number is defined as $Gr_x = (\beta \Delta T) g x^3 / \nu^2$, where $\Delta T = T_i - T_e$, g is the gravitational acceleration [m/s^2], x is the distance from the bottom of the surface where the boundary layer starts, and β is the volumetric coefficient of expansion, which for an ideal gas is simply $1/T$, where T is absolute temperature [K]. On a vertical wall, transition occurs at $Gr_x \approx 10^9$. For air, at normal temperatures, experiments show that the heat transfer coefficient for natural convection on a vertical wall can be approximated by the following formulas:

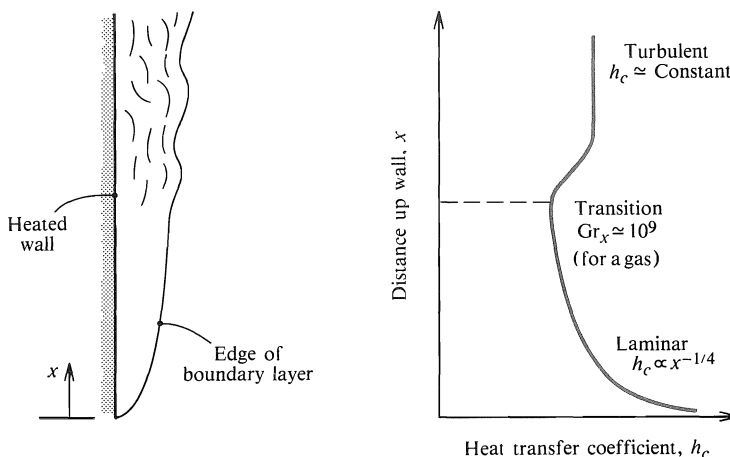


Figure 1.11 A natural-convection boundary layer on a vertical wall, showing the variation of local heat transfer coefficient. For gases, transition from a laminar to turbulent flow occurs at a Grashof number of approximately 10^9 ; hence, $x_u \approx [10^9 \nu^2 / \beta \Delta T]^{1/3}$.

$$\text{Laminar flow: } h_c = 1.07(\Delta T/x)^{1/4} \text{ W/m}^2 \text{ K} \quad 10^4 < \text{Gr}_x < 10^9 \quad (1.23a)$$

$$\text{Turbulent flow: } h_c = 1.3(\Delta T)^{1/3} \text{ W/m}^2 \text{ K} \quad 10^9 < \text{Gr}_x < 10^{12} \quad (1.23b)$$

Since these are dimensional equations, it is necessary to specify the units of h_c , ΔT , and x , which are $[\text{W/m}^2 \text{ K}]$, $[\text{K}]$, and $[\text{m}]$, respectively. Notice that h_c varies as $x^{-1/4}$ in the laminar region but is independent of x in the turbulent region.

Usually the engineer requires the total heat transfer from a surface and is not too interested in the actual variation of heat flux along the surface. For this purpose, it is convenient to define an average heat transfer coefficient $\bar{h}_c$ for an *isothermal* surface of area A by the relation

$$\dot{Q} = \bar{h}_c A (T_s - T_e) \quad (1.24)$$

so that the total heat transfer rate, $\dot{Q}$, can be obtained easily. The relation between $\bar{h}_c$ and h_c is obtained as follows: For flow over a surface of width W and length L , as shown in Fig. 1.12,

$$d\dot{Q} = h_c (T_s - T_e) W dx$$

$$\dot{Q} = \int_0^L h_c (T_s - T_e) W dx$$

or

$$\dot{Q} = \left(\frac{1}{A} \int_0^A h_c dA \right) A (T_s - T_e), \quad \text{where } A = WL, \quad dA = W dx \quad (1.25)$$

if $(T_s - T_e)$ is independent of x . Since T_e is usually constant, this condition requires an isothermal wall. Thus, comparing Eqs. (1.24) and (1.25),

$$\bar{h}_c = \frac{1}{A} \int_0^A h_c dA \quad (1.26)$$

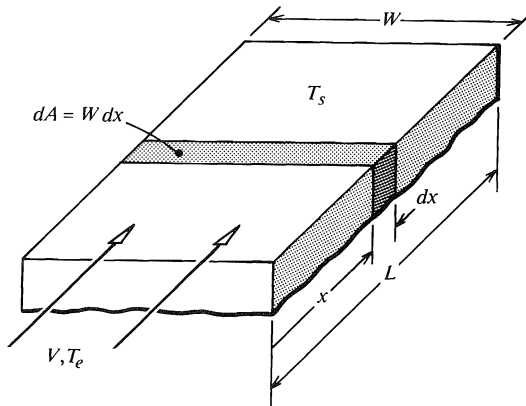


Figure 1.12 An isothermal surface used to define the average convective heat transfer coefficient $\bar{h}_c$.

Table 1.4 Orders of magnitude of average convective heat transfer coefficients.

Flow and Fluid	$\bar{h}_c$ W/m ² K
Free convection, air	3–25
Free convection, water	15–100
Forced convection, air	10–200
Forced convection, water	50–10,000
Forced convection, liquid sodium	10,000–100,000
Condensing steam	5000–50,000
Boiling water	3000–100,000

The surface may not be isothermal; for example, the surface may be electrically heated to give a uniform flux q_s along the surface. In this case, defining an average heat transfer coefficient is more difficult and will be dealt with in Chapter 4. Table 1.4 gives some order-of-magnitude values of average heat transfer coefficients for various situations. In general, high heat transfer coefficients are associated with high fluid thermal conductivities, high flow velocities, and small surfaces. The high heat transfer coefficients shown for boiling water and condensing steam are due to another cause: as we will see in Chapter 7, a large enthalpy of phase change (latent heat) is a contributing factor.

The complexity of most situations involving convective heat transfer precludes exact analysis, and *correlations* of experimental data must be used in engineering practice. For a particular situation, a number of correlations from various sources might be available, for example, from research laboratories in different countries. Also, as time goes by, older correlations may be superseded by newer correlations based on more accurate or more extensive experimental data. Heat transfer coefficients calculated from various available correlations usually do not differ by more than about 20%, but in more complex situations, much larger discrepancies may be encountered. Such is the nature of engineering calculations of convective heat transfer, in contrast to the more exact nature of the analysis of heat conduction or of elementary mechanics, for example.

EXAMPLE 1.3 Heat Loss through Glass Doors

The living room of a ski chalet has a pair of glass doors 2.3 m high and 4.0 m wide. On a cold morning, the air in the room is at 10°C, and frost partially covers the inner surface of the glass. Estimate the convective heat loss to the doors. Would you expect to see the frost form initially near the top or the bottom of the doors? Take $\nu = 14 \times 10^{-6}$ m²/s for the air.

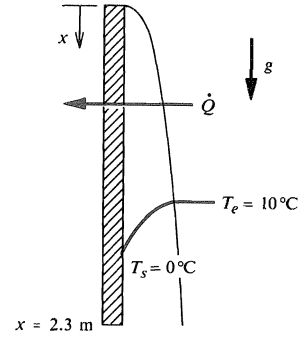
Solution

Given: Glass doors, width $W = 4$ m, height $L = 2.3$ m.

Required: Estimate of convective heat loss to the doors.

Assumptions: 1. Inner surface isothermal at $T_i \approx 0^\circ\text{C}$
 2. The laminar to turbulent flow transition occurs at $\text{Gr}_i \approx 10^9$.

Equation (1.24) will be used to estimate the heat loss. The inner surface will be at approximately 0°C since it is only partially covered with frost. If it were warmer, frost couldn't form; and if it were much colder, frost would cover the glass completely. There is a natural convection flow down the door since $T_e = 10^\circ\text{C}$ is greater than $T_i = 0^\circ\text{C}$. Transition from a laminar boundary layer to a turbulent boundary layer occurs when the Grashof number is about 10^9 . For transition at $x = x_{tr}$,



$$\text{Gr} = 10^9 = \frac{(\beta \Delta T) g x_{tr}^3}{\nu^2}; \quad \beta = 1/T \text{ for an ideal gas}$$

$$x_{tr} = \left[\frac{10^9 \nu^2}{(\Delta T/T)g} \right]^{1/3} = \left[\frac{(10^9)(14 \times 10^{-6} \text{ m}^2/\text{s})^2}{(10/278)(9.81 \text{ m/s}^2)} \right]^{1/3} = 0.82 \text{ m}$$

where the average of T_i and T_e has been used to evaluate β . The transition is seen to take place about one third of the way down the door.

We find the average heat transfer coefficient, $\bar{h}_c$, by substituting Eqs. (1.23a,b) in Eq. (1.26):

$$\begin{aligned} \bar{h}_c &= \frac{1}{A} \int_0^A h_c dA; \quad A = WL, \quad dA = W dx \\ &= \frac{1}{L} \int_0^L h_c dx \\ &= \frac{1}{L} \left[\int_0^{x_{tr}} 1.07(\Delta T/x)^{1/4} dx + \int_{x_{tr}}^L 1.3(\Delta T)^{1/3} dx \right] \\ &= (1/L) \left[(1.07)(4/3) \Delta T^{1/4} x_{tr}^{3/4} + (1.3)(\Delta T)^{1/3} (L - x_{tr}) \right] \\ &= (1/2.3) \left[(1.07)(4/3)(10)^{1/4} (0.82)^{3/4} + (1.3)(10)^{1/3} (2.3 - 0.82) \right] \\ &= (1/2.3) [2.19 + 4.15] \\ &= 2.75 \text{ W/m}^2 \text{ K} \end{aligned}$$

Then, from Eq. (1.24), the total heat loss to the door is

$$\dot{Q} = \bar{h}_c A \Delta T = (2.75 \text{ W/m}^2 \text{ K})(2.3 \times 4.0 \text{ m}^2)(10 \text{ K}) = 253 \text{ W}$$

Comments

1. The local heat transfer coefficient is larger near the top of the door, so that the relatively warm room air will tend to cause the glass there to be at a higher temperature than further down the door. Thus, frost should initially form near the bottom of the door.
2. In addition, interior surfaces in the room will lose heat by radiation through the glass doors.

1.4 COMBINED MODES OF HEAT TRANSFER

Heat transfer problems encountered by the design engineer almost always involve more than one mode of heat transfer occurring simultaneously. For example, consider the nighttime heat loss through the roof of the house shown in Fig. 1.3. Heat is transferred to the ceiling by convection from the warm room air, and by radiation from the walls, furniture, and occupants. Heat transfer across the ceiling and its insulation is by conduction, across the attic crawlspace by convection and radiation, and across the roof tile by conduction. Finally, the heat is transferred by convection to the cold ambient air, and by radiation to the nighttime sky. To consider realistic engineering problems, it is necessary at the outset to develop the theory required to handle *combined modes* of heat transfer.

1.4.1 Thermal Circuits

The electrical circuit analogy for conduction through a composite wall was introduced in Section 1.3.1. We now extend this concept to include convection and radiation as well. Figure 1.13 shows a two-layer composite wall of cross-sectional area A with the layers A and B having thickness and conductivity L_A , k_A and L_B , k_B , respectively. Heat is transferred from a hot fluid at temperature T_i to the inside of the wall with a convective heat transfer coefficient $h_{c,i}$, and away from the outside of the wall to a cold fluid at temperature T_o with heat transfer coefficient $h_{c,o}$.

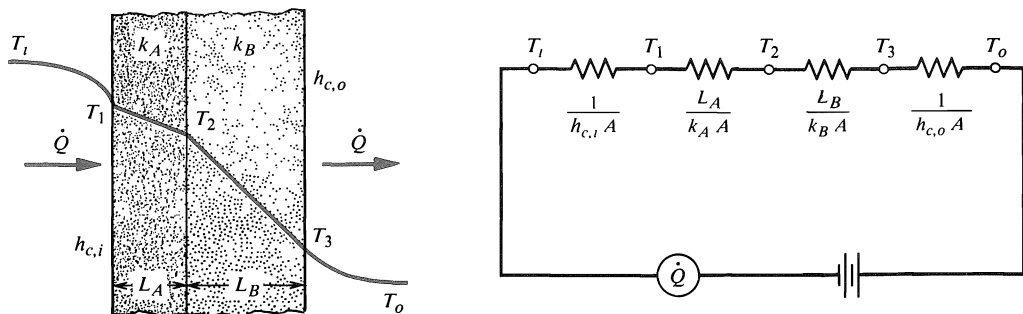


Figure 1.13 The temperature distribution for steady heat transfer across a composite plane wall, and the corresponding thermal circuit.

Newton's law of cooling, Eq. (1.20), can be rewritten as

$$\dot{Q} = \frac{\Delta T}{1/h_c A} \quad (1.27)$$

with $1/h_c A$ identified as a convective thermal resistance. At steady state, the heat flow through the wall is constant. Referring to Fig. 1.13 for the intermediate temperatures,

$$\dot{Q} = \frac{T_i - T_1}{1/h_{c,i} A} = \frac{T_1 - T_2}{L_A/k_A A} = \frac{T_2 - T_3}{L_B/k_B A} = \frac{T_3 - T_o}{1/h_{c,o} A} \quad (1.28)$$

Equation (1.28) is the basis of the thermal circuit shown in Fig. 1.13. The total resistance is the sum of four resistances in series. If we define the **overall heat transfer coefficient** U by the relation

$$\dot{Q} = UA(T_i - T_o) \quad (1.29)$$

then $1/UA$ is an overall resistance given by

$$\frac{1}{UA} = \frac{1}{h_{c,i} A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{1}{h_{c,o} A} \quad (1.30a)$$

or, since the cross-sectional area A is constant for a plane wall,

$$\frac{1}{U} = \frac{1}{h_{c,i}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{1}{h_{c,o}} \quad (1.30b)$$

Equation (1.29) is simple and convenient for use in engineering calculations. Typical values of U [$\text{W}/\text{m}^2 \text{K}$] vary over a wide range for different types of walls and convective flows.

Figure 1.14 shows a wall whose outer surface loses heat by both convection and radiation. For simplicity, assume that the fluid is at the same temperature as the surrounding surfaces, T_o . Using the approximate linearized Eq. (1.19),

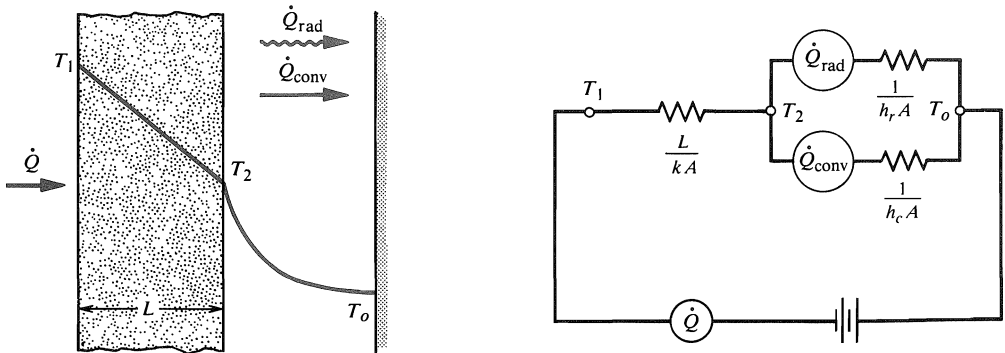


Figure 1.14 A wall that loses heat by both conduction and radiation; the thermal circuit shows resistances in parallel.

$$\dot{Q}_{\text{rad}} = \frac{\Delta T}{1/h_r A} \quad (1.31)$$

with $1/h_r A$ identified as a radiative thermal resistance. We now have two resistances in parallel, as shown in Fig. 1.14. The sum of the resistances is

$$\sum R = \frac{L}{kA} + \frac{1}{h_c A + h_r A}$$

or

$$\frac{1}{UA} = \frac{L}{kA} + \frac{1}{(h_c + h_r)A} \quad (1.32)$$

so that the convective and radiative heat transfer coefficients can simply be added. However, often the fluid and surrounding temperatures are not the same, or the simple linearized representation of radiative transfer [Eq. (1.19)] is invalid, so the thermal circuit is then more complex. When appropriate, we will write $h = h_c + h_r$ to account for combined convection and radiation.⁵

EXAMPLE 1.4 Heat Loss through a Composite Wall

The walls of a sparsely furnished single-room cabin in a forest consist of two layers of pine wood, each 2 cm thick, sandwiching 5 cm of fiberglass insulation. The cabin interior is maintained at 20°C when the ambient air temperature is 2°C. If the interior and exterior convective heat transfer coefficients are 3 and 6 W/m² K, respectively, and the exterior surface is finished with a white acrylic paint, estimate the heat flux through the wall.

Solution

Given: Pine wood cabin wall insulated with 5 cm of fiberglass.

Required: Estimate of heat loss through wall.

Assumptions: 1. Forest trees and shrubs are at the ambient air temperature, $T_e = 2^\circ\text{C}$.
2. Radiation transfer inside cabin is negligible since inner surfaces of walls, roof, and floor are at approximately the same temperature.

From Eq. (1.29), the heat flux through the wall is

$$q = \frac{\dot{Q}}{A} = U(T_i - T_o)$$

From Eqs. (1.30) and (1.32), the overall heat transfer coefficient is given by

$$\frac{1}{U} = \frac{1}{h_{c,i}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{(h_{c,o} + h_{r,o})}$$

⁵ Notice that the notation used for this combined heat transfer coefficient, h , is the same as that used for enthalpy. The student must be careful not to confuse these two quantities. Other notation is also in common use, for example, α for the heat transfer coefficient and i for enthalpy.

The thermal conductivities of pine wood, perpendicular to the grain, and of fiberglass are given in Table 1.1 as 0.10 and 0.038 W/m K, respectively. The exterior radiation heat transfer coefficient is given by Eq. (1.19) as

$$h_{r,o} = 4\varepsilon\sigma T_m^3$$

where $\varepsilon = 0.9$ for white acrylic paint, from Table 1.3, and $T_m \approx 2^\circ\text{C} = 275\text{ K}$ (since we expect the exterior resistance to be small). Thus,

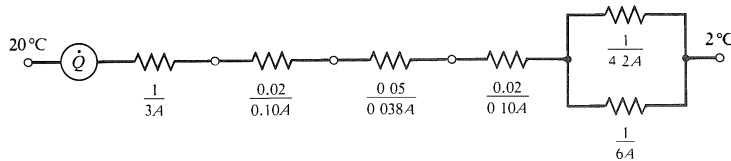
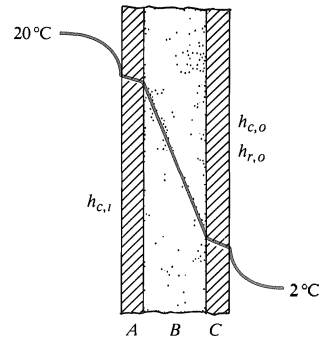
$$\begin{aligned} h_{i,o} &= 4(0.9)(5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(275 \text{ K})^3 \\ &= 4.2 \text{ W/m}^2 \text{ K} \end{aligned}$$

$$\begin{aligned} \frac{1}{U} &= \frac{1}{3} + \frac{0.02}{0.10} + \frac{0.05}{0.038} + \frac{0.02}{0.10} + \frac{1}{6 + 4.2} \\ &= 0.333 + 0.200 + 1.316 + 0.200 + 0.098 \\ &= 2.15 \text{ (W/m}^2 \text{ K)}^{-1} \end{aligned}$$

$$U = 0.466 \text{ W/m}^2 \text{ K}$$

Then the heat flux $q = U(T_i - T_o) = 0.466(20 - 2) = 8.38 \text{ W/m}^2$.

The thermal circuit is shown below.



Comments

1. The outside resistance is seen to be $0.098/2.15 \approx 5\%$ of the total resistance; hence, the outside wall of the cabin is only about 1 K above the ambient air, and our assumption of $T_m = 275\text{ K}$ for the evaluation of $h_{r,o}$ is adequate.
2. The dominant resistance is that of the fiberglass insulation; therefore, an accurate calculation of q depends mainly on having accurate values for the fiberglass thickness and thermal conductivity. Poor data or poor assumptions for the other resistances have little impact on the result.

1.4.2 Surface Energy Balances

Section 1.4.1 assumed that the energy flow $\dot{Q}$ across the wall surfaces is continuous. In fact, we used a procedure commonly called a *surface energy balance*, which is used in various ways. Some examples follow. Figure 1.15 shows an opaque solid that is losing heat by convection and radiation to its surroundings. Two imaginary surfaces are located on each side of the real solid-fluid interface: an *s*-surface

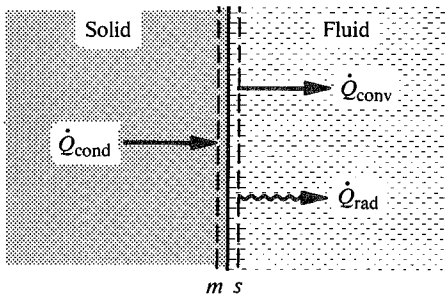


Figure 1.15 Schematic of a surface energy balance, showing the m - and s -surface in the solid and fluid, respectively.

in the fluid just adjacent to the interface, and an m -surface in the solid located such that all radiation is emitted or absorbed between the m -surface and the interface. Thus, energy is transferred across the m -surface by conduction only. (The choice of s and m to designate these surfaces follows an established practice. In particular, the use of the s prefix is consistent with the use of the subscript s to denote a surface temperature T_s in convection analysis.) The first law as applied to the closed system located between m - and s -surfaces requires that $\sum \dot{Q} = 0$; thus,

$$\dot{Q}_{\text{cond}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{rad}} = 0 \quad (1.33)$$

or, for a unit area,

$$q_{\text{cond}} - q_{\text{conv}} - q_{\text{rad}} = 0 \quad (1.34)$$

where the sign convention for the fluxes is shown in Fig. 1.15. If the solid is isothermal, Eq. (1.33) reduces to

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 0 \quad (1.35)$$

which is a simple energy balance on the solid. Notice that these surface energy balances remain valid for unsteady conditions, in which temperatures change with time, because the mass contained between the s - and m -surfaces is negligible and cannot store energy.

EXAMPLE 1.5 Air Temperature Measurement

A machine operator in a workshop complains that the air-heating system is not keeping the air at the required minimum temperature of 20°C. To support his claim, he shows that a mercury-in-glass thermometer suspended from a roof truss reads only 17°C. The roof and walls of the workshop are made of corrugated iron and are not insulated; when the thermometer is held against the wall, it reads only 5°C. If the average convective heat transfer coefficient for the suspended thermometer is estimated to be 10 W/m² K, what is the true air temperature?

Solution

Given: Thermometer reading a temperature of 17°C.

Required: True air temperature.

Assumptions: Thermometer can be modeled as a small gray body in large, nearly black surroundings at 5°C.

Let T_t be the thermometer reading, T_e the air temperature, and T_w the wall temperature. Equation (1.35) applies,

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 0$$

since at steady state there is no conduction within the thermometer. Substituting from Eqs. (1.24) and (1.18),

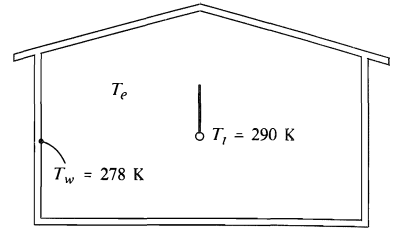
$$\bar{h}_c A (T_t - T_e) + \varepsilon \sigma A (T_t^4 - T_w^4) = 0$$

From Table 1.3, $\varepsilon = 0.8$ for pyrex glass. Canceling A ,

$$10(290 - T_e) + (0.8)(5.67)(2.90^4 - 2.78^4) = 0$$

Solving,

$$T_e = 295 \text{ K} \approx 22^\circ\text{C}$$



Comments

1. Since $T_e > 20^\circ\text{C}$, the air-heating system appears to be working satisfactorily.
2. Our model assumes that the thermometer is completely surrounded by a surface at 5°C : actually, the thermometer also receives radiation from machines, workers, and other sources at temperatures higher than 5°C , so that our calculated value of $T_e = 22^\circ\text{C}$ is somewhat high.

1.5 TRANSIENT THERMAL RESPONSE

The heat transfer problems described in Examples 1.1 through 1.5 were *steady-state* problems; that is, temperatures were not changing in time. In Example 1.2, the transistor temperature was steady with the resistance (I^2R) heating balanced by the radiation heat loss. *Unsteady-state* or *transient* problems occur when temperatures change with time. Such problems are often encountered in engineering practice, and the engineer may be required to predict the temperature-time response of a system involved in a heat transfer process. If the system, or a component of the system, can be assumed to have a spatially uniform temperature, analysis involves a relatively simple application of the energy conservation principle, as will now be demonstrated.

1.5.1 The Lumped Thermal Capacity Model

If a system undergoing a transient thermal response to a heat transfer process has a nearly uniform temperature, we may ignore small differences of temperature within the system. Changes in internal energy of the system can then be specified in terms of changes of the assumed uniform (or average) temperature of the system. This approximation is called the **lumped thermal capacity** model.⁶ The system might be

⁶ The term *capacitance* is also used, in analogy to an equivalent electrical circuit.

a small solid component of high thermal conductivity that loses heat slowly to its surroundings via a large external thermal resistance. Since the thermal resistance to conduction in the solid is small compared to the external resistance, the assumption of a uniform temperature is justified. Alternatively, the system might be a well-stirred liquid in an insulated tank losing heat to its surroundings, in which case it is the mixing of the liquid by the stirrer that ensures a nearly uniform temperature. In either case, once we have assumed uniformity of temperature, we have no further need for details of the heat transfer within the system—that is, of the conduction in the solid component or the convection in the stirred liquid. Instead, the heat transfer process of concern is the interaction of the system with the surroundings, which might be by conduction, radiation, or convection.

Governing Equation and Initial Condition

For purposes of analysis, consider a metal forging removed from a furnace at temperature T_0 and suddenly immersed in an oil bath at temperature T_e , as shown in Fig. 1.16. The forging is a closed system, so the energy conservation principle in the form of Eq. (1.2) applies. Heat is transferred out of the system by convection. Using Eq. (1.24) the rate of heat transfer is $\bar{h}_c A(T - T_e)$, where $\bar{h}_c$ is the heat transfer coefficient averaged over the forging surface area A , and T is the forging temperature. There is no heat generated within the forging, so that $\dot{Q}_v = 0$. Substituting in Eq. (1.2):

$$\begin{aligned} \rho V c \frac{dT}{dt} &= -\bar{h}_c A (T - T_e) \\ \frac{dT}{dt} &= -\frac{\bar{h}_c A}{\rho V c} (T - T_e) \end{aligned} \quad (1.36)$$

which is a first-order ordinary differential equation for the forging temperature, T , as a function of time, t . One initial condition is required:

$$t = 0: \quad T = T_0 \quad (1.37)$$

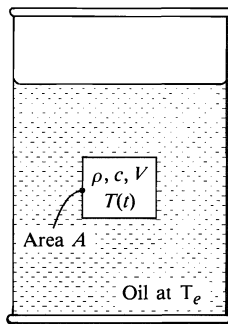


Figure 1.16 A forging immersed in an oil bath for quenching.

Solution for the Temperature Response

A simple analytical solution can be obtained provided we assume that the bath is large, so T_e is independent of time, and that $\bar{h}_c A / \rho V c$ is approximated by a constant value independent of temperature. The variables in Eq. (1.36) can then be separated:

$$\frac{dT}{T - T_e} = -\frac{\bar{h}_c A}{\rho V c} dt$$

Writing $dT = d(T - T_e)$, since T_e is constant, and integrating with $T = T_0$ at $t = 0$,

$$\int_{T_0}^T \frac{d(T - T_e)}{T - T_e} = -\frac{\bar{h}_c A}{\rho V c} \int_0^t dt$$

$$\ln \frac{T - T_e}{T_0 - T_e} = -\frac{\bar{h}_c A}{\rho V c} t$$

$$\frac{T - T_e}{T_0 - T_e} = e^{-(\bar{h}_c A / \rho V c) t} = e^{-t/t_c} \quad (1.38)$$

where $t_c = \rho V c / \bar{h}_c A$ [s] is called the **time constant** of the process. When $t = t_c$, the temperature difference $(T - T_e)$ has dropped to be 36.8% of the initial difference $(T_0 - T_e)$. Our result, Eq. (1.38), is a relation between two dimensionless parameters: a dimensionless temperature, $T^* = (T - T_e) / (T_0 - T_e)$, which varies from 1 to 0; and a dimensionless time, $t^* = t / t_c = \bar{h}_c A t / \rho V c$, which varies from 0 to ∞ . Equation (1.38) can be written simply as

$$T^* = e^{-t^*} \quad (1.39)$$

and a graph of T^* versus t^* is a single curve, as illustrated in Fig. 1.17.

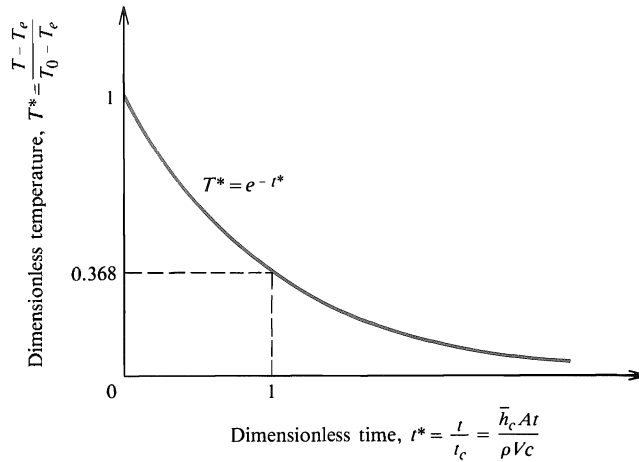


Figure 1.17 Lumped thermal capacity temperature response in terms of dimensionless variables T^* and t^* .

Methods introduced in Chapter 2 can be used to deduce directly from Eqs. (1.36) and (1.37) that T^* must be a function of t^* alone [i.e., $T^* = f(t^*)$] without solving the equation. Of course, the solution also gives us the form of the function. Thus, the various parameters, $\bar{h}_c$, c , ρ , and so on, only affect the temperature response in the combination t^* , and not independently. If both $\bar{h}_c$ and c are doubled, the temperature at time t is unchanged. This dimensionless parameter t^* is a dimensionless group in the same sense as the Reynolds number, but it does not have a commonly used name.

Validity of the Model

We would expect our assumption of negligible temperature gradients within the system to be valid when the internal resistance to heat transfer is small compared with the external resistance. If L is some appropriate characteristic length of a solid body, for example, V/A (which for a plate is half its thickness), then

$$\frac{\text{Internal conduction resistance}}{\text{External convection resistance}} \approx \frac{L/k_s A}{1/\bar{h}_c A} = \frac{\bar{h}_c L}{k_s} \quad (1.40)$$

where k_s is the thermal conductivity of the solid material. The quantity $\bar{h}_c L/k_s$ [$\text{W/m}^2 \text{ K}][\text{m}]/[\text{W/m K}]$ is a dimensionless group called the **Biot number**, Bi . More exact analyses of transient thermal response of solids indicate that, for bodies resembling a plate, cylinder, or sphere, $\text{Bi} < 0.1$ ensures that the temperature at the center will not differ from that at the surface by more than 5%; thus, $\text{Bi} < 0.1$ is a suitable criterion for determining if the assumption that the body has a uniform temperature is justified. If the heat transfer is by radiation, the convective heat transfer coefficient in Eq. (1.40) can be replaced by the approximate radiation heat transfer coefficient h_r defined in Eq. (1.19).

In the case of the well-stirred liquid in an insulated tank, it will be necessary to evaluate the ratio

$$\frac{\text{Internal convection resistance}}{\text{External resistance}} \approx \frac{1/h_{c,i} A}{1/UA} = \frac{U}{h_{c,i}} \quad (1.41)$$

where U is the overall heat transfer coefficient, for heat transfer from the inner surface of the tank, across the tank wall and insulation, and into the surroundings. If this ratio is small relative to unity, the assumption of a uniform temperature in the liquid is justified.

The approximation or model used in the preceding analysis is called a lumped thermal capacity approximation since the thermal capacity is associated with a single temperature. There is an electrical analogy to the lumped thermal capacity model, owing to the mathematical equivalence of Eq. (1.36) to the equation governing the voltage in the simple resistance-capacitance electrical circuit shown

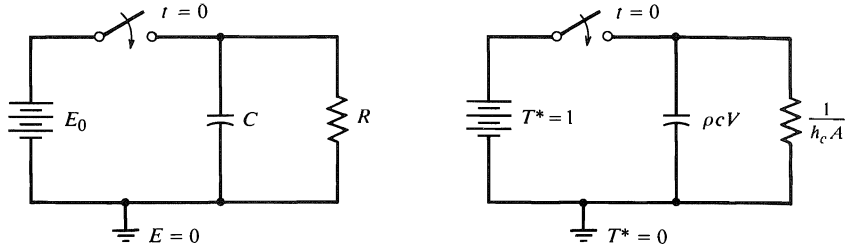


Figure 1.18 Equivalent electrical and thermal circuits for the lumped thermal capacity model of temperature response.

in Fig. 1.18,

$$\frac{dE}{dt} = -\frac{E}{RC} \quad (1.42)$$

with the initial condition $E = E_0$ at $t = 0$ if the capacitor is initially charged to a voltage E_0 . The solution is identical in form to Eq. (1.38),

$$\frac{E}{E_0} = e^{-t/RC}$$

and the time constant is RC , the product of the resistance and capacitance [or $C/(1/R)$, the ratio of capacitance to conductance, to be exactly analogous to Eq. (1.38)].

EXAMPLE 1.6 Quenching of a Steel Plate

A steel plate 1 cm thick is taken from a furnace at 600°C and quenched in a bath of oil at 30°C. If the heat transfer coefficient is estimated to be 400 W/m² K, how long will it take for the plate to cool to 100°C? Take k , ρ , and c for the steel as 50 W/m K, 7800 kg/m³, and 450 J/kg K, respectively.

Solution

Given: Steel plate quenched in an oil bath.

Required: Time to cool from 600°C to 100°C.

Assumptions: Lumped thermal capacity model valid.

First the Biot number will be checked to see if the lumped thermal capacity approximation is valid. For a plate of width W , height H , and thickness L ,

$$\frac{V}{A} \approx \frac{WHL}{2WH} = \frac{L}{2}$$

where the surface area of the edges has been neglected.

$$\begin{aligned}
 \text{Bi} &= \frac{\bar{h}_c(L/2)}{k_s} \\
 &= \frac{(400 \text{ W/m}^2 \text{ K})(0.005 \text{ m})}{50 \text{ W/m K}} \\
 &= 0.04 < 0.1
 \end{aligned}$$

so the lumped thermal capacity model is applicable. The time constant t_c is

$$t_c = \frac{\rho V c}{\bar{h}_c A} = \frac{\rho(L/2)c}{\bar{h}_c} = \frac{(7800 \text{ kg/m}^3)(0.005 \text{ m})(450 \text{ J/kg K})}{(400 \text{ W/m}^2 \text{ K})} = 43.9 \text{ s}$$

Substituting $T_e = 30^\circ\text{C}$, $T_0 = 600^\circ\text{C}$, $T = 100^\circ\text{C}$ in Eq. (1.38) gives

$$\frac{100 - 30}{600 - 30} = e^{-t/43.9}$$

Solving,

$$t = 92 \text{ s}$$

Comments

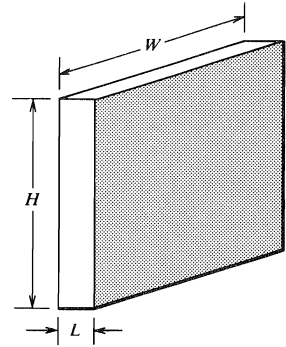
The use of a constant value of h_c may be inappropriate for heat transfer by natural convection or radiation (see Section 1.5.2).

1.5.2 Combined Convection and Radiation

The analysis of Section 1.5.1 assumes that the heat transfer coefficient was constant during the cooling period. This assumption is adequate for forced convection but is less appropriate for natural convection, and when thermal radiation is significant. Equation (1.23) shows that the natural convection heat transfer coefficient $\bar{h}_c$ is proportional to $\Delta T^{1/4}$ for laminar flow and to $\Delta T^{1/3}$ for turbulent flow. The temperature difference $\Delta T = T - T_e$ decreases as the body cools, as does $\bar{h}_c$. Radiation heat transfer is proportional to $(T^4 - T_e^4)$ and hence cannot be represented exactly by Newton's law of cooling. We now extend our lumped thermal capacity analysis to allow both for a variable convective heat transfer coefficient and for situations where both convection and radiation are important.

Governing Equation and Initial Condition

Figure 1.19 shows a body that loses heat by both convection and radiation. For a small gray body in large, nearly black surroundings also at temperature T_e , the radiation heat transfer is obtained from Eq. (1.18) as $\dot{Q} = \varepsilon A \sigma (T^4 - T_e^4)$. As in



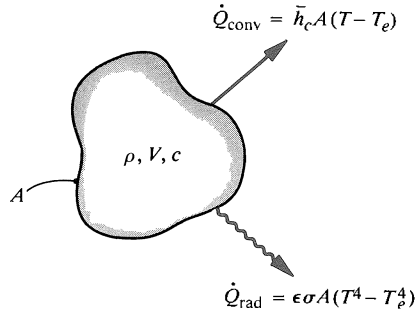


Figure 1.19 Schematic of a body losing heat by convection and radiation for a lumped thermal capacity model.

Section 1.5.1, the energy conservation principle Eq. (1.2) becomes

$$\rho V c \frac{dT}{dt} = -\bar{h}_c A (T - T_e) - \varepsilon A \sigma (T^4 - T_e^4)$$

or

$$\frac{dT}{dt} = -\frac{\bar{h}_c A}{\rho V c} (T - T_e) - \frac{\varepsilon A \sigma}{\rho V c} (T^4 - T_e^4) \quad (1.43)$$

The initial condition is again

$$t = 0: \quad T = T_0 \quad (1.44)$$

This first-order ordinary differential equation has no closed-form analytical solution even when the convective heat transfer coefficient $\bar{h}_c$ is constant as in forced convection. However, Eq. (1.43) can be solved easily using a numerical integration procedure. For this purpose, it can be rearranged as

$$\frac{dT}{dt} + \frac{hA}{\rho V c} (T - T_e) = 0 \quad (1.45)$$

$$h = \bar{h}_c + h_r = B(T - T_e)^n + \sigma \varepsilon (T^2 + T_e^2)(T + T_e) \quad (1.46)$$

where $(T^4 - T_e^4)$ has been factored, as was done in deriving Eq. (1.19). For forced convection, $n = 0$, $B = \bar{h}_c$; for laminar natural convection $n = 1/4$ and B is a constant [for example, for a plate of height L , Eq. (1.23a) gives $B = (4/3)(1.07)/L^{1/4}$]. Equation (1.46) defines a total heat transfer coefficient that accounts for both convection and radiation and changes continuously as the body cools. To put Eq. (1.45) in dimensionless form, we use the dimensionless variables introduced in Section 1.5.1:

$$T^* = \frac{T - T_e}{T_0 - T_e}, \quad t^* = \frac{t}{t_c} \quad (1.47a,b)$$

The definition of the time constant t_c poses a problem since h is not a constant as before. We choose to define t_c in terms of the value of h at time $t = 0$, when the body temperature is T_0 .

$$t_c = \frac{\rho V c}{h_0 A} = \frac{\rho V c}{[B(T_0 - T_e)^n + \sigma \varepsilon (T_0^2 + T_e^2)(T_0 + T_e)]A} \quad (1.48)$$

Equation (1.45) then becomes

$$\frac{dT^*}{dt^*} + \frac{h}{h_0} T^* = 0 \quad (1.49)$$

with the initial condition

$$t^* = 0: T^* = 1 \quad (1.50)$$

Computer Program LUMP

Numerical integration is appropriate for this problem. The computer program LUMP has been prepared accordingly. LUMP solves Eq. (1.49), that is, it obtains the temperature response of a body that loses heat by convection and/or radiation, based on the lumped thermal capacity model. The required input constant B is defined in Eq. (1.46). Any consistent system of units can be used. The output can be obtained either as a graph or as numerical data.

EXAMPLE 1.7 Quenching of an Alloy Sphere

A materials processing experiment under microgravity conditions on the space shuttle requires quenching in a forced flow of an inert gas. A 1 cm-diameter metal alloy sphere is removed from a furnace at 800°C and is to be cooled to 500°C by a flow of nitrogen gas at 25°C. Determine the effect of the convective heat transfer coefficient on cooling time for $10 < \bar{h}_c < 100 \text{ W/m}^2 \text{ K}$. Properties of the alloy include: $\rho = 14,000 \text{ kg/m}^3$; $c = 140 \text{ J/kg K}$; $\varepsilon = 0.1$. The surrounds can be taken as nearly black at 25°C.

Solution

Given: A metal alloy sphere to be quenched.

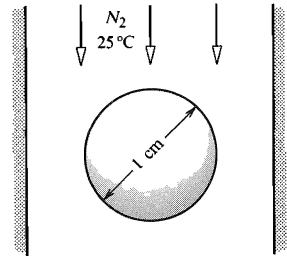
Required: Effect of convective heat transfer coefficient on cooling time.

Assumptions: 1. Lumped thermal capacity model valid.
2. Constant convective heat transfer coefficient.

The computer code LUMP can be used to solve this problem.

The required inputs are:

T_0 and $T_e = 1073, 298$
 $B = \bar{h}_c = 10$ (repeat for 20, 30, 50, 100)
 $n = 0$
 $\sigma = 5.67 \times 10^{-8}$
 $\varepsilon = 0.1$
 Final value of t^* : try $t^* = 1$



The required dimensionless temperature is

$$T^* = \frac{T - T_e}{T_0 - T_e} = \frac{773 - 298}{1073 - 298} = 0.613$$

and the code is used to obtain the corresponding dimensionless time t^* . For a sphere $V/A = (\pi D^3/6)/(\pi D^2) = D/6$, so that the time constant is

$$\begin{aligned} t_c &= \frac{\rho V c}{h_0 A} = \frac{\rho(D/6)c}{h_0} = \frac{(14,000)(0.01/6)(140)}{\bar{h}_c + (5.67 \times 10^{-8})(0.1)(1073^2 + 298^2)(1073 + 298)} \\ &= \frac{3267}{\bar{h}_c + 9.64} \text{ s} \end{aligned}$$

The actual time is $t = t^* t_c$. Results obtained using LUMP are tabulated below.

$\bar{h}_c$ W/m ² K	t^*	t_c s	t s
10	0.59	166	98
20	0.55	110	61
30	0.53	82	43
50	0.52	55	29
100	0.51	30	15

Comments

1. Only two significant figures have been given since high accuracy is not warranted for the problem.
2. The heat transfer coefficient does not have a strong effect on t^* . Why?
3. For the lumped thermal capacity model to be valid, the Biot number should be less than 0.1. The worst case is with $\bar{h}_c = 100 \text{ W/m}^2 \text{ K}$ at time $t = 0$, giving $h_0 = 109.6$ and $0.1 > (109.6)(0.01/6)/k_c$, that is, $k_c > 1.8 \text{ W/m K}$, which certainly will be true for a metal alloy.

1.6 HEAT EXCHANGERS

In Section 1.5, we considered problems in which the temperature of a system changed with time as a result of heat transfer between the system and its surroundings. We now consider problems in which the temperature of a fluid changes as it flows through a passage as a result of heat transfer between the passage walls and the fluid. These problems are encountered in the analysis of heat exchanger performance. A *heat exchanger* is a device that facilitates transfer of heat from one fluid stream to another. Power production, refrigeration, heating and air conditioning, food processing, chemical processing, oil refining, and the operation of almost all vehicles depends on heat exchangers of various types. The analysis and design of heat exchangers is the subject of Chapter 8. The analysis of a very simple heat exchanger configuration is presented here to introduce some of the basic concepts underlying heat exchanger analysis and associated terminology. These concepts will prove useful in the development and application of heat transfer theory in chapters preceding Chapter 8—particularly in Chapters 4 and 5, which deal with convection.

1.6.1 Single- and Two-Stream Exchangers

One important classification of heat exchangers is into **single-stream exchangers** and **two-stream exchangers**. A single-stream exchanger is one in which the temperature of only one stream changes in the exchanger; examples include many types of evaporators and condensers found in power plants and refrigeration systems. A power plant condenser is shown in Fig. 1.20. A two-stream exchanger is one in which the temperatures of both streams change in the exchanger; examples include radiators and intercoolers for automobile engines, and oil coolers for aircraft engines. Figure 1.21 shows an oil cooler, which has a **counterflow** configuration; that is, the streams flow in opposite directions in the exchanger.

In the analysis of heat exchangers, a useful first step is to draw a sketch of the expected fluid temperature variations along the exchanger. Figure 1.22a is such a sketch for the power plant condenser. The hot stream is steam returning from the turbines, which condenses at a constant temperature T_H . This is the saturation temperature corresponding to the pressure maintained in the condenser shell. The cold stream is cold water from a river, ocean, or cooling tower, and its temperature

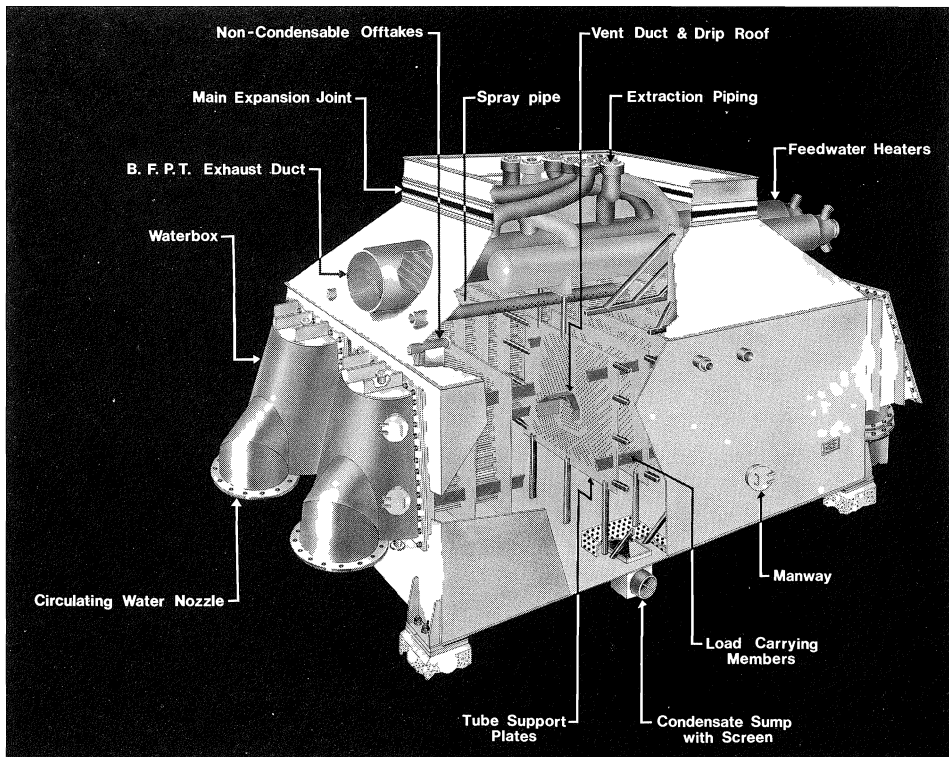


Figure 1.20 A power plant condenser. (Courtesy Senior Engineering Co. [formerly Southwestern Engineering], Los Angeles, California.)

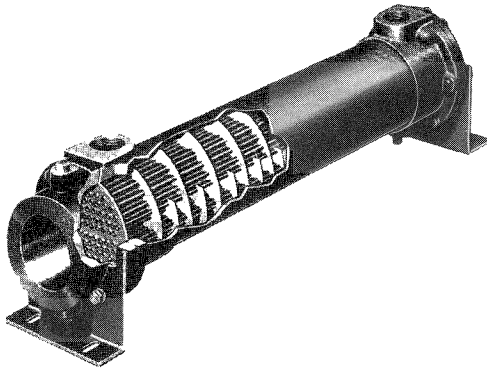


Figure 1.21 A small single-pass shell-and-tube two-stream heat exchanger, typically used for cooling oil or water. (Photograph courtesy of the Young Radiator Company, Racine, Wis.)

T_C increases as it flows through the exchanger. Figure 1.22*b* shows the sketch for the oil cooler. The hot stream is oil from the engine, and the cold stream is coolant water. Notice that in this counterflow configuration, the cold stream can leave the exchanger at a higher temperature than the hot stream!

A point that might confuse the beginning student is that there are actually two streams in many single-stream exchangers. The definition simply requires that the temperature of only one stream changes in a single-stream exchanger. It is this feature that makes the analysis of single-stream exchangers particularly simple, as will now be demonstrated. In Section 1.5, the system analysis was based on the

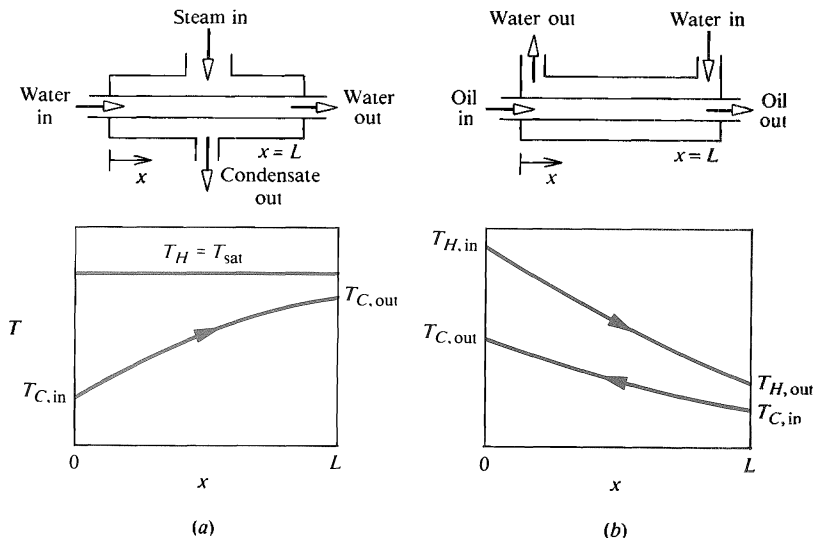


Figure 1.22 Temperature variations along heat exchangers. (a) A power plant condenser. (b) A counterflow oil cooler.

energy conservation principle in the form of the first law of thermodynamics applied to a *closed system*. In contrast, the system analysis that follows is based on the first law applied to an *open system*.

1.6.2 Analysis of a Condenser

Figure 1.23a shows a simple single-tube condenser. Pure saturated vapor enters the shell at the top and condenses on a single horizontal tube. The condensate forms a thin film on the outside of the tube, drops off the bottom, and leaves the shell through a drain. The vapor condenses at the saturation temperature corresponding to the pressure in the shell. Hence, the condensate film surface temperature is $T_{\text{sat}}(P)$. Figure 1.23b shows the temperature variation across the tube wall and the corresponding thermal circuit. The enthalpy of condensation is transferred by conduction across the thin condensate film, by conduction across the tube wall, and by convection into the coolant. As a result, the coolant temperature rises as it gains energy flowing along the tube. The vapor flow rate is denoted $\dot{m}_H$ [kg/s] and the coolant flow rate $\dot{m}_C$ (the *hot* and *cold* streams, respectively).

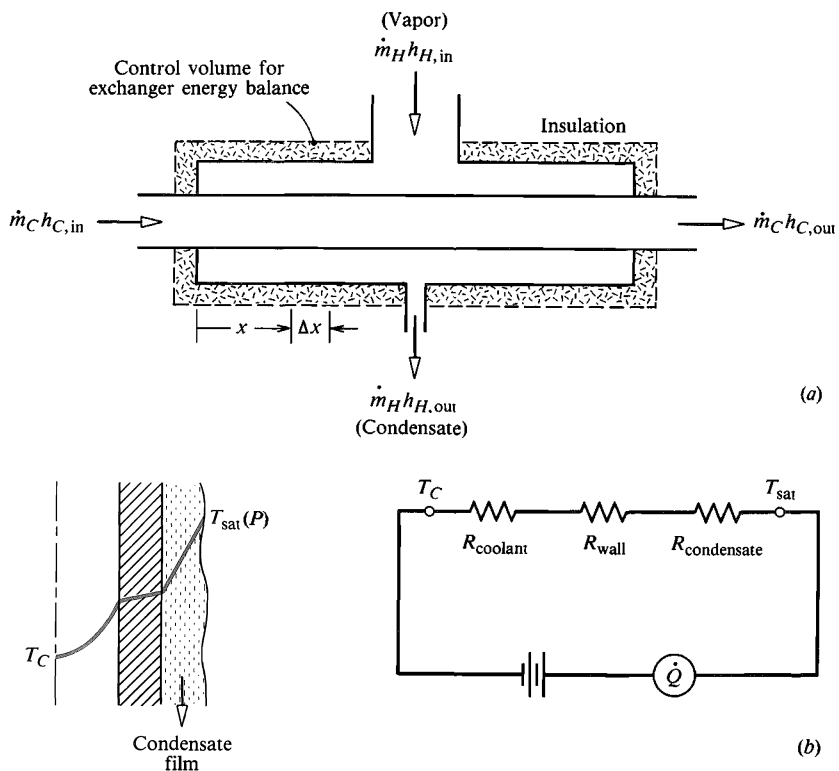


Figure 1.23 (a) Schematic of a single-tube condenser. (b) The temperature variation across the tube wall and the thermal circuit for heat transfer across the tube wall.

The Exchanger Energy Balance

An energy balance on the exchanger as a whole is formulated by writing down the steady-flow energy equation for a control volume enclosing the exchanger (the dashed line in Fig. 1.23a). If the exchanger is well insulated, there is no heat loss to the surroundings, and Eq. (1.4) requires that the enthalpy inflow equal the enthalpy outflow:

$$\dot{m}_H h_{H,\text{in}} + \dot{m}_C h_{C,\text{in}} = \dot{m}_H h_{H,\text{out}} + \dot{m}_C h_{C,\text{out}}$$

where h is specific enthalpy [J/kg] and subscripts “in” and “out” denote inlet and outlet values, respectively. Rearranging gives

$$\dot{m}_C (h_{C,\text{out}} - h_{C,\text{in}}) = \dot{m}_H (h_{H,\text{in}} - h_{H,\text{out}}) \quad (1.51)$$

If we assume a constant specific heat for the coolant and that the condensate leaves at the saturation temperature, Eq. (1.51) becomes

$$\dot{m}_C c_{pC} (T_{C,\text{out}} - T_{C,\text{in}}) = \dot{m}_H h_{\text{fg}} \quad (1.52)$$

where h_{fg} is the enthalpy of vaporization for the vapor. When the coolant flow rate $\dot{m}_C$ and inlet temperature $T_{C,\text{in}}$ are known, Eq. (1.52) relates the coolant outlet temperature $T_{C,\text{out}}$ to the amount of vapor condensed $\dot{m}_H$.

Governing Equation and Boundary Condition

To determine the variation of coolant temperature along the exchanger, we make an energy balance on a differential element of the exchanger Δx long and so derive a differential equation with x as the independent variable and T_C as the dependent variable. When the steady-flow energy equation, Eq. (1.4), is applied to the control volume of length Δx , shown in Fig. 1.24 as a dotted line, the contribution to $\dot{Q}$ due to x -direction conduction in the coolant is small and can be neglected. Thus, the

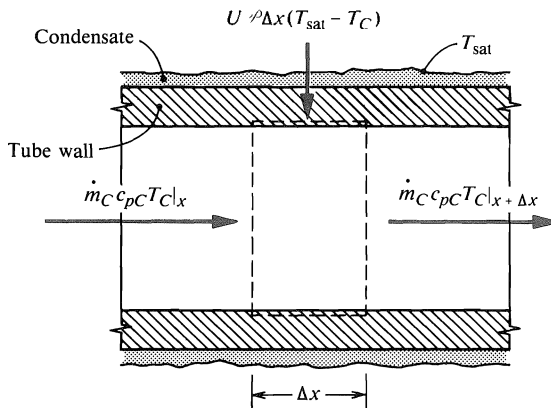


Figure 1.24 An elemental control volume Δx long for application of the steady-flow energy equation to a condenser coolant stream.

coolant flow rate times its enthalpy increase must equal the heat transfer across the tube wall:

$$\dot{m}_C c_{pC} (T_C|_{x+\Delta x} - T_C|_x) = U \mathcal{P} \Delta x (T_{\text{sat}} - T_C)$$

where U [W/m² K] is the overall heat transfer coefficient for heat transfer from the vapor to the coolant, and $\mathcal{P}$ [m] is the perimeter of the tube wall. Thus, $\mathcal{P} \Delta x$ is the surface area of the tube element with length Δx . For a circular tube $\mathcal{P} = \pi D$ where D is the pipe diameter. Dividing by Δx ,

$$\dot{m}_C c_{pC} \left(\frac{T_C|_{x+\Delta x} - T_C|_x}{\Delta x} \right) = U \mathcal{P} (T_{\text{sat}} - T_C)$$

and letting $\Delta x \rightarrow 0$, gives

$$\dot{m}_C c_{pC} \frac{dT_C}{dx} = U \mathcal{P} (T_{\text{sat}} - T_C)$$

Rearranging,

$$\frac{dT_C}{dx} - \frac{U \mathcal{P}}{\dot{m}_C c_{pC}} (T_{\text{sat}} - T_C) = 0 \quad (1.53)$$

Equation (1.53) is a first-order ordinary differential equation for $T_C(x)$; it requires one boundary condition, which is

$$x = 0: \quad T_C = T_{C,\text{in}} \quad (1.54)$$

Temperature Variation

To integrate Eq. (1.53), let $\theta = T_{\text{sat}} - T_C$; then $dT_C/dx = -d\theta/dx$, and the equation becomes

$$\frac{d\theta}{dx} + \frac{U \mathcal{P}}{\dot{m}_C c_{pC}} \theta = 0$$

If U is assumed constant along the exchanger, the solution is

$$\theta = A e^{-(U \mathcal{P} / \dot{m}_C c_{pC}) x}$$

where A is the integration constant. Substituting for θ and using the boundary condition, Eq. (1.54) gives the integration constant:

$$T_{\text{sat}} - T_{C,\text{in}} = A e^0 = A$$

Thus, the solution of Eq. (1.53) is

$$T_{\text{sat}} - T_C = (T_{\text{sat}} - T_{C,\text{in}}) e^{-(U \mathcal{P} / \dot{m}_C c_{pC}) x} \quad (1.55)$$

which is the desired relation $T_C(x)$, showing an exponential variation along the exchanger. Of particular interest is the coolant outlet temperature $T_{C,\text{out}}$ which is obtained by letting $x = L$, the length of the exchanger, in Eq. (1.55):

$$T_{\text{sat}} - T_{C,\text{out}} = (T_{\text{sat}} - T_{C,\text{in}}) e^{-(U \mathcal{P} L / \dot{m}_C c_{pC})} \quad (1.56)$$

Exchanger Performance Parameters

The product of perimeter and length $\mathcal{P}L$ is the area of the heat transfer surface. The exponent in Eq. (1.56) is, of course, dimensionless,

$$\left[\frac{U \mathcal{P}L}{\dot{m}_C c_{pC}} \right] = \frac{[\text{W/m}^2 \text{ K}][\text{m}][\text{m}]}{[\text{kg/s}][\text{J/kg K}]} = \left[\frac{\text{W s}}{\text{J}} \right] = 1$$

since a watt is a joule per second. This dimensionless group is called the **number of transfer units**, with abbreviation NTU and symbol N_{tu} .⁷ For a given $\dot{m}_C c_{pC}$, the larger U , $\mathcal{P}$, or L , the greater the NTU of the exchanger. Thus, the NTU can be viewed as a measure of the heat transfer “size” of the exchanger. Equation (1.56) can then be rearranged as

$$\frac{T_{\text{sat}} - T_{C,\text{out}}}{T_{\text{sat}} - T_{C,\text{in}}} = e^{-N_{tu}} \quad (1.57)$$

Thus, if T_{sat} , $T_{C,\text{in}}$, and the NTU of the exchanger are known, $T_{C,\text{out}}$ can be calculated. But we find it convenient to rearrange Eq. (1.57) by subtracting each side from unity to obtain

$$1 - \frac{T_{\text{sat}} - T_{C,\text{out}}}{T_{\text{sat}} - T_{C,\text{in}}} = 1 - e^{-N_{tu}}$$

or

$$\frac{T_{C,\text{out}} - T_{C,\text{in}}}{T_{\text{sat}} - T_{C,\text{in}}} = 1 - e^{-N_{tu}} \quad (1.58)$$

Now, even if the exchanger were infinitely long, the maximum outlet temperature of the coolant would be T_{sat} (see Fig. 1.22a). Thus, the left-hand side of Eq. (1.58) is the ratio of the actual temperature rise of the coolant ($T_{C,\text{out}} - T_{C,\text{in}}$) divided by the maximum possible rise for an infinitely long exchanger ($T_{\text{sat}} - T_{C,\text{in}}$) and can be viewed as the **effectiveness** of the exchanger, for which we use the symbol ε . Our result is therefore

$$\varepsilon = 1 - e^{-N_{tu}} \quad (1.59)$$

Equation (1.59) indicates that the larger the number of transfer units of the exchanger, the higher its effectiveness. Although a high effectiveness is desirable, as the length of an exchanger increases, so does the cost of materials for its construction and the pumping power required by the coolant flow. Thus, the goal of the design engineer is to maximize the effectiveness subject to the constraints of construction (capital) costs and power (operating) costs. In practice, values of ε between 0.6 and 0.9 are typical.

⁷ NTU is also widely used as the symbol for number of transfer units.

EXAMPLE 1.8 Performance of a Steam Condenser

A steam condenser is 4 m long and contains 2000, 5/8 inch nominal-size, 18 gage brass tubes (1.59 cm O.D., 1.25 mm wall thickness). In a test 120 kg/s of coolant water at 300 K is supplied to the condenser, and when the steam pressure in the shell is 10,540 Pa, condensate is produced at a rate of 3.02 kg/s. Determine the effectiveness of the exchanger and the overall heat transfer coefficient. Take the specific heat of the water to be 4174 J/kg K.

Solution

Given: A shell-and-tube steam condenser.

Required: The effectiveness, ε , and overall heat transfer coefficient, U .

Assumptions: U is constant along the exchanger so that Eq. (1.59) applies.

The hot-stream temperature T_H is the saturation temperature corresponding to the given steam pressure of 10,540 Pa; from steam tables (see Table A.12a in Appendix A of this text) $T_{\text{sat}} = 320.0$ K. We first find the coolant water outlet temperature from the exchanger energy balance Eq. (1.52):

$$\dot{m}_C c_{pC} (T_{C,\text{out}} - T_{C,\text{in}}) = \dot{m}_H h_g$$

From steam tables, the enthalpy of vaporization at $T_{\text{sat}} = 320$ K is $h_g = 2.389 \times 10^6$ J/kg.

$$(120 \text{ kg/s})(4174 \text{ J/kg K})(T_{C,\text{out}} - 300 \text{ K}) = (3.02 \text{ kg/s})(2.389 \times 10^6 \text{ J/kg})$$

Solving gives $T_{C,\text{out}} = 314.4$ K.

The effectiveness, ε , is then obtained from Eq. (1.58) as

$$\varepsilon = \frac{T_{C,\text{out}} - T_{C,\text{in}}}{T_{\text{sat}} - T_{C,\text{in}}} = \frac{314.4 - 300}{320 - 300} = 0.720$$

and the number of transfer units, from Eq. (1.59), is

$$N_{\text{tu}} = \ln \frac{1}{1 - \varepsilon} = \ln \frac{1}{1 - 0.720} = 1.27 = \frac{U \mathcal{P} L}{\dot{m}_C c_{pC}}$$

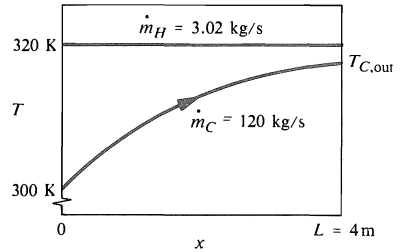
Solving for the $U \mathcal{P} L$ product,

$$U \mathcal{P} L = 1.27 \dot{m}_C c_{pC} = (1.27)(120 \text{ kg/s})(4174 \text{ J/kg K}) = 6.36 \times 10^5 \text{ W/K}$$

If we choose to base the overall heat transfer coefficient on the outside of the tubes, then, for N tubes, the heat transfer area $\mathcal{P} L$ is

$$\mathcal{P} L = N \pi D L = (2000)(\pi)(1.59 \times 10^{-2} \text{ m})(4 \text{ m}) = 400 \text{ m}^2$$

Hence, $U = U \mathcal{P} L / \mathcal{P} L = 6.36 \times 10^5 / 400 = 1590 \text{ W/m}^2 \text{ K}$



Comments

We could have performed these calculations by considering a single tube of the tube bundle, for which the coolant flow is $(120/2000)$ kg/s and the heat transfer area is simply πDL . But common practice is always to consider the exchanger as a whole, as we have done here.

1.6.3 Other Single-Stream Exchangers

Simple evaporators and boilers are also single-stream exchangers, where the cold stream is an evaporating or boiling liquid and the hot stream supplies the enthalpy of vaporization. Such exchangers will be analyzed in Chapter 8. Heat transfer to a fluid stream may also be a concern in problems that do not involve heat exchangers. The exhaust gas stack cooled by a crosswind, shown in Fig. 1.25, can also be viewed as a single-stream heat exchanger, since only the exhaust gas temperature changes with location up the stack. Thus, the analysis of Section 1.6.2, properly interpreted, applies (see Exercise 1–52). Single-stream heat exchanger theory also will be used in Chapters 4 and 5 in the examination of convective heat transfer in internal flows.

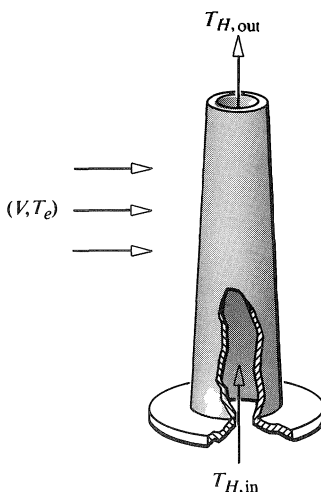


Figure 1.25 An exhaust gas stack cooled by a crosswind.

1.7 DIMENSIONS AND UNITS

Dimensions are physical properties that are measurable—for example, length, time, mass, and temperature. A system of *units* is used to give numerical values to dimensions. The system most widely used throughout the world in science and industry is the International System of units (SI), from the French name *Système International*

d'Unités. This system was recommended at the General Conference on Weights and Measures of the International Academy of Sciences in 1960 and was adopted by the U.S. National Bureau of Standards in 1964. In the United States, the transition from the older English system of units to the SI system has been slow and is not complete. The SI system is used in science education, by engineering professional societies, and by many industries. However, engineers in some more mature industries still prefer to use English units, and, of course, commerce and trade in the United States remains dominated by the English system. We buy pounds of vegetables, quarts of milk, drive miles to work, and say that it is a hot day when the temperature exceeds 80°F. (Wine is now sold in 750 ml bottles, though, which is a modest step forward!)

In this text, we will use the SI system, with which the student has become familiar from physics courses. For convenience, this system is summarized in the tables of Appendix B. Base and supplementary units, such as length, time, and plane angle, are given in Table B.1*a*; and derived units, such as force and energy, are given in Table B.1*b*. Recognized non-SI units (e.g., hour, bar) that are acceptable for use with the SI system are listed in Table B.1*c*. Multiples of SI units (e.g., kilo, micro) are defined in Table B.1*d*. Accordingly, the property data given in the tables of Appendix A are in SI units. The student should review this material and is urged to be careful when writing down units. For example, notice that the unit of temperature is a kelvin (not Kelvin) and has the symbol K (not °K). Likewise, the unit of power is the watt (not Watt). The symbol for a kilogram is kg (not KG). An issue that often confuses the student is the correct use of Celsius temperature. Celsius temperature is defined as $(T - 273.15)$ where T is in kelvins. However, the unit “degree Celsius” is equal to the unit “kelvin” ($1^{\circ}\text{C} = 1\text{ K}$).

Notwithstanding the wide acceptance of the SI system of units, there remains a need to communicate with those engineers (or lawyers!) who are still using English units. Also, component dimensions, or data for physical properties, may be available only in English or cgs units. For example, most pipes and tubes used in the United States conform to standard sizes originally specified in English units. A 1 inch nominal-size tube has an outside diameter of 1 in. For convenience, selected dimensions of U.S. commercial standard pipes and tubes are given in SI units in Appendix A as Tables A.14*a* and A.14*b*, respectively. The engineer must be able to convert dimensions from one system of units to another. Table B.2 in Appendix B gives the conversion factors required for most heat transfer applications. The program UNITS is based on Table B.2 and contains all the conversion factors in the table. With the input of a quantity in one system of units, the output is the same quantity in the alternative units listed in Table B.2. It is recommended that the student or engineer perform all problem solving using the SI system so as to efficiently use the Appendix A property data and the computer software. If a problem is stated in English units, the data should be converted to SI units using UNITS; if a customer requires results in units other than SI, UNITS will give the required values.

1.8 CLOSURE

Chapter 1 had two main objectives:

1. To introduce the three important modes of heat transfer, namely, conduction, radiation, and convection.
2. To demonstrate how the first law of thermodynamics is applied to engineering systems to obtain the consequences of a heat transfer process.

For each mode of heat transfer, some working equations were developed, which, though simple, allow heat transfer calculations to be made for a wide variety of problems. Equations (1.9), (1.18), and (1.20) are probably the most frequently used equations for thermal design. An electric circuit analogy was shown to be a useful aid for problem solving when more than one mode of heat transfer is involved. In applying the first law to engineering systems, both a closed system and an open system were considered. In the first case, the variation of temperature with time was determined for a solid of high conductivity or a well-stirred fluid. In the second case, the variation of fluid temperature with position along a heat exchanger tube was determined.

The student should be familiar with some of the Chapter 1 concepts from previous physics, thermodynamics, and fluid mechanics courses. A review of texts for such courses is appropriate at this time. Many new concepts were introduced, however, which will take a little time and effort to master. Fortunately, the mathematics in this chapter is simple, involving only algebra, calculus, and the simplest first-order differential equation, and should present no difficulties to the student. After successfully completing a selection of the following exercises, the student will be well equipped to tackle subsequent chapters.

A feature of this text is an emphasis on real engineering problems as examples and exercises. Thus, Chapter 1 has somewhat greater scope and detail than the introductory chapters found in most similar texts. With the additional material, more realistic problems can be treated in subsequent chapters. Conduction problems in Chapters 2 and 3 have more realistic convection and radiation boundary conditions. Convective heat transfer coefficients for flow over tube bundles in Chapter 4 are calculated in the appropriate context of a heat exchanger. Similarly, condensation heat transfer coefficients in Chapter 7 can be discussed in the context of condenser performance. Throughout the text are exercises that require application of the first law to engineering systems, for it is always the consequences of a heat transfer process that motivate the engineer's concern with the subject.

Two computer programs accompany Chapter 1. The program LUMP calculates temperature response using the lumped thermal capacity model of Section 1.5. When heat loss is by convection and radiation simultaneously, the problem does not have an analytical solution. However, a numerical solution is easily obtained; LUMP demonstrates the value of writing a computer program in such situations. It is most important that the

engineer be aware of the potential of the PC as an engineering tool and take the initiative to use the PC when appropriate. The program UNITS is a simple units conversion tool that allows unit conversions to be made quickly and reliably.

EXERCISES

1-1. Solve the following ordinary differential equations:

(i) $\frac{dy}{dx} + \beta y = 0$

(ii) $\frac{dy}{dx} + \beta y + \alpha = 0$

(iii) $\frac{d^2y}{dx^2} - \lambda^2 y = 0$

(iv) $\frac{d^2y}{dx^2} + \lambda^2 y = 0$

(v) $\frac{d^2y}{dx^2} - \lambda^2 y + \alpha = 0$

where α , β , and λ are constants.

1-2. A low-pressure heat exchanger transfers heat between two helium streams, each with a flow rate of $\dot{m} = 5 \times 10^{-3}$ kg/s. In a performance test the cold stream enters at a pressure of 1000 Pa and a temperature of 50 K, and exits at 730 Pa and 350 K.

- (i) If the flow cross-sectional area for the cold stream is 0.019 m², calculate the inlet and outlet velocities.
- (ii) If the exchanger can be assumed to be perfectly insulated, determine the heat transfer in the exchanger. For helium, $c_p = 5200$ J/kg K.

1-3. A shell-and-tube condenser for an ocean thermal energy conversion and fresh water plant is tested with a water feed rate to the tubes of 4000 kg/s. The water inlet and outlet conditions are measured to be $P_1 = 129$ kPa, $T_1 = 280$ K; and $P_2 = 108$ kPa, $T_2 = 285$ K.

- (i) Calculate the heat transferred to the water.
- (ii) If saturated steam condenses in the shell at 1482 Pa, calculate the steam condensation rate.

For the feed water, take $\rho = 1000$ kg/m³, $c_v = 4192$ J/kg K. (Steam tables are given as Table A.12a in Appendix A.)

1-4. A pyrex glass vessel has a 5 mm-thick wall and is protected with a 1 cm-thick layer of neoprene rubber. If the inner and outer surface temperatures are 40°C

and 20°C, respectively, and the total surface area of the vessel is 400 cm², calculate the rate of heat loss from the vessel. Also calculate the temperature of the interface between the glass and the rubber, and carefully sketch the temperature profile through the composite wall.

- 1-5.** In the United States, insulations are often specified in terms of their thermal resistance in $[\text{Btu/hr ft}^2 \text{ } ^\circ\text{F}]^{-1}$, called the “R” value.
- (i) What is the R value of a 10 cm-thick layer of fiberglass insulation?
 - (ii) How thick a layer of cork is required to give an R value of 18?
 - (iii) What is the R value of a 2 cm-thick board of white pine?
- 1-6.** A picnic icebox is 40 cm long, is 20 cm high and deep, and is insulated with 2 cm-thick polystyrene foam insulation. If the ambient air temperature is 30°C, estimate how much ice will melt in 8 hours. Use an enthalpy of melting for water of 335 kJ/kg.
- 1-7.** A composite wall has a 6 cm layer of fiberglass insulation sandwiched between 2 cm-thick white pine boards. If the inner and outer surface temperatures are 20°C and 0°C, respectively, calculate the heat flow per unit area across the wall. Also calculate the wood-fiberglass interface temperatures, and accurately draw the temperature profile through the wall.
- 1-8.** A freezer is 1 m wide and deep and 2 m high, and must operate at -10°C when the ambient air is at 30°C. What thickness of polystyrene is required if the load on the refrigeration unit should not exceed 200 W? Assume that the outer surface of the insulation is approximately at the ambient air temperature and that the base of the freezer is perfectly insulated.
- 1-9.** A very effective insulation can be made from multiple layers of thin aluminized plastic film separated by rayon mesh and evacuated to a very low pressure ($\sim 10^{-5}$ torr). Such “superinsulation” can be used for insulating storage tanks holding cryogenic liquids. On a space station, a 1 m-O.D. spherical tank contains saturated nitrogen at 1 atm pressure. What thickness of a superinsulation having an effective thermal conductivity of 9×10^{-6} W/m K is required to have a boil-off rate of less than 2 mg/s when the ambient temperature is 250 K? The boiling point of nitrogen is 77.4 K, and its enthalpy of vaporization is 0.200×10^6 J/kg.
- 1-10.** A blackbody radiates to a surrounding black enclosure. If the body is maintained at 100 K above the enclosure temperature, calculate the net radiative heat flux leaving the body when the enclosure is at 80 K, 300 K, 1000 K, and 5000 K.
- 1-11.** An astronaut is at work in the service bay of a space shuttle and is surrounded by walls that are at -100°C . The outer surface of her space suit has an area of 3 m² and is aluminized with an emittance of 0.05. Calculate her rate of heat loss when the suit’s outer temperature is 0°C. Express your answer in watts and kcal/hr.

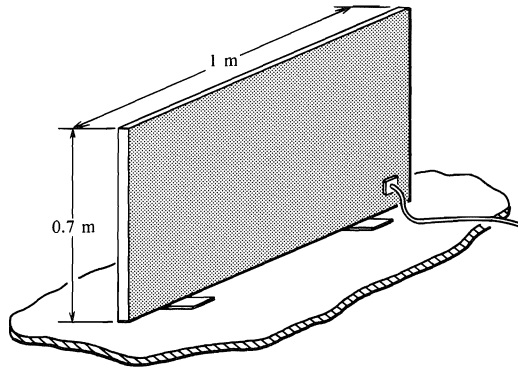
- 1-12.** An electronic device is contained in a cylinder 10 cm in diameter and 30 cm long. It operates inside an unpressurized module of an orbiting space station. The device dissipates 60 W, and its temperature must not exceed 80°C when the module walls are at -80°C . What value of emittance should be specified for the surface coating of the cylinder?
- 1-13.** A high-vacuum chamber has its walls cooled to -190°C by liquid nitrogen. A sensor in the chamber has a surface area of 10 cm^2 and must be maintained at a temperature of 25°C . Plot a graph of the power required versus emittance of the sensor surface.
- 1-14.** Consider a 3 m length of tube with a 1.26 cm inside diameter. Determine the convective heat transfer coefficient when
- (i) water flows at 2 m/s.
 - (ii) oil (SAE 50) flows at 2 m/s.
 - (iii) air at atmospheric pressure flows at 20 m/s.

Thermophysical property data at 300 K are as follows:

	ρ kg/m ³	ν m ² /s	k W/m K	c_p J/kg K
Water	996	0.87×10^{-6}	0.611	4178
SAE 50 oil	883	570×10^{-6}	0.145	1900
Air at 1 atm	1.177	15.7×10^{-6}	0.0267	1005

- 1-15.** Consider flow of water at 300 K in a long pipe of 1 cm inside diameter. Plot a graph of the heat transfer coefficient versus velocity over the range 0.01 to 100 m/s. Repeat for air at 1 atm and 300 K. Use the property values given in Exercise 1-14.
- 1-16.** A 1 m-high vertical wall is maintained at 310 K, when the surrounding air is at 1 atm and 290 K. Plot the local heat transfer coefficient as a function of location up the wall. Take $\nu = 15.7 \times 10^{-6}\text{ m}^2/\text{s}$ for air. Also calculate the convective heat loss per meter width of wall.
- 1-17.** A 2 m-high vertical surface is maintained at 15°C when exposed to stagnant air at 1 atm and 25°C . Plot a graph showing the variation of the local heat transfer coefficient, and calculate the convective heat transfer for a 3 m width of wall. Take $\nu = 15.0 \times 10^{-6}\text{ m}^2/\text{s}$ for air.
- 1-18.** A thermistor is used to measure the temperature of an air stream leaving an air heater. It is located in a 30 cm square duct and records a temperature of 42.6°C when the walls of the duct are at 38.1°C . What is the true temperature of the air? The thermistor can be modeled as a 3 mm-diameter sphere of emittance 0.7. The convective heat transfer coefficient from the air stream to the thermistor is estimated to be $31\text{ W/m}^2\text{ K}$.
- 1-19.** A room heater is in the form of a thin vertical panel 1 m long and 0.7 m high, with air allowed to circulate freely on both sides. If its rating is 800 W, what

will the average panel surface temperature be when the room air temperature is 20°C ? The emittance of the surface is 0.85. Take $\nu_{\text{air}} = 17.5 \times 10^{-6} \text{ m}^2/\text{s}$.

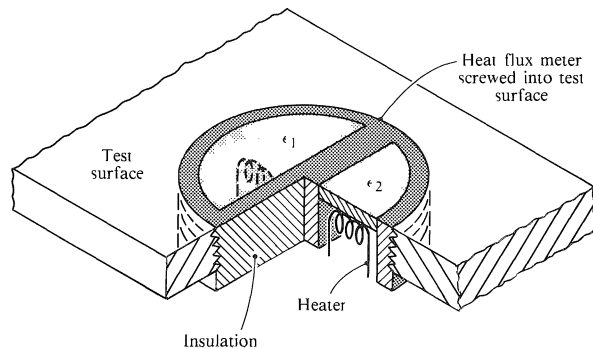


- 1–20.** An electric water heater has a diameter of 1 m and a height of 2 m. It is insulated with 6 cm of medium-density fiberglass, and the outside heat transfer coefficient is estimated to be $8 \text{ W/m}^2 \text{ K}$. If the water is maintained at 65°C and the ambient temperature is 20°C , determine
- the rate of heat loss.
 - the monthly cost attributed to heat loss if electricity costs 8 cents/kilowatt hour.
- 1–21.** A 1 cm-diameter sphere is maintained at 60°C in an enclosure with walls at 35°C through which air at 40°C circulates. If the convective heat transfer coefficient is $11 \text{ W/m}^2 \text{ K}$, estimate the rate of heat loss from the sphere when its emittance is
- 0.05.
 - 0.85.
- 1–22.** Estimate the heating load for a building in a cold climate when the outside temperature is -10°C and the air inside is maintained at 20°C . The 350 m^2 of walls and ceiling are a composite of 1 cm-thick wallboard ($k = 0.2 \text{ W/m K}$), 10 cm of vermiculite insulation ($k = 0.06 \text{ W/m K}$), and 3 cm of wood ($k = 0.15 \text{ W/m K}$). Take the inside and outside heat transfer coefficients as 7 and $35 \text{ W/m}^2 \text{ K}$, respectively.
- 1–23.** If a $2.5 \times 10 \text{ m}$ shaded wall in the building of Exercise 1–22 is replaced by a window, compare the heat loss through the wall if it is
- 0.3 cm-thick glass ($k = 0.88 \text{ W/m K}$).
 - double-glazed with a 0.6 cm air gap between two 0.3 cm-thick glass panes.
 - the original wall.
- 1–24.** Rework Exercise 1–19 for a panel 0.7 m high and 1.5 m long that is rated at 1 kW.

- 1-25. A mercury-in-glass thermometer used to measure the air temperature in an enclosure reads 15°C . The enclosure walls are all at 0°C . Estimate the true air temperature if the convective heat transfer coefficient for the thermometer bulb is estimated to be $12\text{ W/m}^2\text{ K}$.
- 1-26. A tent is pitched on a mountain in an exposed location. The tent walls are opaque to thermal radiation. On a clear night the outside air temperature is -1°C , and the effective temperature of the sky as a black radiation sink is -60°C . The convective heat transfer coefficient between the tent and the ambient air can be taken to be $8\text{ W/m}^2\text{ K}$. If the temperature of the outer surface of a sleeping bag on the tent floor is measured to be 10°C , estimate the heat loss from the bag in W/m^2 .
- (i) if the emittance of the tent material is 0.7.
 (ii) if the outer surface of the tent is aluminized to give an emittance of 0.2.

For the sleeping bag, take an emittance of 0.8 and a convective heat transfer coefficient of $4\text{ W/m}^2\text{ K}$. Assume that the ambient air circulates through the tent.

- 1-27. A schematic of a convective heat transfer coefficient meter is as shown. This meter is intended for use in situations where the air temperature T_e is known and the surrounding surfaces are also at temperature T_e . The two sensors that make up the meter are identical except that one has a surface coating of emittance $\epsilon_1 = 0.9$, and the other has an emittance of $\epsilon_2 = 0.1$. For equal power inputs to each sensor and $T_e = 300\text{ K}$, the measured surface temperatures are $T_{s1} = 321\text{ K}$ and $T_{s2} = 336\text{ K}$. Determine h_c .

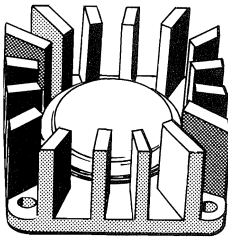


- 1-28. The horizontal roof of a building is surfaced with black tar paper of emittance 0.96. On a clear, still night the air temperature is 5°C , and the effective temperature of the sky as a black radiation sink is -60°C . The underside of the roof is well insulated.
- (i) Estimate the roof surface temperature for a convective heat transfer coefficient of $5\text{ W/m}^2\text{ K}$.

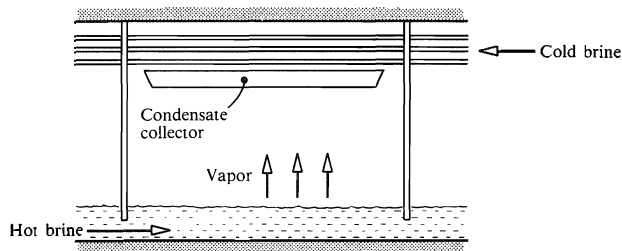
- (ii) If the wind starts blowing, giving a convective heat transfer coefficient of $20 \text{ W/m}^2 \text{ K}$, what is the new roof temperature?
 - (iii) Repeat the preceding calculations for aluminum roofing of emittance 0.15.
- 1–29.** A chemical reactor has a 5 mm-thick mild steel wall and is lined inside with a 2 mm-thick layer of polyvinylchloride. The contents are at 80°C , and the ambient air is at 20°C . The inside thermal resistance is negligible ($h_{c,i}$ very large), and the outside heat transfer coefficient for combined convection and radiation is $7 \text{ W/m}^2 \text{ K}$.
- (i) Draw the thermal circuit.
 - (ii) Plot a graph of the temperature profile through the wall.
 - (iii) Calculate the rate of heat loss for a surface area of 10 m^2 .
- 1–30.** To prevent misting of the windscreen of an automobile, recirculated warm air at 37°C is blown over the inner surface. The windscreen glass ($k = 1.0 \text{ W/m K}$) is 4 mm thick, and the ambient temperature is 5°C . The outside and inside heat transfer coefficients are 70 and $35 \text{ W/m}^2 \text{ K}$, respectively.
- (i) Determine the temperature of the inside surface of the glass.
 - (ii) If the air inside the automobile is at 20°C , 1 atm, and 80% relative humidity, will misting occur? (Refer to your thermodynamics text for the principles of psychrometry.)
- 1–31.** The horizontal roof of a building is coated with tar of emittance 0.94. On a cloudy, still night the air temperature is 5°C , and the convective heat transfer coefficient between the air and the roof is estimated to be $4 \text{ W/m}^2 \text{ K}$.
- (i) If the effective temperature of the sky as a black radiation sink is -10°C , determine the roof temperature. Assume that the under surface of the roof is well insulated.
 - (ii) If a wind starts blowing, resulting in a convective heat transfer coefficient of $12 \text{ W/m}^2 \text{ K}$, what is the new roof temperature?
 - (iii) Repeat the preceding calculations for aluminum roofing of emittance 0.15.
- 1–32.**  An alloy cylinder 3 cm in diameter and 2 m high is removed from an oven at 200°C and stood on its end to cool in ambient air at 20°C . Give a rough estimate of the time for the cylinder to cool to 100°C . For the alloy, take $\rho = 8600 \text{ kg/m}^3$, $c = 340 \text{ J/kg K}$, $k = 110 \text{ W/m K}$, and $\varepsilon = 0.74$; for air, take $\nu = 23 \times 10^{-6} \text{ m}^2/\text{s}$.
- 1–33.** A thermometer is used to check the temperature in a freezer that is set to operate at -5°C . If the thermometer initially reads 25°C , how long will it take for the reading to be within 1°C of the true temperature? Model the thermometer bulb as a 4 mm-diameter mercury sphere surrounded by a 2 mm-thick shell of glass. For mercury, take $\rho = 13,530 \text{ kg/m}^3$, $c = 140 \text{ J/kg K}$; and for glass $\rho = 2640 \text{ kg/m}^3$, $c = 800 \text{ J/kg K}$. Use a heat transfer coefficient of $15 \text{ W/m}^2 \text{ K}$.

- 1–34.** A thermocouple junction bead is modeled as a 1 mm–diameter lead sphere ($\rho = 11,340 \text{ kg/m}^3$, $c = 129 \text{ J/kg K}$) and is initially at a room temperature of 20°C . If the thermocouple is suddenly immersed in ice water to serve as a reference junction, what will be the error in indicated temperature corresponding to 1, 2, and 3 times the time constant of the thermocouple? If the heat transfer coefficient is calculated to be $2140 \text{ W/m}^2 \text{ K}$, what are the corresponding times?
- 1–35.** A hot-water cylinder contains 150 liters of water. It is insulated, and its outer surface has an area of 3.5 m^2 . It is located in an area where the ambient air is 25°C , and the overall heat transfer coefficient between the water and the surroundings is $1.0 \text{ W/m}^2 \text{ K}$, based on outer surface area. If there is a power failure, how long will it take the water to cool from 65°C to 40°C ? Take the density of water as 980 kg/m^3 and its specific heat as 4180 J/kg K .
- 1–36.** An aluminum plate 10 cm square and 1 cm thick is immersed in a chemical bath at 50°C for cleaning. On removal, the plate is shiny bright and is allowed to cool in a vertical position in still air at 20°C . Estimate how long the plate will take to cool to 30°C by
- (i) assuming a constant heat transfer coefficient evaluated at the average ΔT of 20 K.
 - (ii) allowing exactly for the $\Delta T^{1/4}$ dependence of h_c given by Eq. (1.23a).
- For air, take $\nu = 16.5 \times 10^{-6} \text{ m}^2/\text{s}$, and for aluminum, take $k = 204 \text{ W/m K}$, $\rho = 2710 \text{ kg/m}^3$, $c = 896 \text{ J/kg K}$.
- 1–37.** A 2 cm–diameter copper sphere with a thermocouple at its center is suddenly immersed in liquid nitrogen contained in a Dewar flask. The temperature response is determined using a digital data acquisition system that records the temperature every 0.05 s. The maximum rate of temperature change dT/dt is found to occur when $T = 92.5 \text{ K}$, with a value of 19.8 K/s .
- (i) Using the lumped thermal capacity model, determine the corresponding heat transfer coefficient.
 - (ii) Check the Biot number to ensure that the model is valid.
 - (iii) The fact that the cooling rate is a maximum toward the end of the cool-down period is unusual; what must be the reason?
- The saturation temperature of nitrogen at 1 atm pressure is 77.4 K . Take $\rho = 8930 \text{ kg/m}^3$, $c = 235 \text{ J/kg K}$, and $k = 450 \text{ W/m K}$ for copper at 92.5 K .
- 1–38.**  A 1 cm–diameter alloy sphere is to be heated in a furnace maintained at 1000°C . If the initial temperature of the sphere is 25°C , calculate the time required for the sphere to reach 800°C
- (i) if the gas in the furnace is circulated to give a convective heat transfer coefficient of $100 \text{ W/m}^2 \text{ K}$.
 - (ii) if there is no forced convection, and the free-convection heat transfer coefficient is given by $\bar{h}_c \sim 5\Delta T^{1/4} \text{ W/m}^2 \text{ K}$ for ΔT in kelvins.
- Properties of the alloy include $\rho = 4900 \text{ kg/m}^3$, $c = 400 \text{ J/kg K}$, $\varepsilon = 0.45$.

- 1-39.** A material sample, in the form of a 1 cm-diameter cylinder 10 cm long, is removed from a boiling water bath at 100°C and allowed to cool in air at 20°C. If the free-convection heat transfer coefficient can be approximated as $\bar{h}_c = 3.6\Delta T^{1/4}$ W/m² K for ΔT in kelvins, estimate the time required for the sample to cool to 25°C. For the sample properties take $\rho = 2260$ kg/m³, $c = 830$ J/kg K, $\varepsilon = 0.77$.
- 1-40.** Two small blackened spheres of identical size—one of aluminum, the other of an unknown alloy of high conductivity—are suspended by thin wires inside a large cavity in a block of melting ice. It is found that it takes 4.8 minutes for the temperature of the aluminum sphere to drop from 3°C to 1°C, and 9.6 minutes for the alloy sphere to undergo the same change. If the specific gravities of the aluminum and alloy are 2.7 and 5.4, respectively, and the specific heat of the aluminum is 900 J/kg K, what is the specific heat of the alloy?
- 1-41.** A mercury-in-glass thermometer is to be used to measure the temperature of a high-velocity air stream. If the air temperature increases linearly with time, $T_e = \alpha t + \text{constant}$, perform an analysis to determine the error in the thermometer reading due to its thermal “inertia.” Evaluate the error if the inside diameter of the mercury reservoir is 3 mm, its length is 1 cm, and the glass wall thickness is 0.5 mm, when the heat transfer coefficient is 60 W/m² K and the air temperature increases at a rate of
- (i) 1°C per minute.
 - (ii) 1°C per second.
- Property values for mercury are $\rho = 13,530$ kg/m³, $c = 140$ J/kg K; for glass $\rho = 2640$ kg/m³, $c = 800$ J/kg K.
- 1-42.** Under high-vacuum conditions in the space shuttle service bay, radiation is the only significant mode of heat transfer. Set $\bar{h}_c = 0$ in Eq. (1.43) and obtain an analytical solution for the lumped thermal capacity model thermal response. Also, identify a dimensionless group analogous to the Biot number that can be used to determine if the model is valid. (*Hint:* A table of standard integrals found in mathematics handbooks may be of assistance.)
- 1-43.** A thermocouple is immersed in an air stream whose temperature varies sinusoidally about an average value with angular frequency ω . The thermocouple is small enough for the Biot number to be less than 0.1, but the convective heat transfer coefficient is high enough for radiation heat transfer to be negligible compared to convection.
- (i) Set up the differential equation governing the temperature of the thermocouple.
 - (ii) Solve the differential equation to obtain the amplitude and phase lag of the thermocouple temperature response.
 - (iii) The thermocouple can be modeled as a 2 mm-diameter lead sphere ($\rho = 11,340$ kg/m³, $c = 129$ J/kg K). If the air temperature varies as $T = 320 + 10 \sin \tau$, for T in kelvins and τ in seconds, calculate the amplitude and phase lag of the thermocouple for heat transfer coefficients of 30 and 100 W/m² K.

- 1-44.** A system consists of a body in which heat is continuously generated at a rate $\dot{Q}_v$, while heat is lost from the body to its surroundings by convection. Using the lumped thermal capacity model, derive the differential equation governing the temperature response of the body. If the body is at temperature T_0 when time $t = 0$, solve the differential equation to obtain $T(t)$. Also determine the steady-state temperature.
- 1-45.** Electronic components are often mounted with good heat conduction paths to a finned aluminum base plate, which is exposed to a stream of cooling air from a fan. The sum of the mass times specific heat products for a base plate and components is 5000 J/K, and the effective heat transfer coefficient times surface area product is 10 W/K. The initial temperature of the plate and the cooling air temperature are 295 K when 300 W of power are switched on. Find the plate temperature after 10 minutes.
- 
- 1-46.** A reactor vessel's contents are initially at 290 K when a reactant is added, leading to an exothermic chemical reaction that releases heat at a rate of 4×10^5 W/m³. The volume and exterior surface area of the vessel are 0.008 m³ and 0.24 m², respectively, and the overall heat transfer coefficient between the vessel contents and the ambient air at 300 K is 5 W/m² K. If the reactants are well stirred, estimate their temperature after
- 1 minute.
 - 10 minutes.
- Take $\rho = 1200$ kg/m³ and $c = 3000$ J/kg K for the reactants.
- 1-47.** A carbon steel butane tank weighs 4.0 kg (empty) and has a surface area of 0.22 m². When full it contains 2 kg of liquified gas. Butane gas is drawn off to a burner at a rate of 0.05 kg/h through a pressure-reducing valve. If the ambient temperature is 55°C, estimate the steady temperature of the tank and the time taken for 80% of the temperature drop to occur. Take the sum of the convective and radiative heat transfer coefficients from the tank to the surroundings as 5 W/m² K. Property values for butane are $c = 2390$ J/kg K and $h_{fg} = 3.86 \times 10^5$ J/kg; for the steel $c = 434$ J/kg K.
- 1-48.** A 2.5 m-diameter, 3.5 m-high milk storage tank is located in a dairy factory in Onehunga, New Zealand, where the ambient temperature is 30°C. The tank has walls of stainless steel 2 mm thick and is insulated with a 7.5 cm-thick layer of polyurethane foam. The tank is filled with milk at 4°C and is continuously stirred by an impeller driven by an electric motor that consumes 400 W of power. What will the milk temperature be after 24 hours? For the milk, take $\rho = 1034$ kg/m³, $c = 3894$ J/kg K; for the insulation, $k = 0.026$ W/m K; and for the outside heat transfer coefficient, $h = 5$ W/m² K. The impeller motor efficiency can be taken as 0.75.

- 1-49.** Steam is condensed on a bundle of 400 tubes through which there is a flow of cold water. The tubes have an outside diameter of 2 cm, and the overall heat transfer coefficient based on outside area is $600 \text{ W/m}^2 \text{ K}$. The water flow rate is 0.2 kg/s per tube, and it enters at 290 K . If the outlet temperature is 350 K when the condenser is operating at atmospheric pressure, how much steam is condensed on each tube, and how long is the tube bundle? The specific heat of water at 320 K is 4174 J/kg K , and the enthalpy of vaporization for steam at atmospheric pressure is $2.257 \times 10^6 \text{ J/kg}$.
- 1-50.** A 3 m-long stage of a multistage flash vaporization desalination plant operates at $3 \times 10^4 \text{ Pa}$. The condenser tube bundle comprises 1300, 3 cm-O.D. titanium tubes through which cooling water flows at 60 kg/s .



If the cooling water (sea water) enters the stage at 335 K , at what temperature does it exit? At what rate does the stage produce fresh water? Take the overall heat transfer coefficient based on the tube outside area as $1200 \text{ W/m}^2 \text{ K}$ and the specific heat of sea water as 4200 J/kg K .

- 1-51.** A power station operating on a mercury-steam binary cycle has mercury condensing at 530 K on the outside of 6 cm-O.D. steel tubes. Water boils inside the tubes at 500 K . If the overall heat transfer coefficient based on tube outside area is $20,000 \text{ W/m}^2 \text{ K}$, how many tubes 3 m long are required for a turbine steam consumption of 25 kg/s ? For the water, take $h_{fg} = 1.827 \times 10^6 \text{ J/kg}$.
- 1-52.** A sheet iron exhaust gas stack is 50 cm in diameter and 10 m high. Exhaust gas at a rate of 1.0 kg/s enters the base of the stack at a temperature of 600 K . A strong wind blows across the stack, and the ambient air temperature is 300 K . If the overall heat transfer coefficient is $12 \text{ W/m}^2 \text{ K}$, find the outlet temperature of the exhaust. Use $c_p = 1200 \text{ J/kg K}$ for the exhaust gas.
- 1-53.** A geothermal power plant uses isobutane as the secondary working fluid. After expanding through the turbine, isobutane vapor condenses in a shell-and-tube condenser at 325 K . The condenser coolant is water at 305 K supplied from a cooling tower at a rate of 500 kg/s . The condenser shell contains 4000 tubes of 25 mm O.D. and 2 mm wall thickness; the overall heat transfer coefficient based on tube outside area is estimated to be $450 \text{ W/m}^2 \text{ K}$. If it is desired to

have a condenser effectiveness of 80%, determine

- (i) the outlet water temperature.
- (ii) the number of transfer units required.
- (iii) the length of the tube bundle.

Also, if the thermal efficiency of the power cycle is 30%, determine the power output of the turbine. For water, take $c_p = 4174 \text{ J/kg K}$, $\rho = 995 \text{ kg/m}^3$.

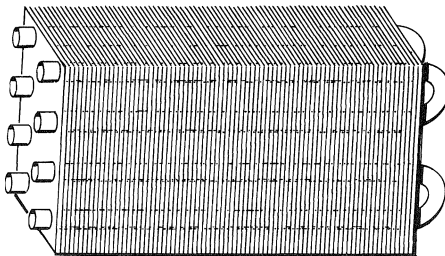
- 1-54.** In a test of a shell-and-tube steam condenser, 140 kg/s of coolant water at 300 K is supplied to the tubes. When the pressure in the shell is maintained at 0.010 MPa, the condensate flow rate is measured to be 3.0 kg/s. Determine

- (i) the outlet water temperature.
- (ii) the heat exchanger effectiveness.
- (iii) the number of transfer units.
- (iv) the $U\mathcal{P}L$ product.

For water, take $c_p = 4174 \text{ J/kg K}$.

- 1-55.** An air-cooled R-12 air-conditioning system condenser is required to condense 0.011 kg/s of saturated R-12 vapor at 320 K. The frontal area of the exchanger is 0.4 m^2 , and air at 295 K enters the exchanger at 2 m/s.

- (i) If the overall heat transfer coefficient is estimated to be $40 \text{ W/m}^2 \text{ K}$, determine the heat transfer area required.
- (ii) Plot a graph of the required heat transfer area for an air inlet temperature in the range 290–305 K.
- (iii) If the overall heat transfer coefficient increases approximately as air velocity to the 0.6 power, plot a graph of the required heat transfer area for an air velocity range of 2–10 m/s.



Take $h_{fg} = 0.1237 \times 10^6 \text{ J/kg}$ for R-12; $\rho = 1.177 \text{ kg/m}^3$ and $c_p = 1005 \text{ J/kg K}$ for air.

- 1-56.** Derive conversion factors for the following units conversions.

- (i) Enthalpy of vaporization, Btu/lb to J/kg
- (ii) Specific heat, Btu/lb °F to J/kg K
- (iii) Density, lb/ft³ to kg/m³
- (iv) Dynamic viscosity, lb/ft hr to kg/m s
- (v) Kinematic viscosity, ft²/hr to m²/s
- (vi) Thermal conductivity, Btu/hr ft °F to W/m K
- (vii) Heat flux, Btu/hr ft² to W/m²

- 1-57.** In the United States, gas and liquid flow rates are commonly expressed in cubic feet per minute (CFM) and gallons per minute (GPM), respectively.
- (i) For air at 1 atm and 300 K ($\rho = 1.177 \text{ kg/m}^3$), prepare a table showing flow rates in m^3/s and kg/s corresponding to 1, 10, 100, 1000, and 10,000 CFM.
 - (ii) For water at 300 K ($\rho = 996 \text{ kg/m}^3$), prepare a table showing flow rates in m^3/s and kg/s corresponding to 1, 10, 100, 1000, and 10,000 GPM.
- 1-58.** In January 1989 the barometric pressure reached 31.84 inches of mercury at Northway, Alaska, a record for North America. On the other hand, a typical barometric pressure for Denver, Colorado, is 24.4 inches of mercury.
- (i) What are these pressures in mbar and pascals?
 - (ii) At what temperature does water boil at these pressures?
- 1-59.** Specify the following in the English system of units (Btu, hr, ft, °F or °R):
- (i) The Stefan-Boltzmann constant
 - (ii) The radiation heat transfer coefficient at 25°C
 - (iii) The free-convection formulas given by Eqs. (1.23a) and (1.23b).
- 1-60.** Convert the problem statement of Example 1.3 to the English system of units, work the problem in English units, and convert your answer back to SI units.
- 1-61.** Convert the problem statement of Example 1.4 to the English system of units, work the problem in English units, and convert your answers back into SI units.
- 1-62.** Convert the problem statement of Example 1.5 to the English system of units, work the problem in English units, and convert your answers back into SI units.
- 1-63.** Convert the problem statement of Example 1.7 to the English system of units, work the problem in English units, and convert your answers back into SI units.
- 1-64.** Convert the problem statement of Example 1.8 to the English system of units, work the problem in English units, and convert your answers back into SI units.
- 1-65.** Check the dimensions of Eq. (1.22) in
- (i) SI units.
 - (ii) English units.
- 1-66.** Write a computer program to solve Eq. (1.45). That is, write your own version of LUMP. (LUMP is based on trapezoidal integration, with Newton's method used to solve the resulting nonlinear equation at each time step. Heun's explicit formula is used for the initial guess of the Newton iteration.)



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STEADY ONE-DIMENSIONAL HEAT CONDUCTION

CONTENTS

- 2.1. INTRODUCTION**
- 2.2. FOURIER'S LAW OF HEAT CONDUCTION**
- 2.3. CONDUCTION ACROSS CYLINDRICAL AND SPHERICAL SHELLS**
- 2.4. FINS**
- 2.5. CLOSURE**

2.1 INTRODUCTION

In this chapter we analyze problems involving steady one-dimensional heat conduction. By **steady** we mean that temperatures are constant with time; as a result, the heat flow is also constant with time. By **one-dimensional** we mean that temperature is a function of a single “dimension” or spatial coordinate. One-dimensional conduction can occur in a number of geometrical shapes. In Section 1.3.1, one-dimensional conduction across a plane wall was examined, with temperature as a function of Cartesian coordinate x only; that is, $T = T(x)$. Conduction in cylinders or spheres is one-dimensional when temperature is a function of only the radial coordinate r and does not vary with polar angle and axial distance, in the case of the cylinder, or with polar or azimuthal angles, in the case of the sphere; that is, $T = T(r)$. Analysis of steady one-dimensional heat conduction problems involves the solution of very simple ordinary differential equations to give algebraic formulas for the temperature variation and heat flow. Thus, if at all possible, engineers like to approximate, or *model*, a practical heat conduction problem as steady and one-dimensional, even though temperatures might vary slowly with time or vary a little in a second coordinate direction.

A wide range of practical heat transfer problems involve steady one-dimensional heat conduction. Examples include most heat insulation problems, such as the refrigerated container of Example 1.1, the prediction of temperatures in a nuclear reactor fuel rod, and the design of cooling fins for electronic gear. Often, complex systems involving two- or three-dimensional conduction can be divided into subsystems in which the conduction is one-dimensional. Cooling of integrated circuit components can often be satisfactorily analyzed in this manner.

In Section 2.2, Fourier’s law of heat conduction is briefly revisited. The physical mechanisms of heat conduction are discussed, and the applicability of Fourier’s law at the interface between two solids is examined. Conduction across plane walls has already been treated in Section 1.3.1; thus, in Section 2.3, we restrict our attention to conduction across cylindrical and spherical shells and include the effect of **heat generation** within the solid. Section 2.4 deals with the class of problems known as **fin** problems, including familiar cooling fins and an interesting variety of mathematically similar problems.

There is a common methodology to the analyses in Chapter 2. Each analysis begins with the application of the first law, Eq. (1.2), to a closed-system volume element and introduction of Fourier’s law, to obtain the governing differential equation. This equation is then integrated to give the temperature distribution, with the constants of integration found from appropriate boundary conditions. Finally, the heat flow is obtained using Fourier’s law.

2.2 FOURIER’S LAW OF HEAT CONDUCTION

Fourier’s law of heat conduction was introduced in Section 1.3.1. A general statement of this law is: The conduction heat flux in a specified direction equals the negative of the product of the medium thermal conductivity and the temperature derivative in that direction. In Chapter 2, we are concerned with one-dimensional conduction. In

Cartesian coordinates, with temperature varying in the x direction only,

$$q = -k \frac{dT}{dx} \quad (2.1)$$

Recall from Section 1.3.1 that the negative sign ensures that the heat flux q is positive in the positive x direction. In cylindrical or spherical coordinates, with temperature varying in the r direction only,

$$q = -k \frac{dT}{dr} \quad (2.2)$$

Equation (2.2) is the form of Fourier's law required for Section 2.3.

2.2.1 Thermal Conductivity

Table 1.1 gave a brief list of thermal conductivities to illustrate typical values for gases, liquids, and solids. Appendix A gives more complete tabulated data. The relevant tables are:

Table A.1	Solid metals
Table A.2	Solid dielectrics (nonmetals)
Table A.3	Insulators and building materials
Table A.4	Solids at cryogenic temperatures
Table A.7	Gases
Table A.8	Dielectric liquids
Table A.9	Liquid metals
Table A.13	Liquid solutions

Additional data may be found in the literature, for example, References [1] through [4]. In the case of commercial products, such as insulations, data can be obtained from the manufacturer.

The engineer needs conductivity data to solve heat conduction problems (as was seen in Chapter 1) and is usually not too concerned about the actual physical mechanism of heat conduction. However, conductivity data for a given substance are often sparse or nonexistent, and then a knowledge of the physics of heat conduction is useful to interpolate or extrapolate what data are available. Unfortunately, the physical mechanisms of conduction are many and complicated, and it is possible to develop simple theoretical models for gases and pure metals only. A brief account of some of the more important aspects of the conduction mechanisms follows.

Gases

The kinetic theory model gives a reliable basis for determining the thermal conductivity of a gas. Molecules are in a state of random motion. When collisions occur, there is an exchange of energy that results in heat being conducted down a temperature

gradient from a hot region to a cold region. The simplest form of kinetic theory gives

$$k = \frac{1}{3} c \lambda \left(m c_v + \frac{1}{2} k \right) \quad [\text{W/m K}] \quad (2.3)$$

where c is the average molecular speed [m/s], λ is the number of molecules per unit volume [m^{-3}], λ is the transport mean free path [m], m is the mass of the molecule [kg], c_v is the constant-volume specific heat [J/kg K], and k is the Boltzmann constant, 1.38054×10^{-23} J/K. The product $c \lambda$ is virtually independent of pressure in the vicinity of atmospheric pressure; thus, conductivity is also independent of pressure. Table A.7 gives data for a nominal pressure at 1 atm but can be used for pressures down to about 1 torr (1/760 of an atmosphere, or 133.3 Pa). Notice also that the average molecular speed is higher and the mean free path is longer for small molecules; thus, conductivities of gases such as hydrogen and helium are much greater than those of xenon and the refrigerant R-12.

Dielectric Liquids

In liquids (excluding liquid metals such as mercury at normal temperatures and sodium at high temperatures), the molecules are relatively closely packed, and heat conduction occurs primarily by longitudinal vibrations, similar to the propagation of sound. The structure of liquids is not well understood at present, and there are no good theoretical formulas for conductivity.

Pure Metals and Alloys

The primary mechanism of heat conduction is the movement of free electrons. A smaller contribution is due to the transfer of atomic motions by lattice vibrations, or waves; however, this contribution is unimportant except at cryogenic temperatures. In alloys, the movement of free electrons is restricted, and thermal conductivity decreases markedly as alloying elements are added. The lattice wave contribution then becomes more important but is difficult to predict owing to the variable effects of heat treatment and cold working.

Dielectric Solids

Heat conduction is almost entirely due to atomic motions being transferred by lattice waves; hence, thermal conductivity is very dependent on the crystalline structure of the material. Many building materials and insulators have anomalously low values of conductivity because of their porous nature. The conductivity is an effective value for the porous medium and is low due to air or a gas filling the interstices and pores, through which heat is transferred rather poorly by conduction and radiation. Expanded plastic insulations, such as polystyrene, have a large molecule refrigerant gas filling the pores to reduce the conductivity.

In general, thermal conductivity is temperature-dependent. Fortunately, the variation in typical engineering problems is small, and it suffices to use an appropriate average value. An exception is solids at cryogenic temperatures, as an examination of Table A.4 will show. Another exception is gases in the vicinity of the critical point. Special care must be taken in such situations.

2.2.2 Contact Resistance

In Section 1.3.1, heat conduction through a composite wall was analyzed. Figure 2.1*a* shows the interface between two layers of a composite wall, with the surface of each layer assumed to be perfectly smooth. Two mathematical surfaces, the u - and s -surfaces, are located on each side of and infinitely close to the real interface, as shown. The first law of thermodynamics applied to the closed system located between the u - and s -surfaces requires that

$$\dot{Q}|_u = \dot{Q}|_s \quad (2.4)$$

since no energy can be stored in the infinitesimal amount of material in the system. Considering a unit area and introducing Fourier's law gives

$$-k_A \left. \frac{dT}{dx} \right|_u = -k_B \left. \frac{dT}{dx} \right|_s \quad (2.5)$$

Also, since the distance between the u - and s -surfaces is negligible, thermodynamic equilibrium requires

$$T_u = T_s \quad (2.6)$$

as shown on the temperature profile (for $k_A > k_B$). For perfectly smooth surfaces, there is no thermal resistance at the interface.

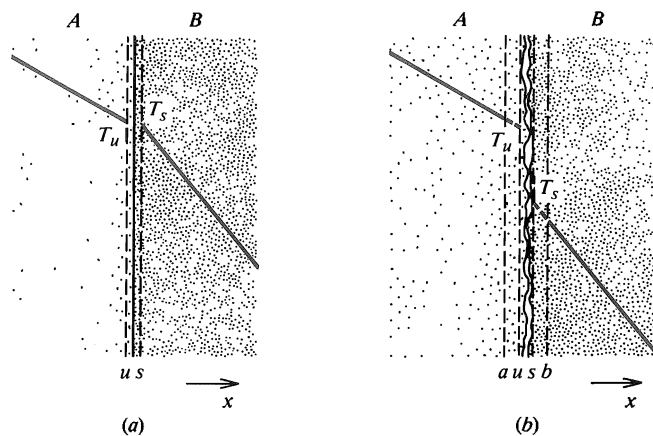


Figure 2.1 Interfaces between two layers of a composite wall. (a) Smooth surfaces. (b) Rough surfaces.

Figure 2.1*b* shows a more realistic situation, in which each surface has some degree of roughness. The solid materials are in contact at relatively few places, and the gaps may contain a fluid or, in some applications, a vacuum. The heat flow in the interface region is complicated: the conduction is three-dimensional as the heat tends to “squeeze” through the contact areas, and there are parallel paths of conduction and radiation through the gaps. The u - and s -surfaces are located just on either side of a somewhat arbitrarily defined interface location. In addition, a - and b -surfaces are located just far enough from the interface for the heat conduction to be one-dimensional. No temperature profile is shown between the a - and b -surfaces since no unique profile $T(x)$ exists there; instead, the temperature profiles are extrapolated from the bulk material to the interface as shown, thereby defining the temperatures and temperature gradients at the u - and s -surfaces. As was the case for the perfectly smooth surfaces, the first law requires

$$-k_A \left. \frac{dT}{dx} \right|_u = -k_B \left. \frac{dT}{dx} \right|_s \quad (2.7)$$

but now there is no continuity of temperature at the interface; that is, $T_u \neq T_s$. The thermal resistance to heat flow at the interface is called the **contact resistance** and is usually expressed in terms of an **interfacial conductance** h_i [$\text{W}/\text{m}^2 \text{ K}$], defined in an analogous manner to Newton’s law of cooling, namely,

$$\dot{Q} = h_i A (T_u - T_s) \quad (2.8)$$

or

$$-k_A \left. \frac{dT}{dx} \right|_u = h_i (T_u - T_s) = -k_B \left. \frac{dT}{dx} \right|_s \quad (2.9)$$

Figure 2.2 shows the contact resistance added to the thermal circuit of Fig. 1.5.

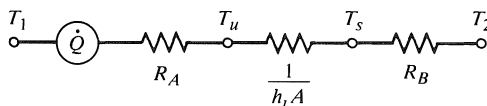


Figure 2.2 A contact resistance in a thermal circuit.

There is always a contact resistance to conduction across real solid-solid interfaces. The contact resistance can be the dominant thermal resistance when high-conductivity metals are involved—for example, in aircraft construction, where aluminum alloys are used extensively. The contact resistance depends on the pressure with which contact is maintained, with a marked decrease once the yield point of one of the materials is reached. Data for contact resistances are, unfortunately, sparse and unreliable. Table 2.1 does, however, show some representative values. Additional data can be found in the literature [5,6,7].

Table 2.1 Typical interfacial conductances (at moderate pressure and usual finishes, unless otherwise stated).

Interface	h_i W/m ² K
Ceramic-ceramic	500–3000
Ceramic-metals	1500–8500
Graphite-metals	3000–6000
Stainless steel–stainless steel	1700–3700
Aluminum-aluminum	2200–12,000
Stainless steel–aluminum	3000–4500
Copper-copper	10,000–25,000
Rough aluminum–aluminum (vacuum conditions)	~150
Iron-aluminum	4000–40,000

2.3 CONDUCTION ACROSS CYLINDRICAL AND SPHERICAL SHELLS

Steady one-dimensional conduction in cylinders or spheres requires that temperature be a function of only the radial coordinate r . The analysis of steady heat flow across a plane wall in Section 1.3.1 was particularly simple because the flow area A did not change in the flow direction. In the case of a cylindrical or spherical shell, the area for heat flow changes in the direction of heat flow. For a cylindrical shell of length L , the area for heat flow is $A = 2\pi rL$; for a spherical shell, it is $A = 4\pi r^2$. In both cases, A increases with increasing r .

2.3.1 Conduction across a Cylindrical Shell

Figure 2.3 shows a cylindrical shell of length L , with inner radius r_1 and outer radius r_2 . The inner surface is maintained at temperature T_1 and the outer surface is maintained at temperature T_2 . An elemental control volume is located between radii r and $r + \Delta r$. If temperatures are unchanging in time and $\dot{Q}_v = 0$, the energy conservation principle, Eq. (1.2), requires that the heat flow across the face at r equal that at the face $r + \Delta r$

$$\dot{Q}|_r = \dot{Q}|_{r+\Delta r}$$

that is,

$$\dot{Q} = \text{Constant, independent of } r$$

Using Fourier's law in the form of Eq. (2.2),

$$\dot{Q} = Aq = 2\pi rL \left(-k \frac{dT}{dr} \right)$$

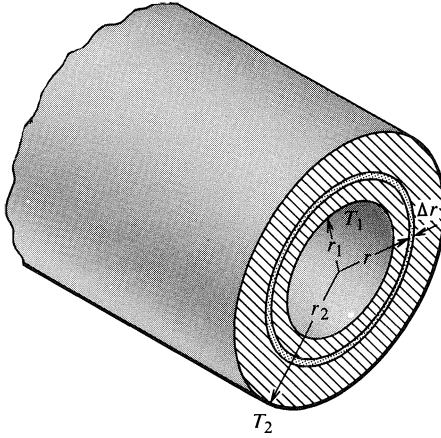


Figure 2.3 A cylindrical shell showing an elemental control volume for application of the energy conservation principle.

Dividing by $2\pi kL$ and assuming that the conductivity k is independent of temperature gives

$$\frac{\dot{Q}}{2\pi kL} = -r \frac{dT}{dr} = \text{Constant} = C_1 \quad (2.10)$$

which is a first-order ordinary differential equation for $T(r)$ and can be integrated easily:

$$\begin{aligned} \frac{dT}{dr} &= -\frac{C_1}{r} \\ T &= -C_1 \ln r + C_2 \end{aligned} \quad (2.11)$$

Two boundary conditions are required to evaluate the two constants; these are

$$r = r_1: \quad T = T_1 \quad (2.12a)$$

$$r = r_2: \quad T = T_2 \quad (2.12b)$$

Substituting in Eq. (2.11) gives

$$T_1 = -C_1 \ln r_1 + C_2$$

$$T_2 = -C_1 \ln r_2 + C_2$$

which are two algebraic equations for the unknowns C_1 and C_2 . Subtracting the second equation from the first:

$$T_1 - T_2 = -C_1 \ln r_1 + C_1 \ln r_2 = C_1 \ln (r_2/r_1)$$

or

$$C_1 = \frac{T_1 - T_2}{\ln (r_2/r_1)}$$

Using either of the two equations then gives

$$C_2 = T_1 + \frac{T_1 - T_2}{\ln(r_2/r_1)} \ln r_1$$

Substituting back in Eq. (2.11) and rearranging gives the temperature distribution as

$$\frac{T_1 - T}{T_1 - T_2} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)} \quad (2.13)$$

which is a logarithmic variation, in contrast to the linear variation found for the plane wall in Section 1.3.1. The heat flow is found from Eq. (2.10) as $\dot{Q} = 2\pi k L C_1$, or

$$\dot{Q} = \frac{2\pi k L (T_1 - T_2)}{\ln(r_2/r_1)} \quad (2.14)$$

Equation (2.14) is again in the form of Ohm's law, and the thermal resistance of the cylindrical shell is

$$R = \frac{\ln(r_2/r_1)}{2\pi k L} \quad (2.15)$$

When $r_2 = r_1 + \delta$ and $\delta/r_1 \ll 1$, Eq. (2.15) reduces to the resistance of a slab, $\delta/2\pi r_1 k L = \delta/kA$.

It is now possible to treat composite cylindrical shells with convection and radiation from either side without any further analysis. Figure 2.4 shows the cross section of an insulated pipe of length L , through which flows superheated steam and

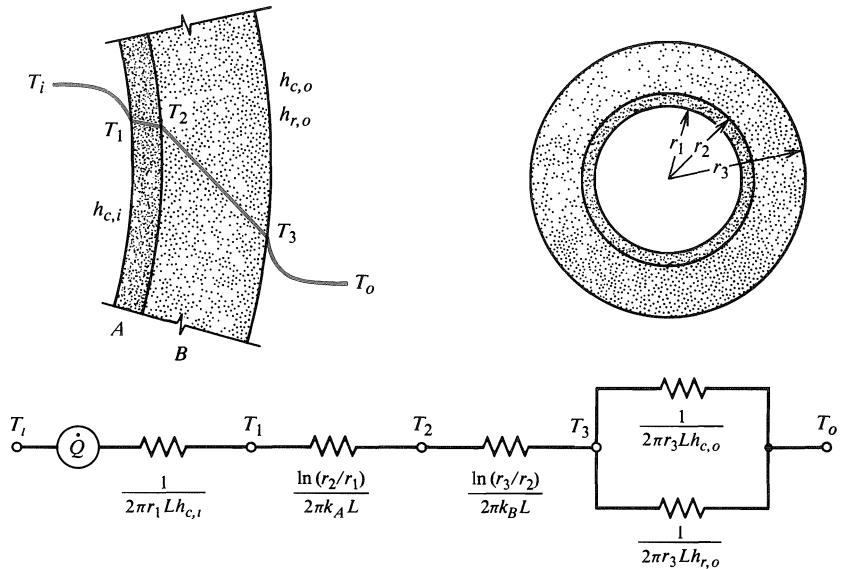


Figure 2.4 An insulated steam pipe showing the temperature distribution and thermal circuit.

which loses heat by convection and radiation to its surroundings. The thermal circuit is also shown, with Eq. (2.15) used for the conductive resistances of the pipe and insulation. Notice that in contrast to the plane wall case, the area for heat flow is different on each side of the composite wall: on the inside it is $2\pi r_1 L$, and on the outside it is $2\pi r_3 L$. Again we define an *overall heat transfer coefficient* by Eq. (1.29):

$$\dot{Q} = UA(T_i - T_o) = \frac{T_i - T_o}{1/UA} \quad (2.16)$$

Then, summing the resistances in the thermal network,

$$\frac{1}{UA} = \frac{1}{2\pi r_1 L h_{c,i}} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{1}{2\pi r_3 L (h_{c,o} + h_{r,o})} \quad (2.17)$$

The area A need not be specified since all we need is the UA product. However, often a value of U will be quoted based on either the inside or outside area; then the appropriate area must be used in Eqs. (2.16) and (2.17).

EXAMPLE 2.1 Heat Loss from an Insulated Steam Pipe

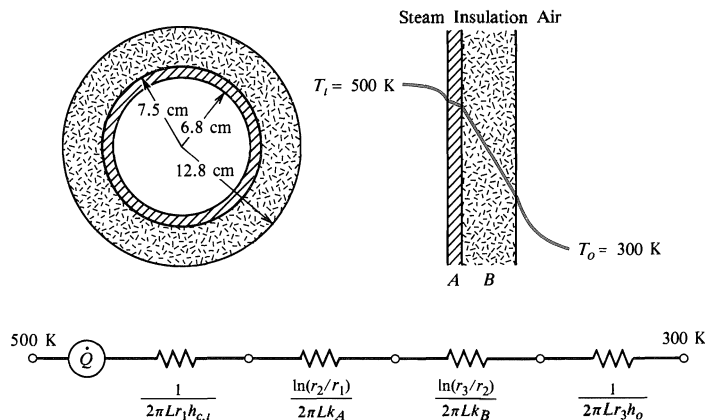
A mild steel steam pipe has an outside diameter of 15 cm and a wall thickness of 0.7 cm. It is insulated with a 5.3 cm-thick layer of 85% magnesia insulation. Superheated steam at 500 K flows through the pipe, and the inside heat transfer coefficient is 35 W/m² K. Heat is lost by convection and radiation to surroundings at 300 K, and the sum of outside convection and radiation coefficients is estimated to be 8 W/m² K. Find the rate of heat loss for a 20 m length of pipe.

Solution

Given: Steam pipe with 85% magnesia insulation.

Required: Heat loss for 20 m length if $h_o = 8$ W/m² K.

Assumptions: Steady one-dimensional heat flow.



Equation (2.16) applies, with Eq. (2.17) used to obtain the UA product,

$$\dot{Q} = UA(T_i - T_o)$$

$$\frac{1}{UA} = \frac{1}{2\pi L} \left(\frac{1}{r_1 h_{c,i}} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{r_3 h_o} \right)$$

Tables A.1b and A.3 in Appendix A give the variation of conductivity with temperature for 1010 steel and magnesia, respectively. As a first step, we guess that the steel is close to the steam temperature (500 K), and since most of the temperature drop will be across the magnesia insulation, its average temperature will be about $(500 + 300)/2 = 400$ K. The corresponding conductivity values are $k_A = 54$ W/m K and $k_B = 0.073$ W/m K.

$$\frac{1}{UA} = \frac{1}{(2)(\pi)(20)} \left(\frac{1}{(0.068)(35)} + \frac{\ln(0.075/0.068)}{54} + \frac{\ln(0.128/0.075)}{0.073} + \frac{1}{(0.128)(8)} \right)$$

$$= \frac{1}{125.7} (0.42 + 0.002 + 7.32 + 0.98)$$

$$UA = 14.4 \text{ W/K}$$

$$\dot{Q} = UA\Delta T = (14.4)(500 - 300) = 2880 \text{ W}$$

Since the resistance of the steel wall is negligible, we do not need to check our guess for its conductivity. For the magnesia insulation, we estimate its average temperature by examining the relevant segment of the thermal circuit. For convenience, the thermal resistance of the insulation is split in half to estimate an average temperature $\bar{T}$:

$$\bar{T} - T_o = \dot{Q} \left[\left(\frac{1}{2} \right) \frac{\ln(r_3/r_2)}{2\pi L k_B} + \frac{1}{2\pi L r_3 h_o} \right]$$

$$\bar{T} - 300 = (2880) \left[\left(\frac{1}{2} \right) \frac{7.32}{125.7} + \frac{0.98}{125.7} \right]$$

$$= 106 \text{ K}$$

$$\bar{T} = 406 \text{ K}$$

A look at Table A.3 shows that our guess of 400 K introduced an error of less than 1%, so there is no need to calculate a new value of $\dot{Q}$ using an improved k value.

Comments

1. After one has gained some experience with this type of calculation, the problem can be simplified by ignoring the small resistance of the steel pipe.
2. In practice, the outside heat transfer coefficient varies somewhat around the circumference of the insulation, and the conduction is not truly one-dimensional. For an engineering calculation, we simply use an average value for h_o .

2.3.2 Critical Thickness of Insulation on a Cylinder

The insulation on the large steam pipe in Example 2.1 was installed because it reduced the heat loss. However, adding a layer of insulation to a cylinder does not necessarily reduce the heat loss. When the outer radius of the insulation r_o is small, there is the possibility that the added thermal resistance of the insulation is less than

the reduction of the outside resistance $1/2\pi r_o L h_o$ due to the larger value of the area for convective and radiative heat transfer, $2\pi r_o L$. This phenomenon is often used for cooling electronic components that must dissipate $I^2 R$ heating. Figure 2.5 shows a resistor with an insulation sheath of inner radius r_i and outer radius r_o . Since the resistor usually has a relatively high thermal conductivity, we will assume it is isothermal at temperature T_i . The ambient air temperature is T_e , and the outside heat transfer coefficient is h_o . There are two resistances in series; denoting the total resistance as R the heat flow is

$$\dot{Q} = \frac{T_i - T_e}{R} = \frac{T_i - T_e}{\ln(r_o/r_i)/2\pi L k + 1/2\pi L r_o h_o} \quad (2.18)$$

$\dot{Q}$ will have a maximum value when the total resistance R has a minimum value; differentiating R with respect to r_o :

$$\frac{dR}{dr_o} = \frac{1}{2\pi L} \left(\frac{1}{r_o k} - \frac{1}{r_o^2 h_o} \right)$$

which equals zero when the outer radius of the insulation equals the **critical radius**,

$$r_o = r_{cr} = \frac{k}{h_o} \quad (2.19)$$

To check whether r_{cr} gives a minimum resistance, we differentiate again and evaluate at $r_o = r_{cr}$:

$$\frac{d^2 R}{dr_o^2} = \frac{1}{2\pi L} \left(-\frac{1}{r_o^2 k} + \frac{2}{r_o^3 h_o} \right)$$

and

$$\left. \frac{d^2 R}{dr_o^2} \right|_{r_o = r_{cr}} = \frac{1}{2\pi L} \left(-\frac{h_o^2}{k^3} + \frac{2h_o^2}{k^3} \right) = \frac{h_o^2}{2\pi L k^3} > 0$$

as required.

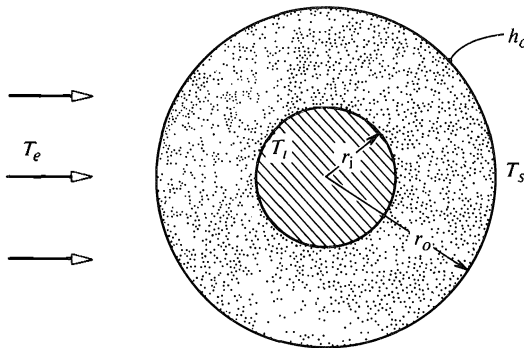


Figure 2.5 An electric resistor with an insulation sheath.

Equation (2.19) is an often-used formula for the critical radius, but it is only an approximate estimate since the heat transfer coefficient was assumed to be independent of r_o . In general, we can write $h_o = \alpha r_o^{-n}$ where, for example, $n = 1/2$ for laminar forced convection. Thus, the total resistance R is more correctly written as

$$R = \frac{\ln(r_o/r_i)}{2\pi Lk} + \frac{1}{2\pi L\alpha r_o^{1-n}}$$

hence,

$$\frac{dR}{dr_o} = \frac{1}{2\pi L} \left(\frac{1}{r_o k} + \frac{(n-1)r_o^{n-2}}{\alpha} \right)$$

which equals zero when

$$r_o = r_{cr} = \left(\frac{\alpha}{(1-n)k} \right)^{1/(n-1)} \quad (2.20)$$

For $n = 1/2$, $r_{cr} = (k/2\alpha)^2$. For natural convection, the situation is more complex: not only do we have $n = 1/4$, but Newton's law of cooling is invalid, with $h_o \propto \Delta T^{1/4}$. In the case of radiation, h_r can be taken to be independent of r_o , but $h_r \propto T_s^3$ [for small $(T_s - T_e)$]. Exercises 2–17 and 2–18 examine these situations. Fortunately, from a practical standpoint, a precise value of r_{cr} is not needed. Because $\dot{Q}$ is a maximum at r_{cr} , the heat loss is not sensitive to the precise value of r when r is in the vicinity of r_{cr} .

EXAMPLE 2.2 Cooling of an Electrical Resistor

A 0.5 W, 1.5 M Ω graphite resistor has a diameter of 1 mm and is 20 mm long; it has a thin glass sheath and is encapsulated in micanite (crushed mica bonded by a phenolic resin). The micanite serves both as additional electrical insulation and to increase the heat loss. It can be assumed that 50% of the I^2R heating is dissipated by combined convection and radiation from the outer surface of the micanite to surroundings at 300 K with $h_o = 16$ W/m² K; the remainder is conducted through copper leads to a circuit board. If the conductivity of micanite is 0.1 W/m K, what radius will give the maximum cooling effect, and what is the corresponding resistor temperature?

Solution

Given: Cylindrical graphite resistor encapsulated in micanite.

Required: Critical radius of micanite insulation, and the resistor temperature.

Assumptions: 1. The resistance of the glass sheath is negligible.
 2. The outside heat transfer coefficient h_o is constant.
 3. The resistor temperature is uniform.

The critical radius is given by Eq. (2.19):

$$r_{cr} = \frac{k}{h_o} = \frac{0.1}{16} = 0.00625 \text{ m}$$

$$= 6.25 \text{ mm}$$

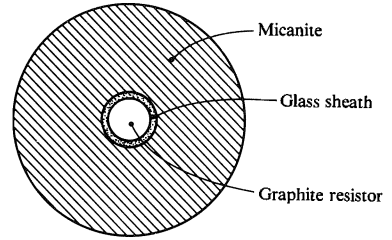
From Eq. (2.18),

$$T_i - T_e = \frac{\dot{Q}}{2\pi L} \left(\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h_o} \right)$$

$$T_i - 300 = \frac{(0.5)(0.5)}{(2)(\pi)(0.020)} \left(\frac{\ln(6.25/0.5)}{0.1} + \frac{1}{(0.00625)(16)} \right)$$

$$= 70.1 \text{ K}$$

Hence, $T_i = 300 + 70.1 = 370.1 \text{ K}$.



Comments

1. To obtain a more accurate result, additional data are required, particularly for the variation of h_o with radius.
2. Section 2.3.4 shows how to check the validity of assumption 3.
3. In general, h_o (and hence r_{cr}) vary around the circumference of the insulation. For engineering purposes, we ignore this complication and use an average value for h_o .

2.3.3 Conduction across a Spherical Shell

Figure 2.6 shows a spherical shell of inner radius r_1 and outer radius r_2 . The inner surface is maintained at temperature T_1 and the outer surface at T_2 . An elemental control volume is located between radii r and $r + \Delta r$. As was shown for the cylindrical shell in Section 2.3.1, energy conservation applied to the control volume requires that the heat flow $\dot{Q}$ be constant, independent of r if the temperatures are unchanging in time and $\dot{Q}_v = 0$. Using Fourier's law, Eq. (2.2),

$$\dot{Q} = Aq = 4\pi r^2 \left(-k \frac{dT}{dr} \right)$$

Dividing by $4\pi k$ and assuming that the conductivity k is independent of temperature gives

$$\frac{\dot{Q}}{4\pi k} = -r^2 \frac{dT}{dr} = \text{Constant} = C_1 \quad (2.21)$$

$$\frac{dT}{dr} = -\frac{C_1}{r^2}$$

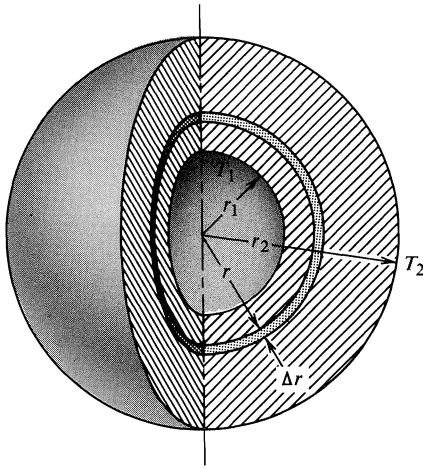


Figure 2.6 A spherical shell showing an elemental control volume for application of the energy conservation principle.

Integrating,

$$T = \frac{C_1}{r} + C_2 \quad (2.22)$$

The boundary conditions required to evaluate the two constants are

$$r = r_1: \quad T = T_1 \quad (2.23a)$$

$$r = r_2: \quad T = T_2 \quad (2.23b)$$

Substituting in Eq. (2.22) and solving for C_1 and C_2 gives

$$C_1 = \frac{T_1 - T_2}{1/r_1 - 1/r_2}; \quad C_2 = T_1 - \frac{T_1 - T_2}{1 - r_1/r_2}$$

Then substituting back in Eq. (2.22) and rearranging gives the temperature distribution as

$$\frac{T_1 - T}{T_1 - T_2} = \frac{1/r_1 - 1/r}{1/r_1 - 1/r_2} \quad (2.24)$$

The heat flow is found from Eq. (2.21) as $\dot{Q} = 4\pi k C_1$, or

$$\dot{Q} = \frac{4\pi k(T_1 - T_2)}{1/r_1 - 1/r_2} \quad (2.25a)$$

Equation (2.25a) can be used to build up thermal circuits for composite spherical shells, as was done for composite cylindrical shells in Section 2.3.1. The thermal resistance of a spherical shell is

$$R = \frac{1/r_1 - 1/r_2}{4\pi k} \quad (2.25b)$$

EXAMPLE 2.3 Determination of Thermal Conductivity

To measure the effective thermal conductivity of an opaque honeycomb material for an aircraft wall, a spherical shell of inner radius 26 cm and outer radius 34 cm was constructed and a 100 W electric light bulb placed in the center. At steady state, the temperatures of the inner and outer surfaces were measured to be 339 and 311 K respectively. What is the effective conductivity of the material?

Solution

Given: Spherical shell containing a 100 W heat source.

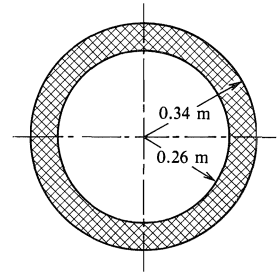
Required: Thermal conductivity of shell material.

Assumptions: 1. Steady state.
2. Spherical symmetry, $T = T(r)$.

Eq. (2.25a) applies, with $\dot{Q}$, T_1 , T_2 known and k the unknown:

$$100 = \frac{4\pi k(339 - 311)}{1/0.26 - 1/0.34}$$

Solving, $k = 0.257 \text{ W/m K}$.

**Comments**

The large thermal resistance of the honeycomb results in a relatively large temperature difference across it, which is easy to measure accurately. The same method would not be practical for determining the conductivity of a metal shell.

2.3.4 Conduction with Internal Heat Generation

In some situations, the thermal behavior of a body is affected by internally generated or absorbed thermal energy. The most common example is I^2R heating associated with the flow of electrical current I in an electrical resistance R . Other examples include fission reactions in the fuel rods of a nuclear reactor, absorption of radiation in a microwave oven, and emission of radiation by a flame. We will use the symbol $\dot{Q}_v'''$ [W/m³] for the heat generation rate per unit volume.¹ As an example, consider internal heat generation in a solid cylinder of outer radius r_1 , as might occur in an electrical wire or a nuclear fuel rod. Figure 2.7 shows an elemental volume located between radii r and $r + \Delta r$. Applying the energy conservation principle, Eq. (1.2), requires

¹ The triple prime indicates "per unit volume" (per length dimension cubed). In Section 1.2, the symbol $\dot{Q}_v$ [W] was used for the heat generated within a system and is related to $\dot{Q}_v'''$ [W/m³] as $\dot{Q}_v = \int_V \dot{Q}_v''' dV$.

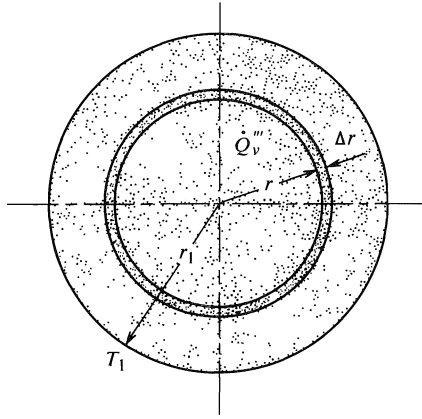


Figure 2.7 A solid cylindrical rod with internal heat generation.

that

$$\dot{Q}|_r - \dot{Q}|_{r+\Delta r} + \dot{Q}_v''' 2\pi r L \Delta r = 0$$

if temperatures are steady. Dividing by Δr and rearranging gives

$$\frac{\dot{Q}|_{r+\Delta r} - \dot{Q}|_r}{\Delta r} = \dot{Q}_v''' 2\pi r L$$

which for $\Delta r \rightarrow 0$ becomes

$$\frac{d\dot{Q}}{dr} = 2\pi r L \dot{Q}_v'''$$

Introducing Fourier's law, $\dot{Q} = Aq = 2\pi r L [-k (dT/dr)]$, and assuming k constant gives

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = -\frac{\dot{Q}_v'''}{k} r \quad (2.26)$$

which is a second-order linear ordinary differential equation for $T(r)$.

Two boundary conditions are required; the first comes from symmetry:

$$r = 0: \quad \frac{dT}{dr} = 0 \quad (2.27a)$$

To obtain a result of some generality, we will take as the second boundary condition a specified temperature on the outer surface of the cylinder:

$$r = r_1: \quad T = T_1 \quad (2.27b)$$

In a typical engineering problem, T_1 might not be specified; however, we shall see that the result will be in a form suitable for problem solving. Integrating Eq. (2.26) once gives

$$r \frac{dT}{dr} = -\frac{1}{2} \frac{\dot{Q}_v'''}{k} r^2 + C_1$$

or

$$\frac{dT}{dr} = -\frac{1}{2} \frac{\dot{Q}_v'''}{k} r + \frac{C_1}{r}$$

Applying the first boundary condition, Eq. (2.27a),

$$0 = 0 + \frac{C_1}{0}, \quad \text{or } C_1 = 0$$

Integrating again,

$$T = -\frac{1}{4} \frac{\dot{Q}_v'''}{k} r^2 + C_2$$

Applying the second boundary condition, Eq. (2.27b) allows C_2 to be evaluated:

$$T_1 = -\frac{1}{4} \frac{\dot{Q}_v'''}{k} r_1^2 + C_2$$

Substituting back gives the desired temperature distribution, $T(r)$:

$$T - T_1 = \frac{1}{4} \frac{\dot{Q}_v'''}{k} (r_1^2 - r^2) \quad (2.28)$$

The maximum temperature is at the centerline of the cylinder. Setting $r = 0$ in Eq. (2.28) gives

$$T_{\max} - T_1 = \frac{1}{4} \frac{\dot{Q}_v''' r_1^2}{k} \quad (2.29)$$

The use of this result is illustrated in the following example.

EXAMPLE 2.4 Temperature Distribution in a Nuclear Reactor Fuel Rod

Uranium oxide fuel is contained inside 0.825 cm–I.D., 0.970 cm–O.D. Zircaloy-4 tubes. The tubes have a 1.75 cm pitch in a square array. The power averaged over the volume including the space between the fuel rods is 152.4 W/cm^3 . At a specific location along the bundle the coolant water is at 400 K and the convective heat transfer coefficient h_c is $1.0 \times 10^4 \text{ W/m}^2 \text{ K}$. If the interfacial conductance between the fuel and the tube, h_i , is $6000 \text{ W/m}^2 \text{ K}$, determine the maximum temperature in the fuel rods.

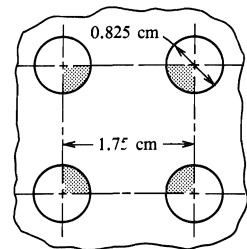
Solution

Given: Nuclear reactor fuel rod.

Required: Maximum rod temperature at location where the coolant water is at $T_c = 400 \text{ K}$.

Assumptions: Steady one-dimensional heat flow.

We cannot immediately use Eq. (2.29) to obtain $T_{\max}$ because the surface temperature of the fuel rod is unknown. We proceed as follows: first we calculate $\dot{Q}_v'''$ in the fuel itself,

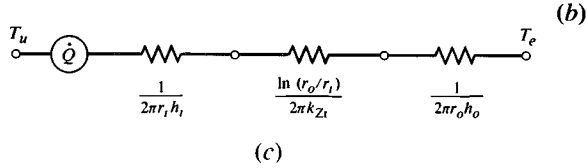
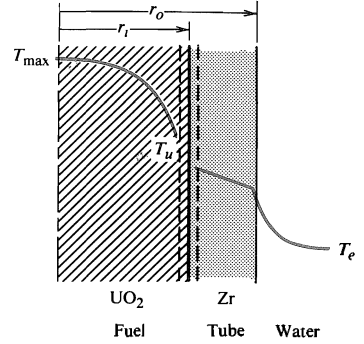


(a)

$$\begin{aligned}
 \dot{Q}_v''' &= 152.4 \frac{\text{Volume of array}}{\text{Volume of fuel}} \\
 &= 152.4 \frac{(1.75)^2}{(\pi/4)(0.825)^2} \\
 &= 873 \text{ W/cm}^3 = 8.73 \times 10^8 \text{ W/m}^3
 \end{aligned}$$

Next we find the temperature of the outer surface of the fuel rod. For unit length of rod, the heat flow across the outer surface of the Zircaloy tube is $\dot{Q} = \dot{Q}_v'''(\pi D^2/4)(1)$:

$$\begin{aligned}
 \dot{Q} &= (8.73 \times 10^8)(\pi/4)(0.825 \times 10^{-2})^2(1) \\
 &= 46,700 \text{ W/m}
 \end{aligned}$$



From the thermal circuit, as shown,

$$\begin{aligned}
 T_u &= T_e + \dot{Q} \sum R \\
 &= 400 + 46,700 \left[\frac{1}{(2\pi)(0.00413)(6000)} + \frac{\ln(0.485/0.413)}{2\pi k_{Zr}} + \frac{1}{(2\pi)(0.00485)(10^4)} \right] \\
 &= 400 + 46,700(0.00642 + 0.0256/k_{Zr} + 0.00328)
 \end{aligned}$$

As a guess, we take the mean temperature of the tube to be 600 K; from Table A.1b, the conductivity of Zircaloy-4 is 17.2 W/m K, and

$$T_u = 400 + 46,700(0.00642 + 0.00149 + 0.00328) = 923 \text{ K}$$

Now Eq. (2.29) can be used to obtain $T_{\max}$. If we guess a mean temperature of 1500 K for the uranium oxide, Table A.2 gives $k_{\text{UO}_2} = 2.6 \text{ W/m K}$, and

$$T_{\max} = T_u + \frac{1}{4} \frac{\dot{Q}_v''' r_i^2}{k_{\text{UO}_2}} = 923 + \frac{(8.73 \times 10^8)(0.00413)^2}{(4)(2.6)} = 2355 \text{ K}$$

To check if our guessed mean temperatures are appropriate, we first determine the mean temperature of the tube. From the thermal circuit,

$$\bar{T}_{\text{tube}} \approx 400 + (923 - 400) \frac{(0.00328 + 0.00149/2)}{(0.00642 + 0.00149 + 0.00328)} = 588 \text{ K}$$

which is close enough to our guess of 600 K. The mean temperature of the fuel rod is

$$\bar{T}_{\text{UO}_2} \approx \frac{923 + 2355}{2} = 1639 \text{ K}$$

and at this temperature, $k_{\text{UO}_2} = 2.5 \text{ W/m}^2 \text{ K}$. The new value of $T_{\max}$ is

$$T_{\max} = 923 + \frac{(8.73 \times 10^8)(0.00413)^2}{(4)(2.5)} = 2412 \approx 2400 \text{ K}$$

Comments

1. Since the k -value of UO_2 is given to only two significant figures, no further iteration is warranted.
2. Notice that the conductivity of the zirconium alloy Zircaloy-4 is lower than that of pure zirconium.
3. The largest thermal resistance in the circuit is at the fuel-cladding interface. The accuracy of the result depends primarily on our ability to obtain a reliable value of h_i . In fact, it could be argued that the second iteration for $T_{\max}$ was unwarranted due to uncertainty in the value of h_i .

2.4 FINS

Heat transfer from a system can be increased by extending the surface area through the addition of fins. Fins are used when the convective heat transfer coefficient h_c is low, as is often the case for gases such as air, particularly under natural-convection conditions. Common examples are the cooling fins on electronics components, on the cylinders of air-cooled motorcycles and lawnmowers, and on the condenser tubes of a home refrigerator. Figure 2.8 shows a variety of fin configurations. A careful examination of an automobile radiator will show how it is designed to provide a large exterior surface.

Fins are added to increase the $h_c A$ product and hence decrease the convective thermal resistance $1/h_c A$. But the added area is not as efficient as the original surface area since there must be a temperature gradient along the fin to conduct the heat. Thus, for cooling, the average temperature difference $(T_s - T_e)$ is lower on a finned surface compared with the unfinned surface, and an appropriate thermal resistance for a fin is $1/h_c A \eta_f$, where A is the surface area of the fin and η_f is the *efficiency* of the fin ($0 < \eta_f < 1$). For short fins of high thermal conductivity, η_f is large, but as the fin length increases, η_f decreases. Our objective here is to analyze heat flow in a fin to determine the temperature variation along the fin and, hence, to evaluate its efficiency η_f . Because fins are thin in one direction, it can be assumed that the temperature variation in this direction is negligible; this key assumption allows the conduction along the fin to be treated as if it were one-dimensional, which greatly simplifies the analysis.

2.4.1 The Pin Fin

Simple *pin fins*, such as those used to cool electronic components, will be analyzed to develop the essential concepts of fin theory. The first law is used to derive the governing differential equation, which, when solved subject to appropriate boundary conditions, gives the temperature distribution along the fin. The heat loss from the fin is then obtained and put in dimensionless form as the fin efficiency.

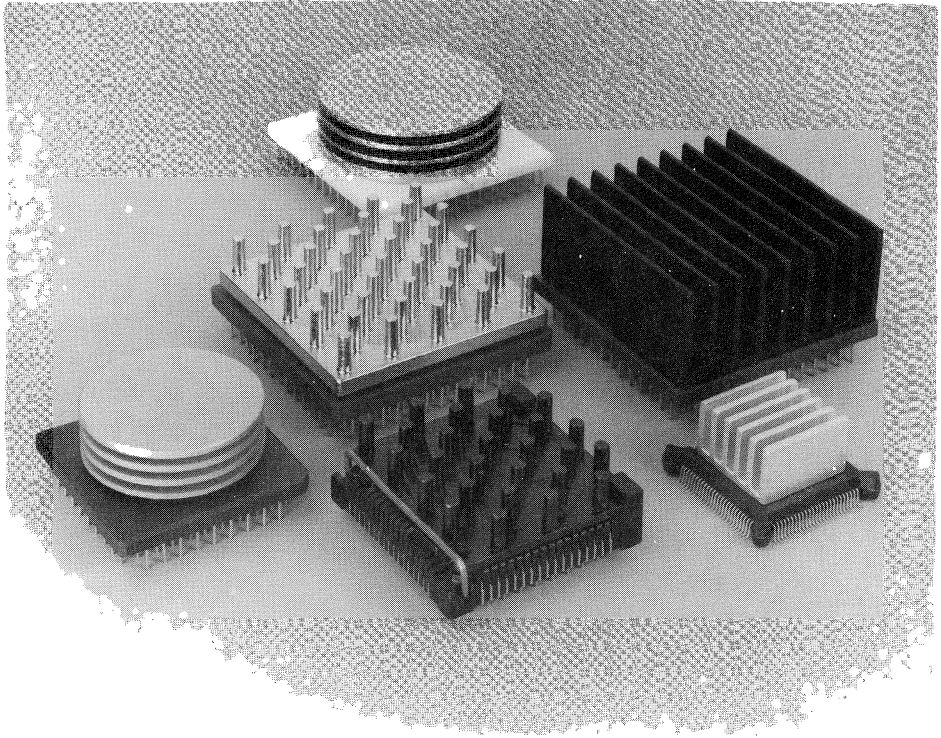


Figure 2.8 Some heat sinks incorporating fins for cooling of standard packages for integrated circuits. (Photograph courtesy of EG&G Wakefield Engineering, Wakefield, Mass.)

Governing Equation and Boundary Conditions

Consider the pin fin shown in Fig. 2.9. The cross-sectional area is $A_c = \pi R^2$ where R is the radius of the pin, and the perimeter $\mathcal{P} = 2\pi R$. Both A_c and R are uniform, that is, they do not vary along the fin in the x direction. The energy conservation principle, Eq. (1.2), is applied to an element of the fin located between x and $x + \Delta x$. Heat can enter and leave the element by conduction along the fin and can also be lost by convection from the surface of the element to the ambient fluid at temperature T_e . The surface area of the element is $\mathcal{P}\Delta x$; thus,

$$qA_c|_x - qA_c|_{x+\Delta x} - h_c\mathcal{P}\Delta x(T - T_e) = 0$$

Dividing by Δx and letting $\Delta x \rightarrow 0$ gives

$$-\frac{d}{dx}(qA_c) - h_c\mathcal{P}(T - T_e) = 0 \quad (2.30)$$

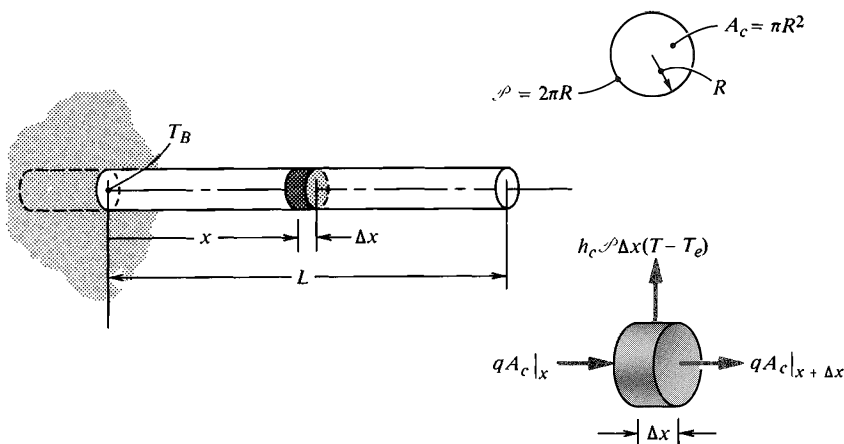


Figure 2.9 A pin fin showing the coordinate system, and an energy balance on a fin element.

For the pin fin, A_c is independent of x ; using Fourier's law $q = -k dT/dx$ with k constant gives

$$kA_c \frac{d^2 T}{dx^2} - h_c \mathcal{P}(T - T_e) = 0 \quad (2.31)$$

which is a second-order ordinary differential equation for $T = T(x)$. Notice that modeling of the conduction along the fin as one-dimensional has caused the convective heat loss from the sides of the fin to appear in the differential equation, in contrast to the problems dealt with in Section 2.3, where convection became involved as a boundary condition.

Next, boundary conditions for Eq. (2.31) must be specified. Since we wish to examine the performance of the fin itself, it is appropriate to take its base temperature as known; that is,

$$T|_{x=0} = T_B \quad (2.32)$$

At the other end, the fin loses heat by Newton's law of cooling:

$$-A_c k \left. \frac{dT}{dx} \right|_{x=L} = A_c h_c (T|_{x=L} - T_e) \quad (2.33a)$$

where the convective heat transfer coefficient here is, in general, different from the one for the sides of the fin because the geometry is different. However, because the area of the end, A_c , is small compared to the side area, $\mathcal{P}L$, the heat loss from the end is correspondingly small and usually can be ignored. Then Eq. (2.33a) becomes

$$\left. \frac{dT}{dx} \right|_{x=L} \simeq 0 \quad (2.33b)$$

and this boundary condition is simpler to use than Eq. (2.33a). An even simpler result

can be obtained if the temperature distribution along the fin is assumed identical to that for an infinitely long fin, for which the appropriate boundary condition is

$$\lim_{x \rightarrow \infty} T = T_e \quad (2.33c)$$

Figure 2.10 illustrates these boundary conditions.

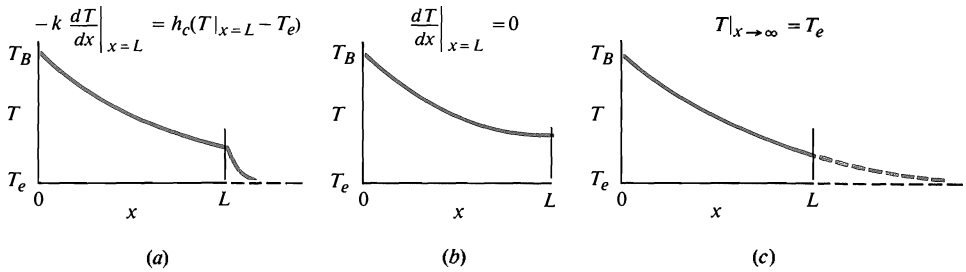


Figure 2.10 Three tip boundary conditions for the pin fin analysis.
(a) Heat loss by convection. (b) Insulated tip. (c) Infinitely long fin.

Temperature Distribution

We will use Eq. (2.33b) for the second boundary condition as a compromise between accuracy and simplicity of the result. For mathematical convenience, let $\theta = T - T_e$ and $\beta^2 = h_c \mathcal{P} / k A_c$; then Eq. (2.31) becomes

$$\frac{d^2 \theta}{dx^2} - \beta^2 \theta = 0 \quad (2.34)$$

For β a constant, Eq. (2.34) has the solution

$$\theta = C_1 e^{\beta x} + C_2 e^{-\beta x}$$

or

$$\theta = B_1 \sinh \beta x + B_2 \cosh \beta x$$

The second form proves more convenient; thus, we have

$$T - T_e = B_1 \sinh \beta x + B_2 \cosh \beta x \quad (2.35)$$

Using the two boundary conditions, Eqs. (2.32) and (2.33b) give two algebraic equations for the unknown constants B_1 and B_2 ,

$$T_B - T_e = B_1 \sinh(0) + B_2 \cosh(0); \quad B_2 = T_B - T_e$$

$$\left. \frac{dT}{dx} \right|_{x=L} = \beta B_1 \cosh \beta L + \beta B_2 \sinh \beta L = 0; \quad B_1 = -B_2 \tanh \beta L$$

Substituting B_1 and B_2 in Eq. (2.35) and rearranging gives the temperature distribution as

$$\frac{T - T_e}{T_B - T_e} = \frac{\cosh \beta(L - x)}{\cosh \beta L}, \quad \text{where } \beta = \left(\frac{h_c \mathcal{P}}{k A_c} \right)^{1/2} \quad (2.36)$$

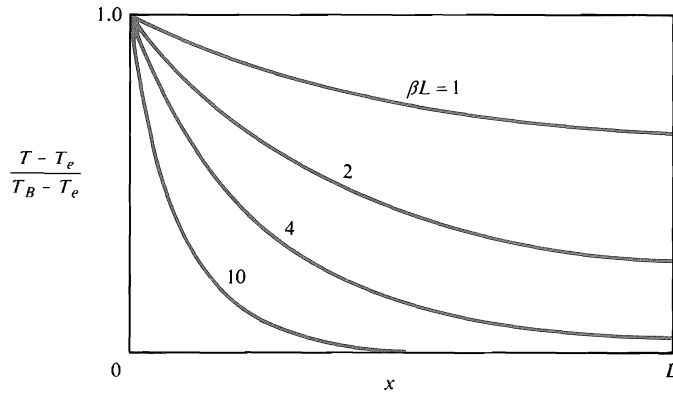


Figure 2.11 Fin temperature distributions calculated from Eq. (2.36).

Figure 2.11 shows a plot of Eq. (2.36). When β is small—for example, if the fin is made of aluminum and has a high thermal conductivity—the temperature T does not drop much below the base temperature T_B . For β large, T approaches the fluid temperature at the tip of the fin.²

Heat Loss

The heat dissipated from the fin can be found by integrating the heat loss over the side surface of the fin (there is no heat loss from the fin tip):

$$\dot{Q} = \int_0^L h_c \mathcal{P} (T - T_e) dx \quad (2.37)$$

with T obtained from Eq. (2.36). Substituting gives

$$\dot{Q} = \frac{h_c \mathcal{P} (T_B - T_e)}{\cosh \beta L} \int_0^L \cosh \beta(L - x) dx$$

To simplify the integration, let $\xi = \beta(L - x)$; then $dx = -d\xi/\beta$ and

² A_c in $\beta = (h_c \mathcal{P}/k A_c)^{1/2}$ is the cross-sectional area of the fin. The subscript c denotes “cross section” and not “convection” as in the heat transfer coefficient h_c . The area for convective heat loss is the surface area of the fin, $\mathcal{P}L$.

$$\begin{aligned}
\dot{Q} &= \frac{(h_c \mathcal{P}/\beta)(T_B - T_e)}{\cosh \beta L} \left[- \int_{\beta L}^0 \cosh \xi \, d\xi \right] \\
&= \frac{h_c \mathcal{P}}{\beta} (T_B - T_e) \left[- \frac{\sinh 0 - \sinh \beta L}{\cosh \beta L} \right] \\
&= \frac{h_c \mathcal{P}}{\beta} (T_B - T_e) \tanh \beta L
\end{aligned} \tag{2.38}$$

A less obvious alternative, but usually a more convenient way to find the heat dissipation, is to apply Fourier's law at the base of the fin:

$$\dot{Q} = -k A_c \left. \frac{dT}{dx} \right|_{x=0} \tag{2.39}$$

Substituting from Eq. (2.36),

$$\begin{aligned}
\dot{Q} &= -k A_c (T_B - T_e) \frac{[(d/dx) \cosh \beta(L-x)]_{x=0}}{\cosh \beta L} \\
&= -k A_c (T_B - T_e) \frac{[-\beta \sinh \beta(L-x)]_{x=0}}{\cosh \beta L} \\
&= k A_c \beta (T_B - T_e) \tanh \beta L
\end{aligned} \tag{2.40}$$

Since $\beta^2 = h_c \mathcal{P}/k A_c$, Eqs. (2.38) and (2.40) give the same result, which is to be expected since there is no heat loss from the end of the fin.

Fin Efficiency

Let us now put Eq. (2.38) in **dimensionless** form by dividing through by $h_c \mathcal{P} L (T_B - T_e)$:

$$\frac{\dot{Q}}{h_c \mathcal{P} L (T_B - T_e)} = \frac{1}{\beta L} \tanh \beta L \tag{2.41}$$

The dimensions of the left-hand side of this equation are $[W]/[W/m^2 K][m][m][K] = 1$, as desired. The right-hand side must also be dimensionless since β has dimensions $[m^{-1}]$ and the group βL has dimensions $[m^{-1}][m] = 1$. (Of course, βL must be dimensionless to be the argument of the $\tanh$ function.) Now $h_c \mathcal{P} L (T_B - T_e)$ is the rate at which heat would be dissipated if the entire fin surface were at the base temperature T_B ; in reality, there is a decrease in temperature along the fin, and the actual heat loss is less. Thus, the left-hand side of Eq. (2.41) can be viewed as the ratio of the actual heat loss to the maximum possible and is termed the **fin efficiency**, η_f . The right-hand side is a function of the dimensionless parameter βL only; we will set $\beta L = \chi$ as a *fin parameter*, and then Eq. (2.41) can be written in the compact form

$$\eta_f = \frac{\tanh \chi}{\chi} \tag{2.42}$$

When χ is small, η_f is near unity; when χ is larger than about 4, $\tanh \chi \approx 1$ and $\eta_f \approx 1/\chi$. Since $\chi = \beta L = (h_c \mathcal{P} L^2 / k A_c)^{1/2}$, a small value of χ corresponds to relatively short, thick fins of high thermal conductivity, whereas large values of χ correspond to relatively long, thin fins of poor thermal conductivity. When χ is small, T does not fall much below T_B , and the fin is an efficient dissipator of heat. However, it is most important to understand that a thick fin with an efficiency of nearly 100% usually is not optimal from the viewpoint of heat transferred per unit weight or unit cost. The concept of fin efficiency refers only to the ability of the fin to transfer heat per unit area of exposed surface. Figure 2.12 shows a plot of Eq. (2.42). Use of dimensionless parameters has allowed the heat dissipation to be given by a single curve: different curves are not required for fins of various materials or lengths or for different values of the heat transfer coefficient. Likewise, storage of this information in a computer software package is efficient.

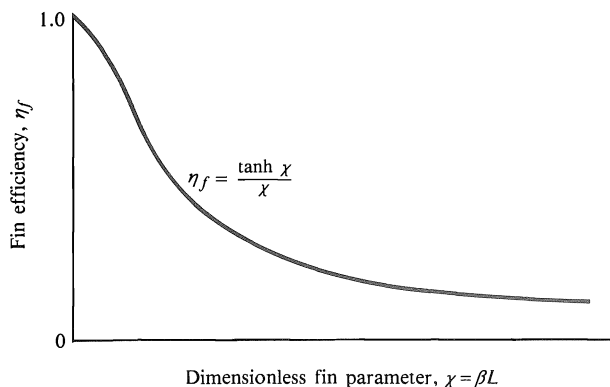


Figure 2.12 Efficiency of a pin fin as given by Eq. (2.42).

Straight Rectangular Fins

Although the pin fin shown in Fig. 2.9 was used for the purposes of this analysis, the results apply to any fin with a cross-sectional area A_c and perimeter $\mathcal{P}$ constant along the fin. The straight rectangular fin shown in Fig. 2.13 has a width W and thickness $2t$. The cross-sectional area is $A_c = 2tW$, and the perimeter is $\mathcal{P} = 2(W + 2t)$. For $W \gg t$, the ratio $\mathcal{P}/A_c$ is simply $1/t$, and $\beta = (h_c/k t)^{1/2}$.

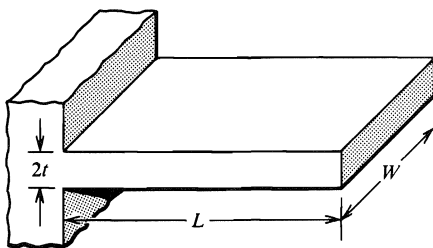


Figure 2.13 A straight rectangular fin.

Computer Program FIN1

The program FIN1 calculates the temperature distribution, fin efficiency, and base heat flow of straight rectangular fins. There are three options for the tip boundary condition: (1) infinitely long fin, (2) insulated, and (3) convective heat loss. The analysis for option 2 was given above; the analyses for options 1 and 3 are given as Exercises 2–30 and 2–31, respectively. For all three options, η_f is defined in terms of an isothermal fin heat loss of $\dot{Q} = h_c \mathcal{P} L(T_B - T_e)$. Use of FIN1 is illustrated in the example that follows.

EXAMPLE 2.5 Fins to Cool a Transistor

An array of eight aluminum alloy fins, each 3 mm wide, 0.4 mm thick, and 40 mm long, is used to cool a transistor. When the base is at 340 K and the ambient air is at 300 K, how much power do they dissipate if the combined convection and radiation heat transfer coefficient is estimated to be 8 W/m² K? The alloy has a conductivity of 175 W/m K.

Solution

Given: Aluminum fins to cool a transistor.

Required: Power dissipated by 8 fins.

Assumptions: 1. Heat transfer coefficient constant along fin.
2. Heat loss from fin tip negligible.

For one fin,

$$A_c = (0.003)(0.0004) = 1.2 \times 10^{-6} \text{ m}^2$$

$$\mathcal{P} = 2(0.003 + 0.0004) = 6.8 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \beta^2 &= \frac{h\mathcal{P}}{kA_c} \\ &= \frac{(8.0 \text{ W/m}^2 \text{ K})(6.8 \times 10^{-3} \text{ m})}{(175 \text{ W/m K})(1.2 \times 10^{-6} \text{ m}^2)} \\ &= 259 \text{ m}^{-2} \end{aligned}$$

$$\beta = 16.1 \text{ m}^{-1}$$

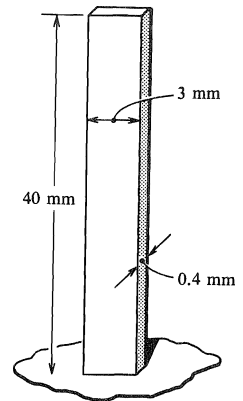
$$\chi = \beta L = (16.1 \text{ m}^{-1})(0.040 \text{ m}) = 0.644$$

Substituting in Eq. (2.42),

$$\eta_f = \frac{1}{0.644} \tanh(0.644) = \frac{1}{0.644} \frac{e^{2(0.644)} - 1}{e^{2(0.644)} + 1} = 0.881$$

The side surface area of one fin is $\mathcal{P}L = (6.8 \times 10^{-3})(0.040) = 2.72 \times 10^{-4} \text{ m}^2$. If each fin were 100% efficient, it would dissipate

$$h(\mathcal{P}L)(T_B - T_e) = (8)(2.72 \times 10^{-4})(340 - 300) = 8.70 \times 10^{-2} \text{ W}$$



Since the fins are only 88.1% efficient,

$$\dot{Q} = (0.881)(8.70 \times 10^{-2}) = 7.67 \times 10^{-2} \text{ W}$$

For 8 fins, $\dot{Q}_{\text{total}} = (8)(7.67 \times 10^{-2}) = 0.613 \text{ W}$.

Solution using FIN1

The required input is:

Boundary condition = 2
 Half-thickness, length, and width = 0.0002, 0.040, 0.003
 Thermal conductivity = 175
 Heat transfer coefficient = 8
 Base temperature and ambient temperature = 340, 300
 x-range for plot = 0.0, 0.04

FIN1 gives the output:

$$\eta_f = 0.881$$

$$\dot{Q} = 7.67 \times 10^{-2} \text{ (watts)}$$

Comments

1. Any consistent system of units can be used with FIN1. Since SI units were used here, the heat flow is in watts.
2. Notice the use of $h = h_c + h_r$ to account for radiation.

2.4.2 Fin Resistance and Surface Efficiency

It is useful to have an expression for the **thermal resistance** of a pin fin for use in thermal circuits. Equation (2.38) can be rewritten as

$$\dot{Q} = \frac{T_B - T_e}{l/[(h_c \mathcal{P}/\beta) \tanh \beta L]} \quad (2.43)$$

Thus, the thermal resistance of a pin fin is

$$R_{\text{fin}} = \frac{1}{(h_c \mathcal{P}/\beta) \tanh \beta L} = \frac{1}{h_c \mathcal{P} L \eta_f} \quad (2.44)$$

Notice that this thermal resistance accounts for both conduction along the fin and convection into the fluid. There are two parallel paths for heat loss from a finned surface—one through the fins and one through the area between the fins, as shown in Fig. 2.14. The respective conductances are thus additive; however, quite often the heat loss through the area between the fins is negligible.

The *total surface efficiency* η_t of a surface with fins of fin efficiency η_f is obtained by adding the unfinned portion of the surface area at 100% efficiency to the surface area of the fins at efficiency η_f :

$$A\eta_t = (A - A_f) + \eta_f A_f \quad (2.45)$$